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ABSTRACT

We use an equilibrium unemployment framework to characterize the optimal unemployment insurance (UI) over the business cycle. We investigate how the classical trade-off between insurance and incentives to search evolves over the business cycle. In a broad class of models the trade-off is resolved by a formula that expresses optimal UI as a function of not only usual statistics—risk aversion and micro-elasticity of unemployment with respect to UI—but also a macro-elasticity that captures the general-equilibrium effect of UI on unemployment. In a model with job rationing the formula implies that optimal UI is countercyclical. In recessions jobs are rationed. Therefore the social value of search is low: aggregate search efforts cannot reduce unemployment much; and individual search efforts create a negative externality by reducing the job-finding rate of other jobseekers as in a rat race. Hence the trade-off tilts towards insurance in recessions. To quantify the cyclical variations of optimal UI, we calibrate and simulate a model with job rationing in which workers insure themselves partially using home production. Optimal UI increases significantly in recessions. This result holds whether the government adjusts the level or the duration of benefits; it holds whether the government balances its budget each period or uses deficit spending.

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1 Introduction

When risk-averse workers cannot insure themselves against unemployment and the search efforts of jobseekers cannot be monitored, the government faces a trade-off between providing insurance against unemployment and providing incentives to search for a job. Most models that study this trade-off assume that unemployment depends solely on individual search efforts [Baily, 1978; Chetty, 2006a; Hopenhayn and Nicolini, 1997; Shavell and Weiss, 1979]. Yet the reality is more complex. During the Great Depression, unemployed workers queued for jobs at factory gates. In a queue, a jobseeker does increase his job-finding probability by searching more and pushing his way up the queue; but those in front of him in the queue fall behind and face a lower job-finding probability because the number of jobs available is limited. In the aggregate, unemployment depends not only on individual search efforts but also on the number of workers that firms choose to hire. Moreover the relationship between search efforts and unemployment evolves over the business cycle because firms are more reluctant to hire workers in recessions than in expansions.

By modeling how unemployed workers search for jobs and how firms hire workers, one can understand more rigorously and more generally how the trade-off between insurance and incentives evolves over the business cycle. In this paper, we adopt the equilibrium unemployment framework of Pissarides [2000] to analyze the optimal unemployment insurance (UI) over the business cycle. The framework offers an appealing description of the labor market: unemployed workers receive benefits funded by a labor tax whose incidence falls on workers; when workers become unemployed, they decide how much to search for a job based on the generosity of UI; firms decide how many vacancies to post based on the state of the economy; not all workers find a job because frictions impede matching of jobseekers with vacancies.

Our analysis rests on a new representation of the labor market equilibrium in a labor supply-labor demand diagram. Because search costs are sunk when a worker and a firm meet, a surplus arises from their match. Any wage sharing the surplus could be an equilibrium wage. Hence wages cannot equalize labor supply to labor demand. Instead labor market tightness—the ratio of vacancies to aggregate search effort—equalizes supply and demand. This property allows us to represent the equilibrium in a labor supply-labor demand diagram in which labor market tightness acts as a price, as depicted in Figure 1(a). The representation is quite general. If labor demand

is perfectly elastic, unemployment depends solely on search efforts as in [Baily \[1978\]](#) and [Chetty \[2006a\]](#). At the polar opposite if labor demand is perfectly inelastic, unemployment is completely independent of search efforts as in a rat race. These two special cases are depicted in [Figure 1\(b\)](#).

We begin in [Section 2](#) by deriving a formula for the optimal replacement rate—the generosity of unemployed benefits as a fraction of the income of employed workers—in a static equilibrium unemployment model. The formula, expressed in sufficient statistics, does not require much structure on the primitives of the model. As in the [Baily \[1978\]-Chetty \[2006a\]](#) formula, a first term captures the trade-off between the need for insurance, measured by a coefficient of risk aversion, and the need for incentives to search, measured by an elasticity of unemployment with respect to UI. But we replace the micro-elasticity used in the Baily-Chetty formula by a macro-elasticity to measure the budgetary costs of UI in an equilibrium unemployment framework. The micro-elasticity ϵ^m is the elasticity of the probability of unemployment for a worker whose individual benefits change, and the macro-elasticity ϵ^M is the elasticity of aggregate unemployment when benefits changes for all workers. Formally ϵ^m takes labor market tightness as given, whereas ϵ^M accounts for the equilibrium adjustment in tightness following a change in UI. Moreover our formula adds to the Baily-Chetty formula a second term proportional to the wedge $\epsilon^m/\epsilon^M - 1$. This wedge captures the welfare effects of the employment change arising from the equilibrium adjustment in tightness after a change in UI. At the end of [Section 2](#), we extend the formula to a model in which workers partially insure themselves against unemployment, and to a model in which wages respond to UI.

In [Section 3](#), we characterize the optimal replacement rate over the business cycle by applying our formula to the equilibrium unemployment model of [Michaillat \[2012\]](#). The model captures two critical elements of the business cycle: (a) unemployment fluctuations, and (b) job rationing—the property that the labor market does not converge to full employment in recessions even when search efforts are arbitrarily large. Feature (a) arises from technology shocks and real wage rigidity. Feature (b) arises from the combination of wage rigidity and diminishing marginal returns to labor. Because of feature (a), the model accommodates periods of high unemployment and periods of low unemployment. Because of feature (b), the model is consistent with the fact that unemployed workers queue for jobs in recessions.¹

¹[Michaillat \[2012\]](#) shows that standard models of equilibrium unemployment do not have job rationing. These models always converge to full employment when job-search efforts are arbitrarily large, even in recessions.

We first prove that the macro-elasticity is lower than the micro-elasticity, creating a wedge $\epsilon^m/\epsilon^M - 1 > 0$. The wedge arises because when the number of jobs available is limited, searching more to increase one's probability of finding a job mechanically decreases others' probability of finding one of the few jobs available. Because of this wedge, our formula calls for a higher replacement rate than the Baily-Chetty formula. As jobseekers search taking the job-finding rate as given, without internalizing their influence on the job-finding rate of others, they impose a negative *rat-race externality*. A higher replacement rate corrects the externality by discouraging job search.

Next, we prove that the wedge $\epsilon^m/\epsilon^M - 1$ is countercyclical and the macro-elasticity ϵ^M procyclical. Recessions are periods of acute job shortage during which job search and matching frictions have little influence on labor market outcomes. The search efforts of jobseekers have little influence on aggregate unemployment and the rat-race externality is exacerbated. Thus the macro-elasticity is small and the wedge between micro-elasticity and macro-elasticity is large.

Finally, we use our formula to prove that the optimal replacement rate is countercyclical. In recessions the macro-elasticity ϵ^M falls. A higher UI only increases unemployment negligibly. Hence the marginal budgetary cost of UI is small. In recessions the wedge $\epsilon^m/\epsilon^M - 1$, which measures the welfare cost of the rat-race externality, increases. Hence the marginal benefit of UI from correcting the externality is high.² At the end of Section 3, we show that optimal UI is also countercyclical in a model in which aggregate demand shocks drive fluctuations, and in a model in which the government provides a wage subsidy to employers to attenuate employment fluctuations.

In Section 4 we calibrate and simulate a dynamic model to quantify the cyclical fluctuations of optimal UI. Workers insure themselves partially against unemployment using home production. When the government balances its budget each period and unemployment benefits never expire, the optimal replacement rate is strongly countercyclical: it increases from 50% when the unemployment rate is 4% to 70% when the unemployment rate reaches 10%. When the government can borrow and save, the government provides higher consumption to all workers in recessions and the optimal replacement rate increases more sharply after an adverse shock. When the government

²In this paper we analyze how the classical trade-off between insurance and incentives to search evolves over the business cycle. Additional mechanisms could justify raising UI in recessions. For instance, if unemployed workers were more likely to exhaust their precautionary savings in recessions, the consumption-smoothing benefits of UI would increase and it would be optimal to raise UI further [Kroft and Notowidigdo, 2011].

adjusts the duration of unemployment benefits, as in the US, the optimal duration is strongly countercyclical: it increases from less than 10 weeks when unemployment is 4%; to 26 weeks when unemployment is 5.9%; and to over 100 weeks when unemployment reaches 8%.

The property that $\varepsilon^m/\varepsilon^M > 1$ distinguishes our model from standard models of equilibrium unemployment: $\varepsilon^m/\varepsilon^M < 1$ in the canonical model with Nash bargaining; and $\varepsilon^m/\varepsilon^M = 1$ if bargaining is replaced by rigid wages. In Section 5, we discuss empirical evidence by Crépon et al. [2012] that $\varepsilon^m/\varepsilon^M > 1$, in support of our model. We also discuss how ε^m and ε^M can be estimated to implement our UI formula. Proofs, derivations, and extensions are collected in the Appendix.

2 Formula for Optimal Unemployment Insurance

In this section we use a static model of equilibrium unemployment to derive a formula expressing the optimal level of UI in terms of sufficient statistics: curvature of the utility function, micro- and macro-elasticity of unemployment with respect to UI. We also propose two extensions of the formula. One accounts for the ability of workers to insure themselves partially against unemployment. The other accounts for the response of wages to UI, through bargaining or labor tax incidence.

2.1 The static model

Labor market. There is a unit mass of workers. Initially, $u \in (0, 1)$ workers are unemployed and search for a job with effort e , while $1 - u$ workers are employed. Firms post o vacancies to recruit unemployed workers. The number of matches h made is given by a constant-returns matching function $h = h(e \cdot u, o)$ of aggregate search effort $e \cdot u$ and vacancies o , differentiable and increasing in both arguments, with the restriction that $h(e \cdot u, o) \leq \min\{u, o\}$. Conditions on the labor market are summarized by labor market tightness $\theta \equiv o/(e \cdot u)$. A jobseeker finds a job at a rate $f(\theta) \equiv h(e \cdot u, o)/(e \cdot u) = h(1, \theta)$ per unit of search effort; a jobseeker searching with effort e finds a job with probability $e \cdot f(\theta)$. A vacancy is filled with probability $q(\theta) \equiv h(e \cdot u, o)/o = h(1/\theta, 1)$. When θ is high, it is easy for jobseekers to find jobs— $f(\theta)$ is high—and difficult for firms to hire— $q(\theta)$ is low. To capture the influence of tightness on these probabilities, we define the tightness elasticities of $f(\theta)$ and $q(\theta)$: $1 - \eta \equiv \theta \cdot f'(\theta)/f(\theta) > 0$ and $-\eta \equiv \theta \cdot q'(\theta)/q(\theta) < 0$.

A fraction $e \cdot f(\theta)$ of the u unemployed workers finds a job during matching. The $h = u \cdot e \cdot f(\theta)$ new hires join $1 - u$ incumbents in firms and the employment level after matching is

$$n^*(e, \theta) = (1 - u) + u \cdot e \cdot f(\theta). \quad (1)$$

Employment $n^*(e, \theta)$ increases mechanically with effort e and tightness θ , as $f(\cdot)$ is increasing.

Workers. A worker's utility is $v(c) - k(e)$, where $v(\cdot)$ is an increasing and concave function of consumption c and $k(\cdot)$ is an increasing and convex function of effort e . The curvature of the utility functions is measured by $\rho \equiv -c^e \cdot v''(c^e)/v'(c^e)$, the coefficient of relative risk aversion evaluated at c^e , and $\kappa \equiv e \cdot k''(e)/k'(e)$, the elasticity of the marginal disutility of effort. Firms pay a wage w . To finance unemployment benefits $b \cdot w$ the government imposes a labor tax t . As in the public finance literature we assume that the incidence of the tax is entirely on the worker's side. We also abstract from possible bargaining effects. Hence the wage w responds neither to the benefit rate b nor to the tax rate t . Workers neither borrow nor save, so consumption is $c^e = w \cdot (1 - t)$ when employed and $c^u = b \cdot w$ when unemployed. Let $\Delta c \equiv c^e - c^u$ and $\Delta v \equiv v(c^e) - v(c^u)$ be the consumption and utility gains from work. Given labor market tightness θ and consumptions $\{c^e, c^u\}$, a jobseeker chooses effort e to maximize expected utility

$$v(c^u) + e \cdot f(\theta) \cdot \Delta v - k(e).$$

The optimal effort is a function $e^*(\theta, \Delta v)$ implicitly defined by the first-order condition

$$k'(e) = f(\theta) \cdot \Delta v. \quad (2)$$

As the disutility $k(\cdot)$ from effort is convex and the job-finding rate $f(\cdot)$ is increasing, the optimal effort $e^*(\theta, \Delta v)$ increases with tightness θ and with the utility gain from work Δv .

Budget constraint. Since the government finances unemployment benefits with a labor tax, its budget constraint is $(1 - n) \cdot b \cdot w = n \cdot t \cdot w$. In terms of consumptions the constraint is

$$n \cdot c^e + (1 - n) \cdot c^u = n \cdot w. \quad (3)$$

As in optimal income tax theory we consider that the government chooses the consumption gain from work Δc , which determines $c^u = n \cdot (w - \Delta c)$ and $c^e = c^u + \Delta c$ through the budget constraint.

Equilibrium. The equilibrium is parameterized by Δc , which characterizes the generosity of UI. It is useful to represent the equilibrium in a labor demand-labor supply framework. Let $e^s(\theta, \Delta c) \equiv e^*(\theta, \Delta v(\Delta c))$ be the effort supply. The labor supply $n^s(\theta, \Delta c) \equiv n^*(e^s(\theta, \Delta c), \theta)$ gives the employment rate after matching when jobseekers search optimally for a given labor market tightness θ . It increases with θ because both $e^s(\theta, \Delta c)$ and $n^*(e, \theta)$ increase with θ . It is concave in θ if and only if $(1 - \eta) \cdot (1 + \kappa) / \kappa < 1$.³ Let $n^d(\theta)$ summarize the firm's demand for labor as a function of θ . In presence of matching frictions the equilibrium wage cannot equalize labor supply to labor demand. Instead, θ acts as a price equilibrating labor supply and labor demand:

$$n^s(\theta, \Delta c) = n^d(\theta). \quad (4)$$

This equilibrium condition is represented in Figure 1(a). Equilibrium employment $n(\Delta c)$ is given by the intersection of the upward-sloping labor supply curve $n^s(\theta, \Delta c)$ with a generic downward-sloping labor demand curve $n^d(\theta)$. Tightness θ equalizes supply and demand. If labor supply is above labor demand, a reduction in θ increases labor demand; it reduces labor supply by reducing the job-finding rate as well as optimal search effort; until labor supply and labor demand are equalized. In Section 3, we impose more structure and derive a downward-sloping labor demand.

Our framework is quite general. It nests the Baily-Chetty model as a special case. In the Baily-Chetty model, employment is solely driven by search efforts. It is a partial-equilibrium model of unemployment in the sense that it fixes labor market tightness θ and job-finding rate $f(\theta)$. As showed in Figure 1(b), the Baily-Chetty model can be represented with a perfectly elastic labor

³Lemma A3 in the Appendix proves the concavity of the labor supply. If jobseekers exert a constant search effort irrespective of labor market tightness ($\kappa = +\infty$), then the labor supply is concave for any parameter values.

demand, which determines θ independently of UI. At the polar opposite, our framework also nests the rat-race model as a special case. In the rat-race model, the number of jobs is fixed. As showed in Figure 1(b), the rat-race mode can be represented with a perfectly inelastic labor demand, which determines n independently of UI.

Government. The government chooses the consumption gain from work Δc to maximize welfare

$$n \cdot v(c^e) + [1 - n] \cdot v(c^u) - u \cdot k(e). \quad (5)$$

Equilibrium effort e and equilibrium employment n are read off the effort supply and labor supply curves: $e = e^s(\theta, \Delta c)$, $n = n^s(\theta, \Delta c)$, where equilibrium tightness θ satisfies condition (4). Consumptions c^e and c^u satisfy the government's budget constraint (3) and $c^e = c^u + \Delta c$.

2.2 Elasticities and optimal unemployment insurance formula

To solve the government's problem we need to characterize the individual response of jobseekers (through a change in effort) and the aggregate response of the labor market (through the response of both jobseekers and firms) to a change in UI. To this end, we define two elasticities:

DEFINITION 1. The *micro-elasticity* of unemployment with respect to consumption gain from work is

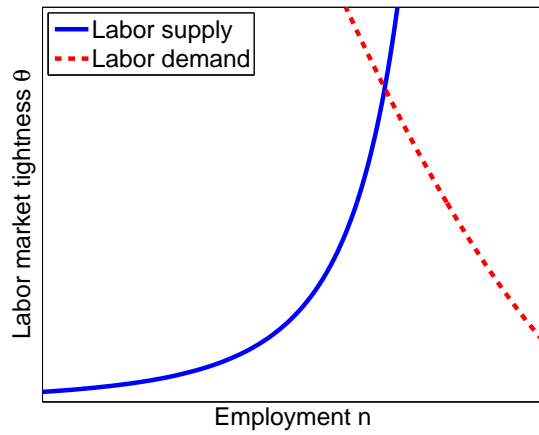
$$\varepsilon^m \equiv \frac{\Delta c}{1 - n} \cdot \frac{\partial n^s}{\partial \Delta c} \Big|_{\theta} = \frac{\Delta c}{1 - n} \cdot \frac{\partial n^*}{\partial e} \Big|_{\theta} \cdot \frac{\partial e^s}{\partial \Delta c} \Big|_{\theta}. \quad (6)$$

The *macro-elasticity* of unemployment with respect to consumption gain from work is

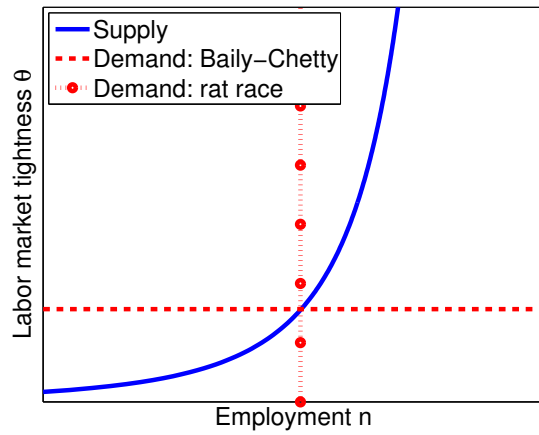
$$\varepsilon^M \equiv \frac{\Delta c}{1 - n} \cdot \frac{dn}{d\Delta c} = \varepsilon^m + \frac{\Delta c}{1 - n} \cdot \left(\frac{\partial n^*}{\partial \theta} \Big|_e + \frac{\partial n^*}{\partial e} \Big|_{\theta} \cdot \frac{\partial e^s}{\partial \theta} \Big|_{\Delta c} \right) \cdot \frac{d\theta}{d\Delta c}. \quad (7)$$

If labor demand is perfectly elastic, θ is determined by firms independently of UI and $\varepsilon^M = \varepsilon^m$.

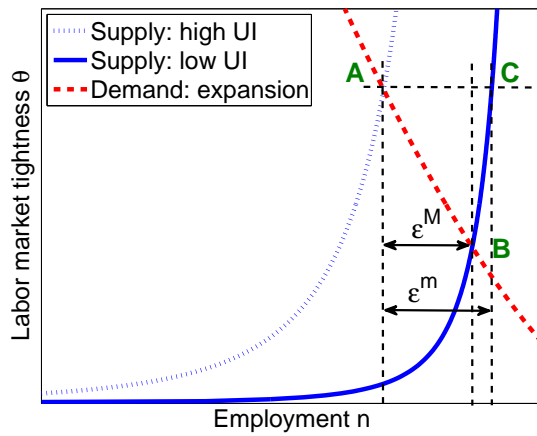
Both elasticities are normalized to be positive and are depicted for various models in Figure 1(c)–1(f). The micro-elasticity measures the percentage increase in unemployment $1 - n$ when the net reward from work Δc decreases by 1%, taking into account the jobseekers' reduction in



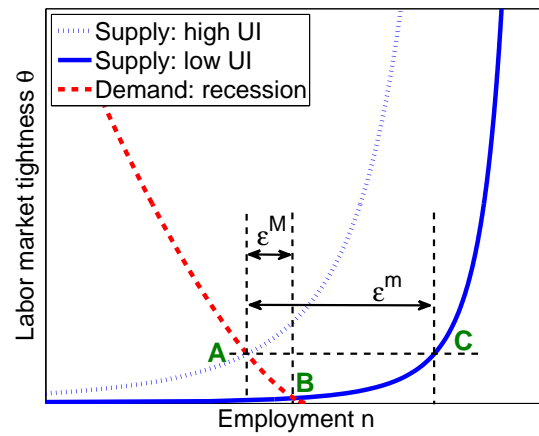
(a) Generic equilibrium



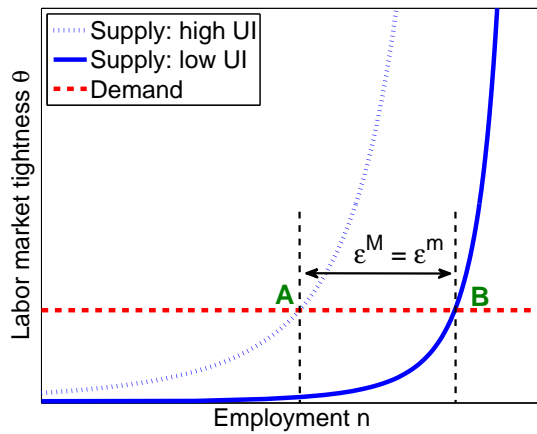
(b) Some special cases of equilibrium



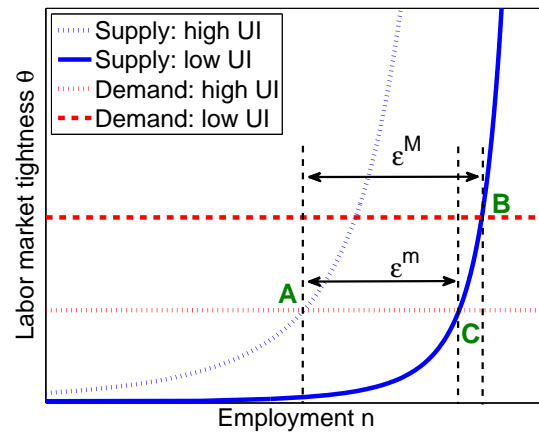
(c) Elasticities in expansion with job rationing



(d) Elasticities in recession with job rationing



(e) Elasticities in Baily-Chetty model



(f) Elasticities in model with Nash bargaining

Figure 1: Labor market equilibria in a price θ -quantity n diagram

search effort e but ignoring the equilibrium adjustment of labor market tightness θ . It can be estimated by measuring the reduction in the job-finding probability of an individual unemployed worker whose unemployment benefits are increased, keeping the benefits of all other workers constant. The macro-elasticity measures the percentage increase in unemployment when the net reward from work decreases by 1%, assuming that both search effort and tightness adjust. It can be estimated by measuring the increase in aggregate unemployment following a general increase in unemployment benefits.

The two elasticities differ in models of equilibrium unemployment as long as labor demand is not perfectly elastic. A rat-race model illustrates the difference. u jobseekers queue in front of $o < u$ vacant jobs. Workers searching more move up the queue and increase their probability of finding a job. Formally, the unconditional probability n^* to be employed after the matching process increases with search effort e : $n^*(e, f) = (1 - u) + u \cdot e \cdot f$, where f is the equilibrium job-finding rate. Searching harder increases the employment probability n^* , so the micro-elasticity ϵ^m is positive. But when a jobseeker moves up the queue by searching more, the jobseekers in front of him in the queue fall behind and face a lower probability of finding a job. Formally, the equilibrium job-finding rate $f = o/(u \cdot e)$ falls when aggregate search effort e rises to equilibrate labor supply $n^*(e, f)$ with the fixed labor demand $1 - u + o$. As a result of the job shortage, the macro-elasticity ϵ^M is smaller than the micro-elasticity ϵ^m . In fact equilibrium employment $n = 1 - u + o < 1$ is fixed, independent of aggregate search effort. Therefore $\epsilon^M = 0$ even though $\epsilon^m > 0$.

To solve the government's problem we use the envelope theorem as workers choose effort e optimally.⁴ The first-order condition of the government's problem (5) with respect to Δc is

$$n \cdot v'(c^e) + \bar{v}' \cdot \frac{dc^u}{d\Delta c} + \Delta v \cdot \left. \frac{\partial n^*}{\partial \theta} \right|_e \cdot \frac{d\theta}{d\Delta c} = 0, \quad (8)$$

where $\bar{v}' \equiv n \cdot v'(c^e) + (1 - n) \cdot v'(c^u)$ denotes the average marginal utility.

We now provide some intuition for (8), and we explain how (8) can be expressed in terms of sufficient statistics. Consider a small increase $d\Delta c > 0$ in the consumption gain from work—equivalent to a cut in unemployment benefits. The first term in (8) captures the utility gain of

⁴To apply the envelope theorem, note that social welfare (5) is $(1 - u) \cdot v(c^e) + u \cdot [v(c^u) + e \cdot f(\theta) \cdot \Delta v - k(e)]$.

the n employed workers, whose consumption $c^e = c^u + \Delta c$ increases by $d\Delta c$: $dS_1 = n \cdot v'(c^e) \cdot d\Delta c$. To satisfy the budget constraint, increasing Δc requires cutting unemployment benefits $c^u = n \cdot (w - \Delta c)$, which reduces by dc^u the consumption of all workers, including the employed as $c^e = c^u + \Delta c$. The second term in (8) captures the associated utility loss $dS_2 = -\bar{v}' \cdot dc^u$. Since $dc^u = -n \cdot d\Delta c + (w - \Delta c) \cdot dn = -\{n - (1 - n) \cdot [(w - \Delta c)/\Delta c] \cdot \epsilon^M\} \cdot d\Delta c$, we can rewrite $dS_2 = -\bar{v}' \cdot \{n - (1 - n) \cdot [(w - \Delta c)/\Delta c] \cdot \epsilon^M\} \cdot d\Delta c$. The macro-elasticity ϵ^M appears in dS_2 to capture the budgetary cost of the increase in unemployment caused by higher UI.

The job-finding rate $f(\theta)$ depends on labor market tightness θ , which is determined in equilibrium by (4) as the intersection of labor demand and labor supply as depicted in Figure 1. The change $d\Delta c > 0$ increases the incentive to search, shifts labor supply $n^s(\theta, \Delta c)$ outwards, and leads to a small equilibrium adjustment $d\theta$ of labor market tightness. The change $d\theta$ in turn leads to a small change dn_θ in employment through two channels: (a) a change $(\partial n^*/\partial e) \cdot (\partial e^s/\partial \theta) \cdot d\theta$ in employment through a reduction in search effort—this reduction, however, does not have any welfare effect by the envelope theorem as workers choose effort to maximize expected utility; and (b) a change $(\partial n^*/\partial \theta) \cdot d\theta$ in employment through a change in job-finding rate $f(\theta)$. Each new job created through (b) generates a first-order utility gain $\Delta v > 0$ as finding a job discretely increases consumption. The third term in (8) captures the welfare change from the equilibrium adjustment $d\theta$. Lemma 1 establishes the relationship between the change $(\partial n^*/\partial \theta) \cdot d\theta$ in employment, which is the only relevant change from a welfare perspective, and the wedge $\epsilon^m - \epsilon^M$:

LEMMA 1. *The derivative of equilibrium labor market tightness satisfies:*

$$\frac{\Delta c}{\theta} \cdot \frac{d\theta}{d\Delta c} = \frac{\kappa}{\kappa + 1} \cdot \frac{1}{1 - \eta} \cdot \frac{1 - n}{h} \cdot [\epsilon^M - \epsilon^m],$$

$$\frac{\Delta c}{1 - n} \cdot \frac{\partial n^*}{\partial \theta} \Big|_e \cdot \frac{d\theta}{d\Delta c} = \frac{\kappa}{\kappa + 1} \cdot [\epsilon^M - \epsilon^m].$$

Using the lemma we rewrite $dS_3 \equiv \Delta v \cdot (\partial n^*/\partial \theta) \cdot d\theta = \Delta v \cdot [\kappa/(1 + \kappa)] \cdot [(1 - n)/\Delta c] \cdot [\epsilon^M - \epsilon^m] \cdot d\Delta c$. At the optimum the sum $dS_1 + dS_2 + dS_3$ is zero, allowing to rewrite (8) as follows.

PROPOSITION 1. *The optimal replacement rate $\tau \equiv c^u/c^e$ satisfies*

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \left[n + (1-n) \cdot \frac{v'(c^u)}{v'(c^e)} \right]^{-1} \cdot \left\{ \frac{n}{\varepsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1 \right] + \frac{\Delta v}{v'(c^e) \cdot \Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \left[\frac{\varepsilon^m}{\varepsilon^M} - 1 \right] \right\}. \quad (9)$$

If $n \approx 1$, and if the third and higher order terms of $v(\cdot)$ are small, the optimal formula simplifies to

$$\frac{\tau}{1-\tau} \approx \frac{\rho}{\varepsilon^M} \cdot (1-\tau) + \left[\frac{\varepsilon^m}{\varepsilon^M} - 1 \right] \cdot \frac{\kappa}{1+\kappa} \cdot \left[1 + \frac{\rho}{2} \cdot (1-\tau) \right]. \quad (10)$$

If labor demand is perfectly elastic, $\varepsilon^m = \varepsilon^M$, the second term in the right-hand side of (9) and (10) vanishes, and the formulas reduce to those in Baily [1978] and Chetty [2006a].

