Optimal Minimum Wage in Competitive Labor Markets[*]

Preliminary and Incomplete

David Lee, Princeton

Emmanuel Saez, UC Berkeley

December 5, 2007

Abstract

This paper presents a theoretical analysis of optimal minimum wage policy in a perfectly competitive labor market. Introducing a minimum wage is desirable if the government values redistribution toward low wage workers. This result remains true in the presence of optimal nonlinear taxes and transfers. In that context, a minimum wage effectively rations low skilled labor which is subsidized by the optimal tax/transfer system, and improves upon the second-best tax/transfer optimum. We derive formulas for the optimal minimum wage (with or without optimal taxes) as a function of the elasticities of labor supply and demand and the redistributive tastes of the government. The optimal minimum wage decreases with the demand elasticity for low skilled labor but increases with the supply elasticity of low skilled labor. The optimal minimum wage follows an inverted U-shape as a function of the strength of the redistributive tastes of the government. We present a number of numerical simulations to illustrate those results. When labor supply is along the extensive margin, a minimum wage should always be associated with in-work subsidies: the co-existence of minimum wages with high participation tax rates for low skilled workers is (second-best) Pareto inefficient.

1 Introduction

The minimum wage is a widely used but controversial policy tool. Minimum wages are a potentially useful tool for redistribution because they increase low skilled workers wages at the expense of other factors of production such as higher skilled labor or capital. Minimum wages, however, may also create involuntary unemployment and hence worsen the welfare of workers who happen to loose their jobs because of the minimum wage. Most OECD countries have adopted minimum wage policies over the course of the 20th century. In a number of OECD countries, and especially in continental Europe, the minimum wage binds for a non-trivial share of the workforce (see e.g., OECD 1998, Immervoll, 2007). A very large empirical literature has analyzed whether minimum wages has an effect on employment and wages of low skilled workers (see e.g., Brown et al. 1982, Card and Krueger 1995, Dolado et al. 1996, Brown 1999, or Neumark and Wascher 2006 for extensive surveys). The normative literature on the minimum wage is much less extensive. This paper proposes a normative analysis of the optimal minimum wage in a simple competitive labor market model and using standard social welfare analysis. We make the important "efficient rationing" assumption that unemployment induced by the minimum wage hits workers with the lowest surplus first. Our main goals are to bring about explicitly the trade-offs involved when setting a minimum wage policy and understand how a minimum wage policy should complement optimal tax and transfer policy.

The first part of our paper focuses on the case with no taxes and transfers. Although this is not a realistic case, it has the advantage of simplicity and it shows very transparently the key forces at play when setting optimal minimum wage policy. Such an exercise amounts to formalizing the pros and cons of the minimum wage that have been discussed primarily in an informal way in the labor literature (see e.g. Stigler 1946 for an early analysis and Freeman 1996, and Dolado, Felgueroso, Jimeno 2000, for recent expositions). We show that a minimum wage is desirable as long as the government values redistribution from high wage to low wage workers and the demand elasticity of low skilled labor is finite and the supply elasticity of low skilled labor is positive. We then derive an optimal minimum wage formula as a function of those demand and supply elasticities of low skilled labor and the redistributive tastes of the government. Unsurprisingly, the optimal minimum wage is decreasing in the demand elasticity because a minimum wage has larger unemployment effects when the demand

elasticity is higher. The optimal minimum wage is increasing in the supply elasticity because a high supply elasticity implies that workers have a low surplus from working (as many would leave the labor force if the wages were slightly reduced). The size of the optimal minimum wage follows an inverted U-shape with the strength of the government redistributive tastes: There is no point in using a minimum wage if the government does not value redistributive. If the government has extreme Rawlsian preferences, then the costs of involuntary unemployment swamp the value of transfers to low skilled workers.

Obviously, the minimum wage is not the only tool the government can use for redistribution. Taxes and transfers play a major redistributive role in OECD countries. Therefore, the second part of our paper analyzes the desirability of the minimum wage when the government also uses taxes and transfers for redistribution. As described below in more detail, our key innovation is to abstract from the hours of work decision and focus only in the job choice and work participation decision. In that context, the government observes only occupation choices and the corresponding wage but not the utility work costs incurred by individuals. Therefore, the informational constraints that the government faces when imposing a minimum wage policy and a nonlinear tax/transfer system are well defined and mutually consistent. In such a model, we show that a minimum wage is desirable as long as the government values redistribution toward low skilled workers (and assuming again that the low skilled work demand elasticity is finite and the supply elasticity is positive). Such a situation is very likely to hold in practice suggesting that the minimum wage is a useful tool to supplement taxes and transfers for redistribution purposes. This result can be seen as an application of the Guesnerie (1981) and Guesnerie and Roberts (1984) theory of quantity controls in second best economies: When the government values redistribution toward low skilled workers, the optimal tax/transfer system over-encourages the supply of low skilled labor. In that context, a minimum wage effectively rations over-supplied low skilled labor which is socially desirable. Put in another way, with a minimum wage rationing low skilled jobs, the government can increase redistribution toward those low skills workers without inducing any adverse supply response.

We also derive optimal formulas for the jointly optimal tax/transfer system and minimum wage. Once a minimum wage is binding, transfers to minimum wage workers do not create labor supply responses and hence are effectively lumpsum, allowing the government to transfer substantial resources to minimum wage workers. When labor supply responses are along the

participation margin only, we show that a minimum wage should always be associated with in-work subsidies. This implies that countries using minimum wages while imposing positive tax rates on work at the bottom are in a second-best Pareto inefficient situation. From such a situation, cutting taxes on low income workers while reducing the (pre-tax) minimum wage leads to a Pareto improvement. Our formulas and numerical simulations show that the optimal minimum wages in such models with optimal taxes are also decreasing in the demand elasticity for low skilled work, are increasing in the supply elasticity for low skilled work, and follow an inverted U-shape pattern with the strength of redistributive tastes. Optimal minimum wages with optimal tax/transfers are actually quantitatively comparable to optimal minimum wages in the no tax situation described above. Importantly, numerical simulations confirm the finding that minimum wages should be associated with larger in-work subsidies than in the optimal tax/transfer system with no minimum wage.

In order to place our paper in its proper context, it is useful to summarize briefly the normative literature on the minimum wage. The basic point that a large demand elasticity for low skilled workers implies that the negative unemployment effects will be large has been recognized for a long time. A well known related point is that, if the demand elasticity is larger than one in absolute value, then a minimum wage reduces total pay going to low skilled workers (see e.g., Freeman 1996 or Danziger, 2006 for recent expositions). It is also well known, at least since Robinson (1933), that if the labor market is monopsonistic, then a minimum wage can actually increase both employment and low skilled wage and hence improve efficiency (see e.g., Card and Krueger 1995 or Manning 2003 for recent expositions). A number of papers have shown that the monopsony logic for the desirability of the minimum wage extends to other models of the labor market with frictions or informational asymmetries such as efficiency wages (Drazen, 1986, Rebitzer and Taylor, 1995), bargaining models (Cahuc, Zylberberg, and Saint-Martin, 2001), signalling models (Lang, 1987) or search models (Swinnerton, 1996, Flinn, 2006). Most of those paper focus solely on efficiency and abstract from redistributive aspects and do not incorporate taxes and transfers.

