Context and Objective

- Key elements to study fiscal policy over the business cycle: AD & unemployment
- Workhorse unemployment model: matching model
- In matching model: AD is not modeled explicitly
- Objective: develop matching model with AD
Four Key Ingredients

- Matching frictions
  - as in Diamond-Mortensen-Pissarides framework

- Rigid price schedules
  - as in Hall [2005]
  - but any schedule admissible (infinite price bands)

- Concave production function and utility function
  - as in Michaillat [2012]

- Nonproduced good
  - common in Keynesian literature of 1970s and 1980s
Results

1. Equilibrium model of AD and unemployment
2. Predictions to separate AD and AS shocks
3. Design of fiscal policy over the business cycle:
   - government consumption
   - transfer payments
   - payroll tax
Outline

1. **Simple model: market for services**
2. Macro model: product market & labor market
3. Conclusion
Idle Labor as Unemployment
Outline

1. Simple model: market for services
   ▶ assumptions and equilibrium
   ▶ aggregate demand & aggregate supply shocks
   ▶ government consumption
   ▶ transfer payments

2. Macro model: product market & labor market

3. Conclusion
Matching Function

$v$ visits to the shop

$y$ slots available
Matching function $M$ with **constant returns to scale**:

$s = M(y, v)$
Matching Function: Market Tightness

Market tightness: \( x = \frac{v}{y} \)

Sales: \( y \cdot M(1, x) = y \cdot f(x) \)

\( s = M(y, v) \)

Purchases: \( v \cdot M\left(\frac{1}{x}, 1\right) = v \cdot q(x) \)
Matching Function

- Market tightness $x$\downarrow$

Selling probability $f(x)$ \downarrow

Quantity $s$ \downarrow

Purchasing probability $q(x)$ \uparrow
Matching Function

Market tightness $x \uparrow$

Selling probability $f(x) \uparrow$

Quantity $s \downarrow$

Purchasing probability $q(x) \downarrow$
A Visit Requires $\rho$ Services

\[
\begin{align*}
\text{s} & \quad \text{purchases of services} & = & \quad \text{c} \quad \text{consumption} & + & \quad \rho \cdot \text{v} \quad \text{matching expenses} \\
\Rightarrow \ s & = \ c + \rho \cdot \frac{s}{q(x)} \\
\Rightarrow \ s \cdot \left(1 - \frac{\rho}{q(x)}\right) & = \ c \\
\Rightarrow \ s & = \left(1 + \frac{\rho}{q(x) - \rho}\right) \cdot \left(1 + \tau(x)\right) \cdot \text{c} \quad \text{consumption}
\end{align*}
\]
Self-Employed Workers

- Amount of services available is $y$
- Probability to sell is $f(x)$
- Consumption is fraction $1/ (1 + \tau(x))$ of sales
- Aggregate supply:

$$c^s(x) = \frac{1}{1 + \tau(x)} \cdot f(x) \cdot y = (f(x) - \rho \cdot x) \cdot y$$
Aggregate Supply

\[ c^s(x) = [f(x) - \rho \cdot x] \cdot y \]
Idle Labor

Market tightness

Amount of services

Aggregate supply

Total sales

Capacity

Consumption

Matching cost

Idle labor
Consumers

- Take price $p$ and market tightness $x$ as given
- Choose $c$, $m$ to maximize utility

$$\left[ \chi \cdot c^{\frac{\epsilon-1}{\epsilon}} + (1 - \chi) \cdot m^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}$$

- Subject to budget constraint

$$m_{\text{numeraire}} + p \cdot (1 + \tau(x)) \cdot c = \mu_{\text{endowment}} + p \cdot f(x) \cdot y_{\text{labour income}}$$
Optimal Consumption Decision

- First-order condition

\[
(1 + \tau(x)) \cdot p \cdot (1 - \chi) \cdot m^{-1/\epsilon} = \chi \cdot c^{-1/\epsilon}
\]