The proposition provides a formula for the optimal replacement rate, defined as the amount transferred to unemployed workers expressed as a fraction of the income of employed workers. The replacement rate measures the generosity of the UI system. Equation (9) provides an exact formula while equation (10) provides a simpler formula using the approximation method of Chetty [2006a]. The approximated formula (10) is expressed in sufficient statistics, which means that the formula is robust to changes in the primitives of the model. Indeed the formula is valid for any utility over consumption with coefficient of relative risk aversion ρ ; any marginal disutility of effort with elasticity κ and associated micro-elasticity ε^m ; any labor demand, function only of labor market tightness and an exogenous shock, yielding a macro-elasticity ε^M ; and any constant-returns matching function. Since the four statistics are estimable, the formula can be used to assess the current UI system.⁵ Admittedly the statistics are endogenous functions of the replacement rate, so we cannot infer directly the optimal replacement rate from estimates of the statistics. Nevertheless, we can infer that increasing the replacement rate τ is desirable if the current $\tau/(1-\tau)$ is lower than the right-hand side of formula (10) evaluated using current estimates of the four statistics.

The first term in the optimal replacement rate (10) increases with the coefficient of relative risk aversion ρ , which measures the value of insurance. If micro- and macro-elasticity are equal ($\varepsilon^m = \varepsilon^M$), our formulas reduce to the Baily-Chetty formula. For instance the approximated for-

⁵Section 5 discusses how to estimate micro- and macro-elasticity. Appendix E.4 explains how to estimate κ from the micro-elasticity of the hazard rate out of unemployment with respect to benefits estimated by Meyer [1990]. Many studies estimate the coefficient of relative risk aversion [for example, Chetty, 2006b].

mula (10) becomes $\tau/(1 - \tau) \approx (\rho/\epsilon^m) \cdot (1 - \tau)$. In the formula the trade-off between the need for insurance (captured by the coefficient of relative risk aversion ρ) and the need for incentives to search (captured by the micro-elasticity ϵ^m) appears transparently. In a model of equilibrium unemployment micro- and macro-elasticity generally differ ($\epsilon^m \neq \epsilon^M$), and our formula presents two departures from the Baily-Chetty formula.

The first term in the right-hand side of formulas (9) and (10) involves the macro-elasticity ϵ^M and not the micro-elasticity ϵ^m , conventionally used to calibrate optimal benefits [Chetty, 2008; Gruber, 1997]. What matters for the government is the budgetary cost of UI from higher aggregate unemployment and higher outlays of unemployment benefits, and only ϵ^M captures this cost in an equilibrium unemployment framework. The optimal replacement rate naturally decreases with ϵ^M .

A second term, increasing with the ratio ϵ^m/ϵ^M , also appears in the right-hand side of formulas (9) and (10) when $\epsilon^m \neq \epsilon^M$. The term is a correction that accounts for the first-order welfare effects of the adjustment of employment that arises from the equilibrium adjustment of labor market tightness after a change in UI. Even in the absence of any concern for insurance—if workers are risk neutral—some unemployment insurance should be provided as long as the correction term is positive ($\epsilon^m/\epsilon^M > 1$).

2.3 Extensions

Self-insurance. We extend the model to include partial self-insurance by workers. Chetty [2006a] shows that the Baily formula carries over to models with savings, borrowing constraints, private insurance, or leisure benefits of unemployment. Similarly, formulas (9) and (10) carry over with minor modifications. Introducing self-insurance through borrowing and saving would require a dynamic model. Instead, we consider the simpler case of self-insurance through home production. In addition to unemployment benefits c^u received from the government, unemployed workers consume an amount y of good produced at home at a utility cost $m(y)$, increasing, convex, and normalized so that $m(0) = 0$. Jobseekers choose effort e and home production y to maximize

$$[1 - e \cdot f(\theta)] \cdot [v(c^u + y) - m(y)] + [e \cdot f(\theta)] \cdot v(c^e) - k(e).$$

Home production y is chosen so that $v'(c^u + y) = m'(y)$. It provides additional insurance that is partially crowded out by UI, as y decreases with c^u . The government chooses Δc to maximize

$$n \cdot v(c^e) + [1 - n] \cdot [v(c^u + y) - m(y)] - u \cdot k(e),$$

where e and y are chosen optimally by individuals, subject to the same constraints as in our original problem. Let $c^h \equiv c^u + y$ be the total consumption when unemployed and $\Delta v^h \equiv v(c^e) - [v(c^u + y) - m(y)]$ be the utility gain from work. Appendix B derives the optimal UI formula

$$\frac{1}{n} \cdot \frac{\tau}{1 - \tau} = \left[n + (1 - n) \cdot \frac{v'(c^h)}{v'(c^e)} \right]^{-1} \cdot \left\{ \frac{n}{\varepsilon^M} \cdot \left[\frac{v'(c^h)}{v'(c^e)} - 1 \right] + \frac{\Delta v^h}{v'(c^e) \cdot \Delta c} \cdot \frac{\kappa}{\kappa + 1} \cdot \left[\frac{\varepsilon^m}{\varepsilon^M} - 1 \right] \right\}.$$

Clearly, formula (9) carries over by replacing $v'(c^u)$ by $v'(c^h)$ and Δv by Δv^h .

Although the structure of the formula does not change, the consumption smoothing benefit $[v'(c^h)/v'(c^e) - 1]$ of UI is smaller if individuals can partially self-insure using home production, because $c^h \geq c^u$. The welfare effect of the equilibrium adjustment of θ is also smaller because $\max_y [v(c^u + y) - m(y)] \geq v(c^u)$ so $\Delta v^h = v(c^e) - [v(c^u + y) - m(y)] \leq \Delta v = v(c^e) - v(c^u)$. If workers can partially smooth consumption on their own, the optimal replacement rate τ is lower than in our original model without self-insurance. As shown in Section 4.2, the extended formula can be implemented using estimates of the consumption-smoothing benefit of UI from Gruber [1997].

Wage response to UI. In the baseline model we assume that wages paid by firms do not respond to UI. But UI may affect the outside options of workers and influence the wage through bargaining. Or the incidence of the labor tax financing UI may fall partly on employers. In this section we show that formula (9) carries over with a minor modification when wages depend on UI.

As UI is parameterized by the consumption gain from work Δc , we assume that the wage is a function $w(\Delta c)$. In that case, a change $d\Delta c$ in the generosity of UI affects the government budget's constraint not only through a change dn in employment, but also through a change dw in wages. The optimal UI formula (9) becomes

$$\frac{1}{n} \cdot \frac{\tau}{1 - \tau} + \frac{1}{\varepsilon^M} \cdot \frac{n}{1 - n} \cdot \frac{dw}{d\Delta c} = \left[n + (1 - n) \frac{v'(c^u)}{v'(c^e)} \right]^{-1} \left\{ \frac{n}{\varepsilon^M} \left[\frac{v'(c^u)}{v'(c^e)} - 1 \right] + \frac{\Delta v}{v'(c^e) \Delta c} \frac{\kappa}{\kappa + 1} \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right) \right\}.$$

A new term appears on the right-hand side of the formula. The term is negative if $dw/d\Delta c < 0$, as lower benefits reduce the outside option of workers and the UI payroll tax leading to lower wages through both bargaining and employer tax incidence channels. At the same time when wages respond to UI in that way, the macro-elasticity ε^M is likely to be higher than in our basic model because higher benefits increase wages, depress labor demand, which increases unemployment further. Overall optimal UI is likely to be lower, despite the additional negative term.⁶

3 Unemployment Insurance in Presence of Job Rationing

In this section we specialize the model to obtain the properties that unemployment is high and jobs are rationed in recessions—periods of low technology. We use formula (9) to prove that optimal UI is countercyclical. At the end of the section we show that the result is quite general. It holds in a model in which aggregate demand, instead of technology, drives fluctuations. It also holds in a model in which the government provides, in addition to UI, a wage subsidy to firms to attenuate unemployment fluctuations. We provide intuitions with the equilibrium diagram of Figure 1.

3.1 Firms

The representative firm takes labor n as input to produce a consumption good according to the production function $a \cdot g(n) = a \cdot n^\alpha$. $\alpha > 0$ measures marginal returns to labor. $a > 0$ is the level of technology, which proxies for the position in the business cycle.

ASSUMPTION 1. The production function has diminishing marginal returns to labor: $\alpha < 1$.

The assumption yields a downward-sloping labor demand curve in a price θ -quantity n diagram, which has important macroeconomic implications. The assumption is motivated by the observation that, at business cycle frequency, some production inputs are slow to adjust.

Wages are set once worker and firm have matched. Since the costs of search are sunk at the time of matching, a surplus arise from each worker-firm match. Any wage sharing this surplus could

⁶In Appendix B we assume that a fraction ψ of the tax burden falls on firms and a fraction $1 - \psi$ falls on workers. The wage w becomes an increasing function of the labor tax rate t . We apply the formula in that case and obtain a formula similar to (9), except that each of the three terms in (9) is corrected with the incidence parameter ψ .

be an equilibrium wage [Hall, 2005]. Given the indeterminacy of wages we use the simple wage schedule of Blanchard and Galí [2010]:

ASSUMPTION 2. The wage schedule is rigid: $w = \omega \cdot a^\gamma$, $\gamma < 1$.

ω is a parameter. The parameter γ captures the rigidity of wages over the business cycle. If $\gamma = 0$, wages do not respond to technology and are completely fixed over the cycle. If $\gamma = 1$, wages are proportional to technology and are fully flexible over the cycle. We assume that wages are rigid in the sense that (a) they only partially adjust to a change in technology, and (b) they do not respond to a change in UI. Rigidity (a) generates unemployment fluctuations over the cycle [Hall, 2005]. Rigidity (b) makes labor demand independent of UI and allows us to focus on the classical trade-off between insurance and incentive to search. Both assumptions are empirically grounded. Many historical, ethnographic, and empirical studies document and explain wage rigidity [for example, Bewley, 1999; Jacoby, 1984; Kramarz, 2001]. Empirical studies consistently find that re-employment wages do not respond to changes in unemployment benefits [Card et al., 2007].

As in Pissarides [2000], it costs $r \cdot a$ to post a vacancy. The parameter $r > 0$ measures the resources spent on recruiting. We assume away randomness at the firm level: a worker is hired with certainty by opening $1/q(\theta)$ vacancies and spending $r \cdot a/q(\theta)$. When the labor market is tighter, a firm posts more vacancies to fill a job, and recruiting is more costly.

The firm takes prices as given. It starts with $1 - u$ workers. Given labor market tightness θ , technology a , and wage w , it decides how many workers to hire such that employment n maximizes real profit:⁷

$$\pi = a \cdot g(n) - w(a) \cdot n - \frac{r \cdot a}{q(\theta)} \cdot [n - (1 - u)].$$

The first-order condition implicitly defines labor demand $n^d(\theta, a)$, which satisfies

$$g'(n) = \frac{w}{a} + \frac{r}{q(\theta)}. \quad (11)$$

Under Assumption 1, $g'(\cdot)$ decreases with n . $q(\cdot)$ decreases with θ . Thus labor demand $n^d(\theta, a)$ decreases with θ . When the labor market is tight, it is expensive for firms to recruit, depressing

⁷We assume that technology is high enough such that it is optimal for the firm to choose positive hiring: $h = n - (1 - u) > 0$. The assumption requires $a > (\omega/\alpha) \cdot (1 - u)^{(1-\alpha)/(1-\gamma)}$.

labor demand. Under Assumption 2, w/a decreases with a so $n^d(\theta, a)$ increases with a . When technology is high, wages are relatively low, stimulating labor demand.

The equilibrium in the labor market is depicted in Figure 1 in a price θ -quantity n plan. The figure plots labor demand curves for high and low technology (panel (c) and panel (d)). It plots labor supply for low and high consumption gains from work Δc (dotted line and solid line). Jobs are rationed in recessions in the sense that the labor market does not clear and some unemployment remains even as unemployed workers exert an arbitrarily large search effort. The mechanism creating a job shortage is quite simple. After a negative technology shock the marginal product of labor falls but rigid wages adjust downwards only partially, so that the labor demand shifts inward (from panel (c) to panel (d)). If the adverse shock is sufficiently large, the marginal product of the least productive workers falls below the wage. It becomes unprofitable for firms to hire these workers even if recruiting is costless at $\theta = 0$: labor demand cuts the x-axis at $n^R < 1$ on the right panel. Even if workers searched infinitely hard, shifting labor supply outwards such that $\theta \rightarrow 0$, firms would never hire more than $n^R < 1$ workers: there is a job shortage. When the shortage is acute in recessions, the social returns to search are small because an increase in search efforts only leads to a negligible increase in employment.

3.2 Elasticity wedge

Formula (9) adds to the Baily-Chetty formula a second term proportional to the wedge $\varepsilon^m/\varepsilon^M - 1$. Proposition 2 establishes that $\varepsilon^m/\varepsilon^M > 1$ in our model with job rationing in which wages are rigid (Assumption 2) and the labor demand is downward sloping (Assumption 1):

PROPOSITION 2. *Under Assumption 2, the wedge $\varepsilon^m/\varepsilon^M$ admits a simple expression:*

$$\frac{\varepsilon^m}{\varepsilon^M} = 1 + \chi \cdot q(\theta) \cdot \frac{h}{n} \cdot n^{\alpha-1},$$

where $\chi \equiv \alpha \cdot (1 - \alpha) \cdot [(1 - \eta)/\eta] \cdot [(1 + \kappa)/\kappa] \cdot (1/r)$. Under Assumption 1, $\varepsilon^m/\varepsilon^M > 1$.

Proposition 2, combined with formula 9, justifies public provision of UI. If $\varepsilon^m/\varepsilon^M > 1$, small private insurers would underprovide UI because they maximize profits by using the Baily-Chetty

formula to determine how much insurance to provide to their clients. Small insurers solely take into account the micro-elasticity of unemployment, and do not internalize search externalities. In that case, the government would improve welfare by complementing the private provision of UI.

To understand why the micro-elasticity ϵ^m is larger than the macro-elasticity ϵ^M , consider the cut in unemployment benefits $d\Delta c > 0$ depicted in Figure 1(c). The change creates variations in all variables $d\Delta c$, dn , $d\theta$, and de , so that all equilibrium conditions continue to be satisfied. The change in effort can be decomposed as $de = de_{\Delta c} + de_{\theta}$, where $de_{\Delta c} = (\partial e^s / \partial \Delta c) d\Delta c$ is a partial-equilibrium variation in response to the change in UI, and de_{θ} is a general-equilibrium adjustment following the change $d\theta$ in labor market tightness. Using the labor supply equation (1) we have $dn = dn_e + dn_{\theta}$ where $dn_e = (\partial n^* / \partial e) de_{\Delta c}$ and $dn_{\theta} = [\partial n^* / \partial \theta + (\partial n^* / \partial e)(\partial e^s / \partial \theta)] d\theta$. Following a cut in benefits an individual jobseeker increases his search effort, increasing his own probability of finding a job by $dn_e > 0$. From the jobseeker's perspective, labor market tightness θ remains constant. The interval A–C in Figure 1(c) represents dn_e . When the jobseeker finds a job, however, he reduces the profitability of the marginal jobs left vacant because (a) the productivity of these jobs falls by diminishing returns to labor, and (b) the prevailing wage does not adjust to the drop in marginal productivity. Thus the firm reduces the number of vacancies posted to fill these less profitable jobs. Labor market tightness falls by $d\theta < 0$, reducing the job-finding rate $f(\theta)$ of jobseekers who are still unemployed. $dn_{\theta} < 0$ is the corresponding reduction in employment, represented by interval C–B in Figure 1(c). As a consequence the equilibrium increase in employment dn following an increase in aggregate search efforts is smaller than the increase dn_e in the individual probability to find a job following an increase in individual search efforts. The interval A–B in Figure 1(c) represents $dn < dn_e$. The difference between micro-effect dn_e and macro-effect dn is $dn_{\theta} < 0$. Equation (7) says that $\epsilon^M = \epsilon^m + [\Delta c / (1 - n)] \cdot dn_{\theta} / d\Delta c$. Since $dn_{\theta} < 0$, $\epsilon^M < \epsilon^m$.

3.3 Optimal replacement rate

Our previous results do not require any assumptions on the functional forms of the utility functions and matching function. They only involve the local elasticities η , ρ , and κ . But to characterize the cyclicity of the micro-elasticity, the macro-elasticity, and the optimal replacement rate, we must control how the local elasticities fluctuate over the business cycle:

ASSUMPTION 3. The utility functions are isoelastic: $v(c) = \ln(c)$, $k(e) = \omega_k \cdot e^{1+\kappa}/(1 + \kappa)$. The matching function is Cobb-Douglas: $h(e \cdot u, o) = \omega_h \cdot (e \cdot u)^\eta \cdot o^{1-\eta}$.

The parameters $\omega_k > 0$ and $\omega_h > 0$ measure the cost of search and the effectiveness of matching.

To determine how the elasticities and the optimal replacement rate vary over the business cycle, we must also specify the initial unemployment u associated with each technology level a :

ASSUMPTION 4. For any a , u is such that in equilibrium $h = n - (1 - u) = s \cdot n$, $s \in (0, 1)$.

The equilibrium is determined given initial unemployment u and technology a . Assumption 4 ensures that in equilibrium, the fraction h/n of new hires in the workforce remains constant over the cycle. The assumption replicates in our static model a feature of dynamic equilibrium unemployment models that assumes a constant job-destruction rate s , independent of technology.⁸

Proposition 3 establishes that the wedge $\varepsilon^m/\varepsilon^M$ is countercyclical and the macro-elasticity ε^M is procyclical in a model with job rationing in recessions (Assumptions 1 and 2):

PROPOSITION 3. *Under Assumptions 1, 2, 3, and 4,*

$$\left. \frac{\partial (\varepsilon^m/\varepsilon^M)}{\partial a} \right|_\tau < 0 \quad \text{and} \quad \left. \frac{\partial \varepsilon^M}{\partial a} \right|_\tau > 0.$$

The proposition says that the macro-elasticity is large in expansions but small in recessions, as illustrated by comparing Figure 1(c) to Figure 1(d). This is because in recessions, jobs are acutely rationed and search efforts have little influence on aggregate unemployment. The proposition also says that the wedge between micro- and macro-elasticity is small in expansions but large in recessions. This is because when jobs are acutely rationed, searching more mechanically increases one's job-finding probability but it decreases others' job-finding probability as in a rat race.

Proposition 4 establishes that the optimal replacement rate τ is countercyclical in a model with job rationing in recessions (Assumptions 1 and 2):

⁸Pissarides [2000] and many others assume a constant job-destruction rate s and balanced labor market flows. When flows are balanced, firms hire each period as many workers as they lose. Therefore the fraction of new hires in the workforce is constant over the cycle.

PROPOSITION 4. *Assume that formula (9) implicitly defines a unique function $\tau(a) \in (0, 1)$, continuous and differentiable. Under Assumptions 1, 2, 3, and 4, if $n > 1/2$ and $(\alpha/\eta) \cdot s \cdot (1 - \eta) \cdot (\kappa + 1)/\kappa \leq 1$, then $d\tau/da < 0$.*

The proposition says that the optimal replacement rate is more generous in recessions than in expansions. The formal proof, relegated in the Appendix, exploits formula (9). But we can sketch the proof informally using the approximated formula (10) and Proposition 3. In recessions the macro-elasticity ε^M falls and the first term in formula (10) increases. In recessions the marginal budgetary cost of UI is small because a higher UI only increases unemployment negligibly. Moreover in recessions the wedge $\varepsilon^m/\varepsilon^M$ increases and the second term in formula (10) increases. The wedge measures the welfare cost of a negative rat-race externality imposed by unemployed workers on others. The externality arises because unemployed workers search taking the job-finding rate as given, and do not internalize their influence on the job-finding rate of others. UI corrects the externality by discouraging job search. In recessions the externality is acute so the marginal benefits of UI are high. Since both terms in formula (10) increase, τ must increase.

The formal proof is more complex because n enters the exact formula (9). The results of Proposition 3 are not sufficient to prove the proposition. We need to prove that ε^M is sufficiently procyclical and that $\varepsilon^m/\varepsilon^M$ is sufficiently countercyclical to compensate the fluctuations in n . To do so, we need two additional assumptions. The assumption $n > 1/2$ is needed because if technology a is so low that most workers become unemployed, it becomes optimal to reduce the replacement rate τ . Suppose all workers are unemployed ($n = 0$, $\theta = 0$). Providing more consumption to employed workers has no budgetary cost but it provides incentives for unemployed workers to search more, which could raise employment. Clearly, it is optimal to reduce the generosity of UI. In fact Lemma A9 in the Appendix establishes that when $a \rightarrow 0$ then $n \rightarrow 0$ and $\tau \rightarrow 0$. This result implies that for very low levels of technology and very low levels of employment, the optimal replacement rate is bound to increase with technology. The assumption that $(\alpha/\eta) \cdot s \cdot (1 - \eta) \cdot (\kappa + 1)/\kappa \leq 1$ is needed to ensure that the labor supply is convex enough. As shown by comparing Figures 1(c) and 1(d), the convexity of labor supply in the (n, θ) plan drives the cyclicalities of elasticities. The assumption is easily satisfied for any reasonable calibration because s , which captures the job-destruction rate, is very small. In the calibration in Table 1,

$$(\alpha/\eta) \cdot s \cdot (1 - \eta) \cdot (\kappa + 1) / \kappa = 0.008 \ll 1.$$

3.4 Relation to the Hosios [1990] condition

To relate our work to the classical efficiency result in Hosios [1990], we confine the analysis to the canonical model of equilibrium unemployment.⁹ The canonical model is characterized by two assumptions that replace Assumptions 1 and 2 in the model with job rationing:¹⁰

ASSUMPTION 5. The production function has constant marginal returns to labor: $\alpha = 1$.

ASSUMPTION 6. The wage w is determined using the generalized Nash solution to the bargaining problem faced by firm-worker pairs. The bargaining power of workers is $\beta \in (0, 1)$.

The Nash bargaining solution allocates a fraction β of the surplus of the match to the worker and the rest to the firm. If the utility function has constant relative risk aversion: $v(c) = (c^{1-\rho} - 1) / (1 - \rho)$, the bargained wage is

$$\frac{w}{a} = \frac{\beta}{1 - \beta} \cdot \frac{1}{v(\tau)} \cdot \frac{r}{q(\theta)}.$$

Substituting the wage w in the firm's profit-maximization condition (11) yields

$$\frac{r}{q(\theta)} = \left[1 - \frac{\beta}{1 - \beta} \cdot \frac{1}{v(\tau)} \right]^{-1}. \quad (12)$$

Equilibrium labor market tightness θ , determined by (12), does not depend on technology a . Therefore, keeping the replacement rate τ constant, there are no fluctuations in tightness over the business cycle.¹¹ As tightness is strongly procyclical in the data, the Nash bargaining solution cannot account for the labor market fluctuations observed over the business cycle.

To make our analysis comparable to the efficiency result in Hosios [1990], we assume that the government assigns a welfare weight equals to the average marginal consumption utility \bar{v}' to profits, as if firm ownership was equally distributed. We then extend formula (9) to a class of model

⁹The results presented in this section are derived formally in Appendix C.

¹⁰See Pissarides [2000] for a complete treatment of the canonical model of equilibrium unemployment.

¹¹Blanchard and Galí [2010] and others have proved similar theoretical results in a variety of settings.

in which Assumption 5 holds and the wage is a function $w(\Delta c)$ of the consumption gain from work Δc , as under Assumption 6. In this class of models, with valuation of profits, the formula becomes

$$\frac{1}{n} \frac{\tau}{1-\tau} = \left[n + (1-n) \frac{v'(c^u)}{v'(c^e)} \right]^{-1} \left\{ \frac{n}{\varepsilon^M} \left[\frac{v'(c^u)}{v'(c^e)} - 1 \right] + \frac{\Delta v}{v'(c^e) \Delta c} \frac{\kappa}{\kappa+1} \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right) \left[1 - \frac{\bar{v}'}{\Delta v} \frac{\eta}{1-\eta} \frac{ra}{q(\theta)} \right] \right\}.$$

The valuation of profits and the response of wages appear simply as a multiplicative correction to the externality term of formula (9).¹² We can apply the formula to the canonical model. If workers are risk neutral and the bargaining power matches the elasticity of the matching function with respect to effort ($\beta = \eta$), the optimal replacement rate implied by the formula is $\tau = 0$. Risk neutrality implies that the first term in the right-hand-side numerator is zero. With $\beta = \eta$ and $\tau = 0$ the bargained wage is $w = \eta / (1 - \eta) \cdot (r \cdot a) / q(\theta) = \Delta v$, so the second term in the right-hand-side numerator is zero. Therefore, our formula conforms to the Hosios [1990] condition for efficiency.

3.5 Robustness

Aggregate demand shocks. A limitation that the model shares with most equilibrium unemployment models is that business cycles are generated by technology shocks only. This is not plausible. For instance, aggregate demand shocks likely contribute to labor market fluctuations. To study optimal UI in a demand-generated business cycle, Appendix F.1 builds a basic model in which recessions are driven by aggregate demand shocks amplified by nominal wage rigidity.

Jobs are rationed in this model as well, albeit through a different mechanism. Firms face a downward-sloping aggregate demand curve in the goods market. The larger the quantity produced by workers, the lower the market price for goods. When aggregate demand is low enough, the production of workers would sell at a price below the nominal wage if all workers were employed. In this situation, firms would not hire all workers in the labor force even if recruiting were costless. Some unemployment would remain if jobseekers searched infinitely hard.

We represent the labor market equilibrium with our labor supply-labor demand diagram in Fig-

¹²In this class of models, profits are $\pi = (1 - u) \cdot (r \cdot a) / q(\theta)$ and wages are $w = a - (r \cdot a) / q(\theta)$. Therefore the response of π and w to a change in UI solely depend on the response of labor market tightness θ . Since the response of θ to a change in UI is captured by the externality term, the response of π and w only enters as a multiplicative correction to the externality term in the formula.

ure A1 in the Appendix. The labor supply is the same, and the labor demand retains the same properties, as in the model of technology-generated business cycle. The labor demand curve is downward sloping in a price θ –quantity n plan because higher employment n implies more production, lower prices in the goods market, higher real wages because of nominal wage rigidity, and requires a lower tightness θ for firms to be willing to hire. When aggregate demand falls, prices fall and real wages rise, so the labor demand shifts inwards.

Given the similarity of the structures of the labor market equilibrium in the two models, it comes as no surprise that all the results derived in the model with technology-generated business cycle also apply in the model with demand-generated business cycles. We prove that the results on the cyclicity of the wedge $\varepsilon^m/\varepsilon^M$ and the macro-elasticity ε^M (Proposition 3), as well as the cyclicity of the optimal replacement rate (Proposition 4) remain valid once derivatives are taken with respect to aggregate demand instead of technology. Hence, the generosity of the optimal UI also increases in recessions caused by low aggregate demand.¹³

Wage subsidies. Because of real wage rigidity, wages are high relative to technology in recessions, which raises unemployment. The government cannot correct wages under our assumption that the incidence of a labor tax is fully on workers. While the assumption is standard in public finance and justified in the long run, the wage rigidity observed in micro-data suggests that changing the payroll tax imposed on employers is likely to change the wages effectively paid by firms in the short run. In this section we explain why our results remain valid when the government can attenuate unemployment fluctuations using wage subsidies. The formal proof is in Appendix D.

The government chooses a rate b of unemployment benefits, a tax rate t imposed on the salary w^* received by employees, and a subsidy rate σ imposed on the salary w^* paid by employers. Firms pay a wage $w = (1 - \sigma) \cdot w^*$, employed workers consume $c^e = (1 - t) \cdot w^*$, and unemployed workers consume $c^u = b \cdot w^*$. Equivalently, we consider that the government chooses directly the wage w and consumptions c^e , and c^u . The government is subject to the budget constraint

¹³To showcase the range of applications of our framework, we entertain another possible source of business cycles in Appendix F.2. We assume that business cycles are generated by a preference shock that affects workers' disutility from job search. In recessions it is unpleasant for unemployed workers to search. Jobseekers reduce their effort, reducing labor supply and increasing unemployment. Simulations suggest that the optimal UI is procyclical in this model. But the model is unrealistic: it has the counterfactual property that labor market tightness is countercyclical.

$(1 - n) \cdot b \cdot w^* + n \cdot \sigma \cdot w^* = t \cdot n \cdot w^*$, but this constraint can be rewritten exactly as the baseline budget constraint (3) that relates w , c^e , and c^u .

If the government could control wages at no cost using wage subsidies, it would be optimal to eliminate unemployment fluctuations and keep UI at a constant level. However, it is improbable that the government could implement a wage subsidy at no cost. Various sources of cost come to mind. First, informational frictions may require the government to collect vast amount of data to devise a subsidy that would eliminate unemployment fluctuations. Second, political constraints may impose a cost on the government to enact the desired subsidy: for instance, trade unions may resist the reduction of the cost of labor incurred by firms. Third, aggregate demand may be low in recessions and firms may be constrained to sell a fixed low quantity of goods. Reducing the marginal cost of labor with a subsidy would not lead firms to hire workers in the short run; it would only be a transfer from the government to firm owners. Since firm ownership is usually concentrated, the marginal propensity to consume of wealthy firm owners is much lower than that of workers, and a subsidy could depress aggregate demand further. Formally, we represent these costs as an increasing convex cost function $C(\sigma)$ included in the objective function of the government. In that case, it is not optimal to eliminate entirely cyclical fluctuations in unemployment because of the cost $C(\sigma)$. Let w be the optimal wage chosen by the government given the cost of a subsidy.

Given w the government chooses Δc to maximize social welfare (5) subject to the budget constraint (3). This is exactly the problem faced by the government in the baseline model. Therefore the optimal UI formula (9) remains valid. Let $\tilde{w} \equiv w/a$ be the optimal wage w normalized by technology a . \tilde{w} is the only source of fluctuations in the economy through the firm's profit-maximization condition (11). Since the government cannot stabilize unemployment completely, \tilde{w} must fluctuate. Once we replace the derivatives with respect to a by derivatives with respect to \tilde{w} , the results on the cyclicity of the elasticities ε^m and ε^M (Proposition 3) and the result on the cyclicity of the optimal replacement rate (Proposition 4) remain valid. The sign of the derivatives naturally changes because an increase in \tilde{w} has the same effect as a decrease in a : it raises unemployment and reduces labor market tightness.