A small literature in public economics has investigated whether minimum wages remain desirable when the government can also use taxes and transfers for redistribution. Most models have adopted the standard Mirrlees (1971) model on optimal taxation or the discrete version

developed by Stiglitz (1982). The general principle, following Allen (1987) and Guesnerie and Roberts (1987), is that a minimum wage is desirable if it expands the redistributive power of the government by relaxing incentive constraints. In the context of the two-skill Stiglitz (1982) model, Allen (1987) shows that a minimum wage can sometimes usefully supplement an optimal linear tax but is never useful to supplement an optimal nonlinear tax.[1] In contrast, Marceau and Boadway (1994) and Boadway and Cuff (2001) develop models where the minimum wage helps relaxing the incentive constraints that the government face, and show that, in that context, a minimum wage is actually desirable. Marceau and Boadway (1994) add the unemployment insurance tool to the Allen (1987) model. As only low skilled workers can become unemployed because of the minimum wage, the unemployment insurance can efficiently tag low skilled workers. Boadway and Cuff (2001), using a continuum of skills model as in Mirrlees (1971), show that a minimum wage policy implicitly allows the government to observe skills at the bottom and this feature can be exploited to improve upon the standard Mirrlees (1971) allocation.

As recognized by Guesnerie and Roberts (1987), all those models create an informational inconsistency within the government. The nonlinear income tax is based on earnings only because hours of work and wage rates are not observable separately. If wage rates were directly observable, the government could base taxes directly on wage rates and attain any first best allocation. However, implementing a minimum wage policy requires observing wage rates of low skilled workers. The negative results of Allen (1987) are the consequence of the fact that the income tax does not try to exploit the information advantage generated by the minimum wage option while the positive results of Marceau and Boadway (1994) and Boadway and Cuff (2001) can be seen as the consequence of twisting or supplementing the income tax in order to take some advantage of the information created by the minimum wage policy.[2] As mentioned above, the most important innovation in our paper is to use a model of job choice that resolves this informational inconsistency: In our model, the minimum wage and the taxes and transfers are based on the same observable information and therefore the government

---

[1]This result is obtained because a minimum wage does not prevent in any way high skilled workers from imitating low skilled workers in the Stiglitz (1982) model. This is in contrast to our occupational model, and we will come back to this important difference.

[2]Some papers have actually explicitly modelled limitations on the use of taxes and transfers using political economy arguments. In that context, a minimum wage can be a useful tool for redistribution (see e.g., Dreze and Gollier 1993 and Bacache and Lehmann 2005).

fully optimizes along both the tax/transfer and the minimum wage dimension using all the information available.

Recently, some papers have explored the issue of joint optimal minimum wages and optimal taxes and transfers in imperfect labor markets. Blumkin and Sadka (2005) consider a signalling model where employers do not observe productivities perfectly and show that a minimum can be desirable to supplement the optimal tax system in that context. Cahuc and Laroque (2007) show that, in a monopsonistic labor market model, with participation labor supply responses only, the minimum wage should not be used when the government can use optimal nonlinear income taxation.[3] Hungerbuhler and Lehmann (2007) analyze a search model and show that a minimum wage can improve welfare even with optimal income taxes if the bargaining power of workers is sufficiently low. However, if the government can directly increase the bargaining power of workers, then the desirability of the minimum wage vanishes. Those latter two papers are closest to ours because they also abstract from the hours of work choice and consider only the participation margin for labor supply. Our paper considers the simpler case of perfect competition with no frictions while those papers consider more complex labor market situations. Therefore, we see our contribution as complementary to those of Cahuc and Laroque (2007) and Hungerbuhler and Lehmann (2007).

The remaining of the paper is organized as follows. Section 2 presents the basic model and optimal minimum wage analysis in a situation with no taxes. Section 3 introduces taxes and transfers and analyzes joint optimal minimum wage policy and taxes and transfers. Section 4 discusses caveats, directions for future work, and concludes.

2 Optimal Minimum Wage with no Taxes/Transfers

2.1 Basic Model

• Demand Side

We consider a simple two sector model where production $F(h_1, h_2)$ depends on the number of low skilled workers h_1 and the number of high skilled workers h_2.[4] We assume perfectly

---

[3]Importantly, Cahuc and Laroque (2007) argue that a minimum wage can be replicated with a sufficiently nonlinear income tax system. We argue below, that this result, although formally correct, is actually misleading to understand the desirability of minimum wages.

[4]We show below that the model can be generalized to include a capital input and more than two labor inputs.

5

competitive markets so that firms take the wages (w_1, w_2) as given. The production sector chooses labor demand (h_1, h_2) to maximize profits: $\Pi = F(h_1, h_2) - w_1 h_1 - w_2 h_2$, which leads to the standard first order conditions where wages are equal to marginal product:

$$w_i = \frac{\partial F}{\partial h_i}, \tag{1}$$

for $i = 1, 2$. We assume that, in any equilibrium, $w_1 < w_2$. We also assume constant returns to scale so that there are no profits in equilibrium: $\Pi = 0$. We denote by $\sigma = d\log(h_1/h_2)/d\log(w_2/w_1)$ the elasticity of substitution between high and low skills in the production function.[5]

- **Supply Side**

We assume that each individual is either low skilled or high skilled. We normalize the population of workers to one and denote by h_1^0 and h_2^0 the fraction of low and high skilled with $h_1^0 + h_2^0 = 1$. We assume that each worker faces a cost of working θ representing disutility of work. In order to generate smooth supply curves, we assume that θ is distributed according to smooth cumulated distributions $P_1(\theta)$ and $P_2(\theta)$ for low and high skill workers respectively. There are three groups of individuals: group 0 for workers (either low or high skilled) out of work (earning 0), group 1 for low skilled workers (earning w_1), and group 2 for high skilled workers (earning w_2). We denote by h_i the fraction of individuals in each group $i = 0, 1, 2$.

In this section, we assume that there are no taxes and transfers. For simplicity and throughout the paper, we assume no income effects in the labor supply decision.[6] An individual with skill i and cost of work θ makes its binary labor supply decision $l = 0, 1$ in order to maximize utility $u = w_i \cdot l - \theta \cdot l$. Therefore, $l = 1$ if and only if $\theta \leq w_i$. Hence, the aggregate labor supply functions for $i = 1, 2$ are:

$$h_i = h_i^0 \cdot P_i(w_i). \tag{2}$$

We denote by e_i the elasticity of labor supply h_i with respect to the wage rate w_i:

$$e_i = \frac{w_i}{h_i} \frac{\partial h_i}{\partial w_i} = \frac{w_i \cdot p_i(w_i)}{P_i(w_i)},$$

where p_i is the density distribution of θ.

---

[5]For example, this elasticity is constant in the case of CES production function $F(h_1, h_2) = (a_1 h_1^{(\sigma-1)/\sigma} + a_2 h_2^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}$, which we will use in numerical simulations.
[6]We discuss in Section XX how our results would be affected in a situation with income effects.

Combining the demand and supply side equations (1) and (2) defines a single undistorted competitive equilibrium denoted by $(w_1^*, w_2^*, h_1^*, h_2^*)$.

