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ABSTRACT

This paper analyzes the optimal income tax treatment of couples. Each couple is modelled as a single rational economic agent supplying labor along two dimensions: primary and secondary earnings. We consider fully general joint income tax systems. Separate taxation is never optimal if social welfare depends on total couple incomes. In a model where secondary earners make only a binary work decision (work or not work), we demonstrate that the marginal tax rate of the primary earner is lower when the spouse works. As a result, the tax distortion on the secondary earner decreases with the earnings of the primary earner and actually vanishes to zero asymptotically. Such negative jointness is optimal because redistribution from two-earner toward one-earner couples is more valuable when primary earner income is lower. We also consider a model where both spouses display intensive labor supply responses. In that context, we show that, starting from the optimal separable tax schedules, introducing some negative jointness is always desirable. Numerical simulations suggest that, in that model, it is also optimal for the marginal tax rate on one earner to decrease with the earnings of his/her spouse. We argue that many actual redistribution systems, featuring family-based transfers combined with individually-based taxes, generate schedules with negative jointness.

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1 Introduction

The tax treatment of couples has been a debating point throughout the existence of the income tax. Actual policies have varied over time and across countries. Over the past three decades, there has been an international trend from joint to individual taxation of husbands and wives, and today the majority of OECD countries use the individual as the basic unit of taxation. Under individual taxation, tax liability is assessed separately for each family member and is therefore independent of the income of other individuals living in the household. By contrast, in a system of fully joint taxation of couples, as operated by for example the United States, tax liability is assessed at the family level and depend on total family income. It is also notable that most countries which have moved to individual income taxation still use joint income to determine welfare benefits and transfers at the bottom end. Two basic points have been noted in the previous informal discussions of the issue (e.g., Rosen, 1977; Pechman, 1987).

First, as the labor supply of secondary earners is more elastic with respect to taxes than the labor supply of primary earners (see Blundell and MaCurdy, 1999, for a recent survey), the traditional Ramsey optimal taxation principle suggests that the labor income of secondary earners should be taxed at a lower rate than labor income of primary earners for efficiency reasons. This is achieved to some extent by a progressive individual income tax since primary earners have higher incomes and hence will face higher marginal tax rates than secondary earners. By contrast, a fully joint income tax generates identical marginal tax rates across members of the same family and hence does not meet this efficiency criterion.

Second, welfare is better measured by family income than individual income. As a result, if the government values redistribution, two married women with the same labor income ought not to be treated identically if their husbands' incomes are very different. This redistributive principle is achieved to some extent by progressive income taxation based on family income, since it imposes higher tax rates on wives married to high-income husbands than on wives married to low-income husbands. By contrast, an individual income tax imposes the same tax burden on wives irrespective of their husbands' earnings and hence does not meet this redistributive criterion.[1]

The purpose of this paper is to explore the optimal income taxation of couples. Following

[1] Another topic which is often discussed is the neutrality of the tax system with respect to marriage decisions (see e.g., Alm et al. 1999). This paper considers only couples and hence will not touch on this issue.

the seminal contribution of Mirrlees (1971), optimal income tax theory has focused almost exclusively on individuals. In contrast to previous work on this topic, we consider fully general income tax systems allowed to depend on the earnings of each spouse in any nonlinear fashion and hence impose no a priori restrictions.[2] Such a problem can be seen as a multi-dimensional screening problem where agents (couples in the present paper) are characterized by a multi-dimensional parameter (ability and taste-for-work parameters of each spouse) that are unobserved by the principal (the government which maximizes social welfare).

Due to the technical difficulties involved, there are very few studies in the optimal taxation literature attempting to deal with multi-dimensional screening problems. Mirrlees (1976, 1986) considered briefly such general screening problems in the context of optimal taxation but did not go beyond obtaining general first-order conditions and did not consider specifically the case of family taxation. More recently, Cremer, Pestieau and Rochet (2001) revisited the issue of commodity versus income taxation in a multi-dimensional screening model in a finite type economy.

The nonlinear pricing literature in the field of Industrial Organization has investigated a number of aspects of multi-dimensional screening problems. Wilson (1993), Armstrong and Rochet (1999), and Rochet and Stole (2003) provide surveys of this literature. Multi-dimensional screening problems are difficult to analyze because, in contrast to the one-dimensional case, first-order conditions are not sufficient to characterize the optimal solution in general. In this paper, we consider primarily models with a discrete number of earnings *outcomes* (instead of types) for the secondary earner which simplifies the theoretical analysis and allows us to characterize optimal solutions using a first-order approach. Furthermore, we are able to derive a number of properties of optimal schedules which are relevant for tax policy analysis and which, to the best of our knowledge, have not been analyzed in nonlinear pricing theory.

As in the nonlinear pricing literature, we have to make certain simplifying assumptions to be able to make progress in our understanding of the optimal schedules. In particular,

[2]Boskin and Sheshinski (1983) considered linear taxation of couples with the possibility of differentiated marginal tax rates on spouses. Their problem is formally identical to a many-person Ramsey optimal tax problem. They analyze the efficiency principle discussed above and provide a number of useful numerical simulations based on empirical labor supply elasticities. However, because they restrict themselves to linear taxation, their tax system is an individual-based (albeit gender specific) income tax by assumption. Hence, they cannot address the central question of how the tax rate on one earner should depend on the earnings of his/her spouse.

we consider a model of family labor supply which assumes no income effects on labor supply, along with separability in the disutility of supplying labor for the two members of the couple. We obtain four main results.

First, we derive optimal tax formulas as a function of labor supply elasticities, the redistributive tastes of the government (measured by social marginal welfare weights), and the distribution of earnings abilities and work costs in the population. We show how the optimal tax formulas can be obtained by considering small reforms around the optimum schedule, which allows us to understand the economic intuition behind each term in the formulas and how they relate to classic individualistic optimal income tax theory. We show that the marginal tax rate faced by primary earners at a given earnings level — averaging over secondary earners — is identical to the marginal tax rate obtained in the standard individualistic Mirrlees model. Thus, the presence of the secondary earner introduces heterogeneity in marginal tax rates faced by primary earners at a given earnings level (depending on their spouses) but does not affect the average.

Second, we analyze the asymptotics of the optimal tax formulas as the earnings of the primary earner become large. Quite strikingly, for a wide class of social welfare objectives, we can show that the tax distortion on the secondary earner vanishes asymptotically when the earnings of the primary earner become very large. In other words, the earnings of spouses married to very high income husbands should be exempted from income taxation.[3] The intuition for the zero optimal tax on secondary earners can be understood as follows. Taxing secondary earners amounts to redistributing from two-earner couples to one-earner couples. For couples with very large primary earner incomes, there is no value in such redistribution as marginal social welfare weights for one- and two-earner couples are about the same in the limit.

Third and most importantly, we show that under some additional regularity assumptions and uncorrelated abilities across spouses, the marginal tax rate on the primary earner is *lower* when his spouse works. As a result, the tax on secondary earners *decreases* with primary earnings. The intuition is an extension of the asymptotic result described previously. When

[3] At first glance, our result may seem reminiscent of the famous result that the top marginal tax rate is zero in the Mirrlees model (Sadka, 1976; Seade, 1977), but the logic is in fact quite different. Indeed, we obtain our zero-tax result for secondary earners under assumptions implying a positive top marginal tax rate on primary earners.

primary earnings are low, secondary earnings make a significant difference for the couple's welfare. Hence, the government would like to compensate one earner-couples for not having secondary earnings relatively more when primary earnings are low. This is equivalent to introducing a tax on secondary earners which decreases with primary earnings.

Fourth, we show that this *negative jointness* result is likely to be robust to more general models where secondary earnings are continuous (instead of binary). In that context, we show that starting from the optimal *separable* schedule, it is desirable to introduce negative jointness at the margin. Although we can only conjecture that negative jointness will be present at the optimum, extensive numerical simulations suggest that negative jointness is indeed a feature of the optimum tax systems.

The desirability of negative jointness seems striking at first glance. Notice that fully joint progressive income taxation, as observed in the United States for example, is characterized by positive jointness, i.e. the marginal tax on one spouse depends positively on the income of the other spouse. Our result suggests that such a system is suboptimal: a move to separate taxation would be a step in the right direction, but this would not go far enough. However, it is important to note that, in practice, transfers programs at the bottom are almost always based on joint family income and the phasing-out of those programs creates implicit taxes on secondary earners which are actually decreasing with primary earnings. For example, the United Kingdom has an individual income tax system but a family-based transfer system. Consider a secondary earner in the United Kingdom with modest earnings. There is a high tax on secondary earnings when primary earnings are low (because secondary earnings reduce transfer payments) and there is a low tax on secondary earnings when primary earnings are high (because the secondary earner then faces solely the individual income tax with low rates for initial earnings). Hence, our optimal tax results are in fact quite consistent with the actual tax and transfer systems of many OECD countries.

The remainder of the paper is organized as follows. Section 2 analyzes the case where secondary earners respond only along the extensive margin (working or not working). Section 3 explores how our results extend to a model where secondary earners respond along the intensive margin. Section 4 presents numerical simulations. Section 5 discusses the implications of alternative models of family decision making and, finally, Section 6 offers concluding remarks and avenues for future work.

2 Extensive Response for the Secondary Earner

2.1 Labor Supply Model

In this section, we consider the simplest possible labor supply model for couples allowing us to derive properties of the fully general optimal joint tax system.

In the model, the primary earner is characterized by a scalar ability parameter n similar to the Mirrlees (1971) model. The cost of earning z for a primary earner with ability n is $n \cdot h(z/n)$, where $h(.)$ is an increasing and convex function of class C^2 and normalized so that $h(0) = 0$ and $h'(1) = 1$. The secondary earner makes a binary decision $l = 0, 1$ of whether or not to work. Secondary earners are characterized by a scalar fixed cost of work parameter q. They earn a uniform amount w when working ($l = 1$) and zero when not working ($l = 0$).

The government cannot observe n and q and hence has to base redistribution solely on observed earnings z and $w \cdot l$. Therefore, the government sets a general non-linear tax system which depends on z and l. We discuss the mechanism design details more formally in Appendix A.1. Hence, the general tax system is characterized by a pair of non-linear tax schedules $T_0(z), T_1(z)$ depending on whether the spouse works or not. The tax system is separable if and only if T_0 and T_1 differ by a constant. Disposable income for a couple with earnings $(z, w \cdot l)$ is given by $c = z + w \cdot l - T_l(z)$. The utility function for a couple whose primary earner has ability n and whose secondary earner has a fixed cost of work q takes the quasi-linear form

$$u(c, z, l) = c - n \cdot h\left(\frac{z}{n}\right) - q \cdot l. \tag{1}$$

The quasi-linear utility specification amounts to ruling out income effects in the labor supply decisions of both spouses. We make this assumption for two reasons. First, as is well known from the Industrial Organization literature on nonlinear pricing (e.g., Wilson, 1993) and as shown more recently by Diamond (1998) in the context of the Mirrlees optimal income tax model, ruling out income effects simplifies substantially the theoretical analysis. Second, since the empirical labor market literature tends to find small income effects (e.g., Blundell and MaCurdy, 1999), the case of no income effects would seem to provide a useful benchmark. The assumption that disutility of work is separable across the two spouses is also made to simplify the analysis.[4]

[4] It would be violated if, for example, spouses prefer to spend leisure time together (if one works more, then

The couple chooses (z, l) so as to maximize utility (1) subject to its budget constraint $c = z + w \cdot l - T_l(z)$. It is important to note that our model is equivalent to a single decision maker optimizing along two dimensions z and l. Thus, there is no conflict in the family about consumption or labor supply choices.[5] The first-order condition for primary earnings z is given by

$$h'\left(\frac{z}{n}\right) = 1 - T_l'(z), \tag{2}$$

where T_l' is the marginal tax rate of the primary earner taking $l = 0, 1$ as given. In the case of no tax distortion, $T_l'(z) = 0$, and our normalization assumption $h'(1) = 1$ implies that $z = n$. That is, primary earnings would be identical to ability n, and it is therefore natural to interpret n as potential earnings. Positive marginal tax rates depress actual earnings z below potential earnings n. If the tax system is not separable (so that T_0' and T_1' are not identical), there will be an interdependence between the labor supply decisions of the two spouses. We denote by z_l the optimal choice of z for a given labor supply choice l of the secondary earner.

We define the elasticity of primary earnings with respect to the net-of-tax rate (one minus the marginal tax rate) as

$$\varepsilon_l = \frac{1 - T_l'}{z_l} \frac{\partial z_l}{\partial(1 - T_l')} = \frac{n h'(z_l/n)}{z_l h''(z_l/n)}. \tag{3}$$

Because we have assumed away income effects, the compensated and uncompensated elasticity of labor supply are of course identical. This elasticity would be constant in the iso-elastic case where $h(x) = x^{1+k}/(1+k)$. In that case, $\varepsilon_l \equiv 1/k$.

We assume that couple characteristics (n, q) are distributed according to a continuous density distribution defined over $[\underline{n}, \bar{n}] \times [0, \infty)$. We normalize the size of the total population to one. We denote by $P(q|n)$ the cumulated distribution function of q conditional on n, $p(q|n)$ the density of q conditional on n, and $f(n)$ the unconditional density of n, so that the density of the joint distribution of (n, q) is given by $p(q|n) \cdot f(n)$.

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leisure is less valuable for the other spouse). The assumption would also be violated if the are economies of scale in household production, for example in child care.

[5]This stands in contrast to the recent literature on collective labor supply (following the seminal contributions by Chiappori 1988, 1992) modelling couples as two individual utility maximizers interacting with each other. The single decision maker hypothesis provides a useful and simpler benchmark for our analysis. We argue in detail in Section 5 that collective labor supply issues matter primarily for redistribution *within* couples and that such within-couple redistribution can be made first best and is largely independent of the second-best redistribution across couples which we consider here.

For the secondary earner to enter the labor market and work, the utility from participation must be greater than or equal to the utility from non-participation. Let us denote by

$$V_l(n) = z_l - T_l(z_l) - nh\left(\frac{z_l}{n}\right) + w \cdot l, \tag{4}$$

the indirect utility of the couple (exclusive of the fixed work cost q). Differentiating with respect to n (which we denote by an upper dot from now on), and using the envelope theorem, we obtain

$$\dot{V}_l(n) = -h\left(\frac{z_l}{n}\right) + \frac{z_l}{n} \cdot h'\left(\frac{z_l}{n}\right). \tag{5}$$

The participation constraint for secondary earners is

$$q \leq V_1(n) - V_0(n) \equiv \bar{q}. \tag{6}$$

As defined in this expression, $\bar{q}$ is the net gain from working exclusive of the fixed work cost q. For families with a fixed cost below the threshold-value $\bar{q}$, the secondary earner works. For families with a fixed cost above the threshold, the secondary earner stays out of the labor force. If the tax function is not separable, the value of $\bar{q}$ and hence the participation decision of the secondary earner will depend on the labor supply decision of the primary earner. The probability of labor force participation for the secondary earner at a given ability level n of the primary earner is given by $P\left(\bar{q}|\, n\right)$.

It is natural to define the participation elasticity with respect to the net gain from working $\bar{q}$ as

$$\eta = \frac{\bar{q}}{P(\bar{q}|n)} \frac{\partial P(\bar{q}|n)}{\partial \bar{q}}. \tag{7}$$

When $\bar{q} = w$, secondary earners for whom $q \leq w$ participate, corresponding to a situation with no tax distortion in the secondary earner labor supply choice. If $\bar{q} = 0$, only spouses with a zero cost of working would participate, representing the case of 100% taxation of secondary earnings. Hence, we can define the tax rate on secondary earners by

$$\tau = \frac{w - \bar{q}}{w}.$$

Note that, when taxation is separate so that $T_0' = T_1'$ and hence $z_0 = z_1$, we have $\tau = (T_1 - T_0)/w$. When taxation is not separate, i.e. $T_0' \neq T_1'$ and hence $z_0 \neq z_1$, the parameter τ captures the tax rate on the secondary earner while $T_1 - T_0$ is the total change in tax liability for the couple when the secondary earner starts working.

Lemma 1 *At any point n, we have:*

- $T_0' > T_1' \iff z_0 < z_1 \iff \dot{\tau} < 0$
- $T_0' = T_1' \iff z_0 = z_1 \iff \dot{\tau} = 0$
- $T_0' < T_1' \iff z_0 > z_1 \iff \dot{\tau} > 0$

The proof follows easily from (5). The lemma is simply another way to restate the theorem of equality of the cross-partial derivatives. We naturally say that a tax system has *positive jointness* if τ is increasing and *negative jointness* if τ is decreasing. If τ is constant, the tax system is separable. Those definitions can be either local (at a given n) or global (for every n).

It is important to note that double-deviation issues are directly taken care off in our model because we always reason along the n-dimension and assume that z adapts optimally. For example, if the secondary earners starts to work, optimal primary earnings shift from $z_0(n)$ to $z_1(n)$ but the key first-order condition (5) continues to apply. More precisely, we can show, exactly as in the Mirrlees (1971) model, that a given path for $(z_0(n), z_1(n))$ can be implemented via a truthful mechanism or equivalently with a non-linear tax system if and only if $z_0(n)$ and $z_1(n)$ are non-negative and non-decreasing in n. We explain these mechanism design issues in more detail in Appendix A.1.

2.2 Deriving the Optimal Income Tax Rates

As in standard optimal income tax models, the government maximizes a social welfare function defined as the sum of a concave and increasing transformation $\Psi(.)$ of the couples' utilities subject to a government budget constraint. Formally, the government maximizes

$$W = \int_{n=\underline{n}}^{\bar{n}} \int_{q=0}^{\infty} \Psi(V_l(n) - q \cdot l) p(q|n) f(n) dq dn, \tag{8}$$

subject to the budget constraint

$$\int_{n=\underline{n}}^{\bar{n}} \int_{q=0}^{\infty} T_l(z_l) p(q|n) f(n) dq dn \geq E, \tag{9}$$

where E is an exogenous per capita revenue requirement. The concavity of $\Psi(.)$ measures the redistributive tastes of the government. We derive formally in appendix A.2 the following optimal tax formulas:

Proposition 1 *The first-order conditions for the optimal marginal tax rates T_0' and T_1' at ability level n can be written as*

$$\frac{T_0'}{1-T_0'} = \frac{1}{\varepsilon_0} \cdot \frac{1}{nf(n)(1-P(\bar{q}|n))} \cdot \int_n^{\bar{n}} \left\{ [1-g_0(n')] \left(1 - P\left(\bar{q}|n'\right)\right) + [T_1 - T_0] p\left(\bar{q}|n'\right) \right\} f(n')dn', \tag{10}$$

$$\frac{T_1'}{1-T_1'} = \frac{1}{\varepsilon_1} \cdot \frac{1}{nf(n)P(\bar{q}|n)} \cdot \int_n^{\bar{n}} \left\{ [1-g_1(n')] P\left(\bar{q}|n'\right) - [T_1 - T_0] p\left(\bar{q}|n'\right) \right\} f(n')dn', \tag{11}$$

where all the terms outside the integral are evaluated at ability level n and all the terms inside the integral are evaluated at n', and where $g_0(n')$ and $g_1(n')$ are the average social marginal welfare weights for couples with primary earners' ability n' and secondary earners not working and working, respectively.

The first-order conditions (10) *and* (11) *apply at any point n where there is no bunching* (*i.e., where $z_l(n)$ is strictly increasing in n*)*. If the conditions generate segments where $z_0(n)$ or $z_1(n)$ are decreasing, then there is bunching and $z_0(n)$ or $z_1(n)$ are constant over a segment.*

Heuristic Proof of Proposition 1

In order to understand the economic intuition behind the formulas in Proposition 1, it is useful to provide a heuristic derivation of the results based on the analysis of a small tax reform around the optimum schedule.