- price of services  MU of nonproduced good  MU of services

- Aggregate demand (as \( m = \mu \)):

\[
c^d(x, p) = \left( \frac{\chi}{1 - \chi} \right)^{\epsilon} \cdot \frac{\mu}{(1 + \tau(x))^{\epsilon} \cdot p^{\epsilon}}
\]
Aggregate Demand

\[ c^d(x, p) = \left( \frac{\chi}{1 - \chi} \right)^\varepsilon \cdot \frac{\mu}{[(1 + \tau(x)) \cdot p]^\varepsilon} \]
Equilibrium with Matching Frictions

- Equilibrium is \((c, x, p)\) such that supply \(=\) demand:

\[
\begin{align*}
  c^s(x) &= c^d(x, p) \\
  c &= c^s(x)
\end{align*}
\]

- 3 variables, 2 equations: indeterminacy

- **Short-run equilibrium:** \(p\) is fixed

- Any \(p\) is acceptable (price band is infinite)
Business Cycle: the Hair Salon Example
Business Cycle: the Hair Salon Example
Short-Run Equilibrium

[Diagram showing the relationship between market tightness, amount of services, total sales, and capacity.]

- Aggregate supply
- Total sales
- Short-run equilibrium
- Idle labor
- Aggregate demand

[Graph with axes: Market tightness on the y-axis, Amount of services on the x-axis, and Capacity on the right side.]
Welfare Properties

Efficient allocation
Market tightness
Consumption
Aggregate supply
Total sales
Price is optimal (Hosios price)
Aggregate demand

Total sales

Efficient allocation
Welfare Properties

Market tightness

Aggregate supply

Slack regime

Aggregate demand

Total sales

Consumption

Price is too high
Welfare Properties

- Aggregate supply
- Total sales
- Aggregate demand
- Price is too low
- Market tightness
- Consumption

- Tight regime
Convergence to Optimum in Dynamic Model?

- **Short run: fixed price**
  - price may not be optimal

- **Medium run: new markets are created**
  - competitive search mechanism of Moen [1997]
  - price increases if market is tight
  - price decreases if market is slack

- **Long run: price at Hosios level**
  - maximizes social welfare
Outline

1. Simple model: market for services
   - assumptions and equilibrium
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   - government consumption
   - transfer payments

2. Macro model: product market & labor market

3. Conclusion
Negative AD Shock: Tightness Falls

Market tightness

Aggregate supply

GDP

Capacity

Amount of services

Aggregate demand
Negative AS Shock: Tightness Rises

Market tightness

Aggregate supply

GDP

Capacity

Amount of services

Aggregate demand
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Slack Regime: \( 0 < \text{Multiplier} < 1 \)

Market tightness

Aggregate supply

Private consumption \( c \) + government consumption \( g \)

Aggregate demand

\[ g > 0 \]
Slack Regime: $0 < \text{Multiplier} < 1$

Market tightness

Private consumption $c +$ government consumption $g$

Aggregate supply

Aggregate demand

crowding out
Slack Regime: $0 < \text{Multiplier} < 1$

Market tightness

Private consumption $c + \text{government consumption } g$

Aggregate supply

Aggregate demand

$0 < \text{multiplier} < 1$
Efficient Regime: Multiplier = 0
Efficient Regime: Multiplier $= 0$

Market tightness

Aggregate supply

one-for-one crowding out

Aggregate demand

Private consumption $c +$ government consumption $g$
Efficient Regime: Multiplier = 0
Tight Regime: Multiplier $< 0$

Private consumption $c +$ government consumption $g$

Market tightness

Aggregate supply

Aggregate demand

Private consumption $c +$ government consumption $g$
Tight Regime: Multiplier < 0

Private consumption $c + \text{government consumption } g$

Market tightness

Aggregate supply

Crowding out

Aggregate demand

Private consumption $c + \text{government consumption } g$
Tight Regime: Multiplier < 0

Private consumption \( c \) + government consumption \( g \)
Normative Implications of the Multiplier