2.2 Desirability of the Minimum Wage

Starting from the market equilibrium $(w_1^*, w_2^*, h_1^*, h_2^*)$ and as illustrated on Figure 1, we introduce a *small* minimum wage just above the low skill wage w_1^*, which we denote by $\bar{w} = w_1^* + d\bar{w}$. Formally, the small minimum wage creates changes dw_1, dw_2, dh_1, dh_2 in our key variables of interest. By definition, $dw_1 = d\bar{w}$.

From $\Pi = F(h_1, h_2) - w_1 h_1 - w_2 h_2$, we have $d\Pi = \sum_i [(\partial F / \partial h_i) dh_i - w_i dh_i - h_i dw_i] = -h_1 dw_1 - h_2 dw_2$ using (1). The no profit condition $\Pi = 0$ implies that $d\Pi = 0$ and hence:

$$h_1 dw_1 + h_2 dw_2 = 0. \tag{3}$$

Equation (3) is fundamental and shows that the earnings gain of low skilled workers $h_1 dw_1 > 0$ due to the small minimum wage is exactly compensated by an earnings loss of high skilled workers $h_2 dw_2 < 0$. If the government values redistribution from high skilled workers to low skilled workers, such a transfer is socially desirable.[7]

However, in addition to this transfer, the minimum wage also creates involuntary unemployment. As depicted on Figure 1, the minimum wage creates excess supply so that equation $h_1 = h_1^0 P_1(w_1)$ no longer holds and h_1 is defined by the demand equation. Hence $dh_1 = -\eta_1 h_1 / w_1$ where η_1 is the low skilled labor demand elasticity (defined with a minus sign so that $\eta_1 > 0$).[8] To evaluate the welfare cost of the involuntary unemployment, we will make the important assumption of *efficient rationing*.

Assumption 1 Efficient Rationing: *Workers who involuntary loose their job because of the minimum wage are those with the highest costs of work (and hence those with the least surplus from working).*

Unfortunately, there is relatively little empirical evidence on this issue which is critical for the welfare analysis of minimum wage policy. Three points can be made to defend assumption

---

[7]More generally, with more labor inputs and capital inputs, the loss will be distributed among all the other production inputs, with the closest substitutes to low skilled workers carrying most of the burden.

[8]Formally, we can derive an expression for η_1 as follows. Using the definition of the substitution elasticity σ, we have $dh_2/h_2 - dh_1/h_1 = \sigma(dw_1/w_1 - dw_2/w_2)$ and using the supply equation for high skilled labor: $dh_2/h_2 = e_2 dw_2/w_2$, and from (3), the fact that $h_2 dw_2 = -h_1 dw_1$, we have: $\eta_1 = \sigma + (e_2 + \sigma)(h_1 w_1)/(h_2 w_2)$. When the low skilled sector is small (relative to the other sectors), the second term is small, and the demand elasticity is closely approximated by the substitution elasticity σ.

1. First, this assumption is more likely to hold in markets where there is frequent turnover and where individuals with the highest surplus from working are willing to spend more effort to increase their chance of getting a job.[9] This is probably a good approximation of the low skilled US labor market (see e.g., Card and Krueger 1995) although this might not hold in European countries where turnover is lower. Second, empirical work on the effects of the minimum wage in the United States has shown that increases in the minimum wages reduces turnover (see e.g., Dube, Naidu, and Reich 2007 for a recent study in the case of citywide minimum wages). The reduction in turnover implies that minimum wage workers derive on average higher surplus when the minimum wage increases. This implies that the rationing induced by the minimum wage is not completely inefficient.[10] Finally, Luttmer (2007) has proposed a recent direct empirical test of efficient rationing in the United States using variation in minimum wage across the US states and finds support for the assumption that the minimum wage does not lead to inefficient allocation of labor.[11] Obviously, the case with efficient rationing is the most favorable to minimum wage policy, a point we will come back to.

Under efficient rationing, as can be seen on Figure 1, the welfare loss due to involuntary unemployment due to the minimum wage is second order (exactly as in the standard Harberger deadweight burden analysis). Therefore, we can state our first result.

Proposition 1 *With no taxes and transfers and under the efficient rationing assumption 1, introducing a minimum wage is desirable if (1) the demand elasticity for low skilled workers is finite, (2) the supply elasticity of low skilled workers is positive, and (3) the government values redistribution from high skilled workers toward low skilled workers.*

It is useful to analyze briefly the desirability of the minimum wage when either of those three conditions breaks down. Obviously, condition (3) is necessary because if the government does not value the transfer created by the minimum wage, the minimum wage only creates deadweight burden and is therefore not desirable. If the government values redistribution, this condition naturally holds, except in the extreme case where the government have Rawlsian

---

[9]Efficient rationing can be justified under a standard Coasian argument: a worker with low surplus would be willing to let an unemployed worker with high surplus from working take his job in exchange for a private transfer.

[10]If workers with the highest surplus lost their jobs first, then the average surplus of workers would not increase with an increase in the minimum wage, as can be seen from Figure 1.

[11]This is in contrast to a situation with low turnover as in the housing market with rent control as in Glaeser and Luttmer (2003).

preferences and cares only about those out of work and hence values equally (at zero) marginal income to low and high skilled workers.

Condition (1) is also necessary. If the demand elasticity is infinite (which in the context of our simple model is equivalent to assuming that low and high skill workers are perfect substitutes, $\sigma = \infty$ and hence $F = a_1 h_1 + a_2 h_2$), then any minimum wage set above w_1^* will shut down entirely the low skilled labor market and hence cannot be desirable.

When condition (2) breaks down and the supply elasticity is zero, then there are no marginal workers with no surplus from working. Therefore, the unemployment welfare loss is no longer second order. In that context, whether a minimum wage is desirable depends on the parameters of the model (reservation wages of low skilled workers and the size of the demand elasticity).[12] Finally, when the efficient rationing assumption does not hold, then the case for the minimum wage depends on the degree of inefficiency in the rationing along with the other parameters of the model.

2.3   Derivation in the Basic Model

Let us now derive the optimal minimum wage when the conditions of Proposition 1 are met. As displayed in Figure 2, with a non infinitesimal minimum wage $\bar{w} > w_1^*$, we can define $\underline{w}$ as the reservation wage (or equivalently the cost of work) of the marginal low skilled worker (i.e., the worker getting the smallest surplus from working). Formally, $\underline{w}$ is defined so that $h_1^0 P_1(\underline{w}) = D_1(\bar{w})$ where $D_1(\bar{w})$ is the demand function for low skilled work when the minimum wage is $\bar{w}$.

We assume that the government evaluates outcomes using a utilitarian social welfare function of the form: $SW = \int G(u)d\nu$ where $u \to G(u)$ is an increasing and concave function. The concavity of $G(.)$ can represent either the individuals risk aversion and/or the redistributive tastes of the government. Given the structure of our basic model, we can write social welfare as:

$$SW = (1 - h_1 - h_2)G(0) + h_1^0 \cdot \int_0^{\underline{w}} G(w_1 - \theta)p_1(\theta)d\theta + h_2^0 \cdot \int_0^{w_2} G(w_2 - \theta)p_2(\theta)d\theta. \quad (4)$$

The government picks $\bar{w}$ to maximize SW subject to the constraints that $w_i = \partial F/\partial h_i$ for $i = 1, 2$ and $h_2 = h_2^0 P_2(w_2)$. In order to obtain a first order condition for the optimal minimum

[12]The well known result that a minimum wage cannot be desirable if $\eta_1 > 1$ is based on such a model with fixed labor supply.

wage $\bar{w}$, we consider a small change $d\bar{w}$ around $\bar{w}$. Figure 2 shows that this change has two effects.