To conclude, the properties of optimal UI are robust to the presence of a wage subsidy that attenuates unemployment fluctuations. Wage subsidies may be a powerful tool to stimulate em-

ployment in recessions. One could model the costs and benefits of wage subsidies to design an optimal wage subsidy over the business cycle that would complement our optimal UI.

4 Simulations

In this section we enrich the static model used for the theoretical analysis to make it more realistic: the static model is cast into a dynamic environment; workers use home production to insure themselves partially against unemployment; and profits enter the budget constraint of the government to capture either ownership of firms by workers or direct taxation of profits by the government. We calibrate the model with US data and solve it numerically to quantify how optimal UI varies over the business cycle. We study various institutional arrangements for the administration of UI that could not be studied in a static environment, such as the adjustment of the duration of unemployment benefits and the recourse to deficit spending in recessions.

4.1 The dynamic model

This section provides an overview of the dynamic model. The solution to the worker's, firm's, and government's problems, as well as the definition of the equilibrium are presented in Appendix E.

Technology follows a stochastic process $\{a_t\}_{t=0}^{+\infty}$. The labor market is similar to that in the static model. The only difference is that at the end of period $t - 1$, a fraction s of the n_{t-1} existing worker-job matches is exogenously destroyed. Workers who lose their job become unemployed, and start searching for a new job at the beginning of period t . At the beginning of period t , $u_t = 1 - (1 - s) \cdot n_{t-1}$ unemployed workers look for a job.

The government fully taxes profits, taxes or subsidizes labor income, and provides unemployed benefits. Its budget must be balanced each period. It is as if, given technology $\{a_t\}_{t=0}^{+\infty}$, the government directly chose consumption $\{c_t^u\}_{t=0}^{+\infty}$ of unemployed workers and consumption $\{c_t^e\}_{t=0}^{+\infty}$ of employed workers to maximize social welfare subject, each period, to the resource constraint

$$a_t \cdot g(n_t) - \frac{r \cdot a_t}{q(\theta_t)} \cdot [n_t - (1 - s) \cdot n_{t-1}] = n_t \cdot c_t^e + (1 - n_t) \cdot c_t^u. \quad (13)$$

Given government policy $\{c_t^e, c_t^u\}_{t=0}^{+\infty}$ and labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$ the representative worker chooses job-search effort and home production $\{e_t, y_t\}_{t=0}^{+\infty}$ to maximize the expected utility

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ (1 - n_t^s) \cdot [v(c_t^u + y_t) - m(y_t)] + n_t^s \cdot v(c_t^e) - [1 - (1 - s) \cdot n_{t-1}^s] \cdot k(e_t) \right\},$$

subject to the law of motion of the employment probability in period t ,

$$n_t^s = (1 - s) \cdot n_{t-1}^s + [1 - (1 - s) \cdot n_{t-1}^s] \cdot e_t \cdot f(\theta_t).$$

\mathbb{E}_0 is the mathematical expectation conditioned on time-0 information, $\delta < 1$ is the discount factor.

The representative firm is owned by risk-neutral entrepreneurs. Given labor market tightness and technology $\{\theta_t, a_t\}_{t=0}^{+\infty}$ the firm chooses employment $\{n_t^d\}_{t=0}^{+\infty}$ to maximize expected profit

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ a_t \cdot g(n_t^d) - w_t \cdot n_t^d - \frac{r \cdot a_t}{q(\theta_t)} \cdot [n_t^d - (1 - s) \cdot n_{t-1}^d] \right\},$$

where $n_t^d - (1 - s) \cdot n_{t-1}^d \geq 0$ is the number of hires in period t .

Wages follow an exogenous process $\{w_t\}_{t=0}^{+\infty}$. Labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$ equalizes labor demand $\{n_t^d\}_{t=0}^{+\infty}$ to labor supply $\{n_t^s\}_{t=0}^{+\infty}$: $n_t \equiv n_t^d = n_t^s$.

4.2 Calibration

We calibrate all parameters of the model at a weekly frequency as shown in Table 1.¹⁴ We calibrate as many parameters as possible directly from micro-evidence and macro-data for the US for the December 2000–June 2010 period. Following [Michaillat \[2012\]](#) we set $\delta = 0.999$, $s = 0.0094$, $r = 0.32 \cdot \omega$. We use a Cobb-Douglas matching function $h(u, o) = \omega_h \cdot u^\eta \cdot o^{1-\eta}$ and set $\eta = 0.7$, in line with empirical evidence [[Petrongolo and Pissarides, 2001](#)]. We choose a coefficient of relative risk aversion $\rho = 1$, on the low side of available estimates [[Chetty, 2006b](#)]. We calibrate the wage flexibility γ based on estimates obtained in micro-data. It is mostly the flexibility of

¹⁴This exercise is only illustrative of the magnitudes of the optimal policy, because our model abstracts from a number of relevant issues and there remains considerable uncertainty about the calibration of some parameters, such as the coefficient of relative risk aversion.

wages in newly created jobs, not in existing jobs, that drives job creation. The best estimate of this flexibility using US data is provided by [Haefke et al. \[2008\]](#). Using panel data following production and supervisory workers over the 1984–2006 period, they estimate an elasticity of total earnings of job movers with respect to productivity of 0.7. If the composition of jobs accepted by workers improves in expansions, 0.7 is an upper bound on the elasticity of wages in newly created jobs. A lower bound on this elasticity is the elasticity of wages in existing jobs, estimated in the 0.1–0.45 range with US data [[Pissarides, 2009](#)]. We set $\gamma = 0.5$, in the range of plausible values.

We calibrate the remaining parameters by matching the steady-state value of the variables in the model to the average of their empirical counterpart. We normalize average technology $\hat{a} = 1$ and average effort $\hat{e} = 1$. We compute average labor market tightness and unemployment using seasonally-adjusted, monthly series for the vacancy level (collected by the Bureau of Labor Statistics (BLS) in the Job Openings and Labor Turnover Survey (JOLTS)) and the unemployment level (computed by the BLS from the Current Population Survey (CPS)) over the 2000–2010 period. We find $\hat{\theta} = \hat{v}/\hat{u} = 0.47$ and $\hat{u} = 5.9\%$, which implies $\hat{n} = 0.950$. In the US, weekly unemployment benefits replace between 50% and 70% of the last weekly pre-tax earnings of a worker [[Pavoni and Violante, 2007](#)]. Following [Chetty \[2008\]](#) we set the benefit rate to 50%. Since earnings are subject to a 7.65% payroll tax, we set the replacement rate to $\hat{\tau} = 0.5/(1 - 0.0765) = 54\%$.

To calibrate the matching efficiency ω_h we exploit the steady-state relationship $u \cdot e \cdot f(\theta) = s \cdot n = s \cdot (1 - u)/(1 - s)$. We find $\omega_h = s/(1 - s) \cdot (1 - \hat{u})/\hat{u} \cdot \hat{\theta}^{\eta-1} = 0.19$. We target the conventional labor share of $\hat{l}s \equiv (\hat{w} \cdot \hat{n})/\hat{y} = 0.66$. The firm’s profit-maximization condition (equation (A24) in the Appendix) implies $\alpha = \hat{l}s \cdot ([1 - \delta \cdot (1 - s)] \cdot 0.32/q(\hat{\theta}) + 1) = 0.67$. The condition also allows us to recover $\omega = 0.70$, and $r = 0.32 \cdot \omega = 0.23$.

We calibrate the parameters of the home-production cost function $m(y) = \omega_m \cdot y^{1+\mu}/(1 + \mu)$.¹⁵ As showed in Appendix E.4 the convexity μ is related to two statistics: ε_2 and ξ , that have been estimated empirically. The ratio $\xi \equiv c^h/c^e = (c^u + y)/c^e$ captures the consumption drop upon unemployment. The statistics ε_2 is the marginal consumption change dc^h/dc^u , which captures the in-

¹⁵We introduce home production to model partial self-insurance by unemployed workers. If self-insurance arises not only from home production but also from saving and borrowing, self-insurance may be less available in recessions, for instance with savings depletion or credit market collapse. [Kroft and Notowidigdo \[2011\]](#) find, however, that the consumption-smoothing benefit of UI is acyclical, which suggests that self-insurance remains available in recessions. Accordingly, we assume a stable home-production technology over the business cycle.

crease in total consumption $dc^h = dc^u + dy > 0$ for an unemployed worker who receives a marginal increase $dc^u > 0$ in benefits. For food consumption f , Gruber [1997] estimates $df^h/dc^u = 0.27$ and $[f^h - f^e]/f^e = -0.068$. As emphasized by Browning and Crossley [2001], however, total consumption is more elastic than food to a change in income. Using the estimates of Hamermesh [1982], we find that the aggregate income elasticity of food consumption for unemployed workers is 0.36, including both food consumed at home and away from home.¹⁶ Accordingly we expect that $dc^h = df^h/0.36$ and $[c^h - c^e]/c^e = ([f^h - f^e]/f^e)/0.36$. Setting $\varepsilon_2 = 0.27/0.36 = 0.75$ and $\xi = 1 - (0.068/0.36) = 0.81$ imply $\mu = 1.01$. In addition the resource constraint (13) yields $\hat{c}^h = 0.79$ and $\hat{y} = 0.26$. We set $\omega_m = 4.84$ such that the worker's optimal choice of home production (equation (A27) in the Appendix) holds for \hat{c}^h and \hat{y} .

We calibrate the parameters of the disutility from search $k(e) = \omega_k \cdot e^{1+\kappa}/(1+\kappa)$. Appendix E.4 shows that the convexity κ is related to a statistics ε_1 that has been estimated empirically. The statistics $\varepsilon_1 \equiv (c^u/\xi) \cdot (\partial\xi/\partial c^u)$ captures the reduction in the hazard rate $\xi \equiv e \cdot f(\theta)$ out of unemployment when an unemployed worker receives an increase $dc^u > 0$ in benefits, keeping labor market tightness θ constant. Meyer [1990] estimates $\varepsilon_1 = 0.9$, which yields $\kappa = 0.97$. We also obtain $\omega_k = 0.58$ to match $\hat{e} = 1$ with the worker's optimal choice of effort (specified in Appendix E).

4.3 Optimal replacement rate

To describe how the optimal replacement rate varies over the business cycle, we compare steady states parameterized by different technology levels.¹⁷ The results are displayed in Figure 2. Panel (a) gives the unemployment rate for the different steady states: unemployment is high in steady states with low technology. Panel (b) is a Beveridge curve, which shows that labor market tightness decreases with unemployment. Panel (c) finds that optimal UI is strongly countercyclical: the optimal replacement rate increases from 51% to 71% when the unemployment rate increases from

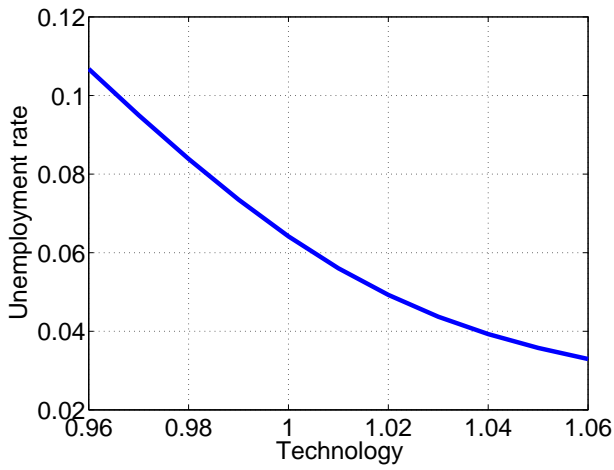
¹⁶Hamermesh [1982] estimates that for unemployed workers the permanent-income elasticity of food consumption at home is 0.24 while that of food consumption away from home is 0.82. He also finds that in the consumption basket of an unemployed worker, the share of food consumption at home is 0.164 while that of food consumption away from home is 0.041. Therefore the aggregate income elasticity of food consumption is $0.24 \times [0.164/(0.164 + 0.41)] + 0.82 \times [0.041/(0.164 + 0.41)] = 0.36$.

¹⁷In steady state, technology remains constant over time: $a_t = a$ for all t . The impulse response function of the optimal replacement rate to a negative technology shock is presented in Figure 3 and discussed in Section 4.4.

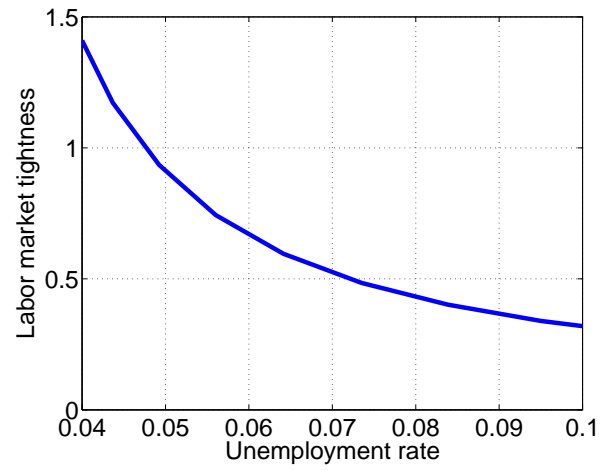
Table 1: Steady-state targets and parameter values used in simulations (weekly frequency)

	Steady-state target	Value	Source
\hat{a}	Technology	1	Normalization
\hat{e}	Effort	1	Normalization
\hat{l}_s	Labor share	0.66	Convention
\hat{u}	Unemployment	5.9%	JOLTS, 2000–2010
$\hat{\theta}$	Labor market tightness	0.47	JOLTS, 2000–2010
τ	Replacement rate c^u/c^e	54%	Pavoni and Violante [2007] , Chetty [2008]
ξ	Consumption drop c^h/c^e	81%	Hamermesh [1982] , Gruber [1997]
ε_2	Marginal consumption change dc^h/dc^u	0.75	Hamermesh [1982] , Gruber [1997]
ε_1	Elasticity of unemployment hazard rate	0.90	Meyer [1990]
	Parameter	Value	Source
δ	Discount factor	0.999	Corresponds to 5% annually
ρ	Relative risk aversion	1	Chetty [2006b]
η	Effort-elasticity of matching	0.7	Petrongolo and Pissarides [2001]
γ	Real wage flexibility	0.5	Pissarides [2009] , Haefke et al. [2008]
r	Recruiting cost	0.21	Barron et al. [1997] , Silva and Toledo [2009]
s	Separation rate	0.94%	JOLTS, 2000–2010
ω_h	Effectiveness of matching	0.19	Matches steady-state targets
α	Marginal returns to labor	0.67	Matches steady-state targets
ω	Steady-state real wage	0.70	Matches steady-state targets
κ	Convexity of disutility of effort	0.97	Matches steady-state targets
ω_k	Steady-state disutility of effort	0.58	Matches steady-state targets
ω_m	Steady-state disutility of home production	4.84	Matches steady-state targets
μ	Convexity of disutility of home production	1.01	Matches steady-state targets

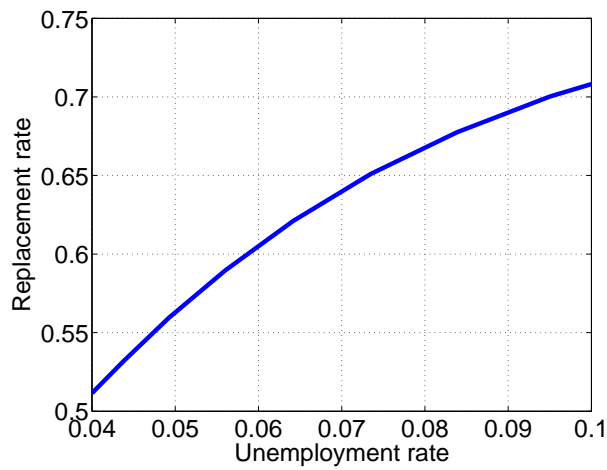
4% to 10%. This result is critical because it confirms that the result of Proposition 4 also holds in a realistic calibrated model. Panel (c) shows that it is optimal to increase the consumption of unemployed workers relative to that of employed workers in recessions. Panel (d) goes one step further. It shows that it is optimal to increase the consumption of unemployed workers in absolute terms in recessions. The gap between benefits and the consumption of unemployed workers in panel (d) corresponds to home production. Panel (e) shows that home production decreases in recessions, when unemployment benefits become more generous. Panel (f) shows that search efforts decrease in recessions, when UI becomes more generous and the job-finding rate falls.



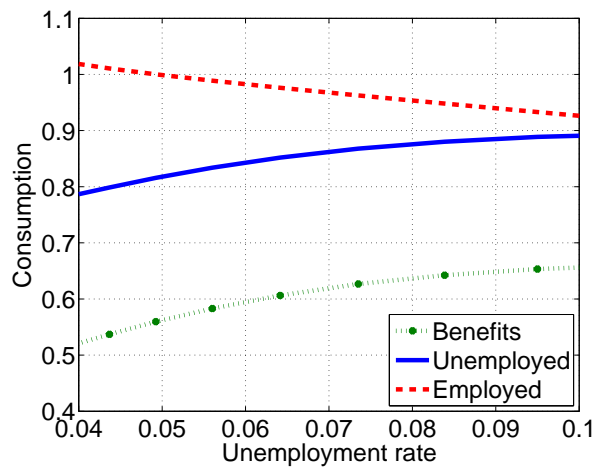
(a) Unemployment rate u



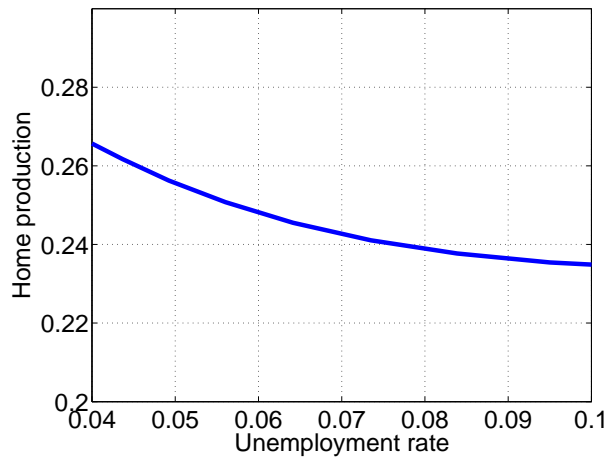
(b) Labor market tightness θ



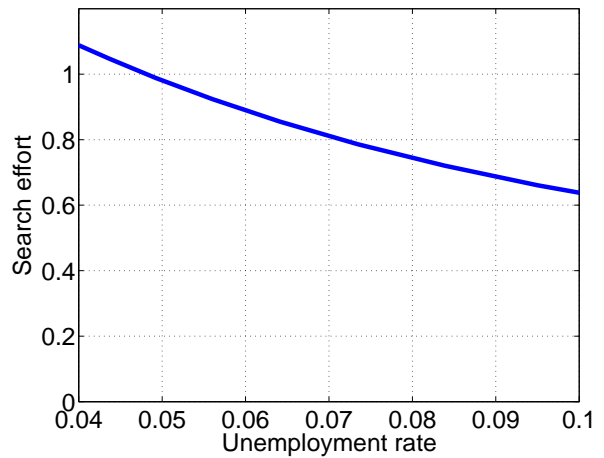
(c) Optimal replacement rate τ



(d) Optimal consumption levels c^u , c^h , and c^e



(e) Home production y



(f) Search effort e

Figure 2: Optimal unemployment insurance over the business cycle

Notes: The simulations, described in the Appendix, are based on the dynamic model calibrated in Table 1.

4.4 Alternative institutional arrangements

This section studies numerically various institutional arrangements for the administration of UI that could not be studied in the static model, such as the adjustment of the duration of benefits (instead of their level), and deficit spending (instead of budget balance).

Deficit spending. In the baseline model, the government must balance its budget each period. It cannot use deficit spending to shift resources intertemporally from expansions to recessions and smooth the consumption of workers. This assumption allows us to focus on the trade-off between insurance and incentives within each period. But in practice the government is able to borrow and save. In this section we assume that the government has access to a complete market for Arrow-Debreu securities, in which it faces risk-neutral investors with discount factor δ . An Arrow-Debreu security pays one unit of consumption good after history a^t . Its price is $\delta^t \cdot p(a^t)$, where $p(a^t)$ is the probability of history a^t based on time-0 information. The government trades securities at time 0 to finance UI in all histories. It faces a single intertemporal budget constraint:

$$0 = \mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ a_t \cdot g(n_t) - \frac{r \cdot a_t}{q(\theta_t)} \cdot [n_t - (1-s) \cdot n_{t-1}] - [n_t \cdot c_t^e + (1-n_t) \cdot c_t^u] \right\}. \quad (14)$$

We solve the government's problem by log-linearization in Appendix E.5. To obtain the co-movements of technology with the optimal replacement rate in a stochastic environment, we compute impulse response functions. Figure 3 depicts the responses to a negative technology shock in two cases: the blue solid lines are responses in the baseline case in which the government is constrained by (13) to balance his budget each period; and the red dashed lines are responses when the government is subject to a single intertemporal budget constraint (14). Unemployment responds similarly in both cases: it builds slowly and peaks after 20 weeks more than 10% above its steady-state value. The response of the optimal replacement rate to an adverse economic shock is qualitatively identical in both cases. But quantitatively the replacement rate increases by 5.6% at the peak under budget balance, whereas it increases by 8.2% under deficit spending. Under budget balance the replacement rate increases because the level of benefits increases by 4.4% and the consumption of employed workers falls by 1.2%. This fall is necessary to finance more gen-

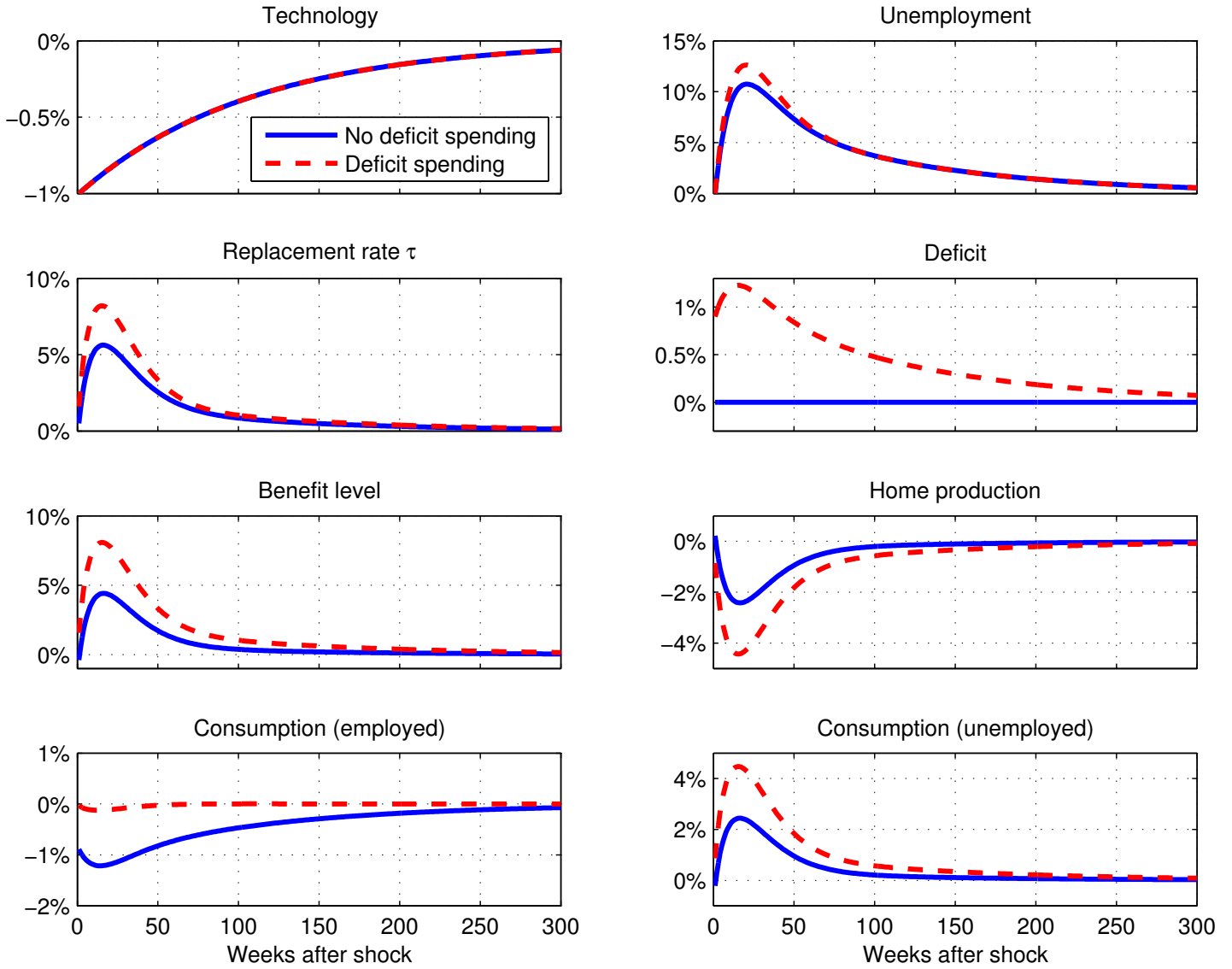


Figure 3: Impulse response of optimal unemployment insurance to a negative technology shock

Notes: The figure displays impulse response functions (IRFs), which represent the percentage-deviation from steady state for each variable. We assume that the log-deviation of technology $\check{a}_t \equiv d \ln(a_t)$ follows an AR(1) process: $\check{a}_{t+1} = \nu \cdot \check{a}_t + z_{t+1}$ where $z_t \sim N(0, \sigma^2)$ is an innovation to technology. [Michaillat \[2012\]](#) estimates the AR(1) process using BLS data for 1964:Q1–2010:Q2 and finds $\nu = 0.991$ and $\sigma = 0.0026$ at weekly frequency. IRFs are obtained by imposing an unexpected negative technology shock $z_1 = -0.01$ to the log-linear dynamic model. The blue solid IRFs are responses of the optimal equilibrium when the government is constrained by (13) to balance its budget each period. The red dashed IRFs are responses of the optimal equilibrium when the government is subject to a single intertemporal budget constraint (14). Log-linear systems and computations are described in the Appendix.

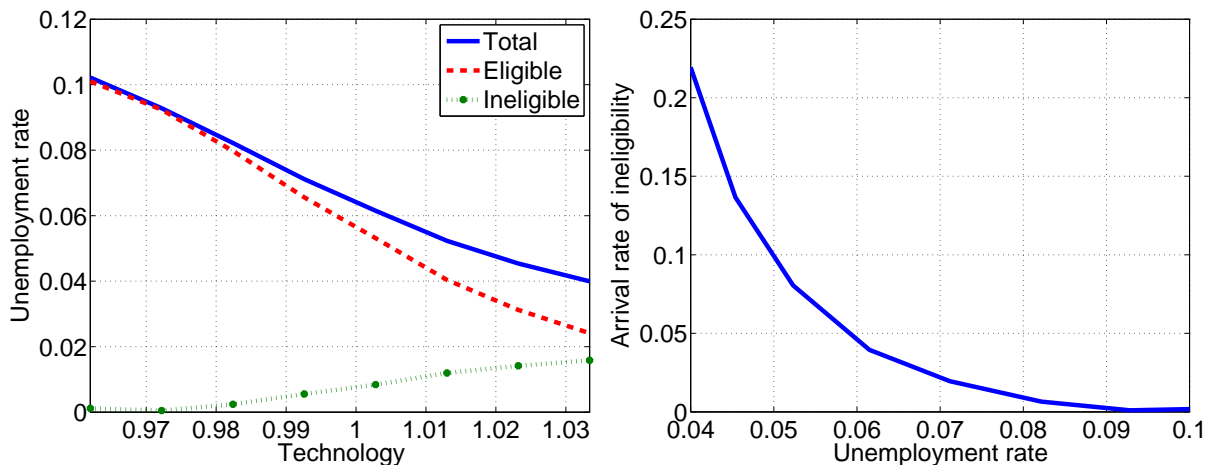


Figure 4: Optimal duration of unemployment insurance over the business cycle

Notes: Panels obtained with the dynamic model in which unemployment benefits have finite duration. The model is calibrated according to Table 1. The Appendix describes the numerical simulations.

erous benefits to a larger number of unemployed workers. Under deficit spending the government smoothes consumption of employed workers almost perfectly, and the benefit level increases by 8.2%. As unemployed workers reduce home production when they receive more generous benefits, the consumption of unemployed workers does not increase as much as unemployment benefits: it increases only by 2.5% under budget balance and 4.5% under deficit spending, because home production falls by 2.4% under budget balance and 4.4% under deficit spending. When the government is able to borrow and save, consumption of both employed and unemployed workers is higher because the government provides additional consumption smoothing in recessions. The budget deficit—benefit outlays minus tax revenue in the period—increases by 1.2% at the peak.

Duration of unemployment benefits. In the baseline model, unemployed workers receive unemployment benefits independent of the length of their unemployment spell. The government adjusts the level of the benefits over the cycle. But in practice benefits have finite duration, and the government modulates the duration of benefits over the cycle.¹⁸ In this section, we assume

¹⁸US unemployment benefits have a maximum duration of 26 weeks in normal times. But under the Extended Benefits program, duration is automatically extended by 13 weeks in states where unemployment is above 6.5% and by 20 weeks in states where unemployment is above 8%. Often duration is further extended by the government in severe recessions. For example, the Emergency Unemployment Compensation program enacted in 2008 extends durations by an additional 53 weeks when state unemployment is above 8.5%.

that unemployment benefits have finite duration, which the government adjusts over the cycle.¹⁹ We follow [Fredriksson and Holmlund \[2001\]](#) and assume that eligible unemployed workers exhaust their unemployment benefits c_t^u with probability λ_t at the end of each period t . Ineligible unemployed workers receive consumption $c_t^a < c_t^u$ from social assistance until they find a job.