- Optimal government consumption $g$ satisfies

$$1 = \frac{\text{MU of } g}{\text{MU of } c} + \text{multiplier}$$

- Samuelson’s rule

- If multiplier $> 0$: optimal $g >$ Samuelson’s rule

- If multiplier $< 0$: optimal $g <$ Samuelson’s rule

- Optimal government consumption is countercyclical
General Optimality Principle

- Optimal government consumption $g$ satisfies

\[ 1 = \frac{\text{MU of } g}{\text{MU of } c} + \frac{c}{x} \cdot \frac{\partial x}{\partial g} \cdot [(1 - \eta) - \eta \cdot \tau(x)] \]

- Economy is slack iff $(1 - \eta) - \eta \cdot \tau(x) > 0$

- Optimal UI satisfies formula with same structure

[Landais, Michaillat, & Saez, 2010]
Outline

1. Simple model: market for services
   ▶ assumptions and equilibrium
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   ▶ government consumption
   ▶ transfer payments

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Wealth Inequality

- $l$ heterogeneous groups of measure $1/l$

- Group $i$’s per person utility:

$$\chi_i \cdot \ln(c_i) + (1 - \chi_i) \cdot \ln(m_i)$$

- Group $i$’s per person budget:

$$m_i + (1 + \tau(x)) \cdot p \cdot c_i = \mu_i + p \cdot f(x) \cdot y$$
AD and Transfer Payments

AD depends on joint distribution of \((\mu_i, \chi_i)\):

\[
c^d(\chi, p) = \frac{\sum_i \mu_i \cdot \chi_i}{\sum_i (1 - \chi_i)} \cdot \frac{1}{p \cdot (1 + \tau(x))}
\]

If transfer endowment \(\mu_i\) from low \(\chi_i\) to high \(\chi_i\):

- AD increases since \(\sum_i \mu_i \cdot \chi_i\) increases
- Market tightness increases
- Consumption increases iff economy is slack
Outline

1. Simple model: market for services
2. Macro model: product market & labor market
   - assumptions and equilibrium
   - more on macro shocks
   - payroll tax
3. Conclusion
Labor Supply and Unemployment

![Diagram showing labor supply, employment, labor force, and labor market tightness. Producers and Recruiters are positioned on the horizontal axis, with Labor supply and Employment on the vertical axis. Unemployment is highlighted where the labor force and employment lines intersect.]
Firms

- Employ producers and recruiters and sell production
- Take real wage $w$ and tightnesses $x$ and $\theta$ as given
- Choose number of producers $n$ to maximize profits

$$f(x) \cdot a \cdot n^\alpha - [1 + \hat{\tau}(\theta)] \cdot w \cdot n$$

- selling probability
- production
- wage of producers + recruiters
Optimal Employment Decision

- First-order condition:

\[
\frac{f(x)}{\text{selling probability}} \cdot a \cdot \alpha \cdot n^{\alpha-1} = \left[1 + \hat{\tau}(\theta)\right] \cdot \frac{1}{1-\alpha} \text{MPL} \cdot \text{matching cost} \cdot \text{real wage}
\]

- Labor demand:

\[
n^d(\theta, x, w) = \left[\frac{f(x) \cdot a \cdot \alpha}{(1 + \hat{\tau}(\theta)) \cdot w}\right]^{\frac{1}{1-\alpha}}
\]
Partial Equilibrium on Labor Market

![Diagram showing labor market equilibrium with labor supply and demand curves intersecting at a point labeled "Partial equilibrium".](image-url)
General Equilibrium \((c, n, x, \theta, p, w)\)

- Supply = demand on product and labor markets

\[
\begin{align*}
  c^s(x, n) &= c^d(x, p) \\
  c &= c^s(x) \\
  n^s(\theta) &= n^d(\theta, x, w) \\
  n &= n^s(\theta)
\end{align*}
\]

- 4 equations, 6 variables: indeterminacy

- Short-run equilibrium: \(p \& w\) are parameters
General Equilibrium Representation