First, it creates a transfer $h_1 d\bar{w}$ toward low skilled workers at the expense of high skilled workers (as $h_2 dw_2 = -h_1 d\bar{w}$). The net social value of this transfer is $dT = [g_1 - g_2]h_1 d\bar{w}$ where $g_i = \int G'(w_i - \theta)p_i d\theta/P_i$ the average social marginal welfare weight on workers of skill i.

Second, the minimum wage increases involuntary unemployment by $dh_1 = -\eta_1 h_1 d\bar{w}/\bar{w}$. By definition, those marginal workers have a reservation wage equal to $\underline{w}$. Therefore, each worker becoming unemployed generates a social welfare cost equal to $G(\bar{w} - \underline{w}) - G(0)$. We can define $g_0^a = [G(\bar{w} - \underline{w}) - G(0)]/(\bar{w} - \underline{w})$ as the marginal welfare weight put on earnings lost due to unemployment. Thus, the welfare cost due to unemployment is $dU = -g_0^a \cdot (\bar{w} - \underline{w}) \cdot \eta_1 \cdot h_1 d\bar{w}$.

Note that the change $dh_2 < 0$ does not generate welfare effects because marginal workers in the high skill sector have no surplus from working and hence the welfare cost is second order. At the optimum, we have $dT + dU = 0$, which implies:

$$\frac{\bar{w} - \underline{w}}{\bar{w}} = \frac{g_1 - g_2}{\eta_1 \cdot g_0^a}. \tag{5}$$

Formula (5) shows that the optimum minimum wage wedge (defined as $(\bar{w} - \underline{w})/\bar{w}$) is decreasing in the labor demand elasticity η_1 as a higher elasticity creates larger negative unemployment effects. As we saw above, when $\eta_1 = \infty$, formula (5) shows that there should be no minimum wage.

The optimum wedge is increasing with $g_1 - g_2$ which measures the net value of transferring $1 from high skilled workers to low skilled workers, and decreasing in g_0^a which measures the social cost of earnings losses due to involuntary unemployment. Obviously g_0^a, g_1, and g_2 are endogenous parameters and depend on the primitive social welfare function $G(.)$ but also on the level of the minimum wage. At the optimum, however, we have $g_a^0 \geq g_1 \geq g_2$. Increasing the redistributive tastes of the government by choosing a more concave $G(.)$ function has an ambiguous effect on the level of the optimum $\bar{w}$ because it is likely to increase both $g_1 - g_2$ and g_a^0. If the government does not value redistribution at all, then $G(u) = u$ and hence $g_1 = g_2 = g_a^0$ and there should be no minimum wage ($\bar{w} = \underline{w}$ from formula (5)). If the government has extreme redistributive tastes with a Rawlsian objective, then $g_1 = g_2 = 0$ and $g_0^a > 0$ which also implies that there should be no minimum wage in the case as well. Therefore, the level of the optimum $\bar{w}$ is expected to follow an inverted U-shape with the level

10

of redistributive tastes.

Formula (5) is not an explicit formula because it depends on $\underline{w}$ which itself depends on $\bar{w}$ through the supply function (as illustrated on Figure 2). It is possible, however, to obtain a more explicit formula if we are willing to make assumptions about the shape of the demand and supply functions. The simplest assumption is that the elasticities of demand η_1 and supply e_1 are constant. In that case $D(w_1) = D_0 \cdot w_1^{-\eta_1}$ and $S(w_1) = S_0 \cdot w_1^{e_1}$ so that $S_0 \cdot w_1^{*e_1} = D_0 \cdot w_1^{*-\eta_1}$ and $S_0 \cdot \underline{w}^{e_1} = D_0 \cdot \bar{w}^{-\eta_1}$. This implies that $\underline{w} = w_1^* \cdot (w_1^*/\bar{w})^{\eta_1/e_1}$, and hence:

$$\frac{\bar{w} - \underline{w}}{\bar{w}} = 1 - \left(\frac{w_1^*}{\bar{w}}\right)^{1+\frac{\eta_1}{e_1}}.$$

This equation shows that for a given wedge $(\bar{w} - \underline{w})/\bar{w}$, the minimum wage markup $\bar{w}/w_1^*$ is *increasing* in e_1 and *decreasing* in η_1. Formula (5) can be rewritten as:

$$\frac{\bar{w}}{w_1^*} = \left(1 - \frac{g_1 - g_2}{g_0^a \cdot \eta_1}\right)^{-\frac{e_1}{e_1+\eta_1}} \simeq 1 + \frac{e_1}{e_1 + \eta_1} \cdot \frac{g_1 - g_2}{g_0^a \cdot \eta_1}, \tag{6}$$

where the approximation holds in the case of a small minimum wage (i.e., when $(g_2 - g_1)/(g_0^a \cdot \eta_1)$ is small). This formula has the advantage of providing an explicit formula for the minimum wage mark-up above the market wage w_1^* expressed in terms of supply and demand elasticities and the social welfare weights. It shows that the optimum minimum wage $\bar{w}$ is decreasing in the supply elasticity e_1. The intuition can be easily understood from Figure 2. A higher supply elasticity, implies a flatter supply curve, and hence lower costs from involuntary unemployment. If the supply elasticity is high, then a small change in w_1 has large effects on supply, implying that many workers derive a very small surplus from working and hence do not loose much when they are forced out of work by a minimum wage. This result is very important because, as is well known, redistribution through taxes and transfers is hampered by a high supply elasticity. As we will see later, situations with high supply elasticities for low skilled workers make it particularly valuable to use the minimum wage even in the presence of optimal taxes. Conversely, when the supply elasticity is low, redistribution through the minimum wage is costly while redistribution through taxes and transfers is efficient.[13]

Formula (6) shows that there are two channels through which a higher demand elasticity η_1 reduces the optimal minimum wage. The first channel is the standard *unemployment level effect*

---

[13]This shows also that the case with no supply elasticity upon which the labor literature has focused is not only unfavorable to the minimum wage but also a case where the minimum wage would be clearly dominated by tax and transfer policy.

developed when discussing (5) that higher demand elasticity creates a larger unemployment response to the minimum wage. The second channel is an *unemployment cost* effect which works through the link between the wedge $(\bar{w} - \underline{w})/\bar{w}$ and the minimum wage markup $\bar{w}/w_1^*$. A higher demand elasticity implies that a given minimum wage markup is associated with a larger wedge, hence higher unemployment costs for the marginal worker. The distinction between those two channels is important because we will see that the first classical unemployment level effect disappears with optimal taxes and transfers but the unemployment cost effect remains.

2.4 Extensions

• More factors of production

It is straightforward to consider a more complex model with many skills $w_1, ..., w_I$ (instead of just two) and a capital input k with price R as well. Formulas (5) and (6) easily generalize to that case with three notable points.