The replacement rates $\tau^{u,e} = c_t^u/c_t^e$ of unemployment benefits and $\tau^{a,e} = c_t^a/c_t^e$ of social assistance are constant over time. The government chooses the rate λ_t at which eligible workers become ineligible to maximize social welfare subject to a budget constraint similar to (13). We solve the model numerically using the calibration in Table 1. We set the replacement rates at $\tau^{u,e} = 0.65/(1 - 0.0765) = 70\%$ and $\tau^{a,e} = 0.52 \cdot \tau^{u,e} = 36\%$ such that an expected duration of 26 weeks is optimal when the unemployment rate is at its average level of 5.9%.²⁰ The left panel in Figure 4 shows how unemployment and its composition varies with technology. When technology increases, total unemployment falls, the number of eligible jobseekers falls, but the number of ineligible jobseekers increases because arrival rate of ineligibility increases drastically. The right panel shows that the optimal arrival rate of ineligibility λ is strongly procyclical. Accordingly the optimal expected duration of unemployment benefits $1/\lambda$ is strongly countercyclical. When unemployment is 4% the optimal arrival rate of ineligibility is 21%, corresponding to an expected benefits duration of less than 5 weeks. When unemployment reaches 5.9% the optimal arrival rate falls to 3.9%, corresponding to an expected benefits duration of 26 weeks. When unemployment reaches 7.0%, the optimal arrival rate drops to 2.0%, corresponding to an expected benefits duration of 50 weeks. The optimal arrival rate is virtually nil when unemployment is above 9%.

5 Empirical Evidence

In this section we present recent empirical evidence that supports our theoretical work. We also discuss how the current UI system could be assessed using formula (10) and empirical estimates

¹⁹This section only provides an overview of the model, whose formal description and analysis is in Appendix E.6.

²⁰We assume that when workers lose their entitlements to unemployment benefits, social assistance provides food stamps. [Pavoni and Violante \[2007\]](#) compute that the median monthly allotment of food stamps for a family of four was \$397 per month in 1996. They also find that the median monthly wage for a worker with at most a high-school diploma, eligible to welfare, is \$1,540. Thus the rate of social assistance is $397/1,540 = 26\%$. As the rate of unemployment benefits is 50%, $\tau^{a,e}/\tau^{u,e} = 0.26/0.5 = 0.52$.

of the micro-elasticity ϵ^m and macro-elasticity ϵ^M .

Proposition 2 shows that the micro-elasticity is always greater than the macro-elasticity in our model with job rationing (characterized by Assumptions 1 and 2): $\epsilon^m/\epsilon^M > 1$. This result is a testable implication that distinguishes our model from standard models of equilibrium unemployment. In the canonical model (characterized by Assumptions 5 and 6), $\epsilon^m/\epsilon^M < 1$.²¹ Figure 1(f) provides some intuition. When UI falls, jobseekers search more. The labor supply shifts outwards, which increases employment by ϵ^m . In addition when UI falls, jobseekers face a worse outside option. The wage obtained by Nash bargaining falls, which raises labor demand and equilibrium labor market tightness. Employment rises further, and the total increase in employment is measured by ϵ^M . Clearly, $\epsilon^M > \epsilon^m$. In the canonical model with rigid wages (characterized by Assumptions 5 and 2), $\epsilon^m/\epsilon^M = 1$.²² This property is illustrated in Figure 1(e). It arises because labor demand is perfectly elastic and independent of UI, such that equilibrium tightness is independent of UI.

To test the validity of these models, we need an empirical estimate of ϵ^m/ϵ^M . Crépon et al. [2012] provide such an estimate by analyzing a large randomized field experiment in France in which some young educated jobseekers are treated by receiving job placement assistance. The experiment has a double-randomization design: some areas are treated and some are not, and within treated areas some jobseekers are treated and some are not. We interpret the treatment as an increase in search effort from e^C for control jobseekers to $e^T > e^C$ for treated jobseekers. We present the results for male workers. Compared to control jobseekers in the same area, treated jobseekers face a higher job-finding probability: $[e^T - e^C] \cdot f(\theta^T) = 11.3\%$. But compared to control jobseekers in control areas, control jobseekers in treated areas face a lower job-finding probability: $e^C \cdot [f(\theta^T) - f(\theta^C)] = -3.9\%$. Therefore the increase in the job-finding probability of treated jobseekers in treated areas compared to control jobseekers in control areas is only $[e^T \cdot f(\theta^T)] - [e^C \cdot f(\theta^C)] = 11.3 - 3.9 = 7.4\%$.²³ By definition, the micro-elasticity ϵ^m is proportional to $[e^T - e^C] \cdot f(\theta^T)$ and the macro-elasticity ϵ^M is proportional $[e^T \cdot f(\theta^T)] - [e^C \cdot f(\theta^C)]$. These empirical results imply a wedge $\epsilon^m/\epsilon^M = 11.3/7.4 = 1.53$. The wedge $\epsilon^m/\epsilon^M > 1$ is evidence of a negative rat-race externality: in the short run, treated jobseekers displace control job-

²¹Proposition A1 in the Appendix establishes the result formally under log utility.

²²The property derives from Proposition 2 applied to the case $\alpha = 1$.

²³See Table 9, column 5 in Crépon et al. [2012].

seekers in queues for jobs. This compelling randomized experiment suggests that our model with job rationing provides a good description of the labor market at business cycle frequency.²⁴

Crépon et al. [2012] find additional evidence supporting our model. Consistent with Proposition 3 they find that the wedge ϵ^m/ϵ^M is larger when the labor market is more slack, in geographical areas or time periods with higher unemployment. For example, the wedge is small in the years preceding the 2008–2009 recession: $\epsilon^m/\epsilon^M = 11.1/(11.1 - 1.4) = 1.14$, while it is much larger during the recession: $\epsilon^m/\epsilon^M = 11.0/(11.0 - 7.5) = 3.14$.²⁵

We can also use empirical estimates of ϵ^m/ϵ^M and ϵ^M to implement formula (10) and assess the UI system in the labor market of interest over the business cycle. The method of Crépon et al. [2012] could provide estimates of ϵ^m/ϵ^M over the business cycle. The ideal experiment to estimate ϵ^M is to offer higher unemployment benefits to all individuals in a randomly selected subset of labor markets and compare unemployment durations across treated and control labor markets. But estimating the macro-elasticity ϵ^M is inherently more difficult than estimating a micro-elasticity ϵ^m because it necessitates exogenous variations in benefits across comparable labor markets, instead of exogenous variations across comparable individuals within a single labor market.²⁶

To circumvent the difficulty of directly estimating ϵ^M , one could combine estimates of ϵ^m with estimates of ϵ^m/ϵ^M and recover estimates of ϵ^M . The ideal experiment to estimate ϵ^m is to compare individuals with different benefits in the same labor market at a given time, while controlling for individual characteristics. Two different methods have been recently designed to implement this approach. Schmieder et al. [2012] use sharp variations in the potential duration of unemployment benefits by age in Germany, population-wide administrative data, and a regression discontinuity method to identify the micro-elasticity of unemployment duration with respect to the potential duration of benefit entitlement. Their estimates are broadly constant over the business cycle (Table 4, column 7).²⁷ Landais [2012] exploits kinks in the level and duration schedules of unemployment

²⁴In the long run bargaining effects may raise the macro-elasticity by raising wages. As long as such effects are constant over the cycle, bargaining effects would affect the optimal *level* but not the optimal *cyclicality* of UI.

²⁵See Table 10, Panel A, column 2 in Crépon et al. [2012].

²⁶Kroft and Notowidigdo [2011] propose to estimate the macro-elasticity by using variations in average benefits within US states over time, controlling for state fixed effects. With this method they find that the elasticity of unemployment duration with respect to benefits is smaller when state unemployment is higher, consistent with the result of Proposition 3 in presence of job rationing (Table 2, columns 1 and 2).

²⁷Schmieder et al. [2012] estimate the effect of potential duration on the duration of both covered unemployment and total non-employment. They find that the elasticity of total non-employment is constant over the business cycle (Table

benefits to conduct a regression kink design. He uses administrative data from the Continuous Wage and Benefit History (CWBH) recording employment and unemployment histories for the universe of workers in 5 US states from 1976 to 1983. He estimates the micro-elasticities of paid unemployment and non-employment durations with respect to both benefit level and potential duration. He finds that micro-elasticities with respect to benefit level and potential duration are broadly constant over the business cycle (Tables 6 and 8).

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Appendix — NOT FOR PUBLICATION

A Proofs

We begin by deriving a few preliminary results.

LEMMA A1. *The derivatives of effort supplies $e^*(\theta, \Delta v)$ and $e^s(\theta, \Delta c)$ satisfy:*

$$\begin{aligned}\frac{\theta}{e^*} \cdot \frac{\partial e^*}{\partial \theta} &= \frac{\theta}{e^s} \cdot \frac{\partial e^s}{\partial \theta} = \frac{1 - \eta}{\kappa} \\ \frac{\Delta v}{e^*} \cdot \frac{\partial e^*}{\partial \Delta v} &= \frac{1}{\kappa}.\end{aligned}$$

Proof. The worker's optimal choice of effort (2) gives $k'(e^*) = f(\theta) \cdot \Delta v$. Differentiating with respect to θ , keeping Δv fixed:

$$\begin{aligned}k''(e^*) \cdot \frac{\partial e^*}{\partial \theta} &= (1 - \eta) \cdot \frac{k'(e^*)}{\theta} \\ \frac{\theta}{e^*} \cdot \frac{\partial e^*}{\partial \theta} &= \frac{1 - \eta}{\kappa},\end{aligned}$$

which yields the first results. We obtain the second result by differentiating the optimal choice of effort (2) with respect to Δv , keeping θ fixed. \square

LEMMA A2. *The derivatives of labor supply $n^*(e, \theta)$ satisfy*

$$\begin{aligned}\frac{\partial n^*}{\partial \theta} &= (1 - \eta) \cdot \frac{n^* - (1 - u)}{\theta} \\ \frac{\partial n^*}{\partial e} &= \frac{n^* - (1 - u)}{e} \\ \frac{\partial n^*}{\partial e} \cdot \frac{\partial e^s}{\partial \theta} &= \frac{\partial n^*}{\partial e} \cdot \frac{\partial e^*}{\partial \theta} = \frac{1}{\kappa} \cdot \frac{\partial n^*}{\partial \theta}.\end{aligned}$$

Proof. Obvious using Lemma A1 and equation (1): $n^*(e, \theta) = (1 - u) + u \cdot e \cdot f(\theta)$. \square

LEMMA A3. *The labor supply $n^s(\theta, \Delta c)$ is concave in θ if and only if $(1 - \eta) \cdot (1 + \kappa) / \kappa < 1$.*

Proof. We prove that $\partial^2 n^s / \partial \theta^2 < 0$. By definition, $n^s(\theta, \Delta c) = n^*(e^s(\theta, \Delta c), \theta)$. Lemmas A1 and A2 imply

$$\begin{aligned}\frac{\partial n^s}{\partial \theta} &= \frac{\partial n^*}{\partial e} \cdot \frac{\partial e^s}{\partial \theta} + \frac{\partial n^*}{\partial \theta} = \frac{1 + \kappa}{\kappa} \cdot \frac{\partial n^*}{\partial \theta} = \frac{1 + \kappa}{\kappa} \cdot (1 - \eta) \cdot \frac{n^* - (1 - u)}{\theta} \\ \frac{\partial^2 n^s}{\partial \theta^2} &= \frac{1 + \kappa}{\kappa} \cdot (1 - \eta) \cdot \frac{n^* - (1 - u)}{\theta^2} \cdot \left[\frac{1 + \kappa}{\kappa} \cdot (1 - \eta) - 1 \right].\end{aligned}$$

\square

A.1 Proof of Lemma 1

According to the Definition 1 of the elasticities ε^m and ε^M ,

$$\frac{1-n}{\Delta c} (\varepsilon^M - \varepsilon^m) = \frac{dn}{d\Delta c} - \frac{\partial n^*}{\partial e} \cdot \frac{\partial e^s}{\partial \Delta c}.$$

Taking the derivative of the equilibrium condition $n = n^s(\theta, \Delta c)$ with respect to Δc ,

$$\begin{aligned} \frac{dn}{d\Delta c} &= \frac{\partial n^*}{\partial e} \cdot \left(\frac{\partial e^s}{\partial \theta} \cdot \frac{d\theta}{d\Delta c} + \frac{\partial e^s}{\partial \Delta c} \right) + \frac{\partial n^*}{\partial \theta} \cdot \frac{d\theta}{d\Delta c} \\ \frac{dn}{d\Delta c} - \frac{\partial n^*}{\partial e} \cdot \frac{\partial e^s}{\partial \Delta c} &= \left(\frac{\partial n^*}{\partial e} \cdot \frac{\partial e^s}{\partial \theta} + \frac{\partial n^*}{\partial \theta} \right) \cdot \frac{d\theta}{d\Delta c}. \end{aligned}$$

Combining both results we obtain

$$\frac{1-n}{\Delta c} (\varepsilon^M - \varepsilon^m) = \left(\frac{\partial n^*}{\partial e} \cdot \frac{\partial e^s}{\partial \theta} + \frac{\partial n^*}{\partial \theta} \right) \cdot \frac{d\theta}{d\Delta c}. \quad (\text{A1})$$

Using Lemma A2 and equation (A1) we obtain

$$\begin{aligned} \frac{1-n}{\Delta c} (\varepsilon^M - \varepsilon^m) &= \frac{1+\kappa}{\kappa} \cdot \frac{\partial n^*}{\partial \theta} \cdot \frac{d\theta}{d\Delta c} \\ \Delta c \cdot \frac{\partial n^*}{\partial \theta} \cdot \frac{d\theta}{d\Delta c} &= \frac{\kappa}{\kappa+1} \cdot (1-n) \cdot (\varepsilon^M - \varepsilon^m) \\ \varepsilon_{\Delta c}^\theta &\equiv \frac{\Delta c}{\theta} \cdot \frac{d\theta}{d\Delta c} = \frac{\kappa}{\kappa+1} \cdot \frac{1}{1-\eta} \cdot \frac{1-n}{h} \cdot (\varepsilon^M - \varepsilon^m), \end{aligned}$$

which is the first result in the lemma. The second result in the lemma is obtained by combining the first result with the result of Lemma A2.

A.2 Proof of Proposition 1

The government chooses Δc to maximize

$$(1-u) \cdot v(c^e) + u \cdot [v(c^u) + e \cdot f(\theta) \cdot \Delta v - k(e)] = n^*(e, \theta) \cdot v(c^u + \Delta c) + (1-n^*(e, \theta)) \cdot v(c^u) - u \cdot k(e)$$

Using the envelope theorem (as workers choose search effort e to maximize $v(c^u) + e \cdot f(\theta) \cdot \Delta v - k(e)$) the first-order condition becomes

$$\begin{aligned} 0 &= [n \cdot v'(c^e) + (1-n)v'(c^u)] \cdot \frac{dc^u}{d\Delta c} + n \cdot v'(c^e) + \Delta v \cdot \frac{\partial n^*}{\partial \theta} \cdot \frac{d\theta}{d\Delta c} \\ 0 &= \bar{v}' \cdot \frac{dc^u}{d\Delta c} + n \cdot v'(c^e) + \Delta v \cdot \frac{\partial n^*}{\partial \theta} \cdot \frac{d\theta}{d\Delta c} \end{aligned} \quad (\text{A2})$$

where we define $\bar{v}' \equiv [n \cdot v'(c^e) + (1-n) \cdot v'(c^u)]$.

First step. Lemma 1 allows us to write

$$\Delta v \cdot \frac{\partial n^*}{\partial \theta} \cdot \frac{d\theta}{d\Delta c} = \frac{\Delta v}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot (1-n) \cdot (\varepsilon^M - \varepsilon^m). \quad (\text{A3})$$

Second step.

LEMMA A4.

$$\frac{dc^u}{d\Delta c} = \frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \cdot \varepsilon^M - n$$

Proof. We start from the budget constraint $c^u = n \cdot [w - \Delta c]$ and differentiate it.

$$\begin{aligned} \frac{dc^u}{d\Delta c} &= \frac{1-n}{\Delta c} \cdot [w - \Delta c] \cdot \varepsilon^M - n \\ \frac{dc^u}{d\Delta c} &= \frac{1-n}{n} \cdot \frac{c^u}{\Delta c} \cdot \varepsilon^M - n \\ \frac{dc^u}{d\Delta c} &= \frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \cdot \varepsilon^M - n. \end{aligned}$$

□

Third step. We come back to the formula, using Lemma A4 and the relationship (A3),

$$\begin{aligned} 0 &= \bar{v}' \left[\frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \varepsilon^M - n \right] + n \cdot v'(c^e) + \frac{\Delta v}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot (1-n) \cdot (\varepsilon^M - \varepsilon^m) \\ 0 &= \bar{v}' \left[\frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \varepsilon^M \right] + n \cdot [v'(c^e) - \bar{v}'] + \frac{\Delta v}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot (1-n) \cdot (\varepsilon^M - \varepsilon^m) \\ 0 &= \bar{v}' \left[\frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \varepsilon^M \right] + n(1-n) [v'(c^e) - v'(c^u)] + \frac{\Delta v}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot (1-n) \cdot (\varepsilon^M - \varepsilon^m). \end{aligned}$$

Dividing the equation by $(1-n) \cdot \varepsilon^M \cdot \bar{v}'$ yields

$$\begin{aligned} \frac{1}{n} \cdot \frac{\tau}{1-\tau} &= \frac{n}{\varepsilon^M} \cdot \frac{1}{\bar{v}'} \cdot [v'(c^u) - v'(c^e)] + \frac{\Delta v}{\bar{v}' \Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right) \\ \frac{1}{n} \cdot \frac{\tau}{1-\tau} &= \frac{v'(c^e)}{\bar{v}'} \left[\frac{n}{\varepsilon^M} \cdot \left\{ \frac{v'(c^u)}{v'(c^e)} - 1 \right\} + \frac{\Delta v}{v'(c^e) \Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right) \right] \\ \frac{1}{n} \cdot \frac{\tau}{1-\tau} &= \left[n + (1-n) \cdot \frac{v'(c^u)}{v'(c^e)} \right]^{-1} \cdot \left[\frac{n}{\varepsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1 \right] + \frac{\Delta v}{v'(c^e) \Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right) \right]. \end{aligned}$$

Approximation. Assuming $n \approx 1$ allows us to simplify the optimal formula to

$$\frac{\tau}{1-\tau} = \frac{1}{\varepsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1 \right] + \frac{\Delta v}{v'(c^e) \cdot \Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right).$$

If the third and higher order terms of $v(\cdot)$ are small ($v'''(c) \approx 0$), we approximate

$$\begin{aligned} \frac{\Delta v}{v'(c^e) \cdot \Delta c} &\approx 1 - \frac{1}{2} \cdot \frac{v''(c^e)}{v'(c^e)} \cdot \frac{c^e}{c^e} \cdot [c^e - c^u] = 1 + \frac{1}{2} \cdot \rho \cdot (1 - \tau) \\ \frac{v'(c^u)}{v'(c^e)} &\approx \frac{v'(c^e) - v''(c^e) \cdot c^e \cdot \frac{\Delta c}{c^e}}{v'(c^e)} = 1 + \rho(1 - \tau). \end{aligned}$$

ρ is the coefficient of relative risk aversion measured at c^e . The optimal UI formula becomes

$$\frac{\tau}{1-\tau} = \frac{1}{\varepsilon^M} \cdot \rho \cdot [1 - \tau] + \frac{\kappa}{\kappa+1} \cdot \left[\frac{\varepsilon^m}{\varepsilon^M} - 1 \right] \cdot \left[1 + \frac{\rho}{2} \cdot (1 - \tau) \right].$$

A.3 Proof of Proposition 2

We differentiate the firm's profit-maximization condition (11) with respect to Δc .

$$(\alpha - 1) \cdot \frac{g'(n)}{n} \cdot \frac{dn}{d\Delta c} = \eta \cdot \frac{r}{q(\theta)} \cdot \frac{1}{\theta} \cdot \frac{d\theta}{d\Delta c}.$$

Using Lemma 1 and the Definition 1 of elasticity ε^M ,

$$\begin{aligned} (\alpha - 1) \cdot g'(n) \cdot \frac{1-n}{n} \cdot \varepsilon^M &= \frac{r}{q(\theta)} \cdot \frac{\kappa}{\kappa+1} \cdot \frac{1-n}{h} \cdot \frac{\eta}{1-\eta} \cdot (\varepsilon^M - \varepsilon^m) \\ -(1-\alpha) \cdot g'(n) &= \frac{r}{q(\theta)} \cdot \frac{\kappa}{\kappa+1} \cdot \frac{n}{h} \cdot \frac{\eta}{1-\eta} \cdot \left(1 - \frac{\varepsilon^m}{\varepsilon^M} \right) \\ \frac{\varepsilon^m}{\varepsilon^M} &= 1 + \left[(1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{1}{r} \cdot \frac{1-\eta}{\eta} \right] \cdot q(\theta) \cdot \left(\frac{h}{n} \right) \cdot n^{\alpha-1}. \end{aligned} \quad (\text{A4})$$

Since $\theta > 0$, $h > 0$, $\eta \in (0, 1)$, $\kappa > 0$: $\varepsilon^m/\varepsilon^M > 1$ if and only if $\alpha \in (0, 1)$.

A.4 Some comparative statics

From now on, we focus on the case with log utility: $v(c) = \ln(c)$. In this case, $\Delta v = \ln(1/\tau)$, and it becomes natural to parameterize the equilibrium with (a, τ) instead of $(a, \Delta c)$. a captures the position of the economy in the business cycle and τ captures the generosity of UI. This parameterization is convenient because when τ remains constant, the supply curve remains in place (as Δv remains constant). All equilibrium variables in the next proofs (such as effort e , tightness θ , or employment n) are implicit functions of (a, τ) .

LEMMA A5. Under Assumptions 1 and 2, if $v(c) = \ln(c)$, we have the following comparative statics for equilibrium variables:

$$\left. \frac{\partial \theta}{\partial a} \right|_{\tau} > 0, \quad \left. \frac{\partial e}{\partial a} \right|_{\tau} > 0, \quad \left. \frac{\partial n}{\partial a} \right|_{\tau} > 0.$$

Proof. We know that $\partial e^*/\partial \theta > 0$, $\partial n^*/\partial \theta > 0$, $\partial n^*/\partial e > 0$. We also know that under Assumptions 1 and 2, $\partial n^d/\partial \theta < 0$, $\partial n^d/\partial a > 0$. We differentiate equilibrium condition (4) with respect to a , keeping τ (and Δv) constant.

$$\begin{aligned} \left[\frac{\partial n^*}{\partial e} \cdot \frac{\partial e^*}{\partial \theta} + \frac{\partial n^*}{\partial \theta} \right] \cdot \frac{\partial \theta}{\partial a} &= \frac{\partial n^d}{\partial a} + \frac{\partial n^d}{\partial \theta} \cdot \frac{\partial \theta}{\partial a} \\ \frac{\partial \theta}{\partial a} &= \underbrace{\frac{\partial n^d}{\partial a}}_{+} \cdot \left[\underbrace{\left\{ \frac{\partial n^*}{\partial e} \cdot \frac{\partial e^*}{\partial \theta} + \frac{\partial n^*}{\partial \theta} \right\}}_{+} - \underbrace{\frac{\partial n^d}{\partial \theta}}_{-} \right]^{-1}. \end{aligned}$$

So $\partial \theta/\partial a > 0$. We conclude using $e(a, \tau) = e^*(\theta(a, \tau), \Delta v(\tau))$ and $n(a, \tau) = n^*(e(a, \tau), \theta(a, \tau))$. \square

A.5 Proof of Proposition 3

Under Assumption 2 we can apply Proposition 2. Under Assumptions 3, 4, and 1, Proposition 2 implies that $\epsilon^m/\epsilon^M = 1 + s \cdot \chi \cdot q(\theta) \cdot n^{\alpha-1}$, where $s > 0$ and $\chi > 0$ are constant. Under Assumptions 2 and 1, Lemma A5 implies that $\partial \theta/\partial a|_{\tau} > 0$ and $\partial n/\partial a|_{\tau} > 0$. Since $q'(\theta) < 0$ and $\alpha \leq 1$, we infer that $\partial [\epsilon^m/\epsilon^M]/\partial a|_{\tau} < 0$, which is the first result in the proposition.

We focus on the second result in the proposition: the cyclicity of ϵ^M . First, we express ϵ^m as a function of the the elasticity $\epsilon_{\Delta c}^{\Delta v} \equiv (\Delta c/\Delta v) \cdot (\partial \Delta v/\partial \Delta c)$. The worker's supply of effort $e^s(\theta, \Delta c, a)$ satisfies the optimal choice of effort (2): $k'(e) = f(\theta) \cdot \Delta v(\Delta c, a)$. Differentiating this condition,

$$\begin{aligned} \kappa \cdot \frac{\Delta c}{e} \cdot \frac{\partial e^s}{\partial \Delta c} &= \epsilon_{\Delta c}^{\Delta v} \\ \epsilon^m &= \frac{\Delta c}{1-n} \cdot \frac{\partial n^*}{\partial e} \cdot \frac{\partial e^s}{\partial \Delta c} = \frac{s}{\kappa} \cdot \frac{n}{1-n} \cdot \epsilon_{\Delta c}^{\Delta v}, \end{aligned} \tag{A5}$$

where we used the Definition 1 of ϵ^m , Lemma A2, and Assumption 4. Next, we derive an expres-

sion for $\varepsilon_{\Delta c}^{\Delta v}$. Using Lemma A4 and the assumption that $v(c) = \ln(c)$,

$$\begin{aligned}\frac{\partial \Delta v}{\partial \Delta c} &= \frac{1}{c^e} + \left[\frac{1}{c^e} - \frac{1}{c^u} \right] \cdot \frac{\partial c^u}{\partial \Delta c} \\ \frac{\partial \Delta v}{\partial \Delta c} &= \left[(1-n) \cdot \frac{1}{c^e} + n \cdot \frac{1}{c^u} \right] + \left[\frac{1}{c^e} - \frac{1}{c^u} \right] \cdot \frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \cdot \varepsilon^M \\ \varepsilon_{\Delta c}^{\Delta v} &= \frac{1-\tau}{\ln(1/\tau)} \cdot \left\{ \left[(1-n) + \frac{n}{\tau} \right] + \left(1 - \frac{1}{\tau} \right) \cdot \frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \cdot \varepsilon^M \right\}.\end{aligned}$$

Combining this result with (A5) we obtain

$$\begin{aligned}\frac{\kappa}{s} \cdot \varepsilon^m &= \frac{1-\tau}{\ln(1/\tau)} \cdot \left[n + \frac{n^2}{1-n} \cdot \frac{1}{\tau} \right] - \frac{1-\tau}{\ln(1/\tau)} \cdot \varepsilon^M \\ \varepsilon^M &= \left[n + \frac{n^2}{1-n} \cdot \frac{1}{\tau} \right] \cdot \left[\frac{\kappa}{s} \cdot \frac{\varepsilon^m}{\varepsilon^M} \cdot \frac{\ln(1/\tau)}{1-\tau} + 1 \right]^{-1}.\end{aligned}\tag{A6}$$

Under Assumption 3 the elasticity κ is constant. According to Lemma A5, valid under Assumptions 1 and 3, $\partial n / \partial a|_{\tau} > 0$. We showed that $\partial [\varepsilon^m / \varepsilon^M] / \partial a|_{\tau} < 0$. We conclude that $\partial \varepsilon^M / \partial a|_{\tau} > 0$ because, keeping τ fixed, the first factor increases with n and therefore with a , and the second factor decreases with $\varepsilon^m / \varepsilon^M$ and therefore increases with a .

A.6 Proof of Proposition 4

The proof requires using elasticities of unemployment “in utility” instead of the elasticities of unemployment “in consumption” used in the text.

DEFINITION A1. The micro-elasticity ε_v^m and macro-elasticity ε_v^M of unemployment with respect to utility gain from work Δv are

$$\varepsilon_v^m \equiv \frac{\Delta v}{1-n} \cdot \frac{\partial n^*}{\partial e} \Big|_{\theta} \cdot \frac{\partial e^*}{\partial \Delta v} \Big|_{\theta}\tag{A7}$$

$$\varepsilon_v^M \equiv \frac{\Delta v}{1-n} \cdot \frac{dn}{d\Delta v}.\tag{A8}$$

We re-derive our optimal UI formula (9) using the elasticities $\varepsilon_v^m, \varepsilon_v^M$.

LEMMA A6. *The optimal replacement rate τ satisfies*

$$\frac{1}{n} \cdot \frac{\tau}{1-\tau} = \frac{\Delta v}{v'(c^e) \cdot \Delta c} \cdot \left\{ \frac{n}{\varepsilon_v^M} \cdot \left[1 - \frac{v'(c^e)}{v'(c^u)} \right] + \left[(1-n) \cdot \frac{v'(c^e)}{v'(c^u)} + n \right] \cdot \frac{\kappa}{\kappa+1} \cdot \left(\frac{\varepsilon_v^m}{\varepsilon_v^M} - 1 \right) \right\}.\tag{A9}$$

Proof.