A useful first step is to present briefly the derivation of the optimal tax rate formula in the standard individualistic case (with no secondary earner). In that case, the model is a classic Mirrlees (1971) optimal income tax model with no income effects as in Diamond (1998). The heuristic derivation of optimal income tax rates has been developed by Piketty (1997) and Saez (2001).

Suppose, as illustrated in Figure 1, that we increase the income tax by dT for individuals with ability above n. This increase in taxes is obtained through a small increase dt in the marginal tax rate in a small band of ability levels $[n, n + dn]$. This tax reform raises more tax revenue from all taxpayers above the small band but decreases their utility. The gain for the government net of the welfare cost is

$$dG = dT \cdot \int_n^{\bar{n}} [1 - g(n')] f(n')dn',$$

where $g(n')$ is the marginal social welfare weight for individuals with ability n', and $f(n')$ is the density distribution of ability.

In the small band $[n, n+dn]$, there is a reduction in earnings due to the higher marginal tax rate dt. This decreases tax revenue collected from taxpayers in this band. An individual in the band reduces earnings by $dz = -z \cdot \varepsilon \cdot dt/(1-T')$ which translates into a tax change of $T'dz$. There are $f(n)dn$ such individuals in the band. Following the same derivation as in Saez (2001), the effect on tax revenue is[6]

$$dL = -dT \cdot nf(n) \cdot \varepsilon \cdot \frac{T'}{1-T'}.$$

At the optimum, this small reform cannot change welfare. Hence, the sum of the behavioral revenue effect dL and the net gain dG must be zero, implying the optimal income tax rate formula

$$\frac{T'}{1-T'} = \frac{1}{\varepsilon} \cdot \frac{1}{nf(n)} \cdot \int_n^{\bar{n}} [1 - g(n')]f(n')dn'. \tag{12}$$

This corresponds to the Mirrlees (1971) formula for optimal marginal tax rates in the case with no income effects as shown in Diamond (1998).[7]

Let us now examine how the introduction of a secondary earner modifies equation (12). With a secondary earner, the tax system can be depicted as a pair of tax schedules, shown in Figure 2, one for couples with working spouses and one for couples with non-working spouses. Note that the vertical distance between the two schedules, $T_1 - T_0$, is the extra tax paid by the couple when the secondary earner enters the labor force.

Let us consider, as illustrated in Figure 2, the same reform as before but only for couples with working spouses. More precisely, all couples with ability above n and a working spouse face a small tax increase dT which is created by increasing the marginal tax rate in the small band $[n, n+dn]$. As above, this tax reform raises more tax revenue from all two-earner couples above the small band but decreases their utility. The gain for the government net of the welfare cost is therefore

$$dG = dT \cdot \int_n^{\bar{n}} [1 - g_1(n')]f(n')P(\bar{q}|n')dn',$$

where $g_1(n')$ is the average marginal social welfare weight for couples with ability n' and a working spouse and $P(\bar{q}|n')$ is the fraction of couples with ability n' for which the secondary

[6] The key point to note is that $dT = dt \cdot dn \cdot z/n$ as the width of the small band in terms of realized earnings is $dn \cdot z/n$.

[7] The Diamond (1998) formula has $1 + 1/\varepsilon$ instead of $1/\varepsilon$ because n is defined as wage rates in the original Mirrlees model used by Diamond (1998). We prefer to define n as potential earnings instead because it simplifies comparative statics in ε (see Saez, 2001).

earner works (those with fixed cost of work below the cut-off level $\bar{q}$).

As above, the increase in the marginal tax rate in the small band creates a negative labor supply response for the primary earner which affects taxes collected by

$$dL = -dT \cdot P(\bar{q}|n) \cdot nf(n) \cdot \varepsilon_1 \cdot \frac{T_1'}{1 - T_1'}.$$

In contrast to the previous case, there is now an additional behavioral effect as the tax reform will induce some working spouses (married to primary earners above n) to drop out of the labor force and fall back on the one-earner tax schedule. At ability level $n' \geq n$, couples with fixed work costs between $\bar{q}$ and $\bar{q} - dT$ (there are $p(\bar{q}|n') \cdot f(n') \cdot dT$ of those couples) will move to the non-working spouse schedule, creating a government revenue effect equal to $-[T_1 - T_0] \cdot p(\bar{q}|n') \cdot f(n') \cdot dT$. Hence, the total effect on tax revenue from participation responses is given by

$$dP = -dT \cdot \int_n^{\bar{n}} [T_1 - T_0] \cdot p(\bar{q}|n') \cdot f(n') dn'.$$

At the optimum, the sum of the three effects dG, dL, and dP will be zero which leads immediately to equation (11) in the Proposition.

Equation (10) can be obtained in a similar way by considering an increase in the tax for one-earner couples above n. In that case, the participation effect goes in the opposite direction: some non-working spouses are induced to start working, which increases government tax revenue (when $T_1 - T_0$ is positive). As a result, the participation term in equation (10) appears with a positive sign.

2.3 Analyzing the Properties of the Optimal Income Tax Rates

2.3.1 Classical Zero Top and Bottom Results

Sadka (1976) and Seade (1977) demonstrated one of the most striking properties of the Mirrlees (1971) model, namely that the marginal tax rate should be zero at the top and at the bottom (provided the bottom skill is positive and everybody works). The same property holds in the two-earner model we are considering.

Proposition 2 *If the distribution of abilities n is bounded, then $T_0' = T_1' = 0$ at the top ability $\bar{n}$. If the bottom ability $\underline{n}$ is positive, then $T_0' = T_1' = 0$ at the bottom.*

The proof follows directly from the transversality conditions (see Appendix A.1).

It is easy to see why these results hold using the heuristic variational method described above. Let us go back to Figure 2 and assume that the increase in the marginal tax rate took place at the very top in the small band $[\bar{n} - dn, \bar{n}]$. In that case, the mechanical effect (net of the welfare cost) is negligible relative to the primary earner labor supply effect because there is nobody above $\bar{n}$ to collect the extra taxes dT from. Similarly the participation effect is negligible relative to the primary earner intensive labor supply effect. Thus, the first-order conditions hold only if $T_0' = T_1' = 0$ at the top skill $\bar{n}$. A similar type of proof can be applied to the bottom ability as well.

Numerical simulations in the context of the Mirrlees model (e.g., Tuomala, 1990) have shown that the top result is not of much use in practice because it is true only at the very top and hence applies only to the top earner. Top tails of the earnings distribution are very well approximated by Pareto distributions and it is therefore much more fruitful to consider infinite tails to obtain useful high-income optimal income tax results (see Saez, 2001, for a discussion of this point). We consider infinite tails below.

2.3.2 The Average Marginal Tax Rate Conditional on Ability n

It is useful to start by noting that the average marginal tax rate over one- and two-earner couples is exactly identical to the marginal tax rate in the individualistic standard case shown in equation (12). By taking the (weighted) sum of (10) and (11), we obtain

$$\varepsilon_0(1 - P(\bar{q}|n))\frac{T_0'}{1 - T_0'} + \varepsilon_1 P(\bar{q}|n)\frac{T_1'}{1 - T_1'} = \frac{1}{nf(n)} \cdot \int_n^{\bar{n}} [1 - \bar{g}(n')]f(n')dn', \qquad (13)$$

where $\bar{g}(n') \equiv P(\bar{q}|n')g_1(n') + (1 - P(\bar{q}|n'))\, g_0(n')$ is the average social marginal welfare weight for couples with ability n'.

This result can be obtained heuristically by increasing slightly the tax for all couples with ability above n. In that case, there is no change in the participation decision of secondary earners and therefore the only behavioral response is a substitution effect for primary earners around n. The result shows that redistribution from high- to low-ability primary earners follows the exact same logic as in the Mirrlees (1971) optimal income tax model. The introduction of a secondary earner does not change the average marginal tax rate faced by primary earners but introduces a difference in the marginal tax rate faced by one- versus two-earner couples, which we now examine in detail.

2.3.3 The Desirability of Joint Taxation

We introduce two assumptions.

Assumption 1 *The function* $V \longrightarrow \Psi'(V)$ *is convex.*

This is a very natural assumption on social preferences, and it will be satisfied for all standard social welfare functions such as the CRRA form $\Psi(V) = V^{1-\gamma}/(1-\gamma)$ with $\gamma > 0$.

Assumption 2 q *and* n *are independently distributed.*

This assumption allows us to isolate the impact on the optimal tax system of the interaction between spouses occurring through the social welfare function. Obviously, we do not expect this assumption to hold in practice and we examine numerically in Section 4 how this assumption affects our results.

To begin with, suppose that the government implements the optimal *separable* tax system, i.e. a tax system where $T_1 - T_0$ is independent of the primary earnings. Then the optimal constrained schedule is characterized by a single set of primary earner marginal tax rates T', a constant tax on the secondary earner $T_1 - T_0$, and an initial condition $T_0(z(\underline{n}))$. In this case, we have that $z_1(n) = z_0(n)$ and that $\bar{q} = w - (T_1 - T_0)$ is constant. Hence Assumption 2 implies that $P(\bar{q})$ is also constant across n. Exactly as in the above heuristic derivation of the average marginal tax rate, it can be shown that the optimal T' is given by the standard Mirrlees (1971) formula:

$$\frac{T'}{1-T'} = \frac{1}{\varepsilon} \cdot \frac{1}{nf(n)} \cdot \int_{n}^{\bar{n}} [1 - \bar{g}(n')] f(n') dn'.$$

The optimal $T_1 - T_0$ can be derived by shifting either the T_1- or the T_0-schedule uniformly by dT. For the T_1-schedule, this generates the formula

$$(T_1 - T_0) \cdot \frac{p(\bar{q})}{P(\bar{q})} = 1 - \int_{\underline{n}}^{\bar{n}} g_1(n) f(n) dn,$$

and for the T_0-schedule, we obtain

$$(T_1 - T_0) \cdot \frac{p(\bar{q})}{1 - P(\bar{q})} = \int_{\underline{n}}^{\bar{n}} g_0(n) f(n) dn - 1.$$

Summing those two equations implies

$$(T_1 - T_0) \cdot \frac{p(\bar{q})}{P(\bar{q}) \cdot (1 - P(\bar{q}))} = \int_{\underline{n}}^{\bar{n}} [g_0(n) - g_1(n)] f(n) dn > 0. \tag{14}$$

The positive sign in (14) can be obtained as follows. By definition,

$$g_0(n) - g_1(n) = \frac{\Psi'(V_0)}{\lambda} - \frac{\int_0^{\bar{q}} \Psi'(V_0 + \bar{q} - q) p(q) dq}{\lambda \cdot P(\bar{q})}. \tag{15}$$

Thus, the fact that Ψ' is decreasing (Ψ concave) implies that $g_0 - g_1 > 0$.

Starting from this separable schedule, let us introduce some negative jointness. We consider an increase in the tax on one-earner couples and a decrease in the tax on two-earner couples above some ability level n as depicted in Figure 3. The change in the tax for two-earner couples is $dT_1 = -dT/P(\bar{q})$ and the change in the tax for one-earner couples is $dT_0 = dT/(1 - P(\bar{q}))$, so that the net effect on taxes collected (absent any behavioral response) is zero.

The net direct welfare effect is

$$dW = dT \cdot \int_n^{\bar{n}} [g_1(n') - g_0(n')] f(n') dn'.$$

There are two behavioral responses to the tax change. First, these tax changes are obtained by raising (lowering) the marginal tax rate on the primary earner in one-earner (two-earner) families around n. The changes in marginal tax rates generate earnings responses for primary earners going in opposite directions in one- and two-earner couples. Since the primary earner elasticity is the same for one- and two-earner couples (from equation (3) as $z_1 = z_0$), these behavioral responses offset each other exactly and the net fiscal effect is zero.

Second, the tax change induces a number of non-working spouses above n to join the labor force. The number of switchers is $(1 - F(n))p(\bar{q})d\bar{q}$ and $d\bar{q} = dT_0 - dT_1 = dT/[P(1-P)]$. Each of these movers pays $T_1 - T_0 > 0$ extra in taxes and hence generate a positive fiscal effect. So the net effect on tax revenue due to the behavioral response is $dB = dT \cdot (1 - F(n)) \cdot (T_1 - T_0) \cdot p(\bar{q})/[P(1-P)]$.

Therefore, the net effect of the reform is given by

$$dB + dW = dT \cdot \left\{ (1 - F(n))(T_1 - T_0) \frac{p(\bar{q})}{P(\bar{q}) \cdot (1 - P(\bar{q}))} - \int_n^{\bar{n}} [g_0(n') - g_1(n')] f(n') dn' \right\}.$$

Using (14), this can be rewritten to

$$dB + dW = dT \cdot \left\{ (1 - F(n)) \int_{\underline{n}}^{\bar{n}} [g_0(n') - g_1(n')] f(n') dn' - \int_{n}^{\bar{n}} [g_0(n') - g_1(n')] f(n') dn' \right\}$$

$$= dT \cdot \left\{ (1 - F(n)) \int_{\underline{n}}^{n} [g_0(n') - g_1(n')] f(n') dn' - F(n) \int_{n}^{\bar{n}} [g_0(n') - g_1(n')] f(n') dn' \right\}. \quad (16)$$

$dB + dW > 0$ will follow from the following Lemma.

Lemma 2 *Under Assumptions 1 and 2 and with a separable tax system,* $g_0(n) - g_1(n)$ *is (weakly) decreasing in* n*.*

Proof:

Because the tax system is separable, we have that $\bar{q} = w - (T_1 - T_0)$ is constant in n. Hence, equation (15) implies:

$$\frac{d(g_0(n) - g_1(n))}{dn} = \left[\frac{\Psi''(V_0)}{\lambda} - \frac{\int_0^{\bar{q}} \Psi''(V_0 + \bar{q} - q) p(q) dq}{\lambda \cdot P(\bar{q})} \right] \cdot \dot{V}_0.$$

Assumption 1 implies that Ψ'' is increasing, thus the expression in square brackets above is negative. Furthermore, V_0 is increasing in n. This demonstrates the Lemma. $\square$

The lemma implies that

$$\frac{\int_{\underline{n}}^{n} [g_0(n') - g_1(n')] f(n') dn'}{F(n)} > g_0(n) - g_1(n) > \frac{\int_{n}^{\bar{n}} [g_0(n') - g_1(n')] f(n') dn'}{1 - F(n)}.$$

This inequality implies that expression (16) above for $dB + dW$ is positive. Therefore, the reform depicted on Figure 3 is desirable, showing in particular that separate taxation is not optimal. We can then state the following proposition.

Proposition 3 *Under Assumptions 1 and 2, starting from the optimal* separable schedule*, introducing some negative jointness in taxes by lowering taxes in* $(n, \bar{n})$ *for two-earner couples and increasing taxes in* $(n, \bar{n})$ *for one-earner couples increases welfare.*

This proposition shows that the desirable direction of the reform is to decrease the tax on secondary earners for high primary earnings or equivalently to increase the marginal tax rate on one-earner couples relative to two-earner couples. This tax reform result is a first step toward establishing this pattern at the full joint optimum which we explore below.

It is important to understand the economic intuition behind this result: the tax on secondary earners, $T_1 - T_0 > 0$, amounts to redistributing from two-earner couples to one-earner couples. This redistributive value is higher for couples with low primary earnings than for couples with high primary earnings. This tax on secondary earnings generates a distortion on the labor supply of the secondary earner which does not depend on primary earnings. Therefore, trading off equity and efficiency, it is desirable for the government to reduce this secondary earner tax when primary earnings are high.

2.3.4 Asymptotic Results for $T_1 - T_0$

Suppose that $\bar{n} = \infty$ so that the ability distribution of primary earners has an infinite tail. For any reasonable welfare function, we would then have that $g_0(n)$ and $g_1(n)$ converge to the same value g^∞, because the additional income generated by the secondary earner becomes infinitesimal relative to primary earner income in the limit.[8] It is also natural to assume that the primary earner elasticities ε_l converge to an asymptotic value ε^∞ as n tends to infinity.

Since top tails of income distributions are well approximated by Pareto distributions, as explained above, we assume that abilities n are Pareto distributed at the top with Pareto parameter a, and that fixed work costs q are distributed independently of n at the top with distribution $P(q)$. Under these assumptions, we can prove the following result:

Proposition 4 *Suppose that $T_1 - T_0$, T_0', T_1', $\bar{q}$ converge to ΔT^∞, T'^∞_0, T'^∞_1, and $\bar{q}^\infty$ when n goes to infinity. Then we have*

- $\Delta T^\infty = 0$, *i.e., the tax on secondary earners goes to zero as the earnings of the primary earner increase to infinity.*
- $T'^\infty_0 = T'^\infty_1 = (1 - g^\infty) \,/\, (1 - g^\infty + a \cdot \varepsilon^\infty) > 0$, *exactly as in the Mirrlees model.*

Proof:

Because $T_1 - T_0$ converges when n goes to infinity, it must be the case that $T'^\infty_0 = T'^\infty_1 = T'^\infty$. Because, $\bar{q}$ converges, $P(\bar{q})$ and $p(\bar{q})$ also converge. Let us denote by P^∞ and p^∞ their limits. The Pareto assumption implies that $(1 - F(n))/(nf(n)) = 1/a$ for n large. Taking the limit when n goes to infinity of the optimal tax formulas (10) and (11) from Proposition 1, we obtain

[8] In the case where $g^\infty = 0$, the optimal tax system extracts as much tax revenue as possible from the very rich ('soaking the rich').

respectively:

$$\frac{T'^{\infty}}{1-T'^{\infty}} = \frac{1}{\varepsilon^{\infty}} \cdot \frac{1}{a} \cdot \left[1 - g^{\infty} + \Delta T^{\infty} \frac{p^{\infty}}{1-P^{\infty}}\right],$$

$$\frac{T'^{\infty}}{1-T'^{\infty}} = \frac{1}{\varepsilon^{\infty}} \cdot \frac{1}{a} \cdot \left[1 - g^{\infty} - \Delta T^{\infty} \frac{p^{\infty}}{P^{\infty}}\right].$$

Hence, it is necessary that $\Delta T^{\infty} = 0$ and the formula for T'^{∞} follows immediately. □

The result in Proposition 4 is quite striking. The earnings of spouses of the highest-income earners should be exempted from taxation, even in the case where the government tries to extract as much tax revenue as possible from high-income couples (the case of $g^{\infty} = 0$). Although the result may seem reminiscent of the classic zero top result of Sadka and Seade discussed above, the logic is completely different. In fact, in the present case where the distribution of abilities n has an infinite tail, the tax on the secondary earner is zero at the top while the marginal tax on the primary earner is actually positive at the top. On the other hand, in the case of a bounded ability distribution, we would obtain a top marginal tax rate on the primary earner equal to zero (cf. Proposition 2), but then the tax on the secondary earner would no longer be zero at the top (a point which we come back to in the following subsection).

The economic intuition of this result can be understood by using Figure 3 again where we increase the tax on one-earner couples and decrease the tax on two-earner couples above some high ability level n. Let us assume that $T_1 - T_0$ were to converge to some limit $\Delta T^{\infty} > 0$ so that the analysis can parallel the analysis of the previous subsection. The mechanical effect on tax revenue is zero as before. Importantly, the direct welfare effect is also zero because the reduced welfare of one-earner couples is exactly compensated for by an increase in the welfare of two-earner couples as the social marginal welfare weights are identical (and equal to g^{∞}) for both groups. As before, the behavioral response along the intensive margin does not affect tax revenue. Finally, the tax change induces a number of non-working spouses above n to join the labor force. Each of these movers would pay $\Delta T^{\infty} > 0$ extra in taxes and hence generate a positive fiscal effect. This positive effect is the net total effect of the reform as all of the previous effects cancelled out. Therefore, $\Delta T^{\infty} > 0$ cannot be optimal.[9] Therefore we must have $\Delta T^{\infty} = 0$ asymptotically as stated in Proposition 4.