First, the welfare weight g_2 should be replaced by an average welfare weight across all the other factors and with weights depending on the elasticity of substitution of each factor with the low skilled factor.[14] The implementation of formula (6) requires knowing g_2 and hence knowing how transfers to minimum wage workers are distributed among each of the other factors.

Second and related, in practice, some sectors such as small firms or agricultural workers are not covered by the minimum wage. Those uncovered sectors should also be included in the non-minimum wage sector. In such a model with an uncovered sector with low wages, it is conceivable that imposing a minimum wage will lead to a decrease in the wages in the uncovered sector which would be undesirable from a social welfare perspective as a higher welfare weight might be attached to uncovered workers with very low wages.

Third, with many labor inputs, it is possible that the optimum minimum wage might end up covering more than one type of labor input. In that case, g_1 needs to be replaced by the average social welfare weight among all minimum wage workers.

• Many Consumption Goods

---

[14]If there are no pure profits, the no-profit condition implies that $h_1 dw_1 + h_2 dw_2 + .. + h_I dw_I + kdR = 0$ following a change in the minimum wage $d\bar{w} = dw_1$. Each change dw_i can be derived as a function of the cross derivatives of the general production function $F(h_1, .., h_I, k)$. If there are pure profits $\Pi > 0$, pure profits might also bear part of the burden but the same analysis goes through.

It is also possible to extend the model to many consumption goods in order to capture the fact that minimum wage workers might be particularly concentrated in some industries (such as fast-food restaurants). In this case, a minimum wage will lead to change in relative consumption prices. Such changes in relative prices are part of the transfer toward minimum wage workers. The formula $\sum_i h_i dw_i = 0$ generalizes to $\sum_i h_i dw_i = \sum_k c_k dp_k$ where c_k denotes consumption of good k and dp_k the (relative) price change of good k due to the minimum wage increase $d\bar{w}$. Again, formulas (5) and (6) are also valid in that case but the average g_2 also includes the marginal welfare weights of consumers affected by the minimum wage increase. For example, if fast-food consumers are low income on average, then that would impact negatively g_2 and reduce the size of the optimal minimum wage.

• Continuum of Wages

The two-skill model we have considered is useful to understand the key economic forces a play in the minimum wage optimization trade-off but is not necessarily well suited to a practical estimate of the optimal minimum wage. Indeed, in reality, low market wages are distributed along a continuum instead of a single point w_1 as the simpler two-skill model assumes. Therefore, it is useful to extend the model to a continuum (or a very large) number of skills in order to capture the actual distribution of wages. In that context, it is interesting to analyze at which percentile of the wage distribution should the optimum minimum wage $\bar{w}$ be set. It is possible to construct a model with a continuum of skills and derive the first order condition for the optimal minimum wage. The drawback is that it does not seem possible to derive an explicit formula for the optimum minimum wage $\bar{w}$. However, it is possible to perform numerical simulations (XX in progress) in such a model.

Let us assume that there is continuum of skills indexed by $i \in (0, \infty)$. Individuals are distributed across skills with density $h^0(i)$ (we normalize population size to one so that $\int h^0(i) di = 1$). Individuals with skill i receive a competitive wage w_i if they work. As above, work costs θ are distributed according to $P_i(\theta)$ among skill i individuals. Hence, the supply of skill i is $h_i = h^0(i) P_i(w_i)$. We denote by $h = (h_i)_{i \in (0, \infty)}$ the labor quantity vector.

Let us assume a CES production function with constant elasticity σ of the form:

$$F(h) = \left(\int_0^\infty a_i h_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}.$$

Wages are set competitively so that,

$$w_i = \frac{\partial F}{\partial h_i} = a_i h_i^{-\frac{1}{\sigma}} \cdot F^{\frac{1}{\sigma}}.$$

The combination of the supply and demand equations at each skill i define a unique equilibrium (h, w). We assume without loss of generality that w_i is increasing in i. In such a model and as illustrated on Figure 3, introducing a minimum wage $\bar{w}$ has three consequences. First, there is a skill level i^* (a function of $\bar{w}$) so that all skills $i \leq i^*$ have wages w_i bunched at $\bar{w}$. Second, at each skill $i < i^*$, a fraction of workers are involuntary unemployed. We denote by $\underline{w}_i$ the reservation wage of the marginal worker at skill i and by $D_i(\bar{w})$ the demand for skill i. We have $h_i^0 \cdot P_i(\underline{w}_i) = D_i(\bar{w})$, and we denote by $\eta_i = -\bar{w} D_i'(\bar{w})/D_i(\bar{w})$ the demand elasticity for skill i labor. Third, wages w_i above the minimum wage for $i > i^*$ will be reduced through general equilibrium effects.

The government picks $\bar{w}$ to maximize social welfare:

$$SW(\bar{w}) = h_0 G(0) + \int_{i=0}^{i^*} \int_{\theta=0}^{\underline{w}_i} G(w_i - \theta) p_i(\theta) d\theta \cdot h^0(i) di + \int_{i=i^*}^{\infty} \int_{\theta=0}^{w_i} G(w_i - \theta) p_i(\theta) d\theta \cdot h^0(i) di,$$

where $h_0 = 1 - \int h_i di$ is the total number of non-working individuals. It is possible to show that the first order condition for $\bar{w}$ takes form similar to (5):

$$\int_{i=0}^{i^*} \frac{G(\bar{w} - \underline{w}_i) - G(0)}{\bar{w}} \cdot \eta_i \cdot h(i) di = (\bar{g} - g^2)\bar{h}, \tag{7}$$

where $\bar{h} = \int_0^{i^*} h_i di$ is the total number of minimum wage workers, $\bar{g}$ is the average social marginal welfare weight among minimum wage workers, and g^2 is the average marginal welfare weight among workers above the minimum wage. This formula is also derived by trading off the redistributive benefits of increasing the minimum wage (right-hand-side) and the welfare costs of increasing unemployment (left-hand-side), showing that exactly the same economic forces are at play in the continuum model.[15] We will use formula (7) in numerical simulations.

Interestingly, and as shown on Figure 3, the continuum model predicts that there should be an atom of workers bunched at the minimum wage and it is easy to show that the size of this atom is (negatively) related to the elasticity of substitution σ. The atom disappears when $\sigma = \infty$ (perfect substitution). Empirically, wage distributions do display bunching at

---

[15]The change $d\bar{w}$ generates a change di^8. However, di^* creates only second order welfare effects as the minimum wage hardly binds at skill level i^*.

the minimum wage (see e.g., Card and Krueger, 1995) making it possible to estimate σ using directly the extent of bunching.

- **Wages vs. Earnings**

In practice, minimum wages are based on hourly wage rates. There are two reasons why wage rates might not be a good indicator of earnings or economic welfare.

First, minimum wage workers might belong to families with higher incomes. For example a spouse married to a high income husband or a teenager with high income parents. Johnson and Browning (1983) and Burkhauser, Couch, and Glenn (1996) propose an empirical analysis of this issue in the United States. This is straightforward to incorporate in the analysis by simply altering the social marginal welfare weights. Our optimal formulas continue to hold in that context. For example, if all minimum wage workers were teenagers from middle class families, g_1 could possibly be lower than g_2 and no minimum wage would be desirable.