First step. The government chooses Δv to maximize $v(c^u) + n \cdot \Delta v - u \cdot k(e)$. Using the envelope theorem the first-order condition becomes

$$0 = v'(c^u) \cdot \frac{dc^u}{d\Delta v} + n + \Delta v \cdot \frac{\partial n^*}{\partial \theta} \cdot \frac{d\theta}{d\Delta v}.$$

Second step. We use the budget constraint $c^e + c^u \cdot (1 - n)/n = w$ to rewrite Δv .

$$\begin{aligned} \Delta v &= v\left(w - \frac{1-n}{n} \cdot c^u\right) - v(c^u) \\ 1 &= v'(c^e) \cdot \left[-\frac{1-n}{n} \cdot \frac{dc^u}{d\Delta v} + \frac{1}{n^2} \cdot c^u \frac{dn}{d\Delta v}\right] - v'(c^u) \cdot \frac{dc^u}{d\Delta v} \\ v'(c^e) \cdot \frac{1}{n^2} \cdot c^u \cdot \frac{dn}{d\Delta v} - 1 &= \left[v'(c^e) \cdot \frac{1-n}{n} + v'(c^u)\right] \cdot \frac{dc^u}{d\Delta v} \\ v'(c^e) \cdot \frac{1-n}{n} \cdot \frac{c^u}{\Delta v} \cdot \varepsilon_v^M - n &= \left[(1-n) \cdot \frac{v'(c^e)}{v'(c^u)} + n\right] v'(c^u) \cdot \frac{dc^u}{d\Delta v}. \end{aligned}$$

Third step. We come back to the formula and use Lemma 1, which also applies to ε_v^m and ε_v^M .

$$\begin{aligned} 0 &= \left[(1-n) \frac{v'(c^e)}{v'(c^u)} + n\right] v'(c^u) \frac{dc^u}{d\Delta v} + n \left[(1-n) \frac{v'(c^e)}{v'(c^u)} + n\right] + \left[(1-n) \frac{v'(c^e)}{v'(c^u)} + n\right] \Delta v \frac{\partial n^*}{\partial \theta} \frac{d\theta}{d\Delta v} \\ 0 &= v'(c^e) \frac{1-n}{n} \frac{c^u}{\Delta v} \varepsilon_v^M - n + n \left[(1-n) \frac{v'(c^e)}{v'(c^u)} + n\right] + \left[(1-n) \frac{v'(c^e)}{v'(c^u)} + n\right] \frac{\kappa}{\kappa+1} (1-n) (\varepsilon_v^M - \varepsilon_v^m) \\ 0 &= v'(c^e) \frac{1-n}{n} \frac{c^u}{\Delta v} \varepsilon_v^M + n(1-n) \left[\frac{v'(c^e)}{v'(c^u)} - 1\right] + \left[(1-n) \frac{v'(c^e)}{v'(c^u)} + n\right] \frac{\kappa}{\kappa+1} (1-n) (\varepsilon_v^M - \varepsilon_v^m) \end{aligned}$$

Dividing by $(1-n) \cdot v'(c^e) \cdot \varepsilon_v^M$:

$$\begin{aligned} \frac{1}{n} \frac{c^u}{\Delta v} &= \frac{1}{v'(c^e)} \frac{n}{\varepsilon_v^M} \left[1 - \frac{v'(c^e)}{v'(c^u)}\right] + \frac{1}{v'(c^e)} \left[(1-n) \frac{v'(c^e)}{v'(c^u)} + n\right] \frac{\kappa}{\kappa+1} \left(\frac{\varepsilon_v^m}{\varepsilon_v^M} - 1\right) \\ \frac{1}{n} \frac{c^u}{\Delta c} &= \frac{\Delta v}{\Delta c} \frac{1}{v'(c^e)} \frac{n}{\varepsilon_v^M} \left[1 - \frac{v'(c^e)}{v'(c^u)}\right] + \frac{\Delta v}{\Delta c} \frac{1}{v'(c^e)} \left[(1-n) \frac{v'(c^e)}{v'(c^u)} + n\right] \frac{\kappa}{\kappa+1} \left(\frac{\varepsilon_v^m}{\varepsilon_v^M} - 1\right) \end{aligned}$$

Note that $c^u/\Delta c = \tau/(1-\tau)$ since $\tau = c^u/c^e$. Thus we obtain the exact optimal UI formula in sufficient statistics (A9). \square

Under Assumption 3 the formula becomes

$$\frac{\tau}{\ln(1/\tau)} = \frac{n^2}{\varepsilon_v^M} \cdot (1-\tau) + [(1-n) \cdot \tau + n] \cdot \frac{\kappa}{\kappa+1} \cdot n \cdot \left(\frac{\varepsilon_v^m}{\varepsilon_v^M} - 1\right). \quad (\text{A10})$$

Next, we need to express $(\varepsilon_v^m/\varepsilon_v^M - 1)$ and ε_v^M as a function of equilibrium variables.

LEMMA A7. Under Assumptions 1, 2, 3 and 4 there exists $Z_0(\tau) > 0$ such that in equilibrium,

$$Z(n, \tau) \equiv \frac{\varepsilon_v^m}{\varepsilon_v^M} - 1 = Z_0(\tau) \cdot n^{-\Omega} > 0, \quad (\text{A11})$$

where the constant Ω is defined by

$$\Omega = (1 - \alpha) + \frac{\kappa}{\kappa + 1} \cdot \frac{\eta}{1 - \eta} \cdot \frac{1}{s} > 0.$$

Proof. Under Assumptions 1 and 2 we can use the result from Proposition 2, which remains valid for the ratio of elasticities in utility. Under Assumption 4,

$$\frac{\varepsilon_v^m}{\varepsilon_v^M} - 1 = (1 - \alpha) \cdot \alpha \cdot \frac{\kappa + 1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1 - \eta}{\eta} \cdot q(\theta) \cdot n^{\alpha - 1}. \quad (\text{A12})$$

Using the results from Lemma A1 and Lemma A2 under Assumption 4,

$$\begin{aligned} \left. \frac{\partial n}{\partial \theta} \right|_{\tau} &= \left. \frac{\partial n^*}{\partial \theta} \right|_e + \left. \frac{\partial n^*}{\partial e} \right|_{\theta} \cdot \left. \frac{\partial e^*}{\partial \theta} \right|_{\tau} = \frac{\kappa + 1}{\kappa} \cdot \left. \frac{\partial n^*}{\partial \theta} \right|_e = \frac{\kappa + 1}{\kappa} \cdot (1 - \eta) \cdot \frac{h}{\theta} \\ \frac{\theta}{n} \cdot \left. \frac{\partial n}{\partial \theta} \right|_{\tau} &= \frac{\kappa + 1}{\kappa} \cdot (1 - \eta) \cdot s \\ \varepsilon_n^{\theta} &\equiv \frac{n}{\theta} \cdot \left. \frac{\partial \theta}{\partial n} \right|_{\tau} = \frac{\kappa}{\kappa + 1} \cdot \frac{1}{1 - \eta} \cdot \frac{1}{s}, \end{aligned}$$

Using the relationship (A12),

$$\left. \frac{\partial \ln(\varepsilon_v^m / \varepsilon_v^M - 1)}{\partial \ln(n)} \right|_{\tau} = - \left[(1 - \alpha) + \eta \cdot \varepsilon_n^{\theta} \right] = - \left[(1 - \alpha) + \frac{\kappa}{\kappa + 1} \cdot \frac{\eta}{1 - \eta} \cdot \frac{1}{s} \right] \equiv -\Omega,$$

where $\Omega > 0$ is constant under Assumption 3. Solving the differential equation yields (A11). \square

LEMMA A8. Under Assumption 4 the micro-elasticity “in utility” satisfies

$$\varepsilon_v^m = \frac{n}{1 - n} \cdot \frac{s}{\kappa}.$$

Proof. The definition (A7) of ε_v^m and the results from Lemma A1 and Lemma A2 imply

$$\varepsilon_v^m = \left(\frac{\Delta v}{1 - n} \right) \cdot \left(\frac{h}{e} \right) \cdot \left(\frac{e}{\Delta v} \cdot \frac{1}{\kappa} \right) = \frac{h}{1 - n} \cdot \frac{1}{\kappa}.$$

We conclude by using the property that under Assumption 4, $h = s \cdot n$. \square

Using Lemmas A7 and A8, we obtain

$$\frac{1}{\varepsilon_v^M} = \frac{1}{\varepsilon_v^m} \cdot \left[1 + \left(\frac{\varepsilon_v^m}{\varepsilon_v^M} - 1 \right) \right] = \frac{1-n}{n} \cdot \frac{\kappa}{s} \cdot [1 + Z(n, \tau)]. \quad (\text{A13})$$

Let $S \equiv s/(\kappa + 1) \in (0, 1)$. Using Lemma A7 and (A13), we can now rewrite formula (A10) as

$$\begin{aligned} \frac{\tau}{\ln(1/\tau)} &= n \cdot (1-n) \cdot \frac{\kappa}{s} \cdot [1 + Z(n, \tau)] \cdot (1-\tau) + [(1-n) \cdot \tau + n] \cdot \frac{\kappa}{\kappa+1} \cdot n \cdot Z(n, \tau) \\ \frac{s}{\kappa} \cdot \frac{\tau}{\ln(1/\tau)} &= n \cdot (1-n) \cdot (1-\tau) + n \cdot Z(n, \tau) \cdot [(1-n) \cdot (1-\tau) + [(1-n) \cdot \tau + n] \cdot S] \\ \frac{s}{\kappa} \cdot \frac{\tau}{\ln(1/\tau)} &= n \cdot (1-n) \cdot (1-\tau) + n \cdot Z(n, \tau) \cdot \{[\tau \cdot S + (1-\tau)] - n \cdot (1-\tau) \cdot (1-S)\}. \end{aligned} \quad (\text{A14})$$

Let us define

$$\begin{aligned} F(\tau) &\equiv \frac{s}{\kappa} \cdot \frac{\tau}{\ln(1/\tau)} \\ G(n, \tau) &\equiv n \cdot (1-n) \cdot (1-\tau) + n \cdot Z(n, \tau) \cdot [\tau \cdot S + (1-\tau) - n \cdot (1-\tau) \cdot (1-S)]. \end{aligned}$$

Furthermore, we define $Q(\tau, a) \equiv G(n(\tau, a), \tau)$. We rewrite the optimal UI formula as $F(\tau) = Q(\tau, a)$. We assume that for any $a > 0$, $F(\tau)$ and $Q(\tau, a)$ cross only once at $\tau(a) \in (0, 1)$. The implicit function $\tau(a)$ characterizes the optimal replacement rate for technology a .

LEMMA A9. *Under Assumptions 1 and 2, $\lim_{a \rightarrow 0} n(a, \tau(a)) = 0$ and $\lim_{a \rightarrow 0} \tau(a) = 0$.*

Proof. Under Assumptions 1 and 2, the firm's profit-maximization condition (11) implies that for any $a > 0$, $\alpha \cdot n(a, \tau(a))^{\alpha-1} \geq \omega \cdot a^{\gamma-1}$ and $0 \leq n(a, \tau(a)) \leq N(a) \equiv [(\alpha/\omega) \cdot a^{1-\gamma}]^{1/(1-\alpha)}$. Since $\gamma < 1$ and $0 < \alpha < 1$, $\lim_{a \rightarrow 0} N(a) = 0$. The squeeze theorem implies that $\lim_{a \rightarrow 0} n(a, \tau(a)) = 0$.

By definition, $q(\theta) \leq 1$. Therefore for any n and any τ ,

$$n \cdot Z(n, \tau) = (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1-\eta}{\eta} \cdot q(\theta) \cdot n^\alpha \leq (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1-\eta}{\eta} \cdot n^\alpha.$$

Using the optimal UI formula $F(\tau(a)) = Q(\tau(a), a)$ and the definition of $Q(\cdot, \cdot)$,

$$F(\tau(a)) \leq n(a, \tau(a)) \cdot [1 - n(a, \tau(a))] + (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1-\eta}{\eta} \cdot n(a, \tau(a))^\alpha.$$

We showed that $\lim_{a \rightarrow 0} n(a, \tau(a)) = 0$. So there exists $a_0 > 0$ such that for all $a < a_0$, $n(a, \tau(a)) < 1/2$. For any $a > 0$, $0 \leq n(a, \tau(a)) \leq N(a)$. Thus for any $a < a_0$,

$$0 \leq F(\tau(a)) \leq N(a) \cdot [1 - N(a)] + (1-\alpha) \cdot \alpha \cdot \frac{\kappa+1}{\kappa} \cdot \frac{s}{r} \cdot \frac{1-\eta}{\eta} \cdot N(a)^\alpha.$$

Under Assumptions 1 and 2, the limit of the right-hand-side term when $a \rightarrow 0$ is 0 because

$\lim_{a \rightarrow 0} N(a) = 0$. Using the squeeze theorem, we infer that $\lim_{a \rightarrow 0} F(\tau(a)) = 0$. We conclude that $\lim_{a \rightarrow 0} \tau(a) = 0$ using the continuity of $F(\cdot)$ on $(0, 1)$. \square

Lemma A9 establishes that when employment converges to 0 because technology decreases to 0, then the optimal replacement rate converges to 0. This result implies that for very low levels of technology, and very low levels of employment, the optimal replacement rate is bound to increase with technology.

LEMMA A10. *If $n > 1/2$ and $\Omega \geq 1$ then $\partial G/\partial n < 0$.*

Proof. We differentiate $G(n, \tau)$ with respect to n , keeping τ constant.

$$\frac{\partial G}{\partial n} = - \{ (2 \cdot n - 1) \cdot (1 - \tau) + Z(n, \tau) \cdot [(2 - \Omega) \cdot (1 - S) \cdot (1 - \tau) \cdot n - (1 - \Omega) \cdot [\tau \cdot S + (1 - \tau)]] \}.$$

If $n > 1/2$, the first term $(2 \cdot n - 1) \cdot (1 - \tau) > 0$ since $\tau < 1$. If $\Omega \geq 1$, the second term is nonnegative. To see this, note that $(1 - S) \cdot n < 1$ and rewrite the second term as

$$Z(n, \tau) \cdot [(\Omega - 1) \cdot [\tau \cdot S + (1 - \tau)] \cdot \{1 - (1 - S) \cdot n\}] + (1 - S) \cdot (1 - \tau) \cdot n \geq 0.$$

If $\Omega \in [0, 1)$, the second term may be negative. \square

At technology a , the optimal replacement rate $\tau(a)$ satisfies $F(\tau(a)) = Q(\tau(a), a)$. We consider a marginal change in technology from a to $a^* > a$. Using Lemma A5 under Assumption 3, we know that $n(\tau(a), a^*) > n(\tau(a), a)$. Using Lemma A10 for $n > 1/2$ and $\tau \in (0, 1)$, $G(n(\tau(a), a^*), \tau(a)) < G(n(\tau(a), a), \tau(a))$ such that $Q(\tau(a), a^*) < Q(\tau(a), a) = F(\tau(a))$. Since $F(\tau)$ and $Q(\tau, a)$ cross only once for $\tau \in (0, 1)$, $\lim_{\tau \rightarrow 0} F(\tau) = 0$, and $\lim_{\tau \rightarrow 0} Q(\tau, a) > 0$, it must be that $F(\tau)$ crosses $Q(\tau, a)$ “from below”. Thus it must be that $\tau(a) > \tau(a^*)$ and $d\tau/da < 0$.

B Extensions of the Optimal UI Formula

In this section, we derive the results presented in Section 2.3, in which we describe extensions of our optimal UI formula to various settings. We also present some additional results, especially approximations of the optimal UI formulas.

B.1 Self-insurance

Unemployed workers choose effort e and home production y to maximize

$$[1 - e \cdot f(\theta)] \cdot [v(c^u + y) - m(y)] + [e \cdot f(\theta)] \cdot v(c^e) - k(e)$$

The first-order condition with respect to home production y yields

$$m'(y) = v'(c^u + y), \tag{A15}$$

which implicitly defines optimal home production $y(c^u)$. The first-order condition with respect to search effort e yields $k'(e) = f(\theta) \cdot \Delta v^h$, where we denote $\Delta v^h = v(c^e) - [v(c^u + y(c^u)) - m(y(c^u))]$ the utility difference between being employed and unemployed. The condition implicitly defines optimal effort $e(\theta, \Delta v^h)$.

The government chooses Δc to maximize expected utility

$$n^*(e, \theta) \cdot v(c^u + \Delta c) + [1 - n^*(e, \theta)] \cdot [v(c^u + y) - m(y)] - u \cdot k(e)$$

Using the envelope theorem, as workers choose search effort e and home production y to maximize expected utility, the first-order condition becomes

$$0 = \left[n \cdot v'(c^e) + (1 - n) \cdot v'(c^h) \right] \cdot \frac{dc^u}{d\Delta c} + n \cdot v'(c^e) + \Delta v^h \cdot \frac{\partial n^*}{\partial \theta} \cdot \frac{d\theta}{d\Delta c}.$$

As in the case without self-insurance, we derive the optimal UI formula in three steps. The first two steps remain the same because Lemma 1 carries over. Therefore the formula becomes

$$\frac{1}{n} \cdot \frac{\tau}{1 - \tau} = \left[n + (1 - n) \cdot \frac{v'(c^h)}{v'(c^e)} \right]^{-1} \cdot \left\{ \frac{n}{\varepsilon^M} \cdot \left[\frac{v'(c^h)}{v'(c^e)} - 1 \right] + \frac{\Delta v^h}{v'(c^e) \cdot \Delta c} \cdot \frac{\kappa}{\kappa + 1} \cdot \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right) \right\}.$$

B.2 Wage response to UI

Optimal UI formula. In equilibrium, the wage paid by firms responds to Δc : $w = w(\Delta c)$. It is likely that $w'(\Delta c) < 0$: for instance with bargaining, higher Δc lowers the outside option of workers and reduces wages. If UI influences wages, labor demand is a function of UI: $n^d = n^d(\theta, \Delta c)$, which reflects the influence of UI on firm's recruiting decision through wages. Labor market tightness $\theta(\Delta c)$ is now characterized by the equilibrium condition $n^d(\theta, \Delta c) = n^s(\theta, \Delta c)$. The macro-elasticity captures the influence of UI on employment and labor market tightness through all channels, including possibly wages.

We amend the budget constraint of the government because the wage $w(\Delta c)$ is now a function of Δc . We modify the second and third steps in the proof of Proposition 1 accordingly. We differentiate the budget constraint $c^u = n \cdot [w(\Delta c) - \Delta c]$.

$$\frac{dc^u}{d\Delta c} = \frac{1 - n}{n} \cdot \frac{\tau}{1 - \tau} \cdot \varepsilon^M - n + n \cdot \frac{dw}{d\Delta c}.$$

We come back to the formula (A2) and use (A3).

$$0 = \bar{v}' \cdot \left[\frac{1 - n}{n} \cdot \frac{\tau}{1 - \tau} \cdot \varepsilon^M - n + n \cdot \frac{dw}{d\Delta c} \right] + n \cdot v'(c^e) + \frac{\Delta v}{\Delta c} \cdot \frac{\kappa}{\kappa + 1} \cdot (1 - n) \cdot (\varepsilon^M - \varepsilon^m)$$

$$0 = \bar{v}' \cdot \frac{1 - n}{n} \cdot \frac{\tau}{1 - \tau} \cdot \varepsilon^M + \bar{v}' \cdot n \cdot \frac{dw}{d\Delta c} + n \cdot (1 - n) \cdot [v'(c^e) - v'(c^u)] + \frac{\Delta v}{\Delta c} \cdot \frac{\kappa}{\kappa + 1} \cdot (1 - n) \cdot (\varepsilon^M - \varepsilon^m).$$

Dividing the equation by $(1 - n) \cdot \varepsilon^M \cdot \bar{v}'$ and rearranging the terms yields

$$\frac{1}{n} \frac{\tau}{1 - \tau} + \frac{1}{\varepsilon^M} \frac{n}{1 - n} \frac{dw}{d\Delta c} = \left[n + (1 - n) \frac{v'(c^u)}{v'(c^e)} \right]^{-1} \left[\frac{n}{\varepsilon^M} \left[\frac{v'(c^u)}{v'(c^e)} - 1 \right] + \frac{\Delta v}{v'(c^e) \Delta c} \frac{\kappa}{\kappa + 1} \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right) \right]. \quad (\text{A16})$$

A special form of incidence. The core of the analysis uses the standard assumption that the incidence of the labor tax falls fully on workers. In this section, we consider the nonstandard assumption that the tax burden $w - c^e = t \cdot w$ —the wedge between wages paid by firms and post-tax earnings received by workers, is shared between firms and workers. We assume that firms cover a constant fraction of the burden:

$$\begin{aligned} w &= w^* + \psi \cdot t \cdot w \\ c^e &= w^* - (1 - \psi) \cdot t \cdot w, \end{aligned}$$

where ψ and w^* are parameters. Using the budget constraint $n \cdot t \cdot w = (1 - n) \cdot b \cdot w$, we express w as a function of w^* .

$$\begin{aligned} t \cdot w &= \frac{1 - n}{n} \cdot c^u = (1 - n) \cdot (w - \Delta c) \\ w &= w^* + \psi \cdot (1 - n) \cdot (w - \Delta c) \\ w &= \frac{w^*}{1 - \psi \cdot (1 - n)} - \frac{\psi \cdot (1 - n)}{1 - \psi \cdot (1 - n)} \cdot \Delta c. \end{aligned}$$

To simplify the derivations, let $X \equiv \psi \cdot (1 - n)$.

$$\begin{aligned} w &= \frac{w^*}{1 - X} - \frac{X}{1 - X} \cdot \Delta c \\ \frac{dw}{d\Delta c} &= \frac{w^*}{(1 - X)^2} \cdot \frac{dX}{d\Delta c} - \frac{\Delta c}{(1 - X)^2} \cdot \frac{dX}{d\Delta c} - \frac{X}{1 - X} = \frac{w^* - \Delta c}{(1 - X)^2} \cdot \frac{dX}{d\Delta c} - \frac{X}{1 - X}. \end{aligned}$$

Notice that

$$\begin{aligned} \frac{dX}{d\Delta c} &= -\psi \cdot \frac{1 - n}{\Delta c} \cdot \varepsilon^M = -\frac{X}{\Delta c} \cdot \varepsilon^M \\ (w - \Delta c) &= (w^* - \Delta c) + \psi \cdot (1 - n) \cdot (w - \Delta c) = \frac{w^* - \Delta c}{1 - X}. \end{aligned}$$

Therefore,

$$\frac{dw}{d\Delta c} = \frac{-X}{1 - X} \cdot \left[\frac{w - \Delta c}{\Delta c} \cdot \varepsilon^M + 1 \right].$$

The assumption on tax incidence yields a simple expression for the response of wages to Δc :

$$\frac{dw}{d\Delta c} = \frac{-\psi \cdot (1-n)}{1-\psi \cdot (1-n)} \cdot \left[\frac{\tau}{1-\tau} \cdot \frac{1}{n} \cdot \varepsilon^M + 1 \right].$$

Using the budget constraint (3),

$$\frac{w - \Delta c}{\Delta c} = \frac{1}{n} \cdot \frac{c^u}{\Delta c} = \frac{1}{n} \cdot \frac{\tau}{1-\tau}.$$

We conclude that

$$\begin{aligned} \frac{dw}{d\Delta c} &= \frac{-\psi \cdot (1-n)}{1-\psi \cdot (1-n)} \cdot \left[\frac{\tau}{1-\tau} \cdot \frac{1}{n} \cdot \varepsilon^M + 1 \right] \\ \frac{1}{\varepsilon^M} \cdot \frac{n}{1-n} \cdot \frac{dw}{d\Delta c} &= \frac{-\psi \cdot n}{(1-\psi) + \psi \cdot n} \cdot \left[\frac{\tau}{1-\tau} \cdot \frac{1}{n} + \frac{1}{\varepsilon^M} \right]. \end{aligned}$$

Combining this result with formula (A16) yields

$$\begin{aligned} \frac{1}{n} \cdot \frac{\tau}{1-\tau} \cdot \frac{1-\psi}{(1-\psi) + \psi \cdot n} &= \left[n + (1-n) \frac{v'(c^u)}{v'(c^e)} \right]^{-1} \cdot \\ &\left\{ \frac{n}{\varepsilon^M} \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1 + \frac{\psi}{(1-\psi) + \psi \cdot n} \cdot \left[n + (1-n) \frac{v'(c^u)}{v'(c^e)} \right] \right] + \frac{\Delta v}{v'(c^e)\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right) \right\}. \end{aligned}$$

The formula becomes

$$\frac{1-\psi}{n} \cdot \frac{\tau}{1-\tau} = \left[n + (1-n) \frac{v'(c^u)}{v'(c^e)} \right]^{-1} \left\{ \frac{n}{\varepsilon^M} \left[\frac{v'(c^u)}{v'(c^e)} - (1-\psi) \right] + \frac{\Delta v}{v'(c^e)\Delta c} [(1-\psi) + \psi \cdot n] \frac{\kappa}{\kappa+1} \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right) \right\}.$$

The structure of our optimal UI formula remains the same. We only adjust each of the three terms with functions of the incidence parameter ψ . The formula gives optimal UI taking into account the insurance value of unemployment benefits, the moral-hazard cost of UI, and the employment cost of financing benefits by taxing firms more. In addition to reducing jobseekers' effort, UI also reduces firms' hiring by increasing the marginal cost of labor through higher wages. The effect on labor demand arises because of the assumption on tax incidence. The effect does not appear directly in the formula but appears indirectly through the macro-elasticity ε^M . In recessions, they are two competing effects: a lower cost of UI because of lower labor supply (through the reduction in search effort caused by higher UI), which lowers ε^M , and a higher cost of UI because of lower labor demand (through the increase in labor tax imposed on firms), which raises ε^M . Only simulations can guide the design of UI in that case.

C The Canonical Equilibrium Unemployment Model

This section studies the canonical model of [Pissarides \[2000\]](#), characterized by Assumptions 5 and 6. We assume that utility has constant relative risk aversion ρ : $v(c) = (c^{1-\rho} - 1)/(1 - \rho)$.

C.1 The Nash bargaining solution

We determine the outcome of the bargaining problem faced by a firm-worker pair. \mathcal{E} denotes the value of being employed, and \mathcal{U} the value of being unemployed. Both values are evaluated after the matching process. They satisfy

$$\begin{aligned}\mathcal{E} &= v((1-t) \cdot w) \\ \mathcal{U} &= v(b \cdot w).\end{aligned}$$

Combining both conditions yields the worker's surplus \mathcal{W} from a relationship with a firm.

$$\mathcal{W} = \mathcal{E} - \mathcal{U} = [v((1-t) \cdot w) - v(b \cdot w)]. \quad (\text{A17})$$

When worker and firm bargain, they take the tax rate t and unemployment benefits $b \cdot w$ as given. In the term $b \cdot w$ of \mathcal{W} , w is the equilibrium wage that is taken as given by worker and firm. In the term $(1-t) \cdot w$ of \mathcal{W} , w is the outcome of the wage bargaining between the firm and the worker. Therefore when the worker evaluates the marginal utility $d\mathcal{W}$ of an increase dw in the wage bargained with the firm, he only considers the marginal change of the post-tax earnings $(1-t) \cdot w$. Accordingly,

$$\frac{d\mathcal{W}}{dw} = (1-t) \cdot v'((1-t) \cdot w) = (1-t)^{1-\rho} \cdot v'(w).$$

In equilibrium the firm's surplus from an established relationship is simply given by the hiring cost since a firm can immediately replace a worker at that cost during the matching period: $\mathcal{F} = r \cdot a/q(\theta)$. Since the firm's utility is simply its profits, a wage w brings a utility $-w$ to the firm (or its owners) and $d\mathcal{F}/dw = -1$.

The generalized Nash solution to the bargaining problem faced by a firm-worker pair is the wage w that maximizes

$$\mathcal{W}(w)^\beta \cdot \mathcal{F}(w)^{1-\beta},$$

where β is the worker's bargaining power. The first-order condition of the maximization problem implies that the worker's surplus each period is related to the firm's surplus by

$$\frac{\beta}{1-\beta} \cdot \frac{d\mathcal{W}}{dw} \cdot \mathcal{F} = \mathcal{W}.$$

Substituting the relationship into equation (A17) for the worker's surplus \mathcal{W} , and using the expressions for \mathcal{F} and $d\mathcal{W}/dw$, we obtain the relationship between equilibrium variables imposed

by Nash bargaining over wages.

$$\begin{aligned}
\frac{\beta}{1-\beta} \cdot \frac{r \cdot a}{q(\theta)} \cdot (1-t)^{1-\rho} \cdot v'(w) &= [v((1-t) \cdot w) - v(b \cdot w)] \\
\frac{r \cdot a}{q(\theta)} \cdot w^{-\rho} &= \frac{1-\beta}{\beta} \cdot [1-\tau^{1-\rho}] \cdot \frac{w^{1-\rho}}{1-\rho} \\
[1-\tau^{1-\rho}] \cdot \frac{w}{1-\rho} &= \frac{\beta}{1-\beta} \cdot a \cdot \frac{r}{q(\theta)} \\
\frac{w}{a} &= \frac{\beta}{1-\beta} \cdot \frac{1}{v(\tau)} \cdot \frac{r}{q(\theta)}.
\end{aligned} \tag{A18}$$

Absence of fluctuations with Nash bargaining. Combining the expression for w/a with the firm's profit-maximization condition (11) yields an expression for tightness θ as a function of the parameters of the model only. Keeping τ constant there are no fluctuations in labor market tightness because θ does not depend on technology a . Indeed equilibrium tightness is described by

$$\frac{r}{q(\theta)} = \left[1 - \frac{\beta}{1-\beta} \cdot \frac{1}{v(\tau)} \right]^{-1}. \tag{A19}$$

C.2 An optimal UI formula

This section derives an optimal UI formula in a class of models in which wages may respond to UI, and the production function satisfies Assumption 5. By choosing Δc the government maximizes

$$n \cdot v(c^e) + (1-n) \cdot v(c^u) - u \cdot k(e) + \zeta \cdot \pi.$$

ζ is the social welfare weight placed by the government on the firm's profits π . ζ is taken as given by the government. Combining the firm's profit-maximization condition, the government's constraint, and the resource constraint, we find that aggregate profits are

$$\pi = (1-u) \cdot \frac{r \cdot a}{q(\theta)}.$$

We also assume that the wage $w(\Delta c)$ may respond to Δc .