$\theta = \theta(x, \text{technology, wage, labor force})$

$\theta = \theta(x, \text{demand, wage, labor force})$

Product market tightness

Labor market tightness

General equilibrium
Outline

1. Simple model: market for services
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   - assumptions and equilibrium
   - more on macro shocks
   - payroll tax
3. Conclusion
## General-Equilibrium Correlations

<table>
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## Possibility to Reject Technology Shocks

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1. Simple model: market for services
2. Macro model: product market & labor market
   ▶ assumptions and equilibrium
   ▶ more on macro shocks
   ▶ **payroll tax**
3. Conclusion
Inequality in Profit Income

- Group $i$’s per person budget:

$$m_i + [1 + \tau(x)] \rho c_i = \mu_i + p\sigma_i\pi + p[1 + \hat{\tau}(\theta)] \cdot wn$$

- AD depends on wage:

$$c^d(x, \theta, p) = \left( \frac{\sum_i \mu_i \cdot \chi_i}{\sum_i (1 - \chi_i) \cdot \sigma_i} \cdot \frac{1}{p \cdot (1 + \tau(x))} \right) + \left( \frac{\sum_i \chi_i \cdot (1 - \sigma_i)}{\sum_i (1 - \chi_i) \cdot \sigma_i} \frac{1}{1 + \tau(x)} \cdot w \cdot \hat{f}(\theta) \right)$$

- Increase in $w$ stimulates AD if **savers own firms**
Outline

1. Simple model: market for services
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3. Conclusion
   - connection with existing business-cycle models
   - summary and future research
Perfect Competition

- No matching costs
- Assumptions on price and wage schedules:

\[ w = MPL \text{ at } (n = h) = a \cdot \alpha \cdot h^{\alpha - 1} \]

\[ p = MRS \text{ at } (n = h) = \frac{\chi}{1 - \chi} \cdot \left( \frac{\mu}{a \cdot h^\alpha} \right)^{\frac{1}{\epsilon}} \]

⇒ Tightnesses → +∞

⇒ Unemployment = 0, unsold production = 0

- AD shocks have no effect
Matching-Bargaining

- Linear production function: $y = a \cdot n$

- Linear utility function: $\chi \cdot c + (1 - \chi) \cdot m$

- Assumptions on price and wage schedules:

  \[ w = \text{Nash bargained wage} = \hat{\beta} \cdot a \cdot f(x) \]

  \[ p = \text{Nash bargained price} = \beta \cdot \frac{\chi}{1 - \chi} \]

- AD shocks have no effect
Monopolistic Competition

- Utility function: \( \chi \cdot \ln(c) + (1 - \chi) \cdot \ln(m) - \nu \cdot h^{\frac{1+\xi}{\xi}} \)

- Assumptions on market tightnesses:

\[
\hat{f}(\theta) = \left( \frac{1}{\text{labor markup}} \right)^{\frac{\xi}{\xi+1}}.
\]

\( \Rightarrow \) Same FOCs, same labor wedge = MPL/MRS

- AD shocks have no effect
Fixprice-Fixwage

- No matching costs
- Special matching functions:

\[ s = \min \{ v, y \} \]
\[ l = \min \{ \hat{v}, 1 \} \]

⇒ Replicate fixprice-fixwage with proportional rationing

- AD shocks have effects, but 4 equilibrium systems
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A New Kind of Business-Cycle Model

- Microfoundations for AD and AS
- Unemployment & unsold production in equilibrium
- 3 variables per market: price, quantity, tightness
- Comovements of tightnesses and quantities ⇒ macro shocks
- Gap between actual tightness and optimal tightness ⇒ optimal macro policies
Future Research

1. Optimal macro policies over the business cycle
   - fiscal policy and insurance programs
   - monetary policy

2. Empirical exploration of market tightness
   - measure tightness
   - estimate gap between actual and optimal tightness

3. Quantitative dynamic model
   - adjustment of prices
   - saving and investment