[9]Conversely, if ΔT^{∞} were to be negative, the opposite tax reform would increase welfare.

In summary, this result can be seen as an extension of Proposition 3. For very high primary earnings, secondary earnings are negligible and hence there is no value in redistributing from two-earner couples to one-earner couples. Therefore, there is no point in introducing a tax distortion on secondary earners when primary earnings are very high.

2.3.5 A General Negative Jointness Result

We now turn to the comparison of T_0' and T_1' over the full tax schedule. In order to obtain our central negative jointness result, we need introduce three additional assumptions.

Assumption 3 *The function* $x \longrightarrow (1-h'(x))/(x \cdot h''(x))$ *is decreasing.*

This assumption is satisfied, for example, for iso-elastic utilities $h(x) = x^{1+k}/(1+k)$ where the labor supply elasticity ε is constant and equal to $1/k$.

Assumption 4 *The function* $x \longrightarrow x \cdot p(w-x)/[P(w-x) \cdot (1-P(w-x))]$ *is increasing.*

This assumption is satisfied for iso-elastic cost of work distributions of the type $P(q) = (q/q_{max})^{\eta}$ where the participation elasticity of secondary earners (with respect to the money metric net utility of working $\bar{q} = V_1 - V_0$) defined as $q \cdot p(q)/P(q)$ is constant and equal to η.

Assumption 5 $q \cdot p(q)/P(q) \leq 1$ *for all* q.

This assumption is satisfied when the participation elasticity η is less than or equal to one.

With these assumptions, we can state the following proposition:

Proposition 5 *Under Assumptions 1-5, and assuming there is no bunching at the optimum, we have*

- $T_1' \leq T_0'$ *for all* n. *Equivalently,* τ *is non-increasing in* n *everywhere.*
- $T_1(z_n) - T_0(z_n) \geq T_1(z_{\bar{n}}) - T_0(z_{\bar{n}}) > 0$ *for all* n *(assuming that* $\bar{n} < \infty$*).*

Proof:

Suppose by contradiction that $T_1' > T_0'$ for some n. Then, because T_0' and T_1' are continuous in n (cf. Appendix A.2) and because $T_1' = T_0'$ at the top and bottom skills (cf. Proposition 2), there exists an interval (n_a, n_b) where $T_1' > T_0'$ and where $T_1' = T_0'$ at the end points, n_a and n_b.

This implies that $z_1 < z_0$ on (n_a, n_b) with equality at the end points. Hence, by Assumption 3, we have $\varepsilon_1 T_1'/(1 - T_1') = (1 - h_1')/(h_1'' \cdot z_1/n) > (1 - h_0')/(h_0'' \cdot z_0/n) = \varepsilon_0 T_0'/(1 - T_0')$ on (n_a, n_b). Then, using the first-order conditions (10) and (11) which apply everywhere because of our no bunching assumption, we obtain

$$\Omega_0(n) \equiv \frac{1}{1-P}\int_n^{\bar{n}} [(1-g_0)(1-P)+\Delta T \cdot p] f(n')dn' < \frac{1}{P}\int_n^{\bar{n}} [(1-g_1)P-\Delta T \cdot p] f(n')dn' \equiv \Omega_1(n)$$

on (n_a, n_b) with equality at the end points. This implies that the derivatives of the above expressions with respect to n, at the end points, obey the inequalities $\dot{\Omega}_0(n_a) < \dot{\Omega}_1(n_a)$ and $\dot{\Omega}_0(n_b) > \dot{\Omega}_1(n_b)$. At the end points, we have $T_1' = T_0'$, $z_0 = z_1$, and $\dot{V}_0 = \dot{V}_1$, which implies $\dot{\bar{q}} = 0$ and $\dot{P} = 0$. Hence, the inequalities in derivatives can be written as

$$\begin{aligned} 1 - g_0 + \Delta T \cdot p/(1-P) &> 1 - g_1 - \Delta T \cdot p/P \quad \text{at } n_a, \\ 1 - g_0 + \Delta T \cdot p/(1-P) &< 1 - g_1 - \Delta T \cdot p/P \quad \text{at } n_b. \end{aligned}$$

Combining these inequalities, we obtain

$$\frac{\Delta T \cdot p}{P(1-P)}|_{n_a} > g_0(n_a) - g_1(n_a) > g_0(n_b) - g_1(n_b) > \frac{\Delta T \cdot p}{P(1-P)}|_{n_b}.$$

The middle inequality is intuitive and follows formally from Assumptions 1-5 as shown in Appendix A.3. Using that $\bar{q} = w - \Delta T$ at n_a and n_b, along with Assumption 4, we obtain $\Delta T(n_a) > \Delta T(n_b)$.

However, $T_1' > T_0'$ and hence $z_1 < z_0$ implies that $\dot{\bar{q}} < 0$ on the interval (n_a, n_b). Then we have $\bar{q}(n_a) > \bar{q}(n_b)$ and hence $\Delta T(n_a) < \Delta T(n_b)$. This generates a contradiction, which proves that $T_1' \leq T_0'$ for all n.

The second part of the proposition follows easily from the first part. Since we now have $T_1' \leq T_0'$ on $(\underline{n}, \bar{n})$ with equality at the end points, we obtain $\Omega_0(n) \geq \Omega_1(n)$ on $(\underline{n}, \bar{n})$ with equality at the end points. Then we have that $\dot{\Omega}_0(\bar{n}) \leq \dot{\Omega}_1(\bar{n})$, which implies

$$1 - g_0 + \Delta T \cdot p/(1-P) \geq 1 - g_1 - \Delta T \cdot p/P \quad \text{at } \bar{n}.$$

Because $g_0(\bar{n}) - g_1(\bar{n}) > 0$, we have $\Delta T(\bar{n}) > 0$.

Finally, $T_1' \leq T_0'$ and hence $z_1 \geq z_0$ implies $\dot{\bar{q}} \geq 0$ and $\bar{q} \geq w - \Delta T$ with equality at $\bar{n}$. Therefore, we have $w - \Delta T(n) \leq \bar{q}(n) \leq \bar{q}(\bar{n}) = w - \Delta T(\bar{n})$, and hence $\Delta T(n) \geq \Delta T(\bar{n})$. □

At a given primary earner ability level, secondary earner participation is a signal of small fixed costs of work and being better off than non-participation. This implies $g_0(n) - g_1(n) > 0$ making it optimal to redistribute from two-earner couples to one-earner couples, i.e. $T_1 - T_0 > 0$. This redistribution gives rise to a tax distortion in the entry-exit decision of secondary earners, creating a trade-off between equity and efficiency. The size of the efficiency cost does not depend on the ability of the primary earner because the characteristics of the two spouses, q and n, are independently distributed. An increase in n therefore only influences the optimal secondary earner tax through its impact on the social welfare weights. The value of redistribution in favor of one-earner couples is declining in primary earnings, i.e. $g_0(n) - g_1(n)$ is decreasing in n, due to the fact that the contribution of the secondary earner to household utility is declining. Therefore, the tax on secondary earnings is declining in primary earnings. As shown previously, if the ability distribution of primary earners is unbounded, the secondary earner tax vanishes to zero at the top. The implication of the declining secondary earner tax is that the marginal tax rate on the primary earner is lower when the spouse works. This is what we have termed negative jointness.

Although our results may seem surprising at first glance, they obey a simple redistributive logic. If the tax schedule for two-earner couples is seen as the base schedule, the tax schedule for one-earner couples is obtained from this base schedule by giving a tax break — a dependent spouse allowance — which is larger for couples with low primary earnings than for couples with large primary earnings. In the limit where primary earnings go to infinity, the tax break is zero. The shrinking tax break generates an implicit tax on secondary earners which decreases with primary earnings.

We can prove a simple result on the necessary and sufficient conditions for the optimum tax to be separable in the earnings of each spouse. This result can be seen as a Corollary to the much more general Proposition 5.

Proposition 6 *Under Assumption 2, if $g_0 - g_1$ is constant over n, then the optimum is characterized by $T_0' = T_1'$ and*

$$T_1 - T_0 = [g_0 - g_1] \cdot \frac{P(\bar{q})(1 - P(\bar{q}))}{p(\bar{q})}, \tag{17}$$

which is independent of n. Conversely, if the optimum is such that $T_1' = T_0'$ (implying $T_1 - T_0$ being independent of n), then it must be the case that $g_0 - g_1$ is constant over n.

We present the proof in Appendix A.4. Note that $g_0 - g_1$ constant in n cannot happen with a standard concave social welfare function Ψ. However, one can consider more general social welfare weights where $g_0 - g_1$ constant is possible which makes the result of this proposition useful.

3 Intensive Response for the Secondary Earner

Instead of specifying a binary choice model for the secondary earner labor supply response, we can use a classic intensive labor supply model for the secondary earner. In that case, the primary and secondary earner are modelled symmetrically. There is a distribution of earnings abilities (n_p, n_s) over the population of couples with density $f(n_p, n_s)$ on the domain D. The utility function is given by

$$u(c, z_p, z_s) = c - n_p h_p(z_p/n_p) - n_s h_s(z_s/n_s),$$

with $c = z_p + z_s - T(z_p, z_s)$. This is a two-dimensional screening problem. There is a small literature in optimal tax theory considering this type of multi-dimensional screening models originating with Mirrlees (1976, 1986). There is a larger literature on multi-dimensional screening in nonlinear pricing theory (see McAffee and McMillan, 1988; Wilson, 1993; Armstrong, 1996; Rochet and Choné, 1998; and Rochet and Stole, 2002).

The first-order conditions for each earner are given by

$$h'_p(z_p/n_p) = 1 - T'_p \quad \text{and} \quad h'_s(z_s/n_s) = 1 - T'_s. \tag{18}$$

The indirect utility is denoted by $V(n_p, n_s)$ and satisfies (using the envelope theorem):

$$V'_{n_p} = -h_p + (z_p/n_p)h'_p \quad \text{and} \quad V'_{n_s} = -h_s + (z_s/n_s)h'_s. \tag{19}$$

The objective of the government is to maximize

$$W = \int\int_D \Psi\left(V\left(n_p, n_s\right)\right) f\left(n_p, n_s\right) dn_p dn_s,$$

subject to the budget constraint

$$\int\int_D T\left(z_p, z_s\right) f\left(n_p, n_s\right) dn_p dn_s \geq E.$$

We can then state the following proposition:

Proposition 7 *The first-order conditions for the optimal marginal tax rates T'_p and T'_s at ability level (n_p, n_s) can be written as*

$$\frac{T'_p}{1-T'_p} = \frac{1}{\varepsilon_p} \cdot \frac{1}{n_p f(n_p, n_s)} \cdot \mu_p, \tag{20}$$

$$\frac{T'_s}{1-T'_s} = \frac{1}{\varepsilon_s} \cdot \frac{1}{n_s f(n_p, n_s)} \cdot \mu_s, \tag{21}$$

where μ_p and μ_s are multipliers satisfying the transversality conditions $\mu_p(\underline{n}_p, n_s) = \mu_p(\bar{n}_p, n_s) = 0$ for all n_s and $\mu_s(n_p, \underline{n}_s) = \mu_s(n_p, \bar{n}_s) = 0$ for all n_p, along with the divergence equation

$$\frac{\partial \mu_p}{\partial n_p} + \frac{\partial \mu_s}{\partial n_s} = [g(n_p, n_s) - 1] \cdot f(n_p, n_s), \tag{22}$$

where $g(n_p, n_s)$ is the marginal welfare weight for couples with ability (n_p, n_s). At the optimum, the following equation has to be satisfied everywhere:

$$\frac{z_p}{n_p^2} \frac{\partial z_p}{\partial n_s} h''_p\left(\frac{z_p}{n_p}\right) = \frac{z_s}{n_s^2} \frac{\partial z_s}{\partial n_p} h''_s\left(\frac{z_s}{n_s}\right). \tag{23}$$

The proof is presented in Appendix A.5.

The formulas are obtained from the first-order conditions of the Hamiltonian. The divergence equation (22) has many solutions satisfying the boundary transversality conditions.[10] Equation (23), which follows from the fact that the second-order derivatives of the indirect utility function $V(n_p, n_s)$ has to be symmetric, gives an additional condition making the optimum solution unique generically.

The optimal marginal tax rate formulas can be obtained heuristically as follows. Consider a tax reform increasing by dT the tax for couples (n'_p, n'_s) above (n_p, n_s), i.e., such that $n'_p > n_p$ and $n'_s > n_s$. This change can be obtained by increasing the marginal tax rate on primary earners in a small interval $[n_p, n_p + dn_p]$ with spouses with ability n'_s above n_s. Symmetrically, the marginal tax rate on secondary earners in a small interval $[n_s, n_s + dn_s]$ with spouses with ability n'_p above n_p is also increased. The reform is illustrated in Figure 4.

The reform leads to a mechanical increase in tax revenue and a reduction in welfare for all couples in the shaded area in Figure 4. The net effect is given by

$$dT \int_{n_p}^{\bar{n}_p} \int_{n_s}^{\bar{n}_s} \left[1 - g\left(n'_p, n'_s\right)\right] f(n'_p, n'_s) dn'_p dn'_s.$$

[10]More precisely, if (μ_p, μ_s) is a solution to the divergence equation, then any function $(\mu_s - \partial\varphi/\partial n_s, \mu_p + \partial\varphi/\partial n_p)$ where $\varphi(n_p, n_s)$ is an arbitrary scalar function will also satisfy the divergence equation.

In addition, there will be a labor supply response for individuals in the south and west borders of the shaded area due to changed marginal tax rates. The net loss of tax revenue is

$$dT\int_{n_s}^{\bar{n}_s}\varepsilon_p\frac{T_p'}{1-T_p'}n_p f\left(n_p,n_s'\right)dn_s'+dT\int_{n_p}^{\bar{n}_p}\varepsilon_s\frac{T_s'}{1-T_s'}n_s f\left(n_p',n_s\right)dn_p'.$$

At the optimum, those two effects need to be equal. It is straightforward to check that the resulting equation implies equations (20), (21), and (22) of the Proposition.

It is easy to show that the average T_p' across n_s is the same as in the standard Mirrlees model. We define $f_p(n_p)$ as the unconditional density distribution of n_p. Let us define F_p as the cumulated distribution of n_p:

$$1-F_p(n_p)=\int_{n_p}^{\bar{n}_p}\int_{\underline{n}_s}^{\bar{n}_s}f(n_p',n_s')dn_s'dn_p',$$

and G_p as the average of marginal welfare weights $g(n_p',n_s')$ above n_p:

$$G_p(n_p)\cdot[1-F_p(n_p)]=\int_{n_p}^{\bar{n}_p}\int_{\underline{n}_s}^{\bar{n}_s}g(n_p',n_s')f(n_p',n_s')dn_s'dn_p'.$$

We can then show,

Proposition 8

$$\frac{T_p'}{1-T_p'}=\frac{1}{\varepsilon_p}\frac{(1-F_p)\cdot(1-G_p)+\delta^p(n_p,n_s)}{n_p f_p},\tag{24}$$

where $\delta^p(n_p,n_s)$ averages to zero when summed over n_s, i.e., for all n_p

$$\int_{\underline{n}_s}^{\bar{n}_s}\delta^p(n_p,n_s)f(n_p,n_s)dn_s=0.$$

The symmetric equations hold when substituting p for s.

Proof:

$\delta^p(n_p,n_s)$ is defined as:

$$\delta^p(n_p,n_s)=n_p f_p\cdot\varepsilon_p\cdot\frac{T_p'}{1-T_p'}-(1-F_p)\cdot(1-G_p).$$

Hence, equation (20) implies:

$$\delta^p(n_p,n_s)\cdot f(n_p,n_s)=\mu_p f_p-(1-F_p)\cdot(1-G_p)f(n_p,n_s).$$

Integrating this expression over $(\underline{n}_s,\bar{n}_s)$, we have:

$$\int_{\underline{n}_s}^{\bar{n}_s}\delta^p(n_p,n_s)f(n_p,n_s)dn_s=f_p(n_p)\int_{\underline{n}_s}^{\bar{n}_s}\mu_p(n_p,n_s)dn_s-f_p(n_p)\cdot(1-F_p)\cdot(1-G_p).\tag{25}$$

Integrating the divergence equation (22) over n_s and using the transversality condition, we have:

$$\int_{\underline{n}_s}^{\bar{n}_s} \frac{\partial \mu_p}{\partial n_p} dn_s = \int_{\underline{n}_s}^{\bar{n}_s} [g(n_p, n_s) - 1] \cdot f(n_p, n_s) dn_s,$$

Integrating again from n_p to $\bar{n}_p$, we have:

$$\int_{\underline{n}_s}^{\bar{n}_s} \mu_p(n_p, n_s) dn_s = \int_{n_p}^{\bar{n}_p} \int_{\underline{n}_s}^{\bar{n}_s} [1 - g(n_p, n_s)] \cdot f(n_p, n_s) dn_s = (1 - G_p(n_p)) \cdot (1 - F_p(n_p)).$$

This implies that the expression (25) is zero which completes the proof. □

Desirability of joint taxation

As in the binary case, we make an assumption on the social welfare function (Assumption 1 above) along with an assumption that innate characteristics are independently distributed, that is,

Assumption 2': n_p and n_s are independently distributed.

Suppose the government implements the optimal *separable* tax schedule. It can be shown easily using the standard one dimensional approach that the optimal separable tax schedules will take the form

$$\frac{T_p'}{1 - T_p'} = \frac{1}{\varepsilon_p} \frac{(1 - F_p(n_p)) \cdot (1 - G_p(n_p))}{n_p f_p}, \tag{26}$$

$$\frac{T_s'}{1 - T_s'} = \frac{1}{\varepsilon_s} \frac{(1 - F_s(n_s)) \cdot (1 - G_s(n_s))}{n_s f_s}. \tag{27}$$

where $G_p(n_p)$ is the average welfare weight above n_p (averaged across all n_s) and $G_s(n_s)$ is the average welfare weight above n_s (averaged across all n_p).

Starting from those separable schedules, we can consider introducing some jointness as shown on Figure 5. Let fix a point $n = (n_p, n_s)$. We increase taxes by $dT/[(1 - F_p(n_p))F_s(n_s)]$ in the South-East (SE) quadrant $(n_p, \bar{n}_p) \times (\underline{n}_s, n_s)$. We decrease taxes by $-dT/[(1 - F_p(n_p))(1 - F_s(n_s))]$ in the North-East (NE) quadrant $(n_p, \bar{n}_p) \times (n_s, \bar{n}_s)$. There is no change in taxes in the North-West and South-West quadrants.

This tax change has no effect on taxes collected, absent any behavioral response because the number of couples in the SE quadrant is $(1 - F_p(n_p)) \cdot F_s(n_s)$ and the number of couples in the NE quadrant is $(1 - F_p(n_p)) \cdot (1 - F_s(n_s))$.

This tax change gives rise to a direct welfare effect equal to

$$dW = -dT \cdot [G((n_p, \bar{n}_p) \times (\underline{n}_s, n_s)) - G((n_p, \bar{n}_p) \times (n_s, \bar{n}_s))],$$

where $G(I \times J)$ denotes the average welfare weight on the set $I \times J$.