As in the proof of Proposition 1, the envelope theorem yields the first-order condition

$$0 = \bar{v}' \cdot \frac{dc^u}{d\Delta c} + n \cdot v'(c^e) + \left[\Delta v \cdot \frac{\partial n^*}{\partial \theta} + \zeta \cdot \eta \cdot \frac{r \cdot a}{q(\theta)} \cdot \frac{1-u}{\theta} \right] \cdot \frac{d\theta}{d\Delta c},$$

where we define $\bar{v}' \equiv [n \cdot v'(c^e) + (1-n) \cdot v'(c^u)]$.

First step. Lemma 1 in the text allows us to write

$$\begin{aligned}\Delta v \cdot \frac{\partial n^*}{\partial \theta} \cdot \frac{d\theta}{d\Delta c} &= \frac{\Delta v}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot (1-n) \cdot (\varepsilon^M - \varepsilon^m) \\ \eta \cdot \frac{1-u}{\theta} \cdot \frac{d\theta}{d\Delta c} &= \frac{1}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \frac{\eta}{1-\eta} \cdot \frac{(1-u) \cdot (1-n)}{h} \cdot (\varepsilon^M - \varepsilon^m).\end{aligned}$$

Second step. We differentiate the budget constraint $c^u(\Delta c) = n(\Delta c) \cdot [w(\Delta c) - \Delta c]$.

$$\begin{aligned}\frac{dc^u}{d\Delta c} &= \frac{1-n}{\Delta c} \cdot [w - \Delta c] \cdot \varepsilon^M - n \cdot \left(-\frac{dw}{d\Delta c} + 1\right) \\ \frac{dc^u}{d\Delta c} &= \frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \cdot \varepsilon^M - n + n \cdot \frac{dw}{d\Delta c}.\end{aligned}$$

Third step. The firm's profit-maximization condition (11) allows us to link wage to labor market tightness θ : $w = a - r \cdot a/q(\theta)$. In turn, the condition allows us to express wage changes imposed by UI to labor market tightness changes imposed by UI. The link between $w(\Delta c)$ and $\theta(\Delta c)$ is

$$\begin{aligned}\frac{dw}{d\Delta c} &= -\eta \cdot \frac{r \cdot a}{q(\theta)} \cdot \frac{1}{\theta} \cdot \frac{\partial \theta}{\partial \Delta c} \\ \frac{dw}{d\Delta c} &= -\frac{r \cdot a}{q(\theta)} \cdot \frac{1}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot \frac{\eta}{1-\eta} \cdot \frac{(1-n)}{h} \cdot (\varepsilon^M - \varepsilon^m),\end{aligned}$$

where we used once more the result from Lemma 1 to express $d\theta/d\Delta c$.

Fourth step. We come back to the formula.

$$\begin{aligned}0 &= \bar{v}' \cdot \frac{1-n}{n} \cdot \frac{\tau}{1-\tau} \cdot \varepsilon^M + n \cdot (1-n) [v'(c^e) - v'(c^u)] \\ &\quad + \frac{1}{\Delta c} \cdot \frac{\kappa}{\kappa+1} \cdot (1-n) \cdot (\varepsilon^M - \varepsilon^m) \cdot \left[\Delta v + \frac{\eta}{1-\eta} \cdot \frac{1}{h} \cdot (\zeta \cdot (1-u) - n \cdot \bar{v}') \cdot \frac{r \cdot a}{q(\theta)} \right].\end{aligned}$$

We set $\zeta = \bar{v}'$ (the welfare weight is arbitrary). Since $h = n - (1-u)$, dividing the equation by $(1-n) \cdot \varepsilon^M \cdot \bar{v}'$ yields an optimal UI formula with endogenous wages and valuation of profits:

$$\frac{1}{n} \frac{\tau}{1-\tau} = \left[n + (1-n) \frac{v'(c^u)}{v'(c^e)} \right]^{-1} \left\{ \frac{n}{\varepsilon^M} \left[\frac{v'(c^u)}{v'(c^e)} - 1 \right] + \frac{\Delta v}{v'(c^e) \Delta c} \frac{\kappa}{\kappa+1} \left[\frac{\varepsilon^m}{\varepsilon^M} - 1 \right] \left[1 - \frac{\bar{v}'}{\Delta v} \frac{\eta}{1-\eta} \frac{ra}{q(\theta)} \right] \right\}. \quad (\text{A20})$$

The formula naturally applies to the canonical model of [Pissarides \[2000\]](#).

C.3 Elasticity wedge

This section derives the pendant of Proposition 2 for the canonical model. Proposition A1 establishes that in the canonical model, under log utility, the macro-elasticity ε^M is greater than the micro-elasticity ε^m :

PROPOSITION A1. *Under Assumptions 5 and 6, and if $\rho = 1$, then $\varepsilon^m/\varepsilon^M < 1$.*

Proof. The equilibrium condition (A19), obtained under Assumptions 5 and 6, implies that $d\theta/d\tau < 0$. If $\rho = 1$, $v(c) = \ln(c)$, $\Delta v = -\ln(\tau)$, and $d\theta/d\Delta v > 0$. Lemma 1 is also valid if (a) we replace the elasticities “in consumption” ε^m and ε^M defined by Definition 1 with the elasticities “in utility” ε_v^m and ε_v^M defined by Definition A1, and (b) we replace the derivative $d\theta/d\Delta c$ by $d\theta/\Delta v$. Since $d\theta/d\Delta v > 0$, Lemma 1 implies that $\varepsilon_v^M > \varepsilon_v^m > 0$ (see Lemma A8 for an expression of ε_v^m). Therefore $\varepsilon_v^m/\varepsilon_v^M < 1$. Comparing Definition 1 for ε^m and ε^M with Definition A1 for ε_v^m and ε_v^M it is clear that $\varepsilon_v^m/\varepsilon_v^M = \varepsilon^m/\varepsilon^M$. Thus $\varepsilon^m/\varepsilon^M < 1$. \square

C.4 Deriving an Hosios [1990] condition

If workers are risk neutral, the social planner does not care about c^e and c^u independently but cares about aggregate consumption $c \equiv n \cdot c^e + (1 - n) \cdot c^u$. Given initial unemployment u the planner chooses consumption, effort, labor market tightness, and number of hires $\{c, e, \theta, h\}$ to maximize social welfare $c - u \cdot k(e)$, subject to the matching process

$$h = e \cdot f(\theta) \cdot u \quad (\text{A21})$$

and the resource constraint

$$c = a \cdot g(1 - u + h) - \frac{r \cdot a}{q(\theta)} \cdot h. \quad (\text{A22})$$

The Lagrangian is

$$\mathcal{L} = c - u \cdot k(e) + A \cdot [e \cdot f(\theta) \cdot u - h] + B \cdot \left[a \cdot g(1 - u + h) - \frac{r \cdot a}{q(\theta)} \cdot h - c \right],$$

where A, B are Lagrange multipliers. The first-order conditions with respect to c, e, θ, h are

$$\begin{aligned} 1 &= B \\ k'(e) &= A \cdot f(\theta) \\ 0 &= A \cdot e \cdot u \cdot \frac{f(\theta)}{\theta} \cdot (1 - \eta) - B \cdot \eta \cdot \frac{r \cdot a}{q(\theta)} \cdot \frac{1}{\theta} \cdot h \\ A &= B \cdot \left[a \cdot g'(n) - \frac{r \cdot a}{q(\theta)} \right]. \end{aligned}$$

The ratio of Lagrange multiplier is

$$\frac{A}{B} = \frac{\eta}{1-\eta} \cdot \frac{r \cdot a}{q(\theta)}.$$

Therefore, optimal effort, hiring, tightness, and consumption $\{e, h, \theta, c\}$ are completely described by 4 relationships: constraint (A21), constraint (A22), and the first-order conditions

$$\begin{aligned} k'(e) &= f(\theta) \cdot \left[\frac{\eta}{1-\eta} \cdot \frac{r \cdot a}{q(\theta)} \right] \\ g'(n) - \frac{r}{q(\theta)} &= \frac{\eta}{1-\eta} \cdot \frac{r}{q(\theta)}. \end{aligned}$$

We compare the conditions with equilibrium conditions (2) and (11) for the canonical model. Assume a Cobb-Douglas matching function, such that η is a constant, and risk-neutral workers. The equilibrium wage is the Nash bargained wage (A18). To replicate the efficient allocation in the canonical model, it suffices that $\beta = \eta$ and $\tau = 0$, which is the Hosios [1990] condition.

D Optimal Unemployment Insurance and Wage Subsidies

We start by describing the labor market equilibrium under technology a , when the replacement rate is $\tau = c^e/c^u$ and the normalized wage is $\tilde{w} = w/a$.

Under Assumption 3, $v(c) = \ln(c)$. Equation (2) implicitly defines a function $e^*(\theta, \tau)$, which gives the optimal search effort for tightness θ and replacement rate τ . The law of motion of employment $n^*(e, \theta)$ is defined by (1). We define the labor supply by $n^s(\theta, \tau) \equiv n^*(e^*(\theta, \tau), \theta)$. The firm's profit-maximization condition (11) can be rewritten as

$$g'(n) = \tilde{w} + \frac{r}{q(\theta)},$$

which implicitly defines a labor demand $n^d(\theta, \tilde{w})$ under Assumption 1. The equilibrium condition (4) can be rewritten as

$$n^s(\theta, \tau) = n^d(\theta, \tilde{w}),$$

which implicitly defines equilibrium labor market tightness $\theta(\tau, \tilde{w})$. Furthermore, we define equilibrium employment $n(\tau, \tilde{w}) \equiv n^s(\theta(\tau, \tilde{w}), \tau)$ and equilibrium effort $e(\tau, \tilde{w}) \equiv e^*(\theta(\tau, \tilde{w}), \tau)$. Lemma A11 establishes how equilibrium variables respond to a change in the wage \tilde{w} :

LEMMA A11. *Under Assumptions 1 and 2, if $v(c) = \ln(c)$, we have the following comparative statics for equilibrium variables:*

$$\left. \frac{\partial \theta}{\partial \tilde{w}} \right|_{\tau} < 0, \quad \left. \frac{\partial e}{\partial \tilde{w}} \right|_{\tau} < 0, \quad \left. \frac{\partial n}{\partial \tilde{w}} \right|_{\tau} < 0.$$

Proof. Similar to the proof of Lemma A5. □

The government chooses a rate b of unemployment benefits, a tax rate t imposed on the salary w^* received by employees, and a subsidy rate σ imposed on the salary w^* paid by employers. Effectively, firms pay a wage $w = (1 - \sigma) \cdot w^*$, employed workers consume $c^e = (1 - t) \cdot w^*$, and unemployed workers consume $c^u = b \cdot w^*$. The government is subject to the budget constraint

$$\begin{aligned}(1 - n) \cdot b \cdot w^* + n \cdot \sigma \cdot w^* &= t \cdot n \cdot w^* \\ (1 - n) \cdot b \cdot w^* + n \cdot w^* - n \cdot t \cdot w^* &= n \cdot w^* - n \cdot \sigma \cdot w^* \\ (1 - n) \cdot c^u + n \cdot c^e &= n \cdot w.\end{aligned}$$

The budget constraint remains the same as in the baseline model even though the labor tax is collected from workers and partly redistributed to firms as a wage subsidy. The budget constraint defines a function that gives the consumption of employed workers in equilibrium: $c^e(\tau, \tilde{w}, a) \equiv a \cdot \tilde{c}^e(\tau, \tilde{w})$ where

$$\tilde{c}^e(\tau, \tilde{w}) \equiv \frac{n(\tau, \tilde{w})}{n + [1 - n(\tau, \tilde{w})] \cdot \tau} \cdot \tilde{w}.$$

In equilibrium, the expected utility of a worker is

$$\ln(c^e(\tau, \tilde{w}, a)) + [1 - n(\tau, \tilde{w})] \cdot \ln(\tau) - u \cdot k(e(\tau, \tilde{w})) = \ln(a) + SW(\tau, \tilde{w}),$$

where we define the function

$$SW(\tau, \tilde{w}) \equiv \ln(\tilde{c}^e(\tau, \tilde{w})) + [1 - n(\tau, \tilde{w})] \cdot \ln(\tau) - u \cdot k(e(\tau, \tilde{w})).$$

In Section 3, we maximized $SW(\tau, \tilde{w})$ over $\tau \in (0, 1)$ for $\tilde{w} = \tilde{w}(a) \equiv \omega \cdot a^{\gamma-1}$ (because we made Assumption 2). The result from Proposition 4 in Section 3 tell us something about the properties of SW . Let $\tau^*(\tilde{w})$ be the function implicitly defined by

$$\frac{\partial SW(\tau, \tilde{w})}{\partial \tau} = 0.$$

Furthermore, we define the replacement rate $\tau(a) \equiv \tau^*(\tilde{w}(a))$. Under some conditions, Proposition 4 shows that $d\tau/da < 0$. Since $d\tilde{w}/da < 0$ and

$$\frac{d\tau}{da} = \frac{d\tau^*}{d\tilde{w}} \cdot \frac{d\tilde{w}}{da},$$

we infer that $d\tau^*/d\tilde{w} > 0$ (under the assumptions of Proposition 4).

Let us consider the problem of the government when the government chooses optimally both the wage \tilde{w} and the replacement rate τ . To capture the various costs of implementing a wage subsidy discussed in Section 3.5, we assume that setting a wage \tilde{w} when the technology is a imposes a welfare cost $C(\tilde{w}, a) > 0$. If the salary is a function $w^*(a)$ of a , a possible welfare cost could be an increasing convex function $C(\sigma)$ of the subsidy rate σ . The reason is that $\sigma = [w - w^*(a)]/w^*(a) = [a \cdot \tilde{w} - w^*(a)]/w^*(a)$ so σ is only a function of \tilde{w} and a . A critical assumption is that the welfare

cost C does not depend on the replacement rate τ . The government chooses jointly τ and \tilde{w} to maximize

$$\ln(a) + SW(\tau, \tilde{w}) - C(\tilde{w}, a).$$

The first-order condition with respect to τ is

$$\left. \frac{\partial SW(\tau, \tilde{w})}{\partial \tau} \right|_{\tilde{w}=\tilde{w}^\dagger} = 0$$

where \tilde{w}^\dagger is the optimal wage. Therefore the optimal replacement rate is $\tau^\dagger = \tau^*(\tilde{w}^\dagger)$, where $\tau^*(\cdot)$ is the function defined above. Our study of the government problem in Section 3 tell us that $\tau^*(\tilde{w})$ has the property that $d\tau^*/d\tilde{w} > 0$.

Note that the the optimal wage $\tilde{w}^\dagger(a)$ is defined implicitly by the first-order condition

$$\left. \frac{\partial SW(\tau, \tilde{w})}{\partial \tilde{w}} \right|_{\tau=\tau^*(\tilde{w})} - \left. \frac{\partial C}{\partial \tilde{w}} \right|_a = 0.$$

Assume that the replacement rate τ^\dagger is fixed. There is a technology shock from a to a' such that employment decreases after the optimal wage is adjusted from $\tilde{w}^\dagger(a)$ to $\tilde{w}^\dagger(a')$. Lemma A11 implies that $\tilde{w}^\dagger(a) < \tilde{w}^\dagger(a')$. Since the optimal replacement rate is solely a function of the optimal wage: $\tau^\dagger = \tau^*(\tilde{w}^\dagger)$ with $d\tau^*/d\tilde{w} > 0$, τ^\dagger must increase. Therefore after an adverse shock that increases unemployment, the optimal replacement rate increases. The substantive conclusion of Proposition 4 is robust to the presence of wage subsidies: optimal UI is more generous when unemployment is high.

E The Dynamic Model

This section describes in detail and studies the dynamic model of Section 4. We assume that the disutility of effort is isoelastic: $k(e) = \omega_k \cdot e^{1+\kappa}/(1+\kappa)$; the disutility of home production is isoelastic: $m(y) = \omega_m \cdot y^{1+\mu}/(1+\mu)$; firm's production function satisfies Assumption 1; and the wage satisfies Assumption 2. To simplify notations, we denote by $c_t^h \equiv c_t^u + y_t$ the total consumption of unemployed workers, including consumption of both market good and home-produced good. We denote $\Delta v_t^h \equiv v(c_t^e) - [v(c_t^u) - m(y_t)]$ the utility gain from work. We denote unemployment $u_t = 1 - (1-s) \cdot n_{t-1}$ and number of hires $h_t = n_t - (1-s) \cdot n_{t-1}$.

We assume that technology follows a stochastic process $\{a_t\}_{t=0}^{+\infty}$. Together with initial employment n_{-1} in the representative firm, the history of technology realizations $a^t \equiv (a_0, a_1, \dots, a_t)$ fully describes the state of the economy in period t . The time- t element of the worker's choice, firm's choice, and government policy must be measurable with respect to (a^t, n_{-1}) .

E.1 Equilibrium

Government's budget. The government fully taxes profits, taxes or subsidizes labor income, and subsidizes unemployed workers. The government must balance its budget each period, so it chooses consumptions $\{c_t^u, c_t^e\}_{t=0}^{+\infty}$ subject to the resource constraint

$$a_t \cdot g(n_t) - \frac{r \cdot a_t}{q(\theta_t)} \cdot [n_t - (1-s) \cdot n_{t-1}] = n_t \cdot c_t^e + (1-n_t) \cdot c_t^u. \quad (\text{A23})$$

Firm's problem. Given labor market tightness and technology $\{\theta_t, a_t\}_{t=0}^{+\infty}$ the firm chooses employment $\{n_t^d\}_{t=0}^{+\infty}$ to maximize expected profit

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ a_t \cdot g(n_t^d) - w_t \cdot n_t^d - \frac{r \cdot a_t}{q(\theta_t)} \cdot [n_t^d - (1-s) \cdot n_{t-1}^d] \right\}.$$

The first-order condition with respect to n_t^d implies

$$a_t \cdot g'(n_t^d) = w_t + \frac{r \cdot a_t}{q(\theta_t)} - \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{r \cdot a_{t+1}}{q(\theta_{t+1})} \right]. \quad (\text{A24})$$

Worker's problem. Given government policy $\{c_t^e, c_t^u\}_{t=0}^{+\infty}$ and labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$ the representative worker chooses job-search effort and home production $\{e_t, y_t\}_{t=0}^{+\infty}$ to maximize the expected utility

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ (1-n_t^s) \cdot [v(c_t^u + y_t) - m(y_t)] + n_t^s \cdot v(c_t^e) - [1 - (1-s) \cdot n_{t-1}^s] \cdot k(e_t) \right\}, \quad (\text{A25})$$

subject to the law of motion of the employment probability in period t ,

$$n_t^s = (1-s) \cdot n_{t-1}^s + [1 - (1-s) \cdot n_{t-1}^s] \cdot e_t \cdot f(\theta_t). \quad (\text{A26})$$

The Lagrangian of the worker's problem is

$$\begin{aligned} \mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ - [1 - (1-s) \cdot n_{t-1}^s] \cdot k(e_t) + (1-n_t^s) \cdot [v(c_t^u + y_t) - m(y_t)] + n_t^s \cdot v(c_t^e) \right. \\ \left. + A_t \cdot [[1 - (1-s) \cdot n_{t-1}^s] \cdot e_t \cdot f(\theta_t) + (1-s) \cdot n_{t-1}^s - n_t^s] \right\}, \end{aligned}$$

where $\{A_t(a^t), \forall a^t\}_{t=0}^{+\infty}$ is a sequence of Lagrange multipliers. The first-order condition with respect to home production y_t is

$$m'(y_t) = v'(c_t^h). \quad (\text{A27})$$

The first-order condition with respect to effort e_t is

$$k'(e_t) = f(\theta_t) \cdot A_t.$$

The first-order condition with respect to employment probability n_t^s is

$$A_t = \Delta v_t^h + \delta \cdot (1-s) \cdot \mathbb{E}_t[k(e_{t+1})] + \delta \cdot (1-s) \cdot \mathbb{E}_t[A_{t+1} \cdot (1 - e_{t+1} \cdot f(\theta_{t+1}))].$$

Using the isoelasticity of $k(\cdot)$, the optimal effort satisfies

$$\left[\frac{k'(e_t)}{f(\theta_t)} - \delta \cdot (1-s) \cdot \frac{k'(e_{t+1})}{f(\theta_{t+1})} \right] + \kappa \cdot \delta \cdot (1-s) \cdot k(e_{t+1}) = \Delta v_t^h. \quad (\text{A28})$$

Labor market equilibrium. Wages follow an exogenous stochastic process and cannot equalize labor supply and demand. As in Hall [2005], we only require that wages should neither interfere with the formation of an employment match that generates a positive bilateral surplus, nor cause the destruction of such a match. Since wages cannot equalize labor supply and labor demand, labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$ equalizes labor demand $\{n_t^d\}_{t=0}^{+\infty}$ to labor supply $\{n_t^s\}_{t=0}^{+\infty}$, which defines employment $\{n_t\}_{t=0}^{+\infty}$:

$$n_t^d = n_t^s \equiv n_t. \quad (\text{A29})$$

Equilibrium definition. An *equilibrium with unemployment insurance* is a collection of stochastic processes $\{c_t^e, c_t^u, y_t, e_t, n_t, \theta_t\}_{t=0}^{+\infty}$ that satisfy equations (A23), (A24), (A26), (A27), and (A28). The unemployment insurance program is fully contingent on the history of realizations of shocks, and is taken as given by firms and workers. We assume that the government can fully commit to the policy plan. The government's problem is to choose a government policy $\{c_t^u, c_t^e\}_{t=0}^{+\infty}$ to maximize social welfare (A25) over all equilibria with unemployment insurance. An *optimal equilibrium* is an equilibrium with unemployment insurance that attains the maximum of (A25).

E.2 Optimal equilibrium

Government's problem. The maximization of the government is over a collection of sequences $\{c_t^e(a^t), c_t^u(a^t), y_t(a^t), e_t(a^t), n_t(a^t), \theta_t(a^t), \forall a^t\}_{t=0}^{+\infty}$. We form a Lagrangian

$$\begin{aligned} \mathcal{L} = & \mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ (1 - n_t) \cdot \left[v(c_t^h) - m(y_t) \right] + n_t \cdot v(c_t^e) - [1 - (1 - s)n_{t-1}] \cdot k(e_t) \right. \\ & + A_t \left[a_t \cdot g(n_t) - \frac{r \cdot a_t}{q(\theta_t)} \cdot [n_t - (1 - s) \cdot n_{t-1}] - n_t \cdot c_t^e - (1 - n_t) \cdot c_t^u \right] \\ & + B_t \left[\left[v(c_t^e) - \left[v(c_t^h) - m(y_t) \right] \right] - \frac{k'(e_t)}{f(\theta_t)} \right] + B_{t-1} \cdot (1 - s) \left[\frac{k'(e_t)}{f(\theta_t)} - \kappa \cdot k(e_t) \right] \\ & + Q_t \left[m'(y_t) - v'(c_t^h) \right] + C_t \left[a_t \cdot g'(n_t) - w_t - \frac{r \cdot a_t}{q(\theta_t)} \right] + C_{t-1} \cdot (1 - s) \left[\frac{r \cdot a_t}{q(\theta_t)} \right] \\ & \left. + D_t [(1 - (1 - s) \cdot n_{t-1}) \cdot e_t \cdot f(\theta_t) + (1 - s) \cdot n_{t-1} - n_t] \right\} \end{aligned}$$

where $\{A_t(a^t), B_t(a^t), Q_t(a^t), C_t(a^t), D_t(a^t), \forall a^t\}_{t=0}^{+\infty}$ are sequences of Lagrange multipliers. We define $B_{-1} \equiv 0$ and $C_{-1} \equiv 0$. The first-order conditions with respect to $y_t(a^t)$ for $t \geq 0$ are

$$0 = (1 - n_t) \cdot \left[v'(c_t^h) - m'(y_t) \right] - B_t \cdot \left[v'(c_t^h) - m'(y_t) \right] + Q_t \cdot \left[m''(y_t) - v''(c_t^h) \right]$$

Using the optimal home production condition (A27), we obtain $0 = Q_t \cdot [m''(y_t) - v''(c_t^h)]$. Since $m''(\cdot) > 0$ and $v''(\cdot) < 0$:

$$0 = Q_t. \tag{A30}$$

The first-order conditions with respect to $c_t^e(a^t)$ for $t \geq 0$ are

$$A_t = v'(c_t^e) \cdot \left(1 + \frac{B_t}{n_t} \right). \tag{A31}$$

Using (A30) the first-order conditions with respect to $c_t^u(a^t)$ for $t \geq 0$ are

$$\begin{aligned} 0 &= -(1 - n_t) \cdot A_t + (1 - n_t) \cdot v'(c_t^h) - B_t \cdot v'(c_t^h) - Q_t \cdot v''(c_t^h) \\ A_t &= v'(c_t^h) \cdot \left[1 - \frac{B_t}{(1 - n_t)} \right]. \end{aligned} \tag{A32}$$

The first-order conditions with respect to $e_t(a^t)$ for $t \geq 0$ are

$$\begin{aligned}
0 &= -u_t \cdot k'(e_t) - B_t \cdot \frac{k''(e_t)}{f(\theta_t)} + (1-s) \cdot B_{t-1} \frac{k''(e_t)}{f(\theta_t)} - \kappa \cdot (1-s) \cdot B_{t-1} \cdot k'(e_t) + D_t \cdot u_t \cdot f(\theta_t) \\
0 &= [u_t + \kappa \cdot (1-s) \cdot B_{t-1}] \cdot k'(e_t) + \frac{k''(e_t)}{f(\theta_t)} \cdot [B_t - (1-s) \cdot B_{t-1}] - D_t \cdot \frac{h_t}{e_t} \\
0 &= u_t + \kappa \cdot (1-s) \cdot B_{t-1} + \frac{\kappa}{e_t \cdot f(\theta_t)} \cdot [B_t - (1-s) \cdot B_{t-1}] - \frac{1}{1+\kappa} \cdot \frac{D_t \cdot h_t}{k(e_t)}. \tag{A33}
\end{aligned}$$

The first-order conditions with respect to $\theta_t(a^t)$ for $t \geq 0$ are

$$\begin{aligned}
0 &= -A_t \cdot \eta \cdot \frac{r \cdot a_t}{f(\theta_t)} \cdot h_t + (1-\eta) \cdot B_t \cdot \frac{k'(e_t)}{\theta_t \cdot f(\theta_t)} - (1-\eta) \cdot (1-s) \cdot B_{t-1} \cdot \frac{k'(e_t)}{\theta_t \cdot f(\theta_t)} \\
&\quad - C_t \cdot \eta \cdot \frac{r \cdot a_t}{f(\theta_t)} + C_{t-1} \cdot (1-s) \cdot \eta \cdot \frac{r \cdot a_t}{f(\theta_t)} + D_t \cdot u_t \cdot (1-\eta) \cdot e_t \cdot q(\theta_t) \\
0 &= \frac{1-\eta}{\eta} \frac{k'(e_t)}{f(\theta_t)} [B_t - (1-s)B_{t-1}] - \frac{ra_t}{q(\theta_t)} [C_t - (1-s)C_{t-1}] + D_t \frac{1-\eta}{\eta} u_t e_t f(\theta_t) - A_t \frac{ra_t}{q(\theta_t)} h_t \\
0 &= \frac{1-\eta}{\eta} q(\theta_t) \left[h_t \cdot D_t + \frac{k'(e_t)}{f(\theta_t)} [B_t - (1-s) \cdot B_{t-1}] \right] - r \cdot a_t \cdot [C_t - (1-s) \cdot C_{t-1} + A_t \cdot h_t]. \tag{A34}
\end{aligned}$$

The first-order conditions with respect to $n_t(a^t)$ for $t \geq 0$ are

$$\begin{aligned}
0 &= v(c_t^e) - \left[v(c_t^h) - m(y_t) \right] + \delta \cdot (1-s) \cdot \mathbb{E}_t [k(e_{t+1})] - D_t + (1-s) \cdot \mathbb{E}_t [D_{t+1} \cdot (1 - e_{t+1} \cdot f(\theta_{t+1}))] \\
&\quad + C_t \cdot a_t \cdot g''(n_t) + A_t \cdot \left[a_t \cdot g'(n_t) - \frac{r \cdot a_t}{q(\theta_t)} - (c_t^e - c_t^u) \right] + (1-s) \cdot \delta \cdot \mathbb{E}_t \left[A_{t+1} \cdot \frac{r \cdot a_{t+1}}{q(\theta_{t+1})} \right] \\
D_t &= v(c_t^e) - \left[v(c_t^h) - m(y_t) \right] + \delta \cdot (1-s) \cdot \mathbb{E}_t [k(e_{t+1})] + (1-s) \cdot \mathbb{E}_t [D_{t+1} \cdot (1 - e_{t+1} \cdot f(\theta_{t+1}))] \\
&\quad + C_t \cdot a_t \cdot g''(n_t) + A_t \cdot \left[a_t \cdot g'(n_t) - \frac{r \cdot a_t}{q(\theta_t)} - (c_t^e - c_t^u) \right] + (1-s) \cdot \delta \cdot \mathbb{E}_t \left[A_{t+1} \cdot \frac{r \cdot a_{t+1}}{q(\theta_{t+1})} \right] \\
D_t &= v(c_t^e) - \left[v(c_t^h) - m(y_t) \right] + \delta \cdot (1-s) \cdot \mathbb{E}_t [k(e_{t+1})] + (1-s) \cdot \mathbb{E}_t [D_{t+1} \cdot (1 - e_{t+1} \cdot f(\theta_{t+1}))] \\
&\quad + C_t \cdot a_t \cdot g''(n_t) + A_t \cdot [w(a_t) - (c_t^e - c_t^u)] + (1-s) \cdot \delta \cdot \mathbb{E}_t \left[(A_{t+1} - A_t) \cdot \frac{r \cdot a_{t+1}}{q(\theta_{t+1})} \right], \tag{A35}
\end{aligned}$$

where we used the firm's profit-maximization condition (A24).