Second, we have made the assumption that involuntary unemployment is not shared and falls in a binary fashion on workers. This assumption is reasonable if low income workers who become unemployed do not have buffer stock savings, or do not have access to credit markets or other resources to smooth consumption overtime. If unemployed workers can smooth consumption and unemployment spells are relatively short, a better assumption would be that unemployment is shared from a longer term perspective. In that case, the utility loss due to unemployment is lower. This can be incorporated in the analysis by replacing the weight g_0^a by another weight g_0^s that is actually much closer to g_1. This obviously leads to a higher optimum minimum wage but does not change the fundamental formulas.

2.5 Numerical Simulations

- **Minimum Wage Desirability with Fixed Tax Rates**

We make the following parametric assumptions. (1) We assume a CES production function with elasticity of substitution $\sigma > 0$. (2) We assume constant labor supply elasticities $e_i > 0$ by choosing $P_i(w) = (w/\bar{\theta}_i)^{e_i}$. We assume $(h_1^0, h_2^0) = (1/4, 3/4)$. We assume a CRRA social welfare function $G(u) = (u + B)^{1-\gamma}/(1 - \gamma)$ with risk aversion parameter $\gamma > 0$ and where $B > 0$ is a constant that is used to avoid infinitely negative utility or infinite social marginal

15

utility for non-workers.[16]   We calibrate the production function so that $(w_1^*, w_2^*) = (1, 3)$ and the labor supply functions so that $(h_0^*, h_1^*, h_2^*) = (0.2, 0.2, 0.6)$ at the no minimum wage equilibrium. We always assume that $e_2 = 0.25$ and $B = 0.5$.

Panel A in Table 1 displays the optimum minimum wage markup over the undistorted market wage w_1^* as well as the involuntary unemployment rate (among all low skilled individuals) under various scenarios for e_1, σ, and γ. The table confirms that the optimum minimum wage is increasing in e_1 (comparing columns (1), (2), (3)), decreasing in σ (comparing columns (4), (5), (6)), and has an inverted U-shape pattern with γ (comparing panels A1, A2, and A3). Optimal minimum wage is small for a high $\gamma = 3$ value.

- **Continuum Wage Model**

TO BE DONE

3   Optimal Minimum Wage with Taxes and Transfers

3.1   The Basic Two-Skill Model

Let us now assume that the government uses taxes and transfers. In our simple two-skill model, we assume that the government can observe job outcomes (not working, work in sector 1 paying w_1, or work in sector 2 paying w_2) but does not observe costs of work. Therefore, the government can condition tax and transfers only on those observable work outcomes. Let us denote by T_i the tax (or transfer if $T_i < 0$) on occupation i. We denote by $c_i = w_i - T_i$ the disposable income in occupation $i = 0, 1, 2$. This is a fully general nonlinear income tax on earnings.

As in our previous model without taxes, an individual with skill $i = 1, 2$ who decides to work earns w_i but increases his disposable by $c_i - c_0$. Hence we can naturally define a tax rate τ_i on skill i workers: $1 - \tau_i = (c_i - c_0)/w_i$. An individual of skill $i = 1, 2$ and with costs of work θ works if and only if $\theta \leq c_i - c_0 = (1 - \tau_i)w_i$. Hence, the aggregate labor supply functions for $i = 1, 2$ are:

$$h_i = h_i^0 \cdot P_i((1 - \tau_i)w_i) = h_i^0 \cdot P_i(c_i - c_0). \tag{8}$$

---

[16]B could represent for example home production, charitable support, etc. As we show below, B is not needed in a model with explicit government transfers financed by taxation.

e_i is the elasticity of labor supply with respect to the net-of-tax wage rate $w_i(1-\tau_i) = c_i - c_0$:

$$e_i = \frac{(1-\tau_i)w_i}{h_i} \frac{\partial h_i}{\partial(1-\tau_i)w_i} = \frac{(1-\tau_i)w_i \cdot p_i((1-\tau_i)w_i)}{P_i((1-\tau_i)w_i)},$$

The demand side of the economy is unchanged. For given parameters c_0, τ_1, τ_2 defining a tax and transfer system, the four equations (1) and (8) for $i = 1, 2$ define the competitive equilibrium $(h_1^*, h_2^*, w_1^*, w_2^*)$.

Assuming no exogenous spending requirement, the government budget constraint can be written as:

$$h_0 c_0 + h_1 c_1 + h_2 c_2 \leq h_1 w_1 + h_2 w_2, \tag{9}$$

which can be rewritten as $c_0 = h_1 \tau_1 w_1 + h_2 \tau_2 w_2$.

• Minimum Wage Desirability with Fixed Tax Rates

Let us first analyze how our previous analysis on the desirability of the minimum wage is affected in the presence of taxes and transfers assuming that τ_1, τ_2 are exogenously fixed and that the transfer to non-workers c_0 adjusts automatically to meet the government budget constraint when a small minimum wage $\bar{w} = w_1^* + d\bar{w}$ is introduced. We assume that the minimum wage applies to wages before taxes and transfers.[17] This assumption does not affect the desirability of a minimum wage and is the most convenient convention. Let us define social marginal welfare weights at each occupation level as:

$$g_i = \frac{\int G'(c_i - \theta)dP_i(\theta)}{\lambda \cdot P_i},$$

where the integration takes places over all individuals in occupation $i = 1, 2$. We define $g_0 = G'(c_0)/\lambda$. λ is a normalization parameter defined such that:

$$h_0 g_0 + h_1 g_1 + h_2 g_2 = 1$$

We will show below that λ is the multiplier of the government budget constraint when the government optimizes taxes and transfers.

---

[17]In practice, the legal minimum wage applies to wages net of employer payroll taxes but before employee payroll taxes, income taxes, and transfers. $\bar{w}$ should be interpreted as the minimum wage gross of all employer taxes.

Proposition 2 *With fixed tax rates τ_1, τ_2, under the efficient rationing assumption 1 and assuming $e_1 > 0$ and $\eta_1 < \infty$, introducing a minimum wage is desirable iff*

$$g_1 \cdot (1 - \tau_1) - g_2 \cdot (1 - \tau_2) + \tau_1 - \tau_2(1 + e_2) - \tau_1 \cdot \eta_1 > 0. \tag{10}$$

When $\tau_1 = \tau_2 = 0$, equation (10) boils down to $g_1 - g_2 > 0$ which is fully consistent with proposition 1. Equation (10) shows with taxes and transfers, introducing a minimum wage creates four fiscal effects that need to be taken into account in the welfare analysis: first, transferring \$1 pre-tax from high skilled workers to low skilled workers through the minimum wage implies a \$ $(1 - \tau_1)$ post tax transfer to low skilled workers and a \$ $(1 - \tau_1)$ post tax loss to high skilled workers, hence the factors $(1 - \tau_i)$ multiplying g_1 and g_2 in 10. Second and related, such a \$1 transfer creates a direct net fiscal effect $\tau_1 - \tau_2$. Third, the reduction in w_2 leads to a supply effect which further reduces taxes paid by the high skilled by $e_2 \tau_2$ per dollar transferred. Finally, involuntary unemployment also creates a tax loss equal to $-\tau_1 \cdot \eta_1$ per dollar transferred.