Those changes can be implemented by decreasing T_p' in the band $n_p \times (n_s, \bar{n}_s)$ and by increasing T_p' in the band $n_p \times (\underline{n}_s, n_s)$. Similarly, T_s' decreases in the band $(n_p, \bar{n}_p) \times n_s$. Those changes affect labor supply and hence tax collected (but the welfare effect is second order due to usual envelope theorem argument).

The changes in T_p' compensate each other exactly in terms of labor supply because we are starting from a separable tax schedule. Hence, the tax revenue effect due to behavioral responses is solely due to the T_s' change. The change in T_s' needs to accommodate a jump down in taxes equal to $dT/[(1-F_p)F_s] + dT/[(1-F_p)(1-F_s)] = dT/[(1-F_p)F_s(1-F_s)]$. Hence, the effect on tax revenue can be written as (exactly as in our derivation of the standard Mirrlees formula in Section 3):

$$dB = \frac{dT}{(1-F_p)F_s(1-F_s)} \int_{n_p}^{\bar{n}_p} \frac{T_s'}{1-T_s'} \varepsilon_s n_s f_s \cdot f_p(n_p') dn_p' = \frac{dT}{F_s(1-F_s)} \frac{T_s'}{1-T_s'} \varepsilon_s n_s f_s = dT \cdot \frac{1-G_s(n_s)}{F_s(n_s)},$$

where we use the separability for the first equality and the optimal tax formula (27) for the second equality.

The average social weight is equal to one, hence:

$$G((\underline{n}_p, \bar{n}_p) \times (\underline{n}_s, n_s)) \cdot F_s(n_s) + G((\underline{n}_p, \bar{n}_p) \times (n_s, \bar{n}_s)) \cdot (1 - F_s(n_s)) = 1,$$

which, using the fact that $G_s(n_s) = G((\underline{n}_p, \bar{n}_p) \times (n_s, \bar{n}_s))$, can be written as:

$$\frac{1-G_s(n_s)}{F_s(n_s)} = G((\underline{n}_p, \bar{n}_p) \times (\underline{n}_s, n_s)) - G((\underline{n}_p, \bar{n}_p) \times (n_s, \bar{n}_s)).$$

Plugging this into our expression for dB, we obtain:

$$dB = dT \cdot [G((\underline{n}_p, \bar{n}_p) \times (\underline{n}_s, n_s)) - G((\underline{n}_p, \bar{n}_p) \times (n_s, \bar{n}_s))].$$

Comparing the expressions for dW and dB above, it is clear that $dW + dB > 0$ if we can show that

$$G((\underline{n}_p, \bar{n}_p) \times (\underline{n}_s, n_s)) - G((\underline{n}_p, \bar{n}_p) \times (n_s, \bar{n}_s)) > G((n_p, \bar{n}_p) \times (\underline{n}_s, n_s)) - G((n_p, \bar{n}_p) \times (n_s, \bar{n}_s)) \tag{28}$$

Proposition 9 *Under Assumptions 1 and 2', starting from the optimal separable schedule, introducing some negative jointness in taxes by increasing taxes in* $(n_p, \bar{n}_p) \times (\underline{n}_s, n_s)$ *and decreasing taxes in* $(n_p, \bar{n}_p) \times (n_s, \bar{n}_s)$ *increases welfare.*

Proof:

$V(n_p, n_s)$ is strictly increasing in n_s. As a result, Assumption 1 implies that $\Psi''(V(n_p, n_s')) < \Psi''(V(n_p, n_s)) < \Psi''(V(n_p, n_s''))$ for $n_s' < n_s < n_s''$. Hence averaging across $n_s' \in (\underline{n}_s, n_s)$ and $n_s'' \in (n_s, \bar{n}_s)$, we obtain:

$$\frac{\int_{\underline{n}_s}^{n_s} \Psi''(V(n_p, n_s')) f_s(n_s') dn_s'}{F_s(n_s)} < \frac{\int_{n_s}^{\bar{n}_s} \Psi''(V(n_p, n_s'')) f_s(n_s'') dn_s''}{1 - F_s(n_s)}.$$

Assumptions 2' implies that $V(n_p, n_s)$ is also separable and hence that V'_{n_p} is independent of n_s. Therefore, the inequality above implies that:

$$\Omega(n_p) = \frac{\int_{\underline{n}_s}^{n_s} \Psi'(V(n_p, n_s')) f_s(n_s') dn_s'}{F_s(n_s)} - \frac{\int_{n_s}^{\bar{n}_s} \Psi'(V(n_p, n_s'')) f_s(n_s'') dn_s''}{1 - F_s(n_s)},$$

is decreasing in n_p. Hence, the average of Ω across $(\underline{n}_p, n_p)$ is larger than $\Omega(n_p)$ which is larger than the average of Ω across $(n_p, \bar{n}_p)$. This can be restated as:

$$G((n_p, \bar{n}_p) \times (\underline{n}_s, n_s)) - G((n_p, \bar{n}_p) \times (n_s, \bar{n}_s)) < G((\underline{n}_p, n_p) \times (\underline{n}_s, n_s)) - G((\underline{n}_p, n_p) \times (n_s, \bar{n}_s)).$$

Using the decompositions

$$G((\underline{n}_p, \bar{n}_p) \times (\underline{n}_s, n_s)) = F_p(n_p) \cdot G((\underline{n}_p, n_p) \times (\underline{n}_s, n_s)) + (1 - F_p(n_p)) \cdot G((\bar{n}_p, n_p) \times (\underline{n}_s, n_s)),$$

$$G((\underline{n}_p, \bar{n}_p) \times (n_s, \bar{n}_s)) = F_p(n_p) \cdot G((\underline{n}_p, n_p) \times (n_s, \bar{n}_s)) + (1 - F_p(n_p)) \cdot G((\bar{n}_p, n_p) \times (n_s, \bar{n}_s)),$$

we can obtain immediately the inequality (28) required to complete the proof. $\square$.

This proposition generalizes our previous statement from Section 2.3.3. It shows that in the double intensive model as well, introducing some negative jointness increases welfare. This suggests that our central proposition 5 result from the binary case might generalize to the double intensive model. Under a set of regularity conditions, we should expect that T_p' is decreasing with n_s (and conversely that T_s' is decreasing with n_p).

4 Numerical Simulations

There are two goals in our numerical simulations. First, we want to illustrate our theoretical results. This includes showing that our no bunching assumption applies to a wide set of situations, and demonstrating that negative jointness is optimal in more general models than the binary case considered in Section 2 and carries over to the case where secondary earners respond along the intensive margin. Second, we want to give a sense of the quantitative importance of the negative jointness result, how it depends on the parameters of the model, and how robust it would be to relaxing some of the Assumptions in our basic model.

For the simulations, we make the following simple parametric assumptions. First, we assume that $h(x) = x^{1+k}/(1+k)$, so that we have a constant primary earner elasticity $\varepsilon = 1/k$. Second, we assume that $F(n)$ is distributed over $[\underline{n}, \bar{n}]$ as a truncated Pareto distribution with parameter $a > 1$, implying a cumulative distribution function equal to $F(n) = [1-(\underline{n}/n)^a]/[1-(\underline{n}/\bar{n})^a]$. Third, we assume that q is distributed as a power function on the interval $[0, q_{max}]$ with distribution function $P(q) = (q/q_{max})^\eta$ and density function $p(q) = \eta \cdot (q^{\eta-1})/q_{max}^\eta$. As a result, the elasticity of participation with respect to net gain of working is constant and equal to η. Fourth, we assume that the social welfare function Ψ is CRRA with coefficient of risk aversion $\gamma > 0$, i.e., $\Psi(V) = V^{1-\gamma}/(1-\gamma)$. In the case of $\gamma = 1$, we have $\Psi(V) = \log V$. The combination of a power function for $P(q)$ with a CRRA social welfare function simplifies considerably the numerical simulations because the integrals over q can be expressed directly in terms of the incomplete beta function making computations much faster. Finally, we assume no exogenous revenue requirement so that $E = 0$.

Simulations are based on the optimal marginal tax rate formulas derived in Proposition 1. As described in Appendix A.6, they are performed using an iterative method until the solution converges to a fixed point satisfying the optimal formulas as well as all the transversality conditions and the government budget constraint.

4.1 The Extensive-Intensive Model

In this subsection, we consider the binary case presented in Section 2. In Appendix A.6, we describe the details of the numerical simulations. In the simulations, we set $\underline{n} = 1$, $\bar{n} = 4$, $w = 1$, and $q_{max} = 2 \cdot w$. We assume that n is Pareto distributed with parameter $a = 2$.

For our benchmark case, we assume $\gamma = 2$, $\varepsilon = 0.5$, $\eta = 0.5$. Figure 6 plots the optimal T_0', T_1', and τ as a function of n. Consistent with our theoretical results, we have $T_0' = T_1' = 0$ at the end points and $T_1' < T_0'$ everywhere else. The difference between T_1' and T_0' is about 7 percentage points which makes T_0' about 30% percent larger than T_1'. The graph also shows that the tax on secondary earners τ is decreasing in n from about 37 percent at $\underline{n}$ to 22 percent at $\bar{n}$. This suggests that the negative jointness property is not a negligible phenomenon and that it generates a significant difference in marginal tax rates between one- and two-earner couples.

Figure 7 examines the sensitivity of optimal tax rates with respect to alternative parameter values. It shows optimal tax rates T_0', T_1', and τ in four situations. In Panel A, we increase the participation elasticity η to one. We find that this decreases the level of the tax on secondary earners by about 10 percentage points but the decreasing slope for τ (or, equivalently, the gap between T_0' and T_1') remains significant and fairly close to the benchmark case. In Panel B, we increase the intensive elasticity ε to one. We find that this decreases the level of marginal tax rates on primary earners by about 10 percentage points but again the decreasing slope for τ (and the gap between T_0' and T_1') remains significant as a proportion of tax rate levels. In panel C, we increase both η and ε to one. This reduces T_0', T_1', and τ but the negative jointness pattern remains. Taken together, results from Panels A, B, C show that levels of tax rates obey the traditional Ramsey principle: when the elasticity increases, the corresponding tax rate decreases.

In Panel D, we increase redistributive tastes of the government to $\gamma = 4$. We find that all tax rates increase significantly but, again, the negative jointness pattern remains about the same in proportion to tax rates.

Figure 8 explores two other departures from our benchmark case. Panel A focuses on the Rawlsian case ($\gamma = \infty$). In this case, we have that $g_1(n) = 0$ and that $g_0(n)$ is a Dirac distribution with all mass concentrated at $\underline{n}$. The optimal tax formulas from Proposition 1 continue to apply but the transversality condition $T_0' = 0$ is no longer true at the bottom. Indeed, the simulation shows that $T_0'(\underline{n}) = 59\%$ in this case. Interestingly, the negative jointness result carries over to this case.[11] The Rawlsian case is theoretically very interesting

[11]It is actually possible to present a formal proof of negative jointness in the Rawlsian case following the same steps as in our proof of Proposition 5.

because it is formally equivalent to a multi-product nonlinear pricing problem as analyzed in the Industrial Organization literature.[12] This shows that the negative jointness result would carry over in that case as well. Interestingly, the intensive-binary multi-dimensional screening problem we have considered does not generate singularities at the bottom even when the objective function corresponds to the one considered in the Industrial Organization literature. This is in sharp contrast with the important findings by Armstrong (1996) and Rochet and Choné (1998) who consider multi-intensive models where there is always singularities at the bottom.

Figure 8, Panel B, explores the case with a long tail. In the simulation, we set $\bar{n} = 200$ (which is a close approximation to an infinite tail). The figure shows that in this case, T_0' and T_1' converge to the theoretical asymptotic value of $1/(1 + a \cdot \varepsilon) = 1/2$. We also see that, as expected, τ converges to zero.

Figure 9 examines the implications of introducing positive or negative correlation in spouse characteristics, n and q. If we think of a low q as reflecting a high ability of the secondary earner, a negative correlation in n and q would correspond to a positive correlation in ability, and vice versa. We introduce correlation by making q_{max} a function of n; it will be a decreasing function in the case of positive ability correlation and an increasing function in the case of negative ability correlation. The correlations are calibrated so that the average participation rates of spouses remains approximately the same. Panel C displays the participation rates of spouses by potential earnings in the cases of independent abilities (benchmark), positive correlation in ability, and negative correlation in ability. Panel C shows that we have introduced significant correlation with participation rates doubling from $\underline{n}$ to $\bar{n}$ in the positive correlation case and decreasing by 50% from $\underline{n}$ to $\bar{n}$ in the negative correlation case. Panels A and B display the optimal tax rates in the positive and negative correlation case, respectively. The levels of tax rates are higher in the positive correlation case because inequality is more important in that case and hence redistribution more desirable. However, the negative jointness pattern is very similar to the cases with no correlation. This suggests that the empirical observation of positive correlation in ability across spouses (positive assortative mating) would not overturn

[12] The case where the government minimizes the efficiency costs of raising a given amount of tax revenue subject to a participation constraint (couples cannot pay more than what they earn in taxes for example) is also formally equivalent to the Rawlsian model. In that case, $g_0 = g_1$ is constant over n and the bottom transversality condition $T_0'(\underline{n}) = 0$ does not hold. The pattern of optimal taxes would be identical to Figure 8, Panel A, but with a uniform scaling down.

the negative jointness result we have obtained.

4.2 The Discrete Intensive-Intensive Model

In order to explore the robustness of the negative jointness results to more general models, we extend our binary model from Section 2 to a larger number of possible earnings outcomes for the spouse. We do not try to simulate directly the double intensive model presented in Section 3 because of the considerable technical difficulty involved. Instead, we consider a simpler model where the intensive response of secondary earners occurs along a *discrete* number of earnings outcomes.

The secondary earner chooses among $I+1$ occupations denoted by $i = 0, 1, ..., I$ and paying wages $w_0 < w_1 < ... < w_I$. We assume that occupation 0 is being out of the labor force and hence pays no wage ($w_0 = 0$). Secondary earners can be of type $i = 1, .., I$. We assume that there is an exogenous fraction $\bar{h}_i$ of spouses of type i. A spouse of type i will earn w_{i-1} if she expends no effort but can earn w_i if she expends a cost q_i. The distribution of costs is given by $\Gamma_i(q_i)$, with density $\gamma_i(q_i)$.

This discrete model has been developed in the one-dimensional case by Piketty (1997) and Saez (2002) as an alternative to the Mirrlees (1971) continuous model. Piketty (1997) and Saez (2002) show that optimal tax rate formulas carry over intuitively to that model.[13] Introducing the discrete intensive choice for the spouses is the simplest way to generalize the binary model while keeping tractability, both for deriving optimal tax formulas and implementing numerical simulations.

There are $I + 1$ tax schedules: $T_0(z), .., T_I(z)$, depending on the occupation of the spouse. As shown in appendix, we can define the marginal tax rate from occupation $i-1$ to occupation i for spouses as $\tau_i = [w_i - V_i - (w_{i-1} - V_{i-1})]/(w_i - w_{i-1})$. The generalization of negative jointness to this model can be stated as $T'_0 \geq T'_1 \geq .. \geq T'_I$ for all n which is equivalent to τ_i being decreasing in n for each i. We do not have a general theoretical result on negative jointness in the double-intensive context, but we expect that it holds in a wide set of parametric assumptions which we explore with numerical simulations.[14]

[13]It is important to note that the discreteness is in the outcomes and not in the types. The discrete type case has been extensively analyzed in the literature. However, the discrete type case does not lend itself to a simple generalization in the multi-dimensional screening case (see Armstrong, 1995).

[14]It is easy to show that starting from the optimal separable schedule, introducing some jointness increases welfare exactly as in Section 3.

In the simulations, we assume that $\Gamma_i = (q/q^i_{max})^\eta$ with a constant elasticity η. We assume that $w_i = i$ so that $w_i - w_{i-1} = 1$. An in our binary benchmark case, we assume $\underline{n} = 1$, $a = 2$, $\gamma = 2$, $\varepsilon = 0.5$, and $E = 0$. We pick a higher parameter for $\eta = 1$. In this model, the effective elasticity of spousal earnings is actually significantly smaller than η (which explains the relatively higher rates on spouses in the multi-discrete model).

The top two panels of Figure 10 consider the case with finite $\bar{n} = 4$ while the bottom panels consider the case with infinite tail $\bar{n} = \infty$.

Panel A, displays the optimal marginal tax rates on the primary earner when $I = 3$, i.e. $T'_0, .., T'_3$. The figure shows clearly that $T'_0 > .. > T'_3$ on $(\underline{n}, \bar{n})$ with equality (and the standard zero results) at the end points. The differences in marginal tax rates from T'_0 to T'_3 are large. Panel B displays the marginal tax rates on spouses for each transition (τ_1 for the transition from occupation 0 to 1, etc.). As expected, we see that each τ_i is decreasing in n consistent with our conjecture that negative jointness is optimal.

An interesting point to note is that the slope of τ_i is larger (in absolute value) at the bottom ($i = 1$) than at the top ($i = 3$). The sensitivity of the spouse marginal tax rate with respect to primary earnings is larger for low earnings spouses than for high earnings spouses. This is consistent with our general theme that introducing wedges in the secondary earnings labor supply decision is more desirable when the primary earnings are low because spousal earnings make a significant difference in welfare. This significant difference in welfare is of course higher when spouse earnings are modest (moving from $w_0 = 0$ to $w_1 = 1$) than when spouse earnings are large (moving from $w_2 = 2$ to $w_3 = 3$).

The bottom two panels display the case with an infinite tail for n. As we obtained in the binary case, we see that marginal tax rates on primary earners converge to 0.5 when n grows and that the marginal taxes on spouses all converge toward zero (although relatively slowly).

Another interesting point to note is that, if we refine the grid for w_i by increasing the number I, we should expect the solution to converge to the double intensive model. It is unfortunately impossible for us to simulate optimal tax system with a very fine grid for I because our iterative simulation method is no longer converging in that case. However, we speculate that the optimum solution in the double intensive model might be regular everywhere with no bunching. This again stands in sharp contrast to the analysis in the Industrial Organization literature where marginal welfare weights are constant and where bunching is always part of the solution as shown in the important contributions by Armstrong (1996) and Rochet and Choné (1998). We speculate that the solution in the double-intensive model of Section 3 would also be smooth with no singularities and display global negative jointness as long as the social welfare function is not degenerate as in the Rawlsian case (where the same singularity phenomena uncovered by Armstrong (1996) and others would clearly be present).

4.3 Link to Actual Tax Schedules

The numerical simulations presented here are quite stylized and do not represent a real world calibration attempt. Nevertheless, it is useful to discuss if observed redistribution schedules display negative jointness as our results suggest they should.

Notice first that joint progressive income taxation featuring increasing marginal tax rates on family income, such as the system in the United States, display positive jointness and hence contradict our results. However, the central point to note is that welfare programs offering low-income support are always based on family income the phasing-out of those means-tested programs typically create high marginal tax rates at the bottom of the earnings distribution. As a result, the tax rate on spousal earnings is very high when primary earnings are low enough to bring the family into the phase-out range of transfer programs. On the other hand, the tax on the spouse is lower when primary earnings are high enough that the family is beyond the phase-out range. Hence, transfer programs in OECD countries do create negative jointness in the lower part of the primary earnings distribution. Then, if the income tax itself is individually based, such as the one operated by the United Kingdom, the tax rate on spouses never increases in the upper part of the primary earnings distribution and hence the global tax/transfer system displays negative jointness as our theory predicts is optimal.

It is also interesting to emphasize that the debate on moving from joint to individual taxation is always about the income tax which applies on the middle and upper part of the distribution (thanks to exemptions at the bottom keeping low income earners out of the income tax) and never about transfers which are means-tested and based on total family income. Our theory provides support to the current practice of basing transfers on family income and having an individual income tax system above the bottom in order to avoid positive jointness.