Equilibrium characterization. The optimal equilibrium is a collection of 11 stochastic processes $\{c_t^e, c_t^u, y_t, e_t, n_t, \theta_t, A_t, B_t, C_t, D_t, Q_t\}_{t=0}^{+\infty}$ that satisfy 11 equations $\{(A23), (A24), (A26), (A27), (A28), (A30), (A31), (A32), (A33), (A34), (A35)\}$.

Steady state. In steady state there are no aggregate shocks: $a_t = a$ for all t . The optimal equilibrium is constant, and is characterized by a collection of 11 variables $\{c^e, c^u, y, n, \theta, e, A, B, C, D, Q\}$ characterized by 11 equations $\{(A23), (A24), (A26), (A27), (A28), (A30), (A31), (A32), (A33), (A34), (A35)\}$. It is useful to combine together a few of the first-order conditions and constraints of the government's problem to re-express some Lagrange multipliers in a simpler form. These relationships are useful to solve steady-state optimal equilibria numerically. Combining (A31) and (A32), we obtain expressions for the Lagrange multipliers A and B as a function of equilibrium variables:

$$A = \left[\frac{n}{v'(c^e)} + \frac{1-n}{v'(c^h)} \right]^{-1}$$

$$B = n \cdot (1-n) \cdot \left[\frac{1}{v'(c^e)} - \frac{1}{v'(c^h)} \right] \cdot A.$$

Using the fact that $e/h = 1/(u \cdot f(\theta))$ in steady state, (A33) becomes:

$$D = \frac{k'(e)}{f(\theta) \cdot u} \cdot [u + \kappa \cdot (1-s) \cdot B] + \frac{k''(e) \cdot e}{f(\theta)} \cdot \frac{s}{h} \cdot B$$

$$D = \frac{k'(e)}{f(\theta)} \cdot \left[1 + \frac{B}{n} \cdot \frac{\kappa}{u} \right].$$

Using this expression, equation (A34) becomes:

$$0 = \frac{1-\eta}{\eta} \cdot q(\theta) \left[h \cdot D + \frac{k'(e)}{f(\theta)} \cdot s \cdot B \right] - r \cdot a \cdot s \cdot [C + A \cdot n]$$

$$C = \frac{1-\eta}{\eta} \cdot \frac{k'(e)}{r \cdot a \cdot \theta} \cdot \left[n + B \cdot \left(\frac{\kappa}{u} + 1 \right) \right] - A \cdot n.$$

E.3 Log-linearization

\bar{x} and $\check{x}_t \equiv d \ln(x_t)$ denote steady-state value and logarithmic deviation of variable x_t . In steady state the 6 variables $\{\bar{c}^e, \bar{c}^u, \bar{y}, \bar{n}, \bar{\theta}, \bar{e}\}$ describing the optimal equilibrium and the 5 associated Lagrange multipliers $\{\bar{A}, \bar{B}, \bar{Q}, \bar{C}, \bar{D}\}$ are characterized by the system of 11 equations $\{(A23), (A24), (A26), (A27), (A28), (A30), (A31), (A32), (A33), (A34), (A35)\}$ for technology $a = \bar{a} = 1$.

To simplify notations, we denote $\bar{h} = s \cdot \bar{n}$, $\bar{u} = 1 - (1-s) \cdot \bar{n}$, $\bar{c}^h = \bar{c}^u + \bar{y}$, and $\bar{\Delta v}^h = v(\bar{c}^e) - v(\bar{c}^h) + m(\bar{y})$. By definition the log-deviations $\{\check{u}_t, \check{h}_t, \check{c}_t^u, \check{\Delta v}_t^h\}$ satisfy: $(1-s) \cdot \check{n}_{t-1} + s \cdot \check{h}_t - \check{n}_t = 0$; $\check{u}_t + o_1 \cdot \check{n}_{t-1} = 0$ where $o_1 = (1-\bar{u})/\bar{u}$; $m_0 \cdot \check{y}_t + m_1 \cdot \check{c}_t^u - \check{c}_t^h = 0$, where $m_0 = \bar{y}/\bar{c}^h$ and $m_1 = 1 - m_0$; and $s_1 \cdot \epsilon^e \cdot \check{c}_t^e + s_2 \cdot \epsilon^h \cdot \check{c}_t^h + s_3 \cdot (1+\mu) \cdot \check{y}_t - \check{\Delta v}_t^h = 0$ where $s_1 = v(\bar{c}^e)/\bar{\Delta v}^h$, $s_2 = -v(\bar{c}^h)/\bar{\Delta v}^h$, $s_3 = 1 - s_1 - s_2$, and $\epsilon^i = d \ln(v(x))/d \ln(x)|_{x=\bar{c}^i}$ is the elasticity of the utility function at \bar{c}^i .

The logarithmic deviations of the 6 variables $\{\check{c}^e, \check{c}^u, \check{y}, \check{n}, \check{\theta}, \check{e}\}$ and 5 Lagrange multipliers $\{\check{A}, \check{B}, \check{Q}, \check{C}, \check{D}\}$ describing the optimal equilibrium are characterized by the following system of

11 log-linear equations. The budget constraint (A23) is

$$\check{a}_t + \alpha \cdot \check{n}_t - q_0 \cdot [\check{h}_t + \eta \cdot \check{\theta}_t + \check{a}_t] - q_1 \cdot [p_1 \cdot (\check{n}_t + \check{c}_t^e) + p_2 \cdot (-p_3 \cdot \check{n}_t + \check{c}_t^u)] = 0,$$

with $p_3 = \bar{n}/(1 - \bar{n})$, $p_1 = 1/(1 + (\bar{c}^u/\bar{c}^e) \cdot (1 - \bar{n})/\bar{n})$, $p_2 = 1 - p_1$, $q_0 = [r/q(\bar{\theta})] \cdot [\bar{h}/\bar{n}^\alpha]$, and $q_1 = 1 - q_0$. Worker's optimal job search (A28) is

$$-\frac{t_2}{1 - \delta(1 - s)} \left[[\kappa \check{e}_t - (1 - \eta) \check{\theta}_t] - \delta(1 - s) \mathbb{E}_t [\kappa \check{e}_{t+1} - (1 - \eta) \check{\theta}_{t+1}] \right] - t_1 \cdot (1 + \kappa) \mathbb{E}_t [\check{e}_{t+1}] + \Delta \check{v}_t^h = 0,$$

where $t_1 = \kappa \cdot \delta \cdot (1 - s) \cdot k(\bar{e})/\overline{\Delta v}^h$ and $t_2 = 1 - t_1$. Worker's optimal home production (A27) is

$$\mu \cdot \check{y}_t + \rho \cdot \check{c}_t^h = 0.$$

Firm's optimal hiring (A24) is

$$-\check{a}_t + (1 - \alpha) \cdot \check{n}_t + r_1 \cdot \gamma \cdot \check{a}_t + r_2 \cdot (\eta \cdot \check{\theta}_t + \check{a}_t) + r_3 \cdot \mathbb{E}_t [\eta \cdot \check{\theta}_{t+1} + \check{a}_{t+1}] = 0,$$

with $r_1 = \omega/[\alpha \cdot \bar{n}^{\alpha-1}]$, $r_2 = [r/q(\bar{\theta})]/[\alpha \cdot \bar{n}^{\alpha-1}]$, and $r_3 = 1 - r_1 - r_2$. The law of motion of employment (A26) is

$$\check{u}_t + \check{e}_t + (1 - \eta) \cdot \check{\theta}_t - \check{h}_t = 0.$$

Equation (A30) imposes $\check{Q}_t = 0$. Multipliers A_t and B_t satisfy (A31) and (A32):

$$\begin{aligned} \check{A}_t + \rho \cdot \check{c}_t^e + u_1 \cdot (\check{n}_t - \check{B}_t) &= 0 \\ \check{A}_t + \rho \cdot \check{c}_t^h + u_2 \cdot (u_3 \cdot \check{n}_t + \check{B}_t) &= 0, \end{aligned}$$

where $u_1 = \bar{B}/(\bar{n} + \bar{B})$, $u_2 = \bar{B}/(1 - \bar{n} - \bar{B})$, and $u_3 = \bar{n}/(1 - \bar{n})$. Multiplier D_t satisfies (A33):

$$\check{D}_t + \check{h}_t + (1 + \kappa) \cdot \check{e}_t - w_2 \cdot \check{u}_t - w_3 \cdot \check{B}_{t-1} - w_4 \left[-(1 - \eta) \cdot \check{\theta}_t - \check{e}_t + \left(\frac{1}{s} \cdot \check{B}_t - \frac{1-s}{s} \cdot \check{B}_{t-1} \right) \right] = 0,$$

where $w_1 = [\bar{D} \cdot \bar{h}]/[(1 + \kappa) \cdot k(\bar{e})]$, and $w_2 = \bar{u}/w_1$, $w_3 = \kappa \cdot (1 - s) \cdot \bar{B}/w_1$, $w_4 = 1 - w_2 - w_3$. Multiplier C_t satisfies (A34):

$$\check{h}_t - \eta \cdot \check{\theta}_t + \check{D}_t - x_8 [\check{a}_t + \check{h}_t + \check{A}_t] - x_6 \left[-\check{\theta}_t + \kappa \cdot \check{e}_t + \frac{1}{s} \check{B}_t - \frac{1-s}{s} \check{B}_{t-1} \right] - x_7 \left[\check{a}_t + \frac{1}{s} \check{C}_t - \frac{1-s}{s} \check{C}_{t-1} \right] = 0,$$

where $x_1 = \bar{h} \cdot q(\bar{\theta}) \cdot \bar{D} \cdot (1 - \eta)/\eta$, $x_8 = r \cdot \bar{h} \cdot \bar{A}/x_1$, $x_7 = r \cdot s \cdot \bar{C}/x_1$, and $x_6 = 1 - x_7 - x_8$. The last

first-order condition (A35) of the government's problem is

$$\begin{aligned} \check{D}_t - \left\{ y_1 \cdot \check{\Delta} v_t^h + y_2 \cdot (1 + \kappa) \cdot \mathbb{E}_t [\check{e}_{t+1}] + y_3 \cdot \mathbb{E}_t [\check{D}_{t+1} - z_4 \cdot [\check{e}_{t+1} + (1 - \eta) \cdot \check{\theta}_{t+1}]] \right. \\ \left. + y_4 \cdot [\check{C}_t + \check{a}_t + (\alpha - 2) \cdot \check{n}_t] + y_5 \cdot [\check{A}_t + (z_1 \cdot \gamma \cdot \check{a}_t + z_2 \cdot \check{c}_t^e + z_3 \cdot \check{c}_t^u)] + y_6 \cdot \mathbb{E}_t [\check{A}_{t+1} - \check{A}_t] \right\} = 0, \end{aligned}$$

where $y_1 = \overline{\Delta} v^h / \overline{D}$, $y_2 = \delta \cdot (1 - s) \cdot k(\bar{e}) / \overline{D}$, $y_3 = (1 - s) \cdot (1 - \bar{e} \cdot f(\bar{\theta}))$, $y_4 = -\alpha \cdot (1 - \alpha) \cdot \bar{n}^{\alpha-2} \cdot \overline{C} / \overline{D}$, $y_5 = 1 - y_1 - y_2 - y_3 - y_4$, $z_1 = \omega / [\omega - (\bar{c}^e - \bar{c}^u)]$, $z_2 = -\bar{c}^e / [\omega - (\bar{c}^e - \bar{c}^u)]$, $z_3 = 1 - z_2 - z_3$, $z_4 = \bar{e} \cdot f(\bar{\theta}) / [1 - \bar{e} \cdot f(\bar{\theta})]$, and $y_6 = (1 - s) \cdot \delta \cdot [\overline{A} / \overline{D}] \cdot [r / q(\bar{\theta})]$.

In addition we assume that the log-deviation of technology \check{a}_t follows an AR(1) process: $\check{a}_t = \nu \cdot \check{a}_{t-1} + z_t$, where $z_t \sim N(0, \sigma^2)$ is the innovation to technology driving fluctuations in the log-linear model. We compute the unique stationary rational expectations solution to the log-linear system using the standard method.²⁸ The solution allows us to compute the IRFs of variables to unexpected technology shocks, as in Figure 3.

E.4 Calibration

This section derives the relationships between the convexity μ and κ of the disutility from home production and from job search, and statistics estimated in the literature. The relationships are used to calibrate the dynamic model of Section 4.

Disutility from home production. We estimate the convexity μ of the disutility $m(y) = \omega_m \cdot y^{1+\mu} / (1 + \mu)$ of home production y . The ratio $\xi \equiv c^h / c^e = (c^u + y) / c^e$ captures the consumption drop upon unemployment. Consider a worker who receives a marginal decrease $dc^u < 0$ in benefits, which decreases his total consumption by $dc^h = dc^u + dy < 0$. The marginal consumption drop is $\varepsilon_2 \equiv dc^h / dc^u$. We relate the statistics ε_2 and ξ to μ . Then we use empirical estimates of ε_2 and ξ to calibrate μ . Differentiating the optimal choice of home production (A15),

$$\varepsilon_2 = \frac{dc^h}{dc^u} = \frac{1}{1 - \frac{v''(c^h)}{m''(y)}}.$$

Using the isoelasticity of $m(\cdot)$ and the identity $y = c^h - c^u = c^h \cdot (1 - \tau / \xi)$,

$$m''(y) = \mu \cdot \frac{m'(y)}{y} = \frac{\mu}{1 - \tau / \xi} \cdot \frac{v'(c^h)}{c^h} = -\frac{\mu}{1 - \tau / \xi} \cdot \frac{1}{\rho} \cdot v''(c^h).$$

²⁸See Anderson, Gary and George Moore, "A Linear Algebraic Procedure for Solving Linear Perfect Foresight Models," *Economics Letters*, 1985, 17(3), 247–252.

Combining these two equations gives us an expression for μ as a function of ξ and ε_2 :

$$\mu = \rho \cdot \left(1 - \frac{\tau}{\xi}\right) \cdot \frac{\varepsilon_2}{1 - \varepsilon_2}.$$

Disutility from job search. We estimate the convexity κ of the disutility $k(e) = \omega_k \cdot e^{1+\kappa}/(1+\kappa)$ from search. Let $\xi \equiv e \cdot f(\theta)$ be the hazard rate out of unemployment. Assume that the worker receives an increase $dc^u > 0$ in benefits, reduces home production by $dy < 0$, and reduces search effort by $de < 0$, which leads to a reduction $d\xi = f(\theta) \cdot de < 0$ in the hazard rate (we consider a change in benefits for one worker only, so labor market tightness θ is not affected by the policy experiment). The reduction in hazard rate is captured by the elasticity $\varepsilon_1 \equiv (c^u/\xi) \cdot (d\xi/dc^u)$. We first show how ε_1 relates to κ . Then we use empirical estimates of ε_1 to calibrate κ .

LEMMA A12. *Let $e^*(\theta, \Delta v^h)$ be the effort supply implicitly defined by the worker's utility-maximization condition (A28) in steady state:*

$$[1 - \delta \cdot (1 - s)] \cdot \frac{k'(e)}{f(\theta_t)} + \kappa \cdot \delta \cdot (1 - s) \cdot k(e) = \Delta v^h. \quad (\text{A36})$$

Assume that $\delta \approx 1$. Then the partial derivative of the effort supply $e^*(\theta, \Delta v^h)$ satisfy:

$$\frac{\Delta v^h}{e^s} \cdot \frac{\partial e^*}{\partial \Delta v^h} = \frac{1}{\kappa} \cdot \frac{u + \kappa}{1 + \kappa}.$$

Proof. Assume that $\delta \approx 1$. From the worker's optimality condition (A36):

$$\begin{aligned} \Delta v^h &= s \cdot \frac{k'(e)}{f(\theta)} + (1 - s) \cdot \kappa \cdot \frac{e}{1 + \kappa} k'(e) \\ \Delta v^h &= k'(e) \cdot \left[\frac{s}{f(\theta)} + (1 - s) \cdot \frac{\kappa}{1 + \kappa} \cdot e \right] \\ \Delta v^h &= e \cdot k'(e) \cdot \left[\frac{u}{n} + (1 - s) \cdot \frac{\kappa}{1 + \kappa} \right] \\ \Delta v^h &= e \cdot k'(e) \cdot \left[\frac{u \cdot (1 + \kappa) + (1 - u) \cdot \kappa}{n \cdot (1 + \kappa)} \right] \\ k'(e) &= \Delta v^h \cdot \frac{n}{e} \cdot \frac{1 + \kappa}{u + \kappa}. \end{aligned}$$

Since $k(e) = e \cdot k'(e)/(1 + \kappa)$ and $s/f(\theta) = u \cdot e/n$ in steady state, we get tow relationships:

$$\begin{aligned} k(e) &= \Delta v^h \cdot \frac{n}{u + \kappa} \\ s \cdot \frac{k'(e)}{f(\theta)} &= \Delta v^h \cdot \left[u \cdot \frac{1 + \kappa}{u + \kappa} \right]. \end{aligned}$$

Assuming that $\delta \approx 1$, we differentiate the optimality condition (A36) with respect to Δv^h keeping θ fixed. The log-deviations of e^* and Δv^h satisfy

$$\begin{aligned}\check{\Delta}v^h &= \kappa \cdot \check{e}^* \cdot \frac{s \cdot k'(e)/f(\theta)}{\Delta v^h} + (1 + \kappa) \cdot \check{e}^* \cdot \frac{\kappa \cdot (1 - s) \cdot k(e)}{\Delta v^h} \\ \check{\Delta}v^h &= \kappa \cdot \check{e}^* \cdot \left[u \cdot \frac{1 + \kappa}{u + \kappa} \right] + (1 + \kappa) \cdot \check{e}^* \cdot \kappa \cdot (1 - s) \cdot \frac{n}{u + \kappa} \\ \check{\Delta}v^h &= \kappa \cdot \frac{1 + \kappa}{u + \kappa} \cdot \check{e}^* \cdot [u + (1 - s) \cdot n] \\ \frac{\check{e}^*}{\check{\Delta}v^h} &= \frac{u + \kappa}{\kappa \cdot (1 + \kappa)}.\end{aligned}$$

□

Let \hat{u} be steady-state unemployment. Using Lemma A12,

$$\left. \frac{\partial \ln(\xi)}{\partial \ln(\Delta v^h)} \right|_{\theta} = \left. \frac{\partial \ln(e^*)}{\partial \ln(\Delta v^h)} \right|_{\theta} = \frac{1}{\kappa} \cdot \frac{\hat{u} + \kappa}{1 + \kappa}. \quad (\text{A37})$$

Since c^e is fixed and the provision of home production is optimal,

$$d\Delta v^h = -v'(c^h) \cdot [dc^u + dy] + m'(y) \cdot dy = -v'(c^h) \cdot dc^u.$$

In addition, if the second and higher order terms of $v(\cdot)$ are small,

$$\Delta v^h \approx v'(c^h) \cdot (c^e - c^h) + m(y).$$

Using the isoelasticity of $m(\cdot)$ and the optimality condition (A27),

$$\begin{aligned}m(y) &= y \cdot \frac{m'(y)}{1 + \mu} = \frac{c^h - c^u}{1 + \mu} \cdot v'(c^h) \\ \Delta v^h &\approx v'(c^h) \cdot c^e \cdot \left[(1 - \xi) + \frac{\xi - \tau}{1 + \mu} \right] \\ d\ln(\Delta v^h) &= \frac{d\Delta v^h}{\Delta v^h} \approx -\frac{dc^u}{c^u} \cdot \frac{\tau}{(1 - \xi) + \frac{\xi - \tau}{1 + \mu}}.\end{aligned}$$

Combining this result with (A37) implies

$$-\frac{d\xi}{\xi} \cdot \frac{c^u}{dc^u} \cdot \frac{(1 - \xi) + \frac{\xi - \tau}{1 + \mu}}{\tau} = \frac{1}{\kappa} \cdot \frac{\hat{u} + \kappa}{1 + \kappa}.$$

We conclude that κ is related to ε_1 by

$$-\frac{\varepsilon_1}{\tau} \cdot \left[(1 - \xi) + \frac{\xi - \tau}{1 + \mu} \right] = \frac{1}{\kappa} \cdot \frac{\hat{u} + \kappa}{1 + \kappa}.$$

E.5 Deficit spending

In this section, we assume that the government has access to a complete market for Arrow-Debreu securities, instead of being constrained to balance its budget each period. In the government's problem, we remove the period-by-period budget constraint (13) for each a^t and each t and replace it by a unique intertemporal budget constraint (14).

In the characterization of the optimal equilibrium, we replace the sequence of Lagrange multipliers $\{A_t(a^t), \forall a^t\}_{t=0}^{+\infty}$ that we placed on the period-by-period budget constraint (13) by a unique Lagrange multiplier A placed on the unique intertemporal budget constraint (14). The Lagrangian of the government's problem remains exactly the same, except that the multipliers A_t on the period-by-period budget constraint are replaced by the multiplier A , constant over time t and across histories a^t . The first-order conditions of the government's problem simplify accordingly. In particular the steady state of the model, in which the government faces the unique budget constraint (14), is the same as the steady state of the baseline dynamic model, in which the government faces a sequence of budget constraints (13). We also obtain the log-linear system describing the optimal equilibrium by replacing \check{A}_t by $\check{A} = 0$ in the log-linear system of the baseline model.

To be able to simulate the log-linear model and obtain the IRFs in Figure 3, however, we need to determine the Lagrange multiplier A on the intertemporal budget constraint. A is determined such that the government's intertemporal budget constraint (14) be binding. We define the deficit in period t by

$$\Lambda(S_t) = n_t \cdot c_t^e + (1 - n_t) \cdot c_t^u - n_t \cdot w(a_t).$$

where we define the vector

$$S_t = [a_t, n_t, c_t^e, c_t^u].$$

The intertemporal budget constraint (14) can be rewritten as

$$\sum_{t=0}^{+\infty} \delta^t \cdot \mathbb{E}_0[\Lambda(S_t)] = 0. \quad (\text{A38})$$

We can linearize the deficit around its steady-state value $\Lambda(\bar{S})$:

$$\begin{aligned} \Lambda(S_t) &\approx \Lambda(\bar{S}) + \bar{a} \cdot \frac{\partial \Lambda}{\partial a}(\bar{S}) \cdot \frac{da_t}{\bar{a}} + \bar{n} \cdot \frac{\partial \Lambda}{\partial n}(\bar{S}) \cdot \frac{dn_t}{\bar{n}} + \bar{c}^e \cdot \frac{\partial \Lambda}{\partial c^e}(\bar{S}) \cdot \frac{dc_t^e}{\bar{c}^e} + \bar{c}^u \cdot \frac{\partial \Lambda}{\partial c^u}(\bar{S}) \cdot \frac{dc_t^u}{\bar{c}^u} \\ \Lambda(S_t) &\approx \Lambda(\bar{S}) + \Lambda_1 \cdot \check{a}_t + \Lambda_2 \cdot \check{n}_t + \Lambda_3 \cdot \check{c}_t^e + \Lambda_4 \cdot \check{c}_t^u \\ \mathbb{E}_0[\Lambda(S_t)] &\approx \Lambda(\bar{S}) + \Lambda_1 \cdot \mathbb{E}_0[\check{a}_t] + \Lambda_2 \cdot \mathbb{E}_0[\check{n}_t] + \Lambda_3 \cdot \mathbb{E}_0[\check{c}_t^e] + \Lambda_4 \cdot \mathbb{E}_0[\check{c}_t^u], \end{aligned}$$

where $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4$ are constant. Using (A38), we infer that the intertemporal budget con-

straint (14) is a linear combination of the expected value of the log-deviations $\{\mathbb{E}_0\{\check{n}_t\}, \mathbb{E}_0\{\check{a}_t\}, \mathbb{E}_0\{\check{c}_t^e\}, \mathbb{E}_0\{\check{c}_t^u\}\}_{t=0}^{+\infty}$ and of the steady-state deficit $\Lambda(\bar{S})$.

We compute the unique stationary rational expectations solution to the log-linear system using the standard method. Let $X_t \in \mathbb{R}^k$ be the vector of log-deviations: $X_t = [\check{a}_t, \check{n}_t, \check{c}_t^e, \check{c}_t^u, \dots]'$. Let $Z_{t+1} \in \mathbb{R}^k$ be a vector of innovations at time $t+1$. In our system there is only one exogenous shock, so there is only one non-zero entry in the vector Z_{t+1} : $Z_{t+1} = [0, 0, \dots, z_{t+1}]'$ where $z_{t+1} \sim N(0, \sigma^2)$. The solution to the log-linear system satisfies

$$X_{t+1} = M_1 X_t + M_2 Z_{t+1},$$

where $M_1 \in \mathbb{R}^{k \times k}, M_2 \in \mathbb{R}^{k \times k}$ are matrices that are constant over time. Taking expectations, and using the fact that X_t is stationary: for all $t \geq 0$,

$$\mathbb{E}_0[X_t] = \mathbb{E}_0[X_{t+1}] = M_1 \mathbb{E}_0[X_t] + M_2 \mathbb{E}_0[Z_{t+1}] = M_1 \mathbb{E}_0[X_t].$$

Since all the eigenvalues from M_1 have an absolute value strictly less than one, we infer that for all $t \geq 0$, $\mathbb{E}_0[X_t] = 0$. Hence the log-linear system is such that

$$\mathbb{E}_0[\check{n}_t] = \mathbb{E}_0[\check{a}_t] = \mathbb{E}_0[\check{c}_t^e] = \mathbb{E}_0[\check{c}_t^u] = 0.$$

We conclude that the intertemporal budget constraint is satisfied by the solution to the log-linear system in a stochastic environment as long as it holds in steady-state and $\Lambda(\bar{S}) = 0$. Therefore the Lagrange multiplier A is simply obtained by solving the steady state of the model, which is the same as that of the baseline model with budget balance each period.

E.6 Duration of unemployment benefits

This section describes and studies a dynamic model in which unemployment benefits have finite duration, as in [Fredriksson and Holmlund \[2001\]](#). We introduce three superscripts: e for Employed workers; u for unemployed workers eligible to receive Unemployment benefits; a for unemployed workers whose unemployment benefits expired, and who only receive social Assistance. The consumptions of market good for each type of worker are c_t^e , c_t^u , and c_t^a . The consumptions of home good for unemployed workers are y_t^u and y_t^a . The search efforts of unemployed workers are e_t^u and e_t^a . To simplify notation, we define the following utility gains: $\Delta v_t^{u,e} \equiv v(c_t^e) - [v(c_t^u + y_t^u) - m(y_t^u)]$, $\Delta v_t^{a,e} \equiv v(c_t^e) - [v(c_t^a + y_t^a) - m(y_t^a)]$, $\Delta v_t^{a,u} \equiv \Delta v_t^{a,e} - \Delta v_t^{u,e}$.

Labor market. At the beginning of period t there are x_t^u eligible jobseekers exerting effort e_t^u , and x_t^a ineligible jobseekers exerting effort e_t^a . The number of matches h_t made is given by $h_t = h(e_t^a \cdot x_t^a + e_t^u \cdot x_t^u, o_t)$, where $e_t^a \cdot x_t^a + e_t^u \cdot x_t^u$ is aggregate search effort and o_t is vacancy. We define labor market tightness as $\theta_t \equiv o_t / (e_t^a \cdot x_t^a + e_t^u \cdot x_t^u)$. After matching, z_t^u eligible workers and z_t^a ineligible workers are unemployed. At the end of period t , a fraction λ_t of the z_t^u eligible unemployed workers

become ineligible. The stocks of workers are related by:

$$z_t^u = x_t^u \cdot (1 - e_t^u \cdot f(\theta_t)) \quad (\text{A39})$$

$$z_t^a = x_t^a \cdot (1 - e_t^a \cdot f(\theta_t)) \quad (\text{A40})$$

$$x_t^u = z_{t-1}^u \cdot (1 - \lambda_{t-1}) + s \cdot n_{t-1} \quad (\text{A41})$$

$$x_t^a = z_{t-1}^a + \lambda_{t-1} \cdot z_{t-1}^u \quad (\text{A42})$$

Government's budget. The government must balance its budget each period, so it chooses the arrival rate of ineligibility $\{\lambda_t\}_{t=0}^{+\infty}$ and consumptions $\{c_t^a, c_t^u, c_t^e\}_{t=0}^{+\infty}$ subject to the resource constraint

$$a_t \cdot n_t^\alpha - \frac{r \cdot a_t}{q(\theta_t)} \cdot [n_t - (1 - s) \cdot n_{t-1}] = n_t \cdot c_t^e + z_t^u \cdot c_t^u + z_t^a \cdot c_t^a. \quad (\text{A43})$$

Firm's problem. The firm's problem is as in the baseline model. Optimal hiring satisfies (A24).