Panel B in Table 1 illustrates numerically that, starting from a substantial flat rate tax where $\tau_1 = \tau_2 = 0.35$ (and using the same parametrization as in Panel A), the optimal minimum wage is much lower (and is actually useless when $\sigma = e_1 = 0.25$).

• Optimal Tax Formulas with no Minimum Wage

As shown in Piketty (1997) in the case of a Rawlsian criterion and Saez (2004) more generally, one can ignore demand effects and assume that wage rates w_i are constant when deriving optimal tax rates. Let us denote by λ the multiplier of the budget constraint. Let us denote by g_i the average social marginal welfare weight in group $i = 0, 1, 2$. By definition $g_0 = G'(c_0)/\lambda$ and $g_i = \int_0^{c_i - c_0} G'(c_0 - c_i - \theta) p_i(\theta) d\theta / (\lambda \cdot P_i)$ for $i = 1, 2$. Following Saez (2002), we have the following conditions at the optimum:

$$h_0 \cdot g_0 + h_1 \cdot g_1 + h_2 \cdot g_2 = 1, \tag{11}$$

$$\frac{\tau_i}{1 - \tau_i} = \frac{1 - g_i}{e_i}. \tag{12}$$

for $i = 1, 2$. Those equations are immediately derived from the first order conditions of the government maximization with respect to c_0, and c_i, $i = 1, 2$ respectively. Equation (11) implies that the average of marginal welfare weights across the three groups $i = 0, 1, 2$ is one.

Indeed, the value of distributing one dollar to everybody is exactly the average marginal social weight and the cost of distributing one dollar in terms of revenue lost is also one dollar as we have assumed away income effects.[18]

Equation (12) is a simple inverse elasticity rule. If $g_1 > 1$, then the optimal tax rate on low skilled workers should be negative so that in-work benefits such as the EITC are optimal. This was the key result emphasized by Diamond (1980), Saez (2002), Laroque (2005), Choné and Laroque (2005, 2006): an EITC type transfer for low wage workers can be optimal in a situation where individuals respond only along the extensive margin. This result is illustrated on Figure 4: starting from an allocation (c_0, c_1, c_2), and increasing c_1 by dc_1 leads to a positive direct welfare effect $h_1 g_1 dc_1$, a mechanical loss in tax revenue $-h_1 dc_1$, and a behavioral response increasing work $dh_1 = \eta_1 h_1 (c_1 - c_0) dc_1 > 0$ and creating a fiscal effect equal to $w_1 \tau_1 dh_1$. If $g_1 > 1$, then, at the optimum, the behavioral response fiscal effect has to be negative implying that $\tau_1 < 0$ at the optimum.

Figure 5 illustrates the important point that the optimal tax/transfer derivation is unchanged when w_1, w_2 are endogenous. In that case, the small reform dc_1 leads to changes in h_1 and hence to changes dw_1 and dw_2 through demand side effects. However, assuming that c_2 and $c_1 + dc_1$ are kept unchanged, the effect of dw_1 and dw_2 is fiscally neutral because $h_1 dw_1 + h_2 dw_2 = 0$ through the no-profit condition.

Let us denote by (w_i^T, c_i^T) the tax/transfer optimum with no minimum wage.

• Desirability of a Minimum Wage

As illustrated on Figure 6, starting from the tax/transfer optimum (w_i^T, c_i^T), let us introduce a minimum wage set at $\bar{w} = w_1^T$. Such a minimum wage is just binding and has no direct impact on the allocation. Let us now increase c_1 by dc_1 while keeping c_0 and c_2 constant. As we showed above, such a change provides incentives for some low skilled individuals to start working. However, as we showed on Figure 5, such a labor supply response would reduce w_1 through demand side effects. However, in the presence of a minimum wage set at w_1^T, w_1 cannot fall, which implies that those individuals willing to start working cannot work and actually shift from voluntary to involuntary unemployment. The assumption of efficient rationing is

---

[18]With income effects, distributing one dollar reduces labor supply which in general reduces tax revenue and hence the average marginal social weight is larger than one. See Saez (2002) for a more detailed discussion.

key here as these are precisely the individuals with the lowest surplus from working.

Given that the labor supply channel is effectively shut down by the minimum wage, the dc_1 change is like a lumpsum tax reform and its net welfare effect is simply $[g_1 - 1]h_1 dc_1$. This implies that if $g_1 > 1$, then this reform is desirable which implies that introducing a minimum wage allows to improve upon the tax/transfer optimum allocation. Therefore, we can state:

Proposition 3 *Under the efficient rationing assumption 1, assuming $e_1 > 0$ and $\eta_1 < \infty$, and that the government uses taxes and transfers optimally, introducing a minimum wage is desirable if $g_1 > 1$ at the optimum tax allocation (with no minimum wage).*

Proposition 3 is fairly similar to our initial proposition with no taxes and transfers. The condition $g_1 > 1$ is somewhat weaker than $g_1 > g_2$ (as the g_i average to one and hence $g_2 < 1$). However, if the government has redistributive tastes, then $g_1 > 1$ is a weak condition especially given that the low skilled sector can be chosen to represent very low income workers. In that case, $g_1 > 1$ breaks down only if the weight g_0 on the non-workers is so high as to make the weights on workers, no matter how low their wage, below 1. This happens in the case of the extreme Rawlsian welfare criteria. The condition $g_1 > 1$ means that the government values redistribution toward low wage workers. In that case, the minimum wage helps the government redistribute more to low wage workers by shutting down the behavioral response channel.

This result is line with the theory of optimum quantity controls developed by Guesnerie (1981) and Guesnerie and Roberts (1984) showing that, in an optimum tax model, introducing a quantity control on subsidized goods is desirable. In our model, a minimum wage is a indirect way for the government to introduce rationing on low skilled work.[19] As we will see in Section 3.2, this result generalizes easily to a model general model with many skills and fully general labor supply responses functions.

Cahuc and Laroque (2007) make the point that a minimum wage can be replicated by a knife-edge nonlinear income tax such that $T(w) = \infty$ for $w < \bar{w}$ (as no occupation could pay less than $\bar{w}$ in equilibrium) and conclude therefore that a minimum wage is redundant with a fully general nonlinear income tax. This argument is mathematically correct but such a knife-edge income tax would effectively be a minimum wage. Our model rules out such knife

---

[19]Guesnerie and Roberts (1987) proposed an analysis of optimal minimum wage. However, the model they considered was not directly related to their quota theory.

edge income taxes because we consider tax rates that are occupation specific (rather than wage level specific). However, a fully general knife-edge income tax could not do better than the combination of our occupation specific tax rates combined with a minimum wage. Therefore, we think that the definition of the tax and minimum wage tools we use is the most illuminating to understand the problem of joint minimum wage and tax optimization.