Figure 11 provides an optimum tax simulation example illustrating this. We consider a distribution of abilities that is uniform from $\underline{n} = 0$ to $n = 3$, and Pareto distributed above

$n = 3$ with Pareto parameter $a = 2$. The density distribution is continuous at $n = 3$ (but has a kink). We use $\varepsilon = 2/3$, $\eta = 1/3$, and $\gamma = 5$. The interesting and realistic feature is that marginal tax rates in the standard Mirrlees model are U-shaped in that case (as shown theoretically in Diamond, 1998 and in the calibrated simulations of Saez, 2001). High rates at the bottom correspond to the phasing-out of the lumpsum grant. As can be seen from (12), increasing rates at the top are due to the redistributive tastes of the government combined with the Paretian assumption and constant elasticity. Figure 11 shows that, for the specific choices of parameters, the tax rate τ on spouses is in between the marginal tax rates for primary earners at the bottom of the ability distribution. This means that the optimal schedule would be closely approximated by a family based transfer system at the bottom where transfers are assessed based on total family earnings and are phased out as earnings increase with high and declining marginal tax rates. Obviously, a family based schedule cannot be optimum at high n as τ vanished to zero and T_1' and T_0' converge to $T'^{\infty} = 43\%$. However, an individually based progressive income tax could generate a pattern with increasing marginal tax rates for primary earners and low marginal tax rates for secondary earners. Thus, combining a family based transfer system with a individually based income tax could be a good approximation to the fully optimal system displayed on Figure 11.

It would be important to calibrate carefully the optimal tax model we have developed to a real world situation, allowing for correlation of ability across spouses and replicating closely the actual distribution of joint earnings, and modelling responses along both the extensive and intensive margin calibrated to match the empirical estimated elasticities. Such work would allow an assessment of the quantitative importance of optimal negative jointness and provide a better guide to policy makers founded in optimal tax theory. It would also be interesting to analyze how our couple results interact with the recent studies (Saez, 2002, Immervoll et al. 2007) showing that work subsidies for low income earners are actually optimal in the presence of strong participation effects in the case of individual taxation. This goes well beyond the theoretical exploration attempted here and is left for future work.

5 The Unitary Versus the Collective Approach

We have considered the unitary labor supply model whereby husbands and wives pool their resources and maximize a single utility function subject to a family budget constraint. A number of papers have challenged the unitary approach and have viewed the family as consisting of members with conflicting interests engaging in bargaining over household resources (see Lundberg and Pollak, 1996, for a survey of this literature). Following the seminal contributions by Chiappori (1988, 1992), the collective labor supply model has become especially popular. The collective approach does not model a particular bargaining process — only Pareto efficiency is assumed — and it encompasses the unitary model and cooperative bargaining models as special cases.

In the absence of income pooling, intra-family resource allocation will generally depend on which family member receives or controls income. Empirical studies have supported this hypothesis. For example, the influential study of Lundberg et al. (1997) demonstrated that giving a child allowance directly to the mother instead of to the main income earner as a reduction in withheld taxes significantly increases spending on children.

What would be the implications of abandoning the income-pooling assumption for the question of optimal income redistribution analyzed here? Let us adopt the collective approach, assuming that consumption is allocated across spouses in a Pareto efficient way. The collective decision process is associated with implicit weights on the individual utilities of each spouse, where the weights may depend on factors such as innate characteristics, relative incomes, and on whom receives government transfers. In the government's problem, social preferences will be defined on the individual utilities of husbands and wives rather than a family utility function, and the government attach welfare weights to each family member which may or may not differ from the weights implicit in the family's decision process.

It is natural to distinguish between two cases depending on the government's view on the intra-family distribution. In one case, policy makers respect family sovereignty, i.e., the marginal welfare weight on the husband relative to the wife is exactly identical to the relative bargaining weights implied by the sharing rule in the family. In this case, it is easy to see that changes in intra-household distribution have no consequences for social welfare, implying that all of our optimal tax results would continue to apply.

In the alternative case, policy makers disagree with intra-household distribution. Suppose for example that, from the point of view of the government, husbands have too much power and get too large a fraction of consumption in the family. How can the government get a fairer distribution within families? The findings by Lundberg et al. (1997) show that the government can actually modify within-family consumption allocation at no fiscal cost simply by transferring the child benefit from husband to wife. As shown in the formal analysis of Kroft (2006), by transferring enough resources from husband to wife, the government is able to restore a fair allocation across spouses in the family. In sharp contrast to the previous models we have considered here, this within-family redistribution is first best (it does not create any efficiency costs) as long as the within-family bargaining is Pareto efficient (as assumed in the theory of Chiappori 1988, 1992).

Hence, within-family distributional issues can be solved using such non-distortionary government transfers within families. Once those within-family distributional issues are fully resolved at no efficiency costs, we are essentially back to the problem of redistribution across families which we have analyzed in this paper. Hence, collective labor supply models introduce a new within-family dimension to the redistribution problem which is very interesting and calls for more work but which is largely independent of the across-family redistribution problem we have considered in this paper.

6 Conclusion

This paper has explored the optimal income tax treatment of couples allowing for fully general joint income tax systems. To make progress on this difficult problem, we have considered a simple model with no income effects, separability of labor supply decisions across spouses, and focusing primarily on the case where labor supply of the secondary earner is a binary participation choice. Under additional regularity assumptions and independent abilities across spouses, our central result is that the optimal tax function should have a negative cross partial derivative: the tax rate on secondary earnings should decrease with primary earnings and the marginal tax rate on primary earners should be lower when secondary earnings increase. The intuition for this negative jointness result can be understood as follows.

Redistribution from couples with high primary earnings to couples with low primary earnings takes place according to the logic of the Mirrlees (1971) model. Indeed, in our model, the average marginal tax rate on primary earners at each earnings level is identical to the one obtained in the Mirrlees model. Conditional on primary earnings, redistribution takes place by transferring income from two-earner couples to one-earner couples. Such a transfer creates a tax wedge on secondary earnings. This tax wedge is largest at low primary earnings because this is where redistribution from two-earner couples to one-earner couples is most valuable. Thus, although our results may seem surprising at first sight, they obey a simple redistributive logic. If the tax schedule for two-earner couples is seen as the base schedule, the schedule for one-earner couples is obtained from that base schedule by giving a dependent spouse tax allowance, which is larger for couples with low primary earnings than for couples with high primary earnings.

This seems a surprising result at first sight, and at odds with the actual practice of joint progressive taxation of family income. However, we have argued that the current practice of many European countries — such as the United Kingdom — of having an individual income tax system for middle- and high-income earners in combination with a means-tested family-based transfer system for low-income earners creates such a pattern: at the bottom, secondary earners face a large tax rate due to the phasing-out of transfer benefits, while at the middle and high end, secondary earners face a low tax rate due to the individual income tax.

It would clearly be important to extend the numerical simulations to a carefully calibrated model which is closer to the real world in terms of the distribution of abilities and the correlation of such abilities across couples. Such numerical simulations would allow us to assess the quantitative importance of the negative jointness result relative to the many other factors and parameters that affect optimal tax rates. We leave such an important extension for future work.

A Appendix

A.1 Mechanism Design and Implementation

In our model, agents are characterized by the private information $\theta = (n, q) \in \Theta$. Agents choose the observable action $x = (z, l)$ and receive consumption c. The utility function is

$$u(x, c, \theta) = c - n \cdot h\left(\frac{z}{n}\right) - q \cdot l.$$

The taxes paid to the government are defined as $z + w \cdot l - c$. By the revelation principle, any government mechanism can be decentralized by a truthful mechanism $(x(\theta), c(\theta))_{\theta \in \Theta}$ such that, for any θ, θ':

$$u(x(\theta), c(\theta), \theta) \geq u(x(\theta'), c(\theta'), \theta).$$

Given the binary structure for action l, we have:

Lemma 3 *Any truthful mechanism $(x(\theta), c(\theta))_{\theta \in \Theta}$ can be replaced by a simpler "truthful" mechanism $(z_l(n), c_l(n))_{l \in \{0,1\}, n \in (\underline{n}, \bar{n})}$ such that, for each n, there is a $\bar{q}(n)$ so that:*

- *When $q < \bar{q}(n)$, $(l' = 1, n' = n)$ maximizes $u(z_{l'}(n'), l', c_{l'}(n'), (n, q))$ over all (l', n').*
- *When $q \geq \bar{q}(n)$, $(l' = 0, n' = n)$ maximizes $u(z_{l'}(n'), l', c_{l'}(n'), (n, q))$ over all (l', n').*

For each agent, the new mechanism generates the same utility as the original mechanism and raises at least as much taxes.

Proof:

For each n, the set $Q = (0, \infty)$ is partitioned into 2 sets $Q_0(n)$ and $Q_1(n)$ such that, $q \in Q_0(n)$ implies $l(n, q) = 0$ (spouse does not work) and $q \in Q_1(n)$ implies $l(n, q) = 1$ (spouse works). Let us assume by convention that, in case of indifference between $l = 0$ or $l = 1$, we always have $l(n, q) = 0$. For a given n, and for all $q, q' \in Q_0(n)$, truthfulness implies

$$c(n, q) - nh\left(\frac{z(n, q)}{n}\right) \geq c(n, q') - nh\left(\frac{z(n, q')}{n}\right).$$

Hence $c(n, q) - nh(z(n, q)/n)$ is constant for $q \in Q_0(n)$. Let us denote its value by $V_0(n)$. Let us denote by $Z_0(n) = \{z(n, q), q \in Q_0(n)\}$. Let us denote by $m = \sup_{z \in Z_0(n)} z - nh(z/n) - V_0(n)$. Because $z \to z - nh(z/n)$ is continuous with a maximum at $z = n$ and decreases to $-\infty$ when z goes to infinity, there is some $z_0(n) \in \bar{Z}_0(n)$ (the closure of $Z_0(n)$) such that $m =$

$z_0(n) - nh(z_0(n)/n) - V_0(n)$.[15] We define $c_0(n) = nh(z_0(n)/n) + V_0(n)$. The choice $(c_0(n), z_0(n))$ "maximizes" government taxes $z - c$ over the (closure of the) set $(c(n,q), z(n,q))_{q \in Q_0(n)}$.

Similarly, let us define $V_1(n) = c(n,q) - nh(z(n,q)/n)$ constant over $q \in Q_1(n)$, and $(c_1(n), z_1(n))$ which "maximizes" taxes $z - c$ over the (closure of the) set $(c(n,q), z(n,q))_{q \in Q_1(n)}$.

Let us define $\bar{q}(n) = V_1(n) - V_0(n)$. Truthfulness implies:

$V_1(n) - q \geq V_0(n)$, for all $q \in Q_1(n)$.

$V_0(n) \geq V_1(n) - q$, for all $q \in Q_0(n)$.

Therefore, $Q_1(n) = (0, \bar{q}(n))$ and $Q_0(n) = [\bar{q}(n), \infty)$. If $q < \bar{q}(n)$, the agent chooses $l = 1$ and $(c_1(n), z_1(n))$. If $q > \bar{q}(n)$, the agent chooses $l = 0$ and $(c_0(n), z_0(n))$. If $q = \bar{q}(n)$, the agent is indifferent and we assume by convention that the agent chooses $l = 0$.

Let us show that the new mechanism is truthful. For all $n, n', q < \bar{q}(n), q' < \bar{q}(n')$,

$$u(z_1(n), 1, c_1(n), (n,q)) = V_1(n) - q \geq c(n', q') - nh(z(n', q')/n) - q.$$

Because $(z_1(n'), c_1(n'))$ is in the closure of the set $(c(n', q'), z(n', q'))_{q' \in Q_1(n')}$, and $u(.)$ is continuous, the inequality above implies that, for all $n, n', q < \bar{q}(n)$:

$$u(z_1(n), 1, c_1(n), (n,q)) \geq u(z_1(n'), 1, c_1(n'), (n,q)).$$

Similarly, for all $n, n', q < \bar{q}(n), q' \geq \bar{q}(n')$,

$$u(z_1(n), 1, c_1(n), (n,q)) = V_1(n) - q \geq V_0(n) \geq c(n', q') - nh(z(n', q')/n),$$

which implies, for all $n, n', q < \bar{q}(n)$:

$$u(z_1(n), 1, c_1(n), (n,q)) \geq u(z_0(n'), 0, c_0(n'), (n,q)).$$

For all $n, n', q \geq \bar{q}(n)$, the inequalities:

$$u(z_0(n), 0, c_0(n), (n,q)) \geq u(z_0(n'), 0, c_0(n'), (n,q)),$$

$$u(z_0(n), 0, c_0(n), (n,q)) \geq u(z_1(n'), 1, c_1(n'), (n,q)),$$

can be demonstrated in the same way and complete the proof. $\square$

[15] To see this, take a sequence $z^k \in Z_0(n)$ such that $z^k - nh(z^k/n) - V_0(n)$ converges to m. z^k is bounded above, and hence a subsequence of z^k converges to some limit $z_0(n)$.

Thanks to this lemma, we can restrict ourselves to the simpler mechanism consisting of two standard one-dimensional schedules $(z_0(n), c_0(n))$ and $(z_1(n), c_1(n))$, where agents choose which schedule to use based on their choice for l. As, in the one-dimensional mechanism design theory (see e.g., Guesnerie and Laffont (1987)), we define implementability as follows:

Definition 1 *An action profile $(z_0(n), z_1(n))_{n\in(\underline{n},\bar{n})}$ is implementable if and only if there exists transfer functions $(c_0(n), c_1(n))_{n\in(\underline{n},\bar{n})}$ such that $(z_l(n), c_l(n))_{l\in\{0,1\},n\in(\underline{n},\bar{n})}$ is a simple truthful mechanism.*

The central implementability theorem of the one-dimensional case carries over to our model.

Lemma 4 *An action profile $(z_0(n), z_1(n))_{n\in(\underline{n},\bar{n})}$ is implementable if and only if $z_0(n)$ and $z_1(n)$ are both non-decreasing in n.*

Proof:

The utility function $c-nh(z/n)$ satisfies the classic single crossing (Spence-Mirrlees) condition. Hence, from the one-dimensional case, we know that $z(n)$ is implementable, i.e., there is some $c(n)$ such that $c(n) - nh(z(n)/n)) \geq c(n') - nh(z(n')/n))$ for all n, n', if and only if $z(n)$ is non-decreasing.

Suppose $(z_0(n), z_1(n))$ is implementable, implying that there exists $(c_0(n), c_1(n))$ such that $(z_l(n), c_l(n))_{l\in\{0,1\},n\in(\underline{n},\bar{n})}$ is a simple truthful mechanism. That implies in particular that $c_l(n) - nh(z_l(n)/n)) \geq c_l(n') - nh(z_l(n')/n))$ for all n, n' and for $l = 0, 1$. Hence, the one dimensional result implies that $z_0(n)$ and $z_1(n)$ are non-decreasing.

Conversely, suppose that $z_0(n)$ and $z_1(n)$ are non-decreasing. Because $z_0(n)$ is non decreasing, the one dimensional result implies there is $c_0(n)$ such that $c_0(n)-nh(z_0(n)/n)) \geq c_0(n')-nh(z_0(n')/n))$. Similarly, there is $c_1(n)$ such that $c_1(n)-nh(z_1(n)/n)) \geq c_1(n')-nh(z_1(n')/n))$.

It is easy to show that the mechanism $(z_l(n), c_l(n))_{l\in\{0,1\},n\in(\underline{n},\bar{n})}$ is actually truthful. Define $V_l(n) = c_l(n) - nh(z_l(n)/n))$ for $l = 0, 1$ and $\bar{q}(n) = V_1(n) - V_0(n)$. We only need to prove the cross-inequalities. For all $n, n', q \geq \bar{q}(n)$,

$$u(z_0(n), 0, c_0(n), (n, q)) = V_0(n) \geq V_1(n) - q \geq u(z_1(n'), 1, c_1(n'), (n, q)).$$

For all $n, n', q < \bar{q}(n)$,

$$u(z_1(n), 1, c_1(n), (n, q)) = V_1(n) - q \geq V_0(n) \geq u(z_0(n'), 0, c_0(n'), (n, q)).$$

The key assumption that allows us to obtain those simple results is the fact that q is separable in our utility specification. $\square$

A.2 Solving the Government Maximization Problem and Proposition 1

The government maximizes

$$W = \int_{\underline{n}}^{\bar{n}} \left\{ \int_0^{V_1(n)-V_0(n)} \Psi(V_1(n) - q)p(q|n)dq + \int_{V_1(n)-V_0(n)}^{\infty} \Psi(V_0(n))p(q|n)dq \right\} f(n)dn,$$

subject to the budget constraint

$$\int_{\underline{n}}^{\bar{n}} \int_0^{V_1(n)-V_0(n)} [z_1(n) + w - nh(z_1(n)/n) - V_1(n)]p(q|n)f(n)dqdn+$$

$$\int_{\underline{n}}^{\bar{n}} \int_{V_1(n)-V_0(n)}^{\infty} [z_0(n) - nh(z_0(n)/n) - V_0(n)]p(q|n)f(n)dqdn \geq E,$$

and the constraints arising from the couples utility maximization:

$$\dot{V}_0(n) = -h(z_0(n)/n) + (z_0(n)/n)h'(z_0(n)/n),$$

$$\dot{V}_1(n) = -h(z_1(n)/n) + (z_1(n)/n)h'(z_1(n)/n),$$

and the implementability constraints:

$$\dot{z}_0(n) \geq 0,$$

$$\dot{z}_1(n) \geq 0,$$

Let us denote by λ, $\mu_0(n)$, $\mu_1(n)$, $\rho_0(n)$, and $\rho_1(n)$, the five multipliers associated. The transversality conditions are $\mu_0(\underline{n}) = \mu_1(\underline{n}) = \mu_0(\bar{n}) = \mu_1(\bar{n}, 1) = 0$ and $\rho_0(n) = 0$ at each point of increase of $z_0(n)$ and $\rho_1(n) = 0$ at each point of increase of $z_1(n)$. We abbreviate $h(z_1(n)/n)$ into h_1, etc.