Worker's problem. Given government policy $\{c_t^e, c_t^u, c_t^a, \lambda_t\}_{t=0}^{+\infty}$ and tightness $\{\theta_t\}_{t=0}^{+\infty}$ the representative worker chooses efforts and home productions $\{e_t^u, e_t^a, y_t^u, y_t^a\}_{t=0}^{+\infty}$ to maximize expected utility

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \{v(c_t^e) - [x_t^u \cdot k(e_t^u) + x_t^a \cdot k(e_t^a) + z_t^u \cdot \Delta v_t^{u,e} + z_t^a \cdot \Delta v_t^{a,e}]\}, \quad (\text{A44})$$

subject to the laws of motion (A39), (A40) (A41),and (A42) of the unemployment probabilities $\{x_t^u, x_t^a, z_t^u, z_t^a\}_{t=0}^{+\infty}$. Given $\{\theta_t\}_{t=0}^{+\infty}$, a choice of efforts $\{e_t^u, e_t^a\}_{t=0}^{+\infty}$ determines labor supply $\{n_t^s\}_{t=0}^{+\infty}$, which is the employment rate in period t given by

$$n_t^s = 1 - (z_t^a + z_t^u). \quad (\text{A45})$$

We form the Lagrangian of the worker's problem with multipliers A_t, B_t, C_t, D_t assigned to the laws of motion (A39), (A40) (A41),and (A42). The first-order conditions with respect to home productions y_t^u and y_t^a are

$$m'(y_t^u) = v'(c_t^u + y_t^u) \quad (\text{A46})$$

$$m'(y_t^a) = v'(c_t^a + y_t^a). \quad (\text{A47})$$

The first-order conditions with respect to efforts e_t^u and e_t^a are

$$k'(e_t^u) = f(\theta_t) \cdot A_t$$

$$k'(e_t^a) = f(\theta_t) \cdot B_t.$$

The first-order conditions with respect to unemployment probabilities x_t^u and x_t^a are

$$\begin{aligned} C_t &= k(e_t^u) + A_t \cdot (1 - e_t^u f(\theta_t)) \\ D_t &= k(e_t^a) + B_t \cdot (1 - e_t^a f(\theta_t)). \end{aligned}$$

The first-order conditions with respect to probabilities z_t^u and z_t^a are

$$\begin{aligned} A_t &= \Delta v_t^{u,e} + (1-s) \cdot \delta \cdot \mathbb{E}_t [C_{t+1}] + \lambda_t \cdot \delta \cdot \mathbb{E}_t [D_{t+1} - C_{t+1}] \\ B_t &= \Delta v_t^{a,e} + (1-s) \cdot \delta \cdot \mathbb{E}_t [D_{t+1}] + s \cdot \delta \cdot \mathbb{E}_t [D_{t+1} - C_{t+1}]. \end{aligned}$$

Combining these equations we have

$$\begin{aligned} \frac{\Delta k_t'}{f(\theta_t)} &= \Delta v_t^{a,u} + (1-\lambda_t) \cdot \delta \cdot \mathbb{E}_t [D_{t+1} - C_{t+1}] \\ \mathbb{E}_t [D_{t+1} - C_{t+1}] &= \mathbb{E}_t \left[\frac{\Delta k_{t+1}'}{f(\theta_{t+1})} - \kappa \cdot \Delta k_{t+1} \right], \end{aligned}$$

where $\Delta k_t = k(e_t^a) - k(e_t^u)$ and $\Delta k_t' = k'(e_t^a) - k'(e_t^u)$. Combining the equations once more yields

$$\frac{k'(e_t^u)}{f(\theta_t)} + (1-s) \cdot \delta \cdot \mathbb{E}_t \left[\kappa \cdot k(e_{t+1}^u) - \frac{k'(e_{t+1}^u)}{f(\theta_{t+1})} \right] = \Delta v_t^{u,e} + \lambda_t \cdot \delta \cdot \mathbb{E}_t \left[\frac{\Delta k_{t+1}'}{f(\theta_{t+1})} - \kappa \cdot \Delta k_{t+1} \right] \quad (\text{A48})$$

$$\frac{k'(e_t^a)}{f(\theta_t)} + (1-s) \cdot \delta \cdot \mathbb{E}_t \left[\kappa \cdot k(e_{t+1}^a) - \frac{k'(e_{t+1}^a)}{f(\theta_{t+1})} \right] = \Delta v_t^{a,e} + s \cdot \delta \cdot \mathbb{E}_t \left[\frac{\Delta k_{t+1}'}{f(\theta_{t+1})} - \kappa \cdot \Delta k_{t+1} \right]. \quad (\text{A49})$$

Labor market equilibrium. As in the baseline model, tightness $\{\theta_t\}_{t=0}^{+\infty}$ equalizes labor demand $\{n_t^d\}_{t=0}^{+\infty}$ to labor supply $\{n_t^s\}_{t=0}^{+\infty}$ such that (A29) holds, defining employment $\{n_t\}_{t=0}^{+\infty}$.

Equilibrium definition. An *equilibrium with unemployment insurance* is a collection of stochastic processes $\{\lambda_t, c_t^e, c_t^u, c_t^a, y_t^u, y_t^a, e_t^u, e_t^a, n_t, \theta_t\}_{t=0}^{+\infty}$ that satisfy equations (A39), (A40), (A41), (A42), (A43), (A24), (A45), (A46), (A47), (A49), and (A48).

Steady state. In steady state there are no aggregate shocks: $a_t = a$ for all t . The stocks of workers are constant over time. We can recombine the laws of motion of employment and unemployment probabilities to express $\{z_u, x_u, z_a, x_a, n\}$ as a function of $\{\lambda, \theta, e^a, e^u\}$. These steady-state relationships are useful to solve steady-state equilibria numerically.

In steady state the outflows into and outflows from social assistance are equal.

$$\begin{aligned} x_a \cdot e^a \cdot f(\theta) &= \lambda \cdot x_u \cdot [1 - e^u \cdot f(\theta)] \\ x_a &= x_u \cdot \lambda \cdot \frac{1 - e^u \cdot f(\theta)}{e^a \cdot f(\theta)}. \end{aligned}$$

The outflows from and inflows into employment are equal.

$$\begin{aligned} s \cdot n &= x_a \cdot e^a \cdot f(\theta) + x_u \cdot e^u \cdot f(\theta) \\ n &= \frac{1}{s} \cdot x_u \cdot [e^u \cdot f(\theta) \cdot (1 - \lambda) + \lambda]. \end{aligned}$$

We write the stock of unemployment at the beginning of the period in two different ways.

$$\begin{aligned} 1 - (1 - s) \cdot n &= x_a + x_u \\ 1 - \frac{1-s}{s} \cdot x_u \cdot [e^u \cdot f(\theta) \cdot (1 - \lambda) + \lambda] &= x_u \left[1 + \lambda \cdot \frac{1 - e^u \cdot f(\theta)}{e^a \cdot f(\theta)} \right]. \end{aligned}$$

Combining our previous results, we get the following relationships:

$$\begin{aligned} x_u &= \left[1 + \lambda \cdot [1 - e^u \cdot f(\theta)] \left[\frac{1}{e^a \cdot f(\theta)} + \frac{1-s}{s} \right] + \frac{1-s}{s} \cdot e^u \cdot f(\theta) \right]^{-1} \\ x_a &= \left[1 + \frac{1-s}{s} \cdot e^a \cdot f(\theta) \cdot \left\{ 1 + \frac{1}{\lambda} \cdot \left[\frac{1}{e^u \cdot f(\theta)} - 1 \right]^{-1} \right\} \right]^{-1} \\ z_u &= \left[1 + \lambda \cdot \left[\frac{1}{e^a \cdot f(\theta)} + \frac{1-s}{s} \right] + \frac{1}{s} \cdot \left[\frac{1}{e^u \cdot f(\theta)} - 1 \right]^{-1} \right]^{-1} \\ z_a &= \left[1 + \left[\frac{1}{e^a \cdot f(\theta)} - 1 \right]^{-1} \cdot \frac{1}{s} \cdot \left\{ 1 + \frac{1}{\lambda} \cdot \left[\frac{1}{e^u \cdot f(\theta)} - 1 \right]^{-1} \right\} \right]^{-1} \\ n &= \left[1 + s \cdot \left[\frac{1}{e^a \cdot f(\theta)} - 1 \right] + \frac{s}{(1-\lambda) \cdot e^u \cdot f(\theta) + \lambda} \cdot \left[1 - \frac{e^u}{e^a} \right]^{-1} \right]^{-1}. \end{aligned}$$

Optimal equilibrium. We assume that the generosity of the system of transfers is constant: there exists $\tau^{u,e}, \tau^{a,e}$ such that for all t , $\tau^{u,e} = c_t^u/c_t^e$, $\tau^{a,e} = c_t^a/c_t^e$. The government's problem is to choose a government policy $\{\lambda_t, c_t^e\}_{t=0}^{+\infty}$ to maximize social welfare (A44) over all equilibria with unemployment insurance. An *optimal equilibrium* is an equilibrium with unemployment insurance that attains the maximum of (A44). To determine numerically the optimal arrival rate $\lambda(a)$ in a steady state with technology a , we perform a grid search over a large range of arrival rates $\{\lambda_i\}$ (once we have picked λ , consumption c^e is given by the resource constraint (A43)). We pick the arrival rate λ_i such that the associated steady state maximizes social welfare. We repeat the computation for a sequence of technology $\{a_j\}$ to plot the graphs in Figure 4.

F Alternative Models of Recessions

F.1 Aggregate demand shock

This section characterizes optimal UI in a model in which recessions are caused by the combination of low aggregate demand and nominal wage rigidity. After a negative demand shock, prices fall. Nominal wage rigidity, combined with a lower price level, leads to a higher real wage and a higher marginal cost of labor, which leads to lower hiring and higher unemployment.²⁹

Wage. Assume that nominal wages are rigid. The real wage w follows a simple wage rule

$$w = \frac{\mu}{p}, \quad (\text{A50})$$

where p is the aggregate price level and μ is a parameter. The rule says that the real wage w is constant in nominal terms: $w \cdot p = \mu$.

Firm's problem. The production function is linear in employment: $g(n) = n$. Productivity is constant: workers always produce one unit of good. The firm starts with $1 - u$ workers, and decides how many additional workers to hire such that employment n maximizes real profit:

$$\pi = n - w \cdot n - \frac{r}{q(\theta)} \cdot [n - (1 - u)].$$

The first-order condition implies

$$1 = w + \frac{r}{q(\theta)}, \quad (\text{A51})$$

where w is the real wage—taken as given by the firm.

Money. Because of nominal wage rigidity, it is necessary to define the price-setting mechanism. The firm's production is sold in a perfectly competitive goods market. The firm takes the market price p as given. The aggregate demand curve on the goods market takes the simple form m/p , borrowed from the quantity theory of money. Aggregate demand m proxies for the position of the economy in the business cycle. The firm's production at a given price p determines aggregate supply of goods. When the labor market is in equilibrium the amount of goods produced is n . When the goods market is in equilibrium, the price clears the market:

$$\frac{m}{p} = n. \quad (\text{A52})$$

Equilibrium. Given the aggregate price level determined by (A52) the equilibrium real wage is a function of employment

$$w = \frac{\mu}{m} \cdot n.$$

²⁹The model loosely captures one story of the Great Depression: contractionary monetary policy lead to deflation, which raised real wages above trend in presence of nominal wage rigidity, which in turn depressed employment.

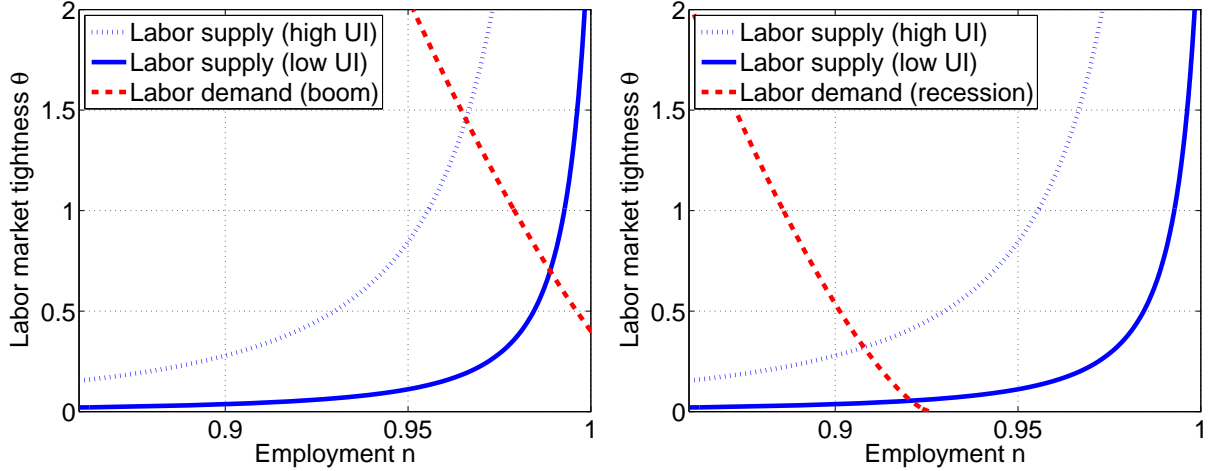


Figure A1: Labor market equilibrium in presence of demand shocks

When aggregate demand m falls, the real wage w tends to rise. Inserting the equilibrium real wage into the firm's profit-maximization condition (A51) yields a labor demand curve

$$n^d(\theta, m) = \frac{m}{\mu} \cdot \left[1 - \frac{r}{q(\theta)} \right]. \quad (\text{A53})$$

The labor supply $n^s(\theta, \Delta c)$ retains the same structure as in the model with technology shocks. Equating labor demand with labor supply curve defines implicitly equilibrium labor market tightness $\theta(m, \Delta c)$ and employment $n(m, \Delta c)$ as a function of aggregate demand m and consumption gain from work Δc . The labor market equilibrium, depicted in Figure A1, shares the same structure as the equilibrium in the text.

Jobs are also rationed in recessions. Higher employment implies more production, lower prices in the goods market, higher real wages because of nominal wage rigidity, and requires a lower tightness for firms to be willing to hire: the aggregate labor demand curve is downward sloping in a price θ -quantity n plan. If demand is low enough ($m < \mu$), the labor demand falls below zero for $n < 1$: jobs are rationed.

Business cycle fluctuations. We focus on the case with log utility: $v(c) = \ln(c)$. We parameterize the equilibrium of the model with (m, τ) . We have the following comparative statics for equilibrium variables:

$$\left. \frac{\partial \theta}{\partial m} \right|_{\tau} > 0, \quad \left. \frac{\partial e}{\partial m} \right|_{\tau} > 0, \quad \left. \frac{\partial n}{\partial m} \right|_{\tau} > 0.$$

The result corresponds to Lemma A5. The proof is identical to that of Lemma A5 because, even if the labor demand is different in the model with demand shocks, it remains true that $\partial n^d / \partial \theta < 0$, $\partial n^d / \partial m > 0$.

Optimal UI formula. In the model, real wages respond to UI because UI affects equilibrium tightness, equilibrium employment, equilibrium price level, and eventually equilibrium real wage because the nominal wage is rigid. In the optimal UI formula, we must account for the impact of UI on the government's budget through wages. For instance if higher UI implies higher wages, then higher UI has an additional beneficial effect because it increases the tax base. Of course the wage increase is partly at the cost of firm's profits. Thus we account for fluctuations in profits for consistency. The appropriate optimal UI formula in this framework is given by (A20). As in the text, assume that $n \approx 1$ and that the third and higher order terms of $v(\cdot)$ are small. The formula simplifies to

$$\frac{\tau}{1-\tau} = \frac{\rho}{\varepsilon^M} \cdot (1-\tau) + \frac{\kappa}{\kappa+1} \cdot \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right) \cdot \left[1 + \frac{\rho}{2} \cdot (1-\tau) - \frac{1}{\Delta c} \cdot \frac{\eta}{1-\eta} \cdot \frac{r}{q(\theta)} \right]. \quad (\text{A54})$$

The government's budget constraint combined with (A51) imposes

$$n \cdot c^e + (1-n) \cdot c^u = n \cdot w = n \cdot \left\{ 1 - \frac{r}{q(\theta)} \right\}.$$

With $n \approx 1$, $c^e \approx 1 - r/q(\theta)$. The formula simplifies to

$$\frac{\tau}{1-\tau} = \frac{\rho}{\varepsilon^M} \cdot (1-\tau) + \frac{\kappa}{\kappa+1} \cdot \left(\frac{\varepsilon^m}{\varepsilon^M} - 1 \right) \cdot \left[1 + \frac{\rho}{2} \cdot (1-\tau) - \frac{1}{1-\tau} \cdot \frac{\eta}{1-\eta} \cdot \frac{1}{[q(\theta)/r] - 1} \right]. \quad (\text{A55})$$

Elasticities. We now study the elasticities ε^m and ε^M in the model with demand shocks. We first examine the elasticity wedge $\varepsilon^m/\varepsilon^M$. We differentiate the labor demand condition (A53).

$$\frac{dn}{d\Delta c} = -\frac{m}{\mu} \cdot \eta \cdot \frac{r}{q(\theta)} \cdot \frac{1}{\theta} \cdot \frac{d\theta}{d\Delta c}.$$

Using Lemma 1, which remains valid because the structure of labor supply has not changed, and Definition 1 of elasticity ε^M :

$$(1-n) \cdot \varepsilon^M = -\frac{m}{\mu} \cdot \frac{r}{q(\theta)} \cdot \frac{\kappa}{\kappa+1} \cdot \frac{1-n}{h} \cdot \frac{\eta}{1-\eta} \cdot (\varepsilon^M - \varepsilon^m)$$

$$\left[\frac{\varepsilon^m}{\varepsilon^M} - 1 \right] = \frac{\mu}{m} \cdot \frac{q(\theta)}{r} \cdot \frac{\kappa+1}{\kappa} \cdot h \cdot \frac{1-\eta}{\eta}.$$

Under Assumption 4 we can write

$$\left[\frac{\varepsilon^m}{\varepsilon^M} - 1 \right] = \varkappa \cdot \frac{q(\theta)}{r} \cdot n \cdot \frac{\mu}{m},$$

where \varkappa is a constant under Assumption 3, defined by

$$\varkappa \equiv \frac{1-\eta}{\eta} \cdot \frac{\kappa+1}{\kappa} \cdot s > 0.$$

Finally, using the labor demand condition (A53),

$$\left[\frac{\varepsilon^m}{\varepsilon^M} - 1 \right] = \varkappa \cdot \left[\frac{q(\theta)}{r} - 1 \right] > 0.$$

There is a positive wedge $\varepsilon^m > \varepsilon^M$ between micro- and macro-elasticity, as in the model with technology shocks (Proposition 2). The wedge widens in recessions. Since $\partial\theta/\partial m|_\tau > 0$ and $q(\cdot)$ is decreasing, $\partial [\varepsilon^m/\varepsilon^M] / \partial m|_\tau < 0$. The result corresponds to Part 1 of Proposition 3.

To determine the cyclicity of the macro-elasticity ε^M , we need to derive an equation equivalent to (A6) in presence of demand shocks ((A6) is only valid in a context in which, unlike here, real wages do not respond to UI). We start from the budget constraint.

$$\begin{aligned} c^u &= n \cdot (w - \Delta c) = n \cdot \left(n \cdot \frac{\mu}{m} - \Delta c \right) \\ \frac{dc^u}{d\Delta c} &= \varepsilon^M \cdot (1-n) \cdot \left[2 \cdot \frac{w}{\Delta c} - 1 \right] - n \\ \frac{dc^u}{d\Delta c} &= \varepsilon^M \cdot (1-n) \cdot \left[\frac{2}{n} \cdot \frac{\tau}{1-\tau} + 1 \right] - n. \end{aligned}$$

Under the assumption that $v(c) = \ln(c)$,

$$\begin{aligned} \frac{d\Delta v}{d\Delta c} &= \frac{1}{c^e} + \left[\frac{1}{c^e} - \frac{1}{c^u} \right] \cdot \frac{dc^u}{d\Delta c} \\ \frac{d\Delta v}{d\Delta c} &= \left[(1-n) \cdot \frac{1}{c^e} + n \cdot \frac{1}{c^u} \right] + \left[\frac{1}{c^e} - \frac{1}{c^u} \right] \cdot \varepsilon^M \cdot (1-n) \cdot \left[\frac{2}{n} \cdot \frac{\tau}{1-\tau} + 1 \right]. \end{aligned}$$

Equation (A5) remains valid. With log utility, it can be written

$$\frac{\ln(1/\tau)}{1-\tau} \cdot \frac{\kappa}{s} \cdot \varepsilon^m = c^e \cdot \frac{n}{1-n} \cdot \frac{d\Delta v}{d\Delta c}.$$

Using these two results,

$$\begin{aligned} \frac{\ln(1/\tau)}{1-\tau} \cdot \frac{\kappa}{s} \cdot \varepsilon^m &= \left[n + \frac{n^2}{1-n} \cdot \frac{1}{\tau} \right] - \frac{1-\tau}{\tau} \cdot \varepsilon^M \cdot \left[2 \cdot \frac{\tau}{1-\tau} + n \right] \\ \varepsilon^M &= \left[1 + \frac{n}{1-n} \cdot \frac{1}{\tau} \right] \cdot \left[\frac{\kappa}{s} \cdot \frac{1}{n} \cdot \frac{\varepsilon^m}{\varepsilon^M} \cdot \frac{\ln(1/\tau)}{1-\tau} + \frac{2}{n} + \frac{1-\tau}{\tau} \right]^{-1}. \end{aligned}$$

$\partial n / \partial m|_\tau > 0$, $\partial [\varepsilon^m/\varepsilon^M] / \partial m|_\tau < 0$, so $\partial \varepsilon^M / \partial m|_\tau > 0$. The result corresponds to Part 2 of Proposi-

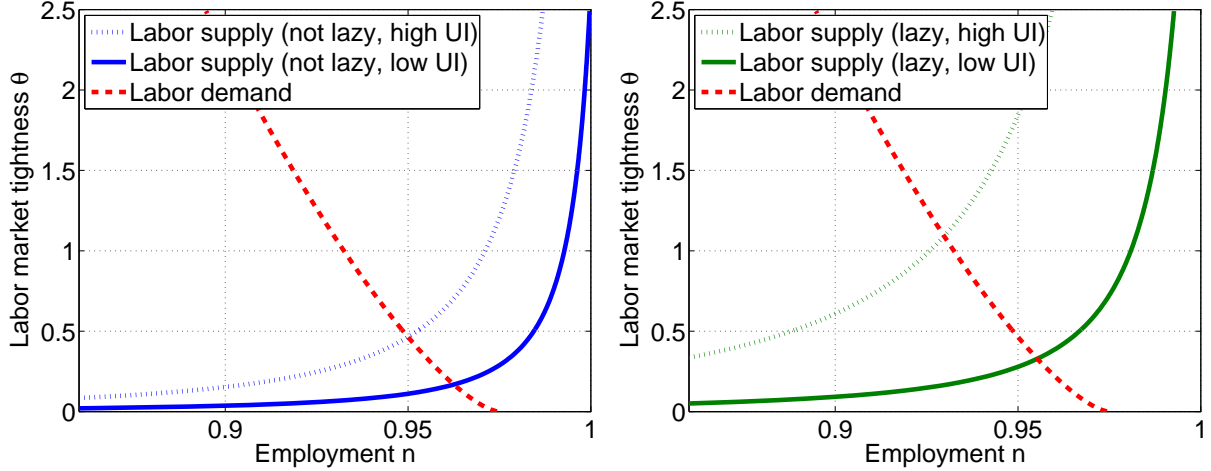


Figure A2: Labor market equilibrium in presence of preference shocks

tion 3.

Optimal replacement rate over the business cycle. Using optimal UI formula (A55), the fact that $\partial q(\theta)/\partial m|_{\tau} < 0$, as well as our results that $\partial [\varepsilon^m/\varepsilon^M]/\partial m|_{\tau} < 0$ and $\partial \varepsilon^M/\partial m|_{\tau} > 0$, we infer that the optimal replacement rate τ is countercyclical: $d\tau/dm < 0$. The economic mechanism is the same as in the model with technology shocks in the text: the moral hazard cost of UI falls in recession, while the value of UI as a correction of the rat-race externality rises; hence it is optimal to increase the generosity of UI in recessions.

F.2 Preference shock

We assume that technology a remains constant at $a = 1$. Instead of technology shocks, recessions are driven by shocks to the disutility from search.

Worker's problem. A worker's utility is $v(c) - \lambda \cdot k(e)$, where λ is a preference parameter that indicates the disutility of search. Fluctuations in λ drive the business cycle. Given labor market tightness θ and consumptions c^e and c^u , a jobseeker chooses effort e to maximize expected utility $v(c^u) + e \cdot f(\theta) \cdot \Delta v - \lambda \cdot k(e)$. The optimal job-search effort satisfies the following first-order condition:

$$k'(e) = f(\theta) \cdot \frac{\Delta v}{\lambda}. \quad (\text{A56})$$

As the disutility from effort $k(\cdot)$ is convex and the job-finding rate $f(\cdot)$ is increasing, the optimal effort e increases with tightness θ , increases with the utility gain from working Δv , and decreases with the preference parameter λ .

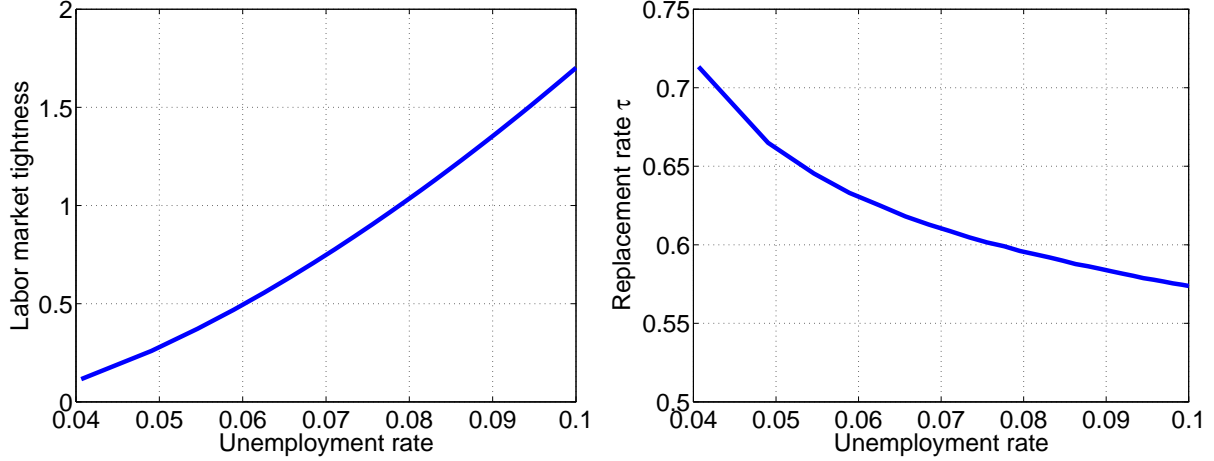


Figure A3: Optimal unemployment insurance over a business cycle driven by preference shocks

Equilibrium. The labor market equilibrium is depicted in Figure A2. It shares the same structure as the labor market equilibrium in the text. The only difference is the response of the economy to a shock. When λ increases, search becomes more costly, effort supply $e^s(\theta, \Delta c, \lambda)$ diminishes for a given θ , and the labor supply curve $n^s(\theta, \Delta c, \lambda) \equiv n^*(e^s(\theta, \Delta c, \lambda), \theta)$ shifts left. Equilibrium employment falls, unemployment increases, and labor market tightness increases. Periods with higher λ are “recessions” because they are periods with higher unemployment. However, these periods are unrealistic because they combine high unemployment with high labor market tightness. In reality tightness falls when unemployment increases.

Optimal unemployment insurance formula. Looking at the derivation of the optimal UI formula in the text (proof of Proposition 1), it appears that replacing the disutility of effort $k(e)$ by $\lambda \cdot k(e)$ does not influence the optimal UI formula because the preference parameter λ does not affect the convexity κ of $k(e)$.

Elasticities. We now study the elasticities ϵ^m and ϵ^M in the model with preference shocks. Our goal is to determine whether optimal UI should be procyclical or countercyclical. Proposition 2 remains valid: under Assumptions 2, 3, and 4 the elasticity wedge is given by

$$\frac{\epsilon^m}{\epsilon^M} = 1 + \chi \cdot q(\theta) \cdot n^{\alpha-1},$$

where $\chi = \alpha \cdot (1 - \alpha) \cdot [(1 - \eta)/\eta] \cdot [(1 + \kappa)/\kappa] \cdot (s/r)$ is constant. In recessions, n decreases while θ increases. So $q(\theta)$ decreases while $n^{\alpha-1}$ increases. To determine the cyclicity of the wedge ϵ^m/ϵ^M , note that the firm’s profit-maximization condition (11) implies

$$q(\theta) \cdot n^{\alpha-1} = \frac{w}{\alpha} \cdot q(\theta) + \frac{r}{\alpha}.$$

In recessions the wage w remains constant so that the right-hand side of the equation decreases as $q(\theta)$ decreases. Hence $q(\theta) \cdot n^{\alpha-1}$ decreases. The elasticity wedge $\varepsilon^m/\varepsilon^M$ becomes procyclical, whereas it was countercyclical in presence of technology shocks (Proposition 3). In general, we cannot conclude on the cyclicity of ε^M given τ . We conjecture that the cyclicity of the optimal replacement rate τ depends on parameter values. Therefore we resort to simulations to describe the optimal replacement rate over the business cycle.

Simulations. The results from the simulation of the model with preference shocks are displayed in Figure A3. All computations are based on the dynamic model calibrated in Table 1 (the calibration does not need to change even if the source of shock is different). The optimal replacement rate is procyclical: it increases from 58% to 72% when the unemployment rate decreases from 10% to 4%. Labor market tightness increases sharply in recessions, making this model of the business cycle unrealistic.