• Optimum Tax/Transfer and Minimum Wage Allocation

Formally, the government chooses $\bar{w}, c_0, c_1, c_2$ to maximize

$$SW = (1 - h_1 - h_2)G(c_0) + h_1^0 \cdot \int_0^{\underline{w}(1-\tau_1)} G(c_1 - \theta)p_1(\theta)d\theta + h_2^0 \cdot \int_0^{c_2-c_0} G(c_2 - \theta)p_2(\theta)d\theta. \quad (13)$$

subject to its budget constraint (with multiplier λ), the supply function constraint (if the minimum wage does not bing), and the fact that wage rates are competitively set. As above, $\underline{w}$ is defined as the reservation wage of the marginal worker: $h_1^0 \cdot P_1(\underline{w}(1-\tau_1)) = D_1(\bar{w})$ where $D_1(\bar{w})$ is the demand for low skilled labor for a given minimum wage $\bar{w}$.

As above, the first order condition with respect to c_0 implies that λ is such that $h_0 g_0 + h_1 g_1 + h_2 g_2 = 1$.

With a binding minimum wage, as illustrated on Figure 6, increasing c_1 is a lumpsum transfer. Therefore, the government will increase c_1 up to point where $g_1 = 1$. Therefore, the first order condition for the optimum c_1 leads to:

Proposition 4 *Under the efficient rationing assumption 1 and if the minimum wage is desirable, at the joint tax/transfer and minimum wage optimum, we have $g_1 = 1$.*

This proposition shows that the minimum wage is a powerful tool to increase redistribution to low wage workers as it can bring their average social marginal welfare weight up to the average among all individuals, effectively reaching the point where the government no longer values further redistribution toward them.

The first order condition with respect to c_2 leads to the standard formula (12): $\tau_2/(1-\tau_2) = (1 - g_2)/e_2$ as the minimum wage does not impact the trade-off for the choice of c_2.

Finally, there is a first order condition for the optimal choice of $\bar{w}$ that is illustrated on Figure 7. Increasing $\bar{w}$ by $d\bar{w}$ and keeping c_0, c_1, c_2 constant leads to an increase in involuntary unemployment: $dh_1 < 0$. Such involuntary unemployment leads to a (negative) welfare effect

21

on those individuals equal to $dh_1[G(c_0 + (\bar{w} - \underline{w})(1 - \tau_1)) - G(c_0)]/\lambda < 0$ and a fiscal effect equal to $dh_1 \cdot \tau_1 \cdot \bar{w}$, which leads to the following first order condition:

$$-\tau_1 \cdot \bar{w} = \frac{G(c_0 + (\bar{w} - \underline{w})(1 - \tau_1)) - G(c_0)}{\lambda}. \tag{14}$$

As we did in Section 2, we can introduce the social marginal weight on earnings losses due to (marginal) involuntary unemployment: $g_0^a = [G(c_0 + (\bar{w} - \underline{w})(1 - \tau_1)) - G(c_0)]/[\lambda(\bar{w} - \underline{w})(1 - \tau_1)]$ in order to rewrite (14) as:

$$\frac{\bar{w} - \underline{w}}{\bar{w}} = -\frac{\tau_1}{1 - \tau_1} \cdot \frac{1}{g_0^a} > 0. \tag{15}$$

Proposition 5 *Under the efficient rationing assumption 1 and if the minimum wage is desirable, our model with extensive labor supply responses implies that the tax rate on low skilled workers τ_1 should be negative. Consequently, a binding minimum wage associated with a positive tax rate at the bottom is second best Pareto inefficient.*

Proposition 5 is illustrated on Figure 8. Suppose that the minimum wage binds and that $\tau_1 > 0$. This is a situation most OECD countries face, especially in continental Europe where traditional transfer programs combined with substantial payroll tax rates creates very large tax rates on low skilled work. Suppose that the government reduces the minimum wage by $d\bar{w} < 0$ while keeping c_0, c_1, c_2 constant.[20] Reducing the minimum wage leads to a positive employment effect $dh_1 > 0$ as involuntary unemployment is reduced which improves the welfare of the newly employed workers and also increases tax revenue as $\tau_1 > 0$. The increase $dh_1 > 0$ also leads to a change $dw_2 > 0$. Because $h_1 d\bar{w} + h_2 dw_2 = 0$, the mechanical fiscal effect of $d\bar{w}$ and dw_2 while keeping c_1 and c_2 constant is zero.[21] Therefore, this reform is a (second-best) Pareto improvement which immediately suggests a double dividend policy reform in countries such as France with high minimum wages combined with high tax rates at the bottom.[22]

Quantitatively, τ_1 is primarily determined to meet the condition $g_1 = 1$. Then, the optimal minimum wage wedge $(\bar{w} - \underline{w})/\bar{w}$ is determined by equation (15) and is increasing in the size of

---

[20]In practice, this could be achieved by cutting the employer payroll taxes for low income workers, which lowers the (gross) minimum wage without affecting the net minimum wage after taxes and transfers.

[21]In practice, keeping c_2 constant while w_2 increases means that it will be necessary to increase the tax rate on higher skilled workers.

[22]Actually, such a double dividend policy reform was started in France in the early 1990s when the employer payroll tax on low income workers was reduced (see Crépon and Desplatz, 2002 for an empirical analysis). However, if our theory is close approximation of the real world, this policy should be pushed further.

the absolute subsidy $|\tau_1|$ and decreasing in the social weight on unemployment earnings losses g_0^a. As we discussed in Section 2, we can define the implicit market wage rate w_1 as the wage rate that would prevail under the same tax rates τ_1, τ_0 but with no minimum wage. In that case, and assuming constant elasticity of supply and demand, we showed that the minimum wage markup over the market wage rate $\bar{w}/w_1$, for a given minimum wage wedge $(\bar{w}-\underline{w})/\bar{w}$ was increasing in e_1 and decreasing in η_1, suggesting that our previous results that the optimum minimum wage increases with e_1 and decreases with η_1 carry over to the case with optimal taxes. It is important to note that a high demand elasticity leads to a smaller minimum wage not because this creates more unemployment but rather because a large demand elasticity makes unemployment more costly by increasing the wedge $(\bar{w} - \underline{w})/\bar{w}$. As we discussed in Section 2, the unemployment level channel vanishes with optimal taxes but the unemployment cost channel remains.

The result that the optimum minimum wage follows an inverted U-shape pattern with the strength of redistributive tastes also carries over to this case. As we have seen, extreme redistributive tastes imply that $g_1 < 1$ and hence no minimum wage is desirable and conversely, no redistributive tastes imply that $g_0 = g_1 = g_2 = 1$, a situation where no minimum wage is desirable either.

• Numerical Simulations

Table 2 provides some numerical simulation illustrations using the same parametrization as in the situation with no taxes and transfers of Table 1. Table 2 shows the optimal tax rates with no minimum wage, and then displays the optimal tax rates, optimal minimum wage markup (and associated unemployment level among the unskilled) in the case of joint minimum wage/tax optimization. The Table confirms our key findings that the minimum wage should be associated with higher low skilled work subsidies than in the case of optimal tax rates with no minimum wage. The table also shows that the optimal minimum wage is increasing with e_1 and decreasing with σ. Finally, the minimum wage is useless in the high redistributive case $\gamma = 3$ as $g_1 < 1$ at the pure tax optimum.[23] Interestingly, comparing Tables 1 and 2 suggests

---

[23]The fact that the minimum wage is zero is in large part the consequence of the two skill model assumption. A model with many skills would generate $g_1 > 1$ at the tax optimum except for extreme Rawlsian redistributive tastes. As discussed below, such a model would allow a better calibration of the