The first order conditions with respect to $z_0(n)$ and $z_1(n)$ are

$$\mu_0 \cdot \frac{z_0}{n^2} h_0'' + \lambda \cdot (1 - h_0') \cdot (1 - P(\bar{q}|n)) \cdot f(n) + \dot{\rho}_0 = 0,$$

$$\mu_1 \cdot \frac{z_1}{n^2} h_1'' + \lambda \cdot (1 - h_1') \cdot P(\bar{q}|n) \cdot f(n) + \dot{\rho}_1 = 0.$$

The first order conditions with respect to $V_0(n)$ and $V_1(n)$ are

$$-\dot{\mu}_0 = \int_{V_1-V_0}^{\infty} \Psi'(V_0(n))p(q|n)f(n)dq - \lambda(1 - P(\bar{q}|n))f(n) - \lambda[T_1 - T_0]p(\bar{q}|n)f(n),$$

$$-\dot{\mu}_1 = \int_0^{V_1-V_0} \Psi'(V_1(n)-q)p(q|n)f(n)dq - \lambda P(\bar{q}|n)f(n) + \lambda[T_1 - T_0]p(\bar{q}|n)f(n).$$

Introducing the social marginal welfare weights

$$g_0(n) = \frac{\int_{V_1-V_0}^{\infty} \Psi'(V_0(n))p(q|n)f(n)dq}{\lambda \cdot (1-P(\bar{q}|n))f(n)},$$

$$g_1(n) = \frac{\int_0^{V_1-V_0} \Psi'(V_1(n)-q)p(q|n)f(n)dq}{\lambda \cdot P(\bar{q}|n)f(n)},$$

we can integrate those two equations using the upper transversality conditions and obtain:

$$-\frac{\mu_0(n)}{\lambda} = \int_n^{\bar{n}} \left\{[1-g_0(n')](1-P(\bar{q}|n'))f(n') + [T_1-T_0]p(\bar{q}|n')f(n')\right\} dn',$$

$$-\frac{\mu_1(n)}{\lambda} = \int_n^{\bar{n}} \left\{[1-g_1(n'_p)]P(\bar{q}|n')f(n') - [T_1-T_0]p(\bar{q}|n')f(n')\right\} dn'.$$

On a segment where z_l is increasing, we have $\rho_l = 0$ and hence $\dot{\rho}_l = 0$. In that case, plugging these two equations into the first order conditions for z_0 and z_1, we obtain:

$$\frac{1-h'_0}{h'_0} \cdot (1-P(\bar{q}|n))f(n)n = \frac{h''_0 \cdot z_0/n}{h'_0} \cdot \int_n^{\bar{n}} \left\{[1-g_0(n')](1-P(\bar{q}|n'))f(n') + [T_1-T_0]p(\bar{q}|n')f(n')\right\} dn',$$

$$\frac{1-h'_1}{h'_1} \cdot P(\bar{q}|n)f(n)n = \frac{h''_1 \cdot z_1/n}{h'_1} \cdot \int_n^{\bar{n}} \left\{[1-g_1(n')]P(\bar{q}|n')f(n') - [T_1-T_0]p(\bar{q}|n')f(n')\right\} dn'.$$

Using the fact that $T'_l = 1 - h'_l$ and the definition of the labor supply intensive elasticity (3), $\varepsilon_l = h'_l/(h''_l \cdot z_l/n)$, we obtain the expressions (10) and (11) in Proposition 1.

If the above expressions generate solutions z_0, z_1 that are non-decreasing everywhere, then there is no bunching and those expressions apply everywhere. However, if the z_0, z_1 coming out of those expressions are decreasing on some portions, then they cannot be the solution and the constraint $\dot{z}_l \geq 0$ has to bind on some segments and there is bunching. On bunching segments, we no longer have $\dot{\rho}_l = 0$ and hence the expressions of Proposition 1 no longer apply.

As in the standard one dimensional model, z_l and T'_l are continuous in n and display no jumps (even when there is bunching) because of our simple quasi-linear specification.

Note that the bottom transversality conditions imply

$$\int_{\underline{n}}^{\bar{n}} \left\{[1-g_0(n')](1-P(\bar{q}|n'))f(n') + [T_1-T_0]p(\bar{q}|n')f(n')\right\} dn' = 0,$$

$$\int_{\underline{n}}^{\bar{n}} \left\{[1-g_1(n')]P(\bar{q}|n')f(n') - [T_1-T_0]p(\bar{q}|n')f(n')\right\} dn' = 0.$$

A.3 Proof of Lemma in Proposition 4

Lemma 5 *If* $T_1' > T_0'$ *on* (n_a, n_b) *with equality at the end points, then* $g_0(n_a) - g_1(n_a) > g_0(n_b) - g_1(n_b)$.

Proof:

We have $\bar{q} = V_1 - V_0$ and

$$g_0(n) - g_1(n) = \frac{\Psi'(V_0)}{\lambda} - \frac{\int_0^{\bar{q}} \Psi'(V_1 - q)p(q)dq}{\lambda \cdot P(\bar{q})} > 0,$$

where positivity follows from Ψ' decreasing. Differentiating the equation with respect to n, we have:

$$\dot{g}_0(n) - \dot{g}_1(n) = \dot{V}_0 \cdot \frac{\Psi''(V_0)}{\lambda} - \dot{V}_1 \cdot \frac{\int_0^{\bar{q}} \Psi''(V_1 - q)p(q)dq}{\lambda \cdot P(\bar{q})}$$
$$+ \frac{p(\bar{q})\dot{\bar{q}}}{P(\bar{q})} \cdot \frac{\int_0^{\bar{q}} \Psi'(V_1 - q)p(q)dq}{\lambda \cdot P(\bar{q})} - \frac{p(\bar{q})\dot{\bar{q}}}{P(\bar{q})} \cdot \frac{\Psi'(V_0)}{\lambda}.$$

Rearranging, we obtain:

$$\dot{g}_0(n) - \dot{g}_1(n) = \dot{V}_0 \cdot \frac{\Psi''(V_0)}{\lambda} - \dot{V}_1 \cdot \frac{\int_0^{\bar{q}} \Psi''(V_1 - q)p(q)dq}{\lambda \cdot P(\bar{q})} - (g_0(n) - g_1(n)) \cdot \frac{p(\bar{q})\dot{\bar{q}}}{P(\bar{q})},$$

and therefore:

$$\dot{g}_0(n) - \dot{g}_1(n) = \dot{V}_1 \cdot \left[\frac{\Psi''(V_0)}{\lambda} - \frac{\int_0^{\bar{q}} \Psi''(V_0 + \bar{q} - q)p(q)dq}{\lambda \cdot P(\bar{q})} \right] +$$
$$\dot{\bar{q}} \cdot \left[-(g_0(n) - g_1(n)) \cdot \frac{p(\bar{q})}{P(\bar{q})} - \frac{\Psi''(V_0)}{\lambda} \right]. \quad (29)$$

The first term in expression is negative because $\dot{V}_1 > 0$ and by Assumption 1, Ψ'' is increasing and hence the term inside the first square brackets is negative.

In the segment (n_a, n_b), $z_1 < z_0$ and hence $\dot{\bar{q}} < 0$. Furthermore,

$$g_0(n) - g_1(n) = \frac{\int_0^{\bar{q}} [\Psi'(V_0) - \Psi'(V_0 + \bar{q} - q)]p(q)dq}{\lambda \cdot P(\bar{q})} = \frac{\int_0^{\bar{q}} -\Psi''(V_q)(\bar{q} - q)p(q)dq}{\lambda \cdot P(\bar{q})}.$$

where $V_0 \le V_q \le V_1 - q$ using the intermediate value theorem. Because, $-\Psi'' > 0$ is decreasing, we have:

$$g_0(n) - g_1(n) \le \frac{-\Psi''(V_0)\bar{q}}{\lambda}.$$

Assumption 5 ($\bar{q} \cdot p(\bar{q})/P(\bar{q}) \le 1$) then implies that:

$$(g_0(n) - g_1(n)) \cdot \frac{p(\bar{q})}{P(\bar{q})} \le \frac{-\Psi''(V_0)}{\lambda}.$$

Therefore, the second term in square brackets in expression (29) above is non-negative. Thus, the second term in (29) is non-negative. As a result, $\dot{g}_0(n) - \dot{g}_1(n) < 0$ on (n_a, n_b) and the Lemma is proven. $\square$

A.4 Proof of Proposition 6

For the first part, we want to show that, if q and n are independent and $g_0 - g_1$ is constant, then $T_0' = T_1'$ and $T_1 - T_0$ given by eq. (17) satisfy the first-order conditions of Proposition 6. Notice that $T_0' = T_1'$ implies $z_0 = z_1$, $\varepsilon_0 = \varepsilon_1$, and $\bar{q} = w - T_1 + T_0$ being independent of n. Then the first-order conditions of Proposition 1 implies that, for every n,

$$\int_n^{\bar{n}} \left\{ [1 - g_0] + [T_1 - T_0] \frac{p(\bar{q})}{1 - P(\bar{q})} \right\} f(n')dn' = \int_n^{\bar{n}} \left\{ [1 - g_1] - [T_1 - T_0] \frac{p(\bar{q})}{P(\bar{q})} \right\} f(n')dn',$$

which may be written as

$$\int_n^{\bar{n}} [g_0 - g_1] f(n')dn' = \int_n^{\bar{n}} [T_1 - T_0] \frac{p(\bar{q})}{P(\bar{q})(1 - P(\bar{q}))} f(n')dn'. \tag{30}$$

If $g_0 - g_1$ is constant, this condition is solved by $T_1 - T_0$ given by (17).

For the second part, with q and n being independent, if $T_0' = T_1'$ we have just shown that the first-order conditions imply (30). Taking the derivative of eq. (30) with respect to n, we obtain

$$g_0 - g_1 = [T_1 - T_0] \frac{p(\bar{q})}{P(\bar{q})(1 - P(\bar{q}))},$$

which is constant in n under the assumptions.

A.5 Proof of Proposition 7

We start by forming the integrated Hamiltonian:

$$\begin{aligned} H = & \int\int_D \Psi(V(n_p, n_s)) f(n_p, n_s) dn_p dn_s \\ & + \int\int_D \lambda [-V + z_p + z_s - n_p h_p (z_p/n_p) - n_s h_s (z_s/n_s)] f(n_p, n_s) dn_p dn_s \\ & + \int\int_D \left[-h_p + (z_p/n_p) h_p' - V_{n_p}\right] \tilde{\mu}_p (n_p, n_s) dn_p dn_s \\ & + \int\int_D \left[-h_s + (z_s/n_s) h_s' - V_{n_s}\right] \tilde{\mu}_s (n_p, n_s) dn_p dn_s, \end{aligned}$$

where λ is the scalar budget constraint multiplier and $\tilde{\mu}_p$ and $\tilde{\mu}_s$ are scalar functions of (n_p, n_s). To simplify the problem, it is useful to introduce the following formula from multi-variable calculus (see Mirrlees, 1976)

$$\int\int_D \left(V_{n_p}\tilde{\mu}_p + V_{n_s}\tilde{\mu}_s\right) dn_p dn_s + \int\int_D V\left(\frac{\partial\tilde{\mu}_p}{\partial n_p} + \frac{\partial\tilde{\mu}_s}{\partial n_s}\right) dn_p dn_s = \int_{\partial D} V\left(\tilde{\mu}\cdot ds\right),$$

where $\tilde{\mu} = (\tilde{\mu}_p, \tilde{\mu}_s)$ and ds denotes the normal outward vector along ∂D, the boundary of D. Using the above expression, we may rewrite the Hamiltonian to

$$\begin{aligned} H \;=\; & \int\int_D \Psi\left(V\left(n_p, n_s\right)\right) f\left(n_p, n_s\right) dn_p dn_s \\ & + \int\int_D \lambda\left[-V + z_p + z_s - n_p h_p\left(z_p/n_p\right) - n_s h_s\left(z_s/n_s\right)\right] f\left(n_p, n_s\right) dn_p dn_s \\ & + \int\int_D \left[-h_p + \left(z_p/n_p\right) h_p'\right] \tilde{\mu}_p\left(n_p, n_s\right) dn_p dn_s \\ & + \int\int_D \left[-h_s + \left(z_s/n_s\right) h_s'\right] \tilde{\mu}_s\left(n_p, n_s\right) dn_p dn_s \\ & + \int\int_D V\left(\frac{\partial\tilde{\mu}_p}{\partial n_p} + \frac{\partial\tilde{\mu}_s}{\partial n_s}\right) dn_p dn_s - \int_{\partial D} V\left(\tilde{\mu}\cdot ds\right). \end{aligned}$$

The transversality condition is that $\tilde{\mu}\cdot dS = 0$ on the boundary ∂D. In words, the scalar product of the normal vector ds to the boundary of D and $\tilde{\mu}$ must be zero at all points along the boundary ∂D. If $D = [\underline{n}_p, \bar{n}_p] \times [\underline{n}_s, \bar{n}_s]$, then $\tilde{\mu}_p = 0$ for $n_p = \underline{n}_p, \bar{n}_p$ and $\tilde{\mu}_s = 0$ for $n_s = \underline{n}_s, \bar{n}_s$.

The first-order conditions in z_p and z_s are:

$$\lambda\left[1 - h_p'\left(z_p/n_p\right)\right] f\left(n_p, n_s\right) + \frac{z_p}{n_p} h_p''\left(z_p/n_p\right)\cdot\frac{\tilde{\mu}_p}{n_p} = 0 \tag{31}$$

$$\lambda[1 - h_s'(z_s/n_s)] f(n_p, n_s) + \frac{z_s}{n_s} h_s''(z_s/n_s)\cdot\frac{\tilde{\mu}_s}{n_s} = 0 \tag{32}$$

After routine rewriting and introducing the elasticity of earnings with respect to $1-T_p'$, denoted by ε_p, for the primary earner, the first-order condition in z_p at (n_p, n_s) becomes

$$\frac{T_p'}{1-T_p'} = \frac{1}{\varepsilon_p}\cdot\frac{1}{n_p f\left(n_p, n_s\right)}\cdot\frac{\tilde{\mu}_p}{\lambda}. \tag{33}$$

Similarly, the first-order condition in z_p at (n_p, n_s) is

$$\frac{T_s'}{1-T_s'} = \frac{1}{\varepsilon_s}\cdot\frac{1}{n_s f\left(n_p, n_s\right)}\cdot\frac{\tilde{\mu}_s}{\lambda}. \tag{34}$$

The first-order condition in V at (n_p, n_s) gives the divergence equation

$$\frac{\partial\tilde{\mu}_p}{\partial n_p} + \frac{\partial\tilde{\mu}_s}{\partial n_s} = \left[\lambda - \Psi'\left(\cdot\right)\right] f\left(n_p, n_s\right). \tag{35}$$

By defining $\mu_i \equiv \tilde{\mu}_i/\lambda$ for $i = p, s$ and $g(n_p, n_s) = \Psi'(\cdot)/\lambda$, we rewrite the first-order conditions above so as to obtain the conditions (20), (21), and (22) in Proposition 5.

The solution to a multi-variable calculus problem must also fulfill the so-called integrability constraint. According to this constraint, the resulting marginal tax rates (T'_p, T'_s) must be a gradient so that the tax function $T(z_p, z_s)$ is well defined. (T'_p, T'_s) is a gradient iff the matrix of second derivative is symmetric, i.e., $T''_{ps} = T''_{sp}$. Similarly, the indirect utility function $V(n_p, n_p)$ needs to satisfy $V_{n_s n_p} = V_{n_p n_s}$. It turns out that those two conditions are equivalent and are satisfied iff condition (23) is fulfilled. From the derivatives of the indirect utility function in (19), we have $V_{n_p n_s} = \left(z_p/n_p^2\right) h''_p(z_p/n_p)\, \partial z_p/\partial n_s$ and $V_{n_s n_p} = \left(z_s/n_s^2\right) h''_s(z_s/n_s)\, \partial z_s/\partial n_p$ thereby obtaining condition (23) directly.

To see that condition (23) is also equivalent to $T''_{ps} = T''_{sp}$ note that $1 - T'_p(z_p, z_s) = h'_p(z_p/n_p)$ and $1 - T'_s(z_p, z_s) = h'_s(z_s/n_s)$. By differentiating each of those two equations with respect to n_p and n_s, we obtain a system of equations which takes the matrix form

$$T'' Dz = Dg'(z/n),$$

from which it is seen that $T''_{ps} = T''_{sp}$ if condition (23) is satisfied.

A.6 Numerical Simulations

A.6.1 Extensive-Intensive Model Simulations

Simulations are performed with Matlab software and our programs are available upon request. We select a grid for n, from $\underline{n} = 1$ to $\bar{n} = 4$ with 1000 elements: $(n_k)_k$. Integration along the n variable is carried out using the trapezoidal approximation. All integration along the q variable is carried out using explicit closed form solutions using the incomplete β function:

$$\int_0^{V_1 - V_0} \Psi'(V_1 - q)p(q)dq = \int_0^{V_1 - V_0} \frac{1}{(V_1 - q)^\gamma} \frac{\eta \cdot q^{\eta - 1}}{q_{max}^\eta} dq$$

$$= \frac{\eta}{q_{max}^\eta} \int_0^{V_1 - V_0} (V_1 - q)^{-\gamma} q^{\eta-1} dq = \frac{\eta \cdot V_1^{\eta-\gamma}}{q_{max}^\eta} \int_0^{1 - \frac{V_0}{V_1}} t^{\eta-1}(1-t)^{-\gamma} dt = \frac{\eta \cdot V_1^{\eta-\gamma}}{q_{max}^\eta} \cdot \beta(1 - \frac{V_0}{V_1}, \eta, 1-\gamma)$$

where the incomplete beta function β is defined as (for $0 \le x \le 1$):

$$\beta(x, a, b) = \int_0^x t^{a-1}(1-t)^{b-1} dt.$$

Matlab does not compute it directly for $\gamma \geq 1$ ($b \leq 0$) but we have used the development in series to compute it very accurately and quickly with a subroutine:

$$\beta(x, a, b) = 1 + \sum_{n=1}^{\infty} \frac{(1-b)(2-b)..(n-b)}{n!} \cdot \frac{x^{n+a}}{n+a}.$$

We pick $q_{max} = 2 \cdot w^{1+1/\eta}$ so that the fraction of spouses working is normalized in the situation with no taxes (when w or η change). We set $w = 1$ in the simulations presented so that $q_{max} = 2$.

Simulations proceed by iteration:

We start with given T_0', T_1' vectors, derive all the vector variables z_0, z_1, V_0, V_1, $\bar{q}$, T_0, T_1, λ, etc. which satisfy the government budget constraint and the transversality conditions.[16] This is done with a sub-iterative routine that adapts T_0 and T_1 as the bottom $\underline{n}$ until those conditions are satisfied. We then use the first order conditions (10), (11) from Proposition 1 to compute new vectors T_0', T_1'. In order to converge, we use adaptive iterations where we take as the new vectors T_0', T_1', a weighted average of the old vectors and newly computed vectors. The weights are adaptively adjusted down when the iteration explodes. We then repeat the algorithm. This procedure converges to a fixed point in most circumstances. The fixed point satisfies all the constraints and the first order conditions. We check that the resulting z_0 and z_1 are non-decreasing so that the fixed point is implementable. So the fixed point is expected to be the optimum.[17]

The central advantage of our method is that the optimal solution can be approximated very closely and quickly. In contrast, brute force simulations where we search the optimum over a large set of parametric tax systems by computing directly social welfare would be much slower and less precise.

A.6.2 Discrete Intensive-Intensive Model Simulations

We can denote again by V_i the indirect utility (before fixed cost of work) of a couple when the spouse works in occupation i: $V_i = z_i - T_i(z_i) + w_i - nh(z_i/n)$, where z_i is chosen optimally such that $h'(z_i/n) = 1 - T_i'$.

[16]The adjust the constants for $T_l(\underline{n})$ until all those constraints are satisfied. This is done using a secondary iterative procedure.

[17]We also compute total social welfare and verify on examples that it is higher than social welfare generated by other tax rates T_1', T_0' satisfying the government budget constraint.

A spouse of type i works in job i (instead of job $i-1$) if and only if her fixed costs of work effort q_i are such that $\bar{q}_i \equiv V_i - V_{i-1} \geq q_i$. Hence, the fraction of spouses of type i who work in job i is $\Gamma_i(\bar{q}_i)$. We denote by $P_i = \bar{h}_i \cdot \Gamma_i(\bar{q}_i) + \bar{h}_{i+1} \cdot [1 - \Gamma_{i+1}(\bar{q}_{i+1})]$ the number of spouses working in job i. We denote by $Q_i(\bar{q}_i) = \bar{h}_i \cdot \Gamma_i(\bar{q}_i) + \bar{h}_{i+1} + .. + \bar{h}_I$ the number of spouses working in jobs $i, i+1, .., I$. The first order conditions of the government problem can be written as follows:

$$\varepsilon_i \cdot \frac{T_i'}{1-T_i'} \cdot nf(n) \cdot P_i = \int_n^{\bar{n}} [(1-g_i)P_i - \Delta T_i \cdot \bar{h}_i \cdot \gamma_i(\bar{q}_i) + \Delta T_{i+1} \cdot \bar{h}_{i+1} \cdot \gamma_{i+1}(\bar{q}_{i+1})] f(n')dn'. \quad (36)$$

The average marginal tax rate across spouses occupations is also given by the classical Mirrlees formula and the transversality conditions imply that $T_i' = 0$ at $\underline{n}$ and $\bar{n}$. For the simulations, we pick the following (equivalent) transversality conditions:

$$\int_{\underline{n}}^{\bar{n}} [(1-G_i)Q_i - \Delta T_i \cdot \bar{h}_i \cdot \gamma_i(\bar{q}_i)] f(n')dn'.$$

where G_i is the average of $g_i, .., g_I$, and $Q_i = P_i + .. + P_I$.

Simulations in that case proceed in exactly the same way as in the binary case. We choose the same functional form $\Gamma_i(q) = (q/q_{max})^\eta$. We choose $w_i = i$ and $I = 3$ in the example. We choose $\bar{h}_i = 1/I$ and we set again $q_{max} = 2$.

We then use the same iterative process starting from a set of vectors $T_0', .., T_I'$, then computing all the vector variables $z_0, .., z_I$, $V_0, .., V_1$, $\bar{q}_1, .., \bar{q}_I$, $T_0, .., T_I$, λ, etc. which satisfy the government budget constraint and the transversality conditions. We then recompute $T_0', .., T_I'$ using the first order conditions (36) and using the same adaptive weighting procedure as above. The iterative process is converging in most cases when I is not too large.

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Figure 1

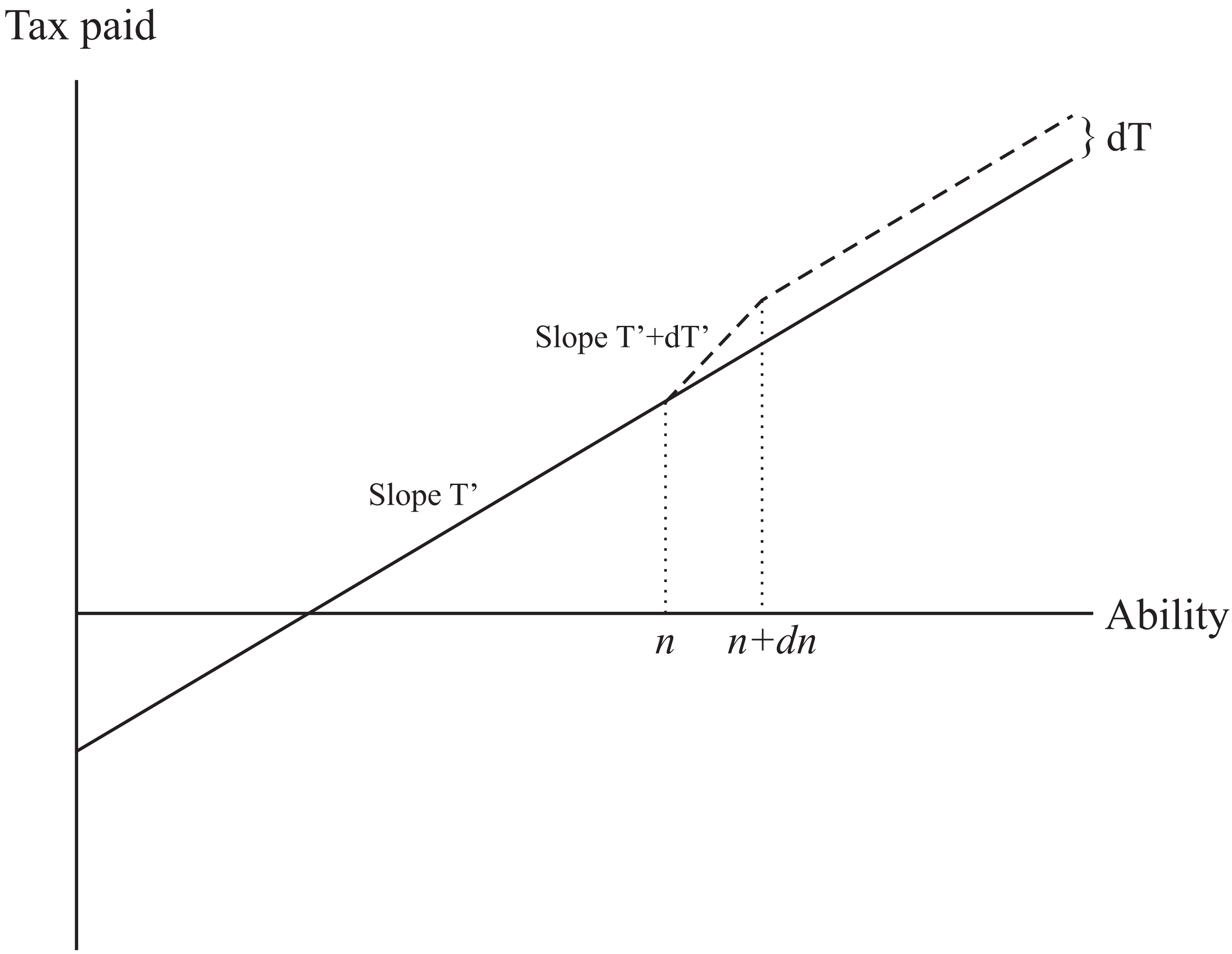
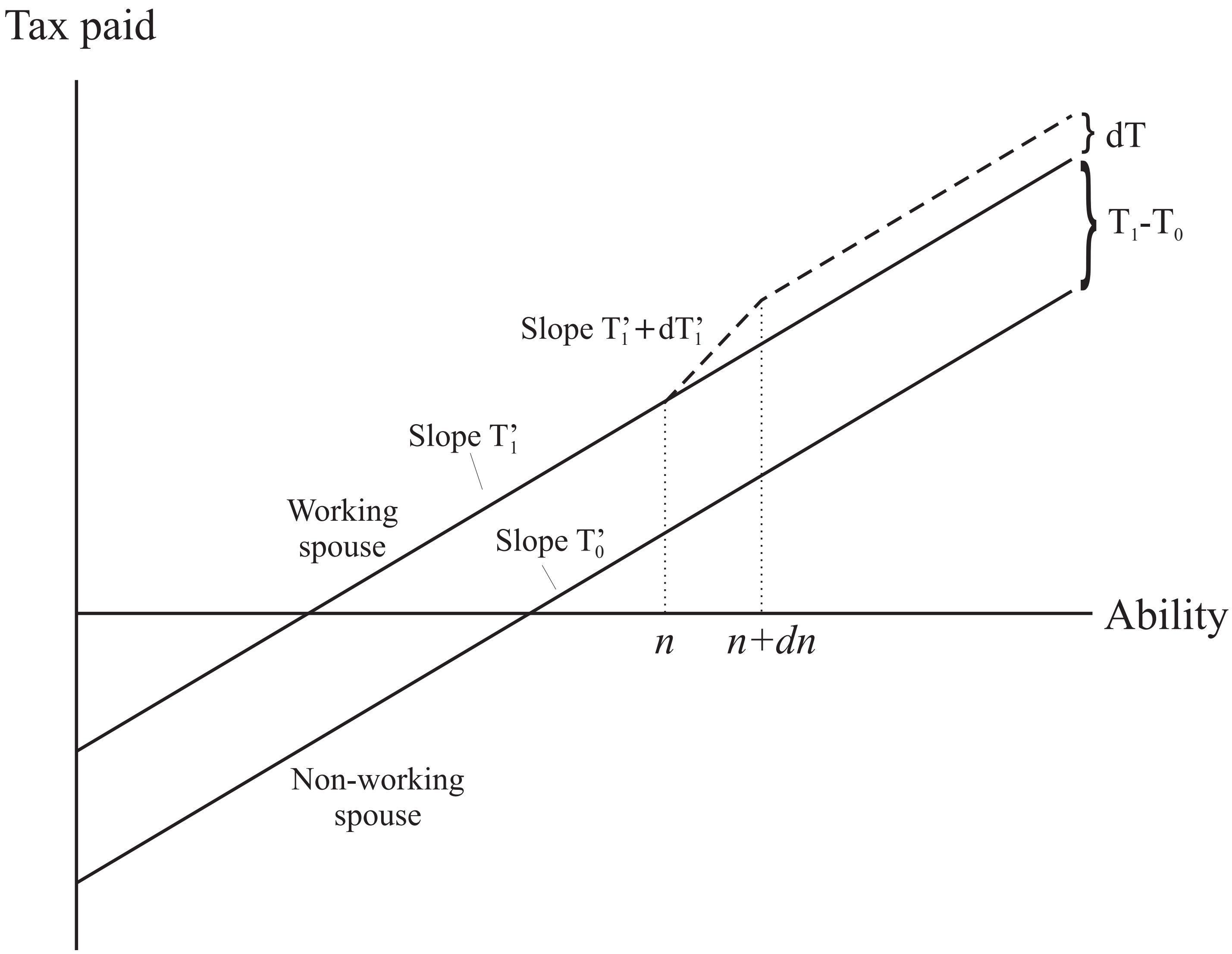
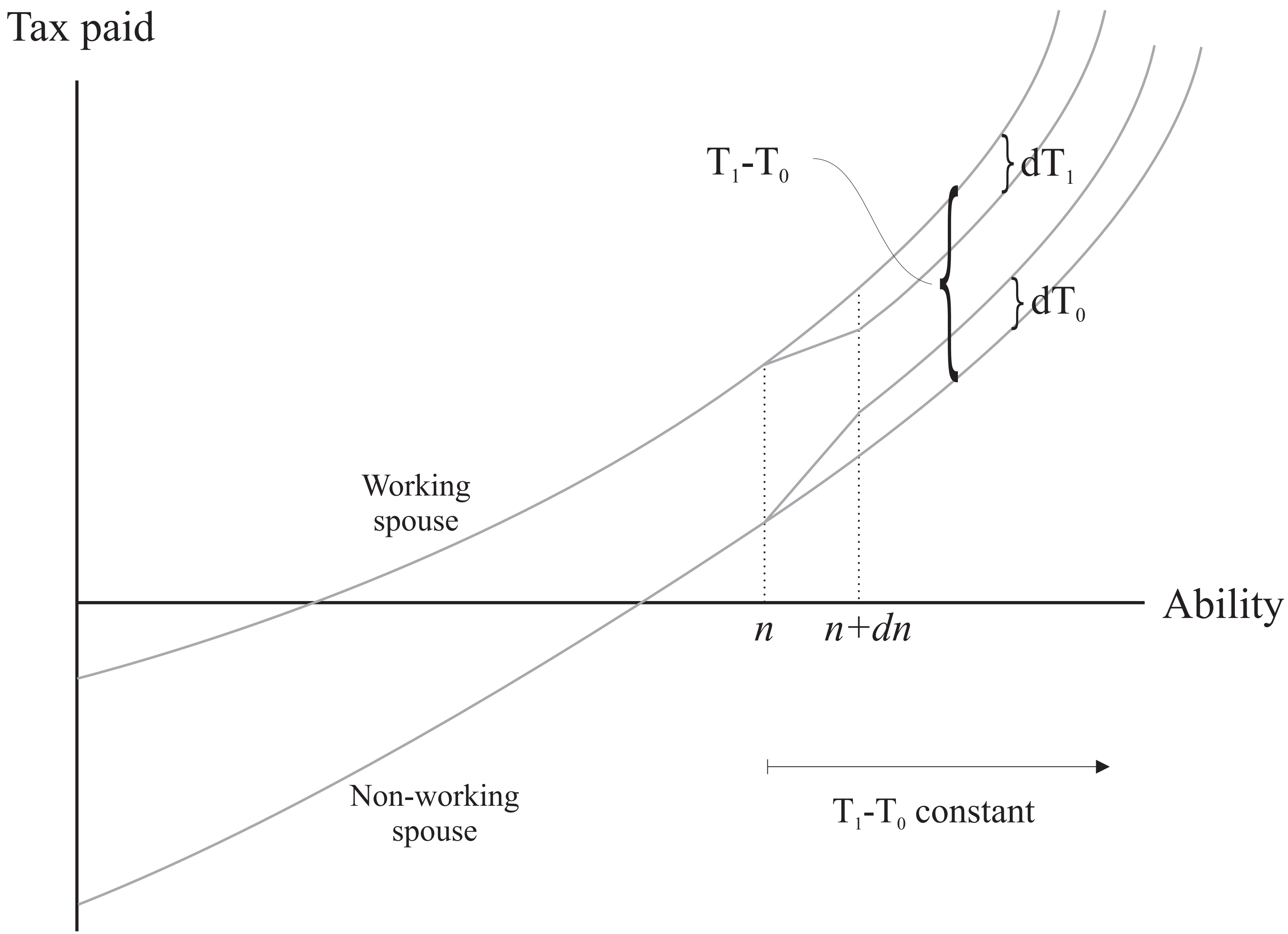


Figure 2



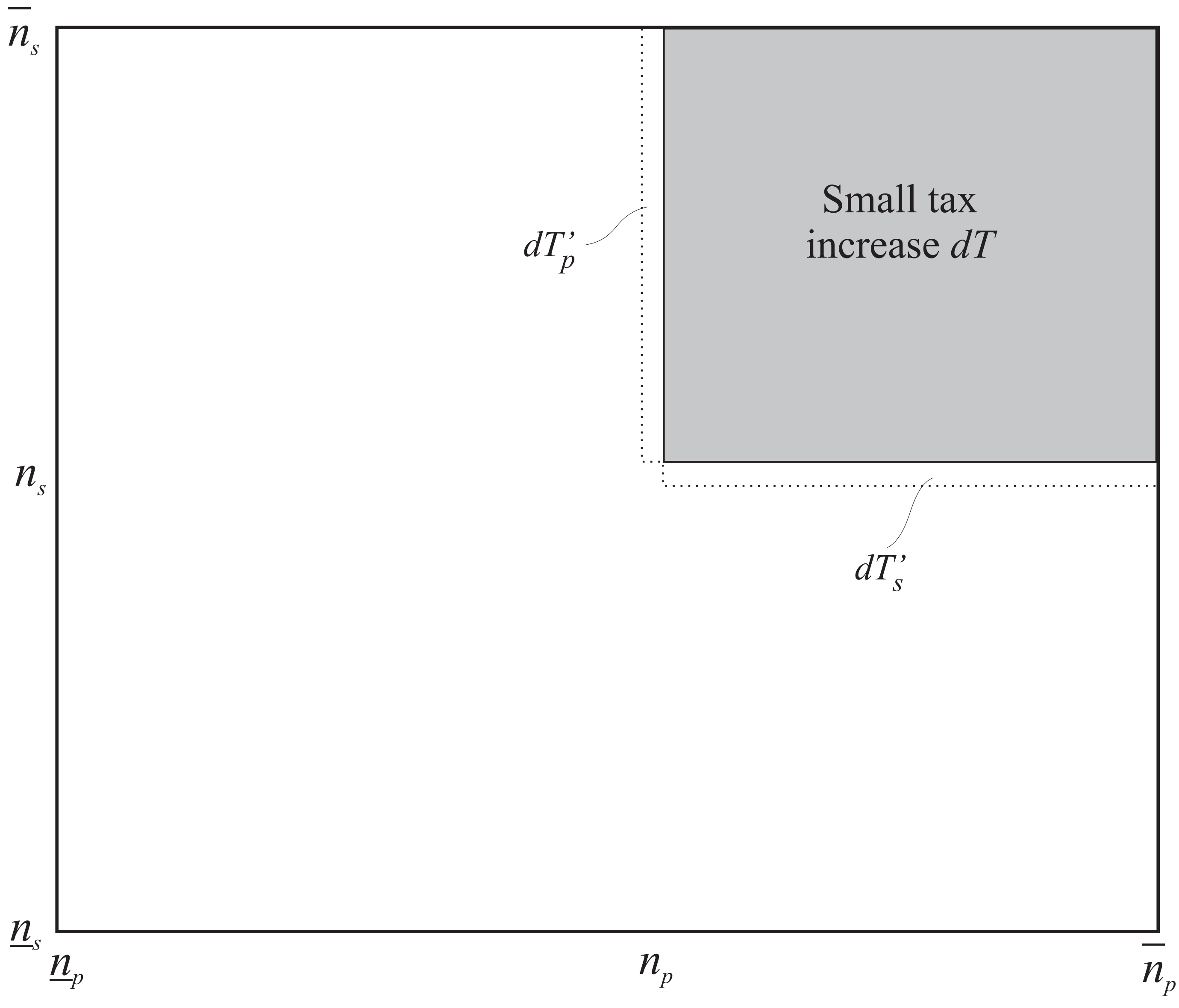
Tax paid
dT
T1-T0
Slope T1'+dT1'
Slope T1'
Working spouse
Slope T0'
Ability
n
n+dn
Non-working spouse

Figure 3



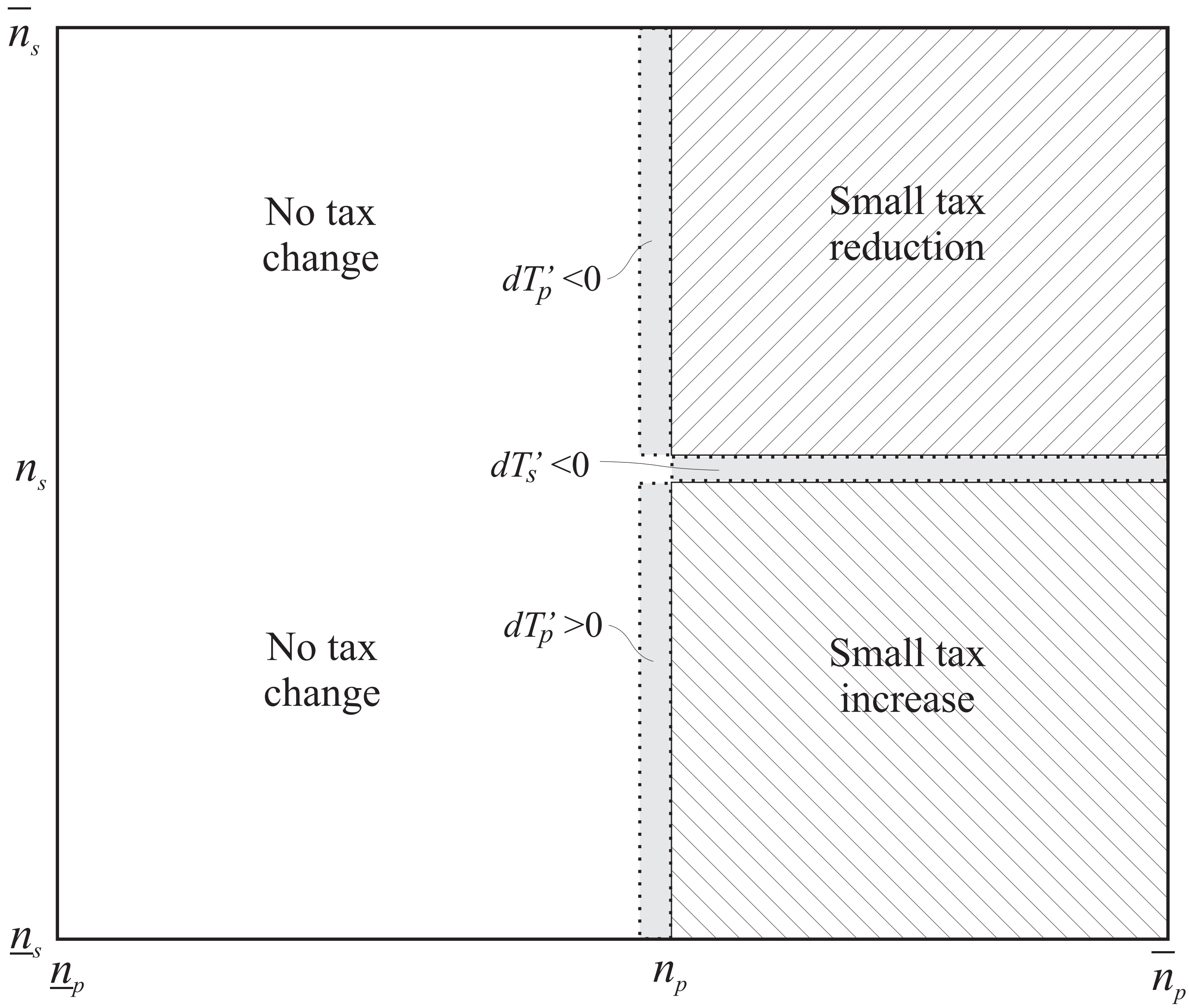
Tax paid
T_1-T_0
dT_1
dT_0
Working spouse
Ability
n
$n+dn$
Non-working spouse
T_1-T_0 constant

Figure 4



$\overline{n}_s$
n_s
$\underline{n}_s$
$\underline{n}_p$
n_p
$\overline{n}_p$
Small tax increase dT
dT'_p
dT'_s

Figure 5



No tax change
Small tax reduction
$dT_p' < 0$
$dT_s' < 0$
$dT_p' > 0$
No tax change
Small tax increase
$\overline{n}_s$
n_s
$\underline{n}_s$
$\underline{n}_p$
n_p
$\overline{n}_p$

Figure 6: Benchmark Simulation: γ=2, η=0.5, ε=0.5

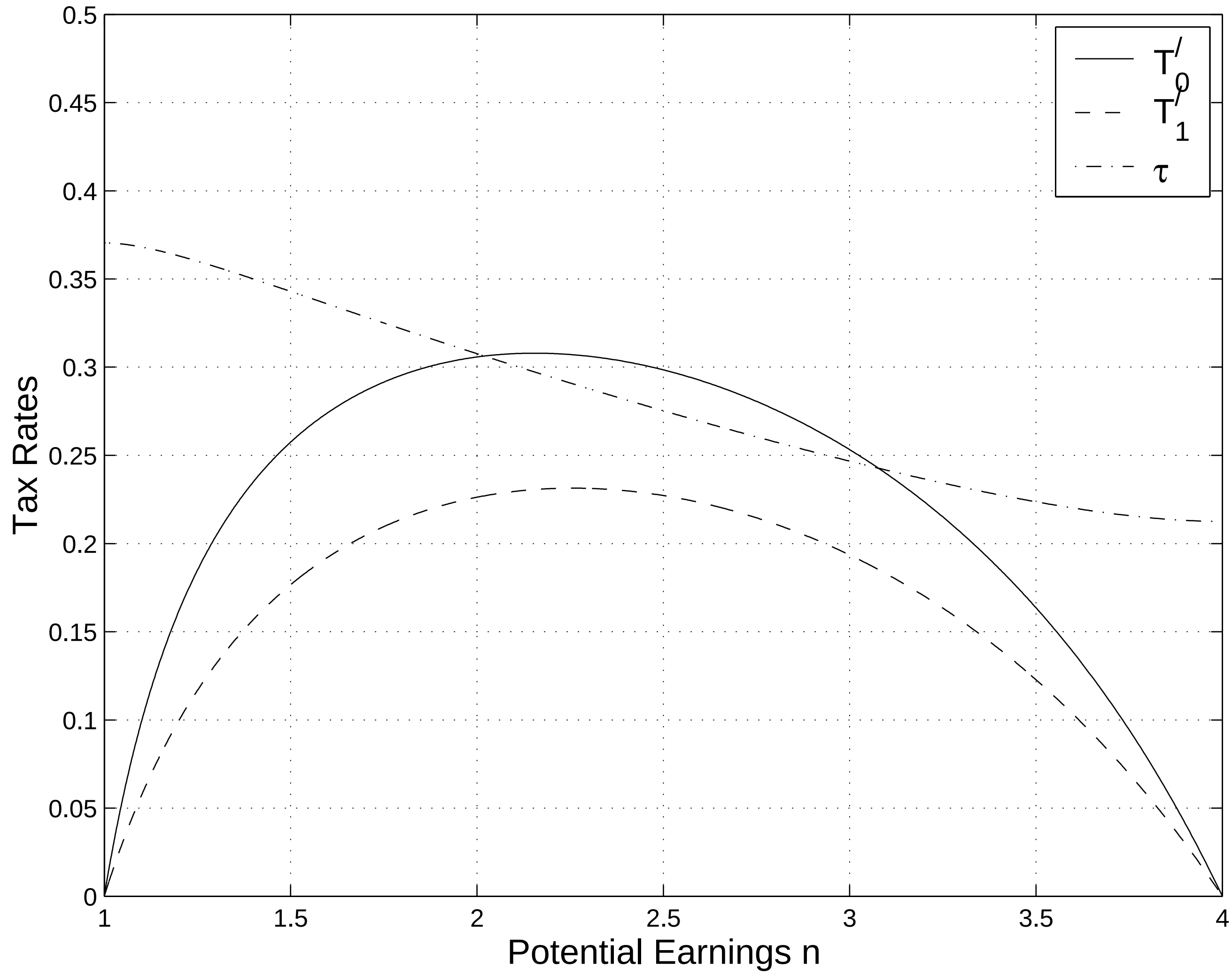


Figure 7: Sensitivity Analysis around Benchmark (γ=2, η=0.5, ε=0.5)

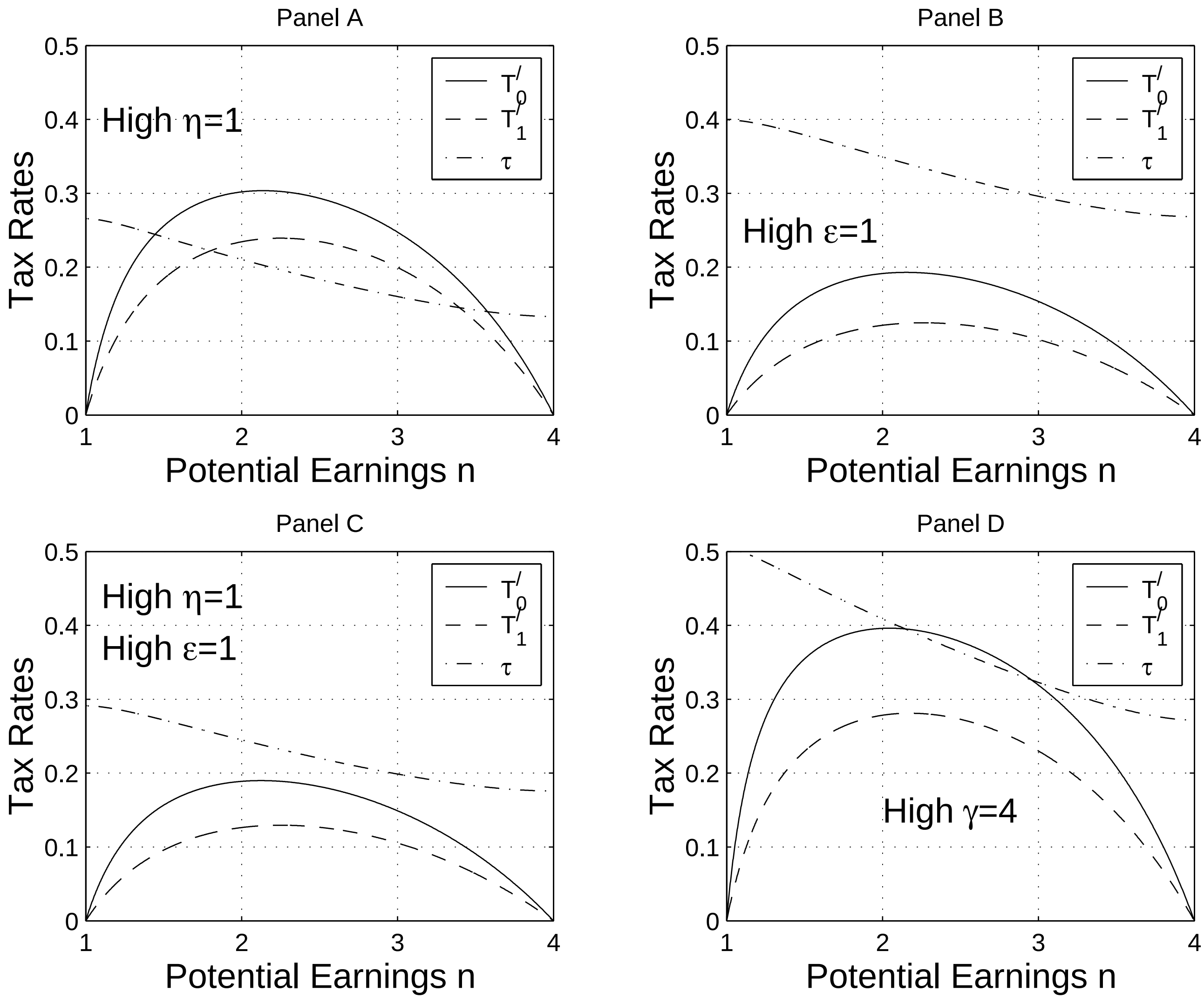


Figure 8: Two Cases of Interest

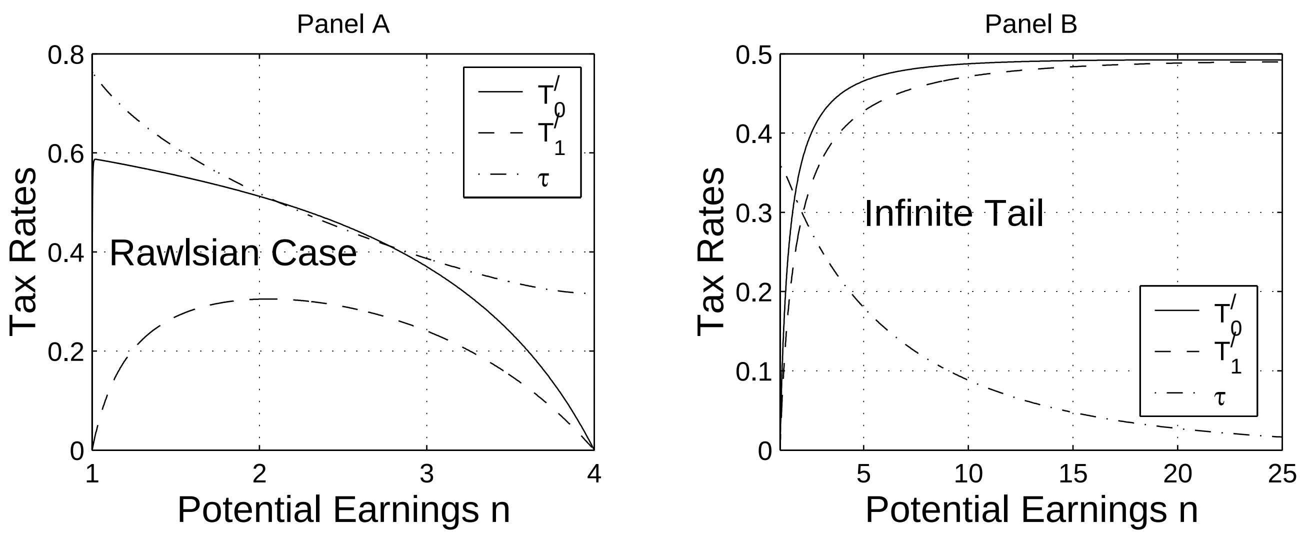


Figure 9: The Effects of Spousal Correlation of Ability

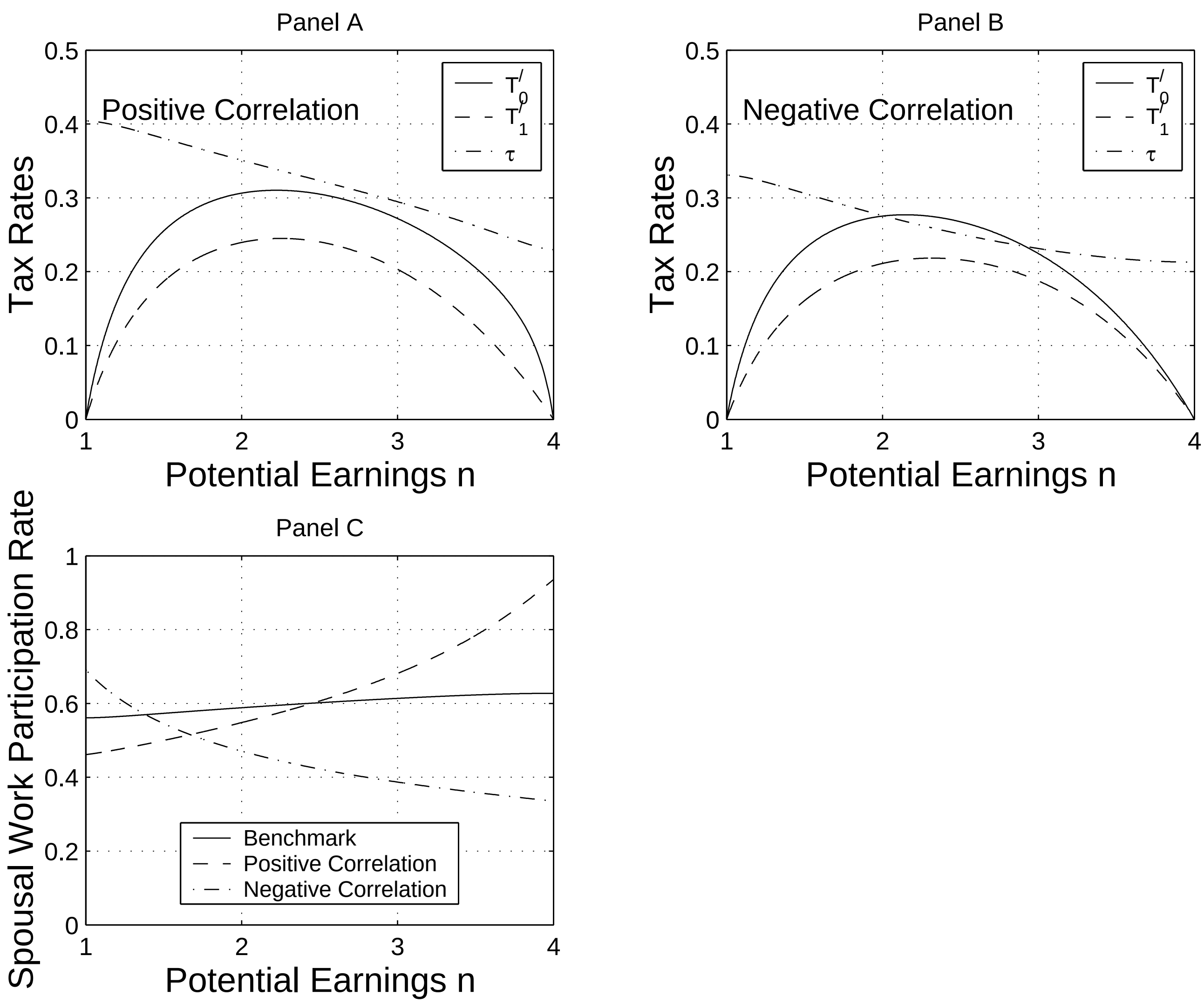


Figure 10: Discrete Intensive Spouse Model

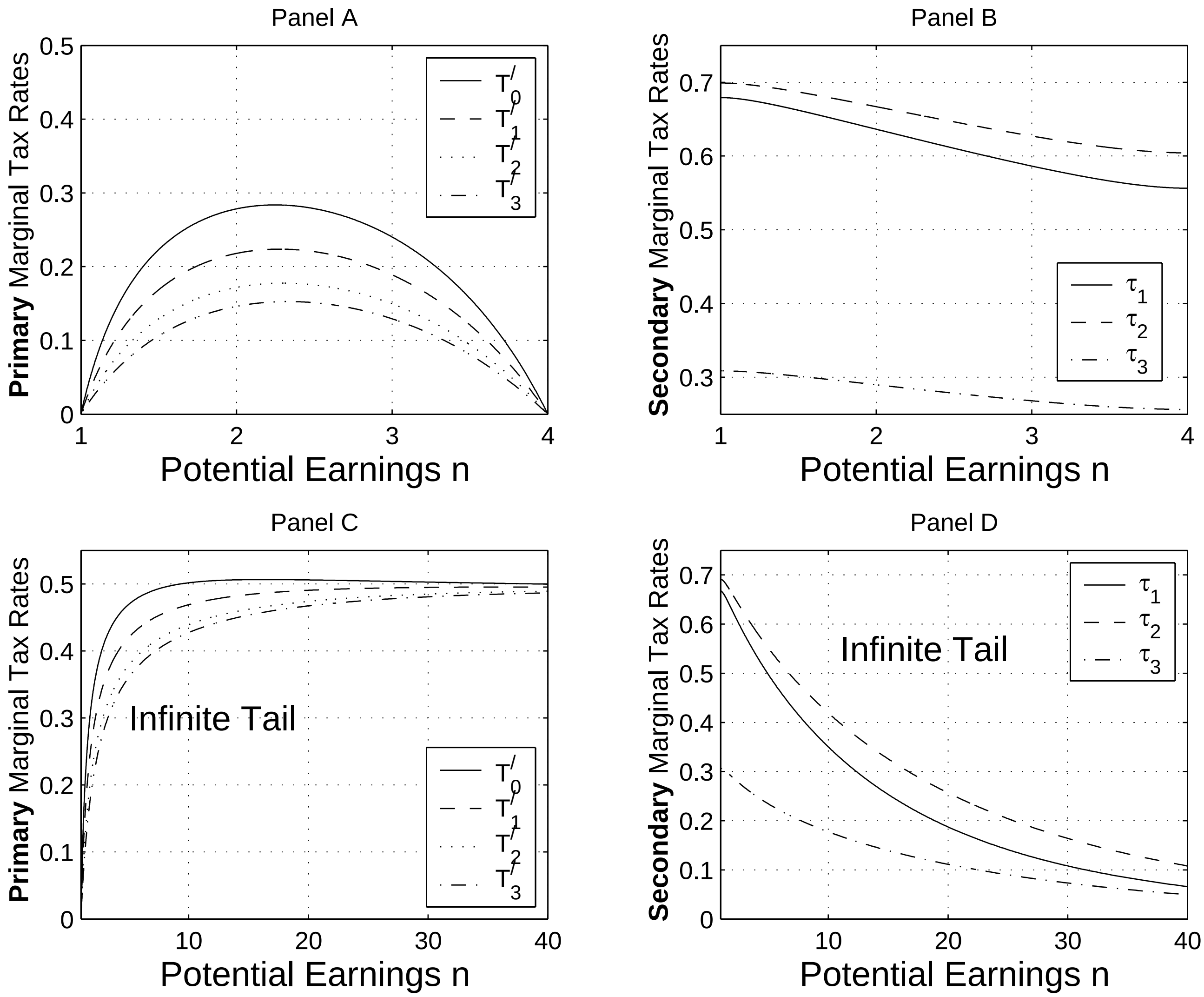


Figure 11: Getting Closer to Family Based Transfers, γ=5, η=1/3, ε=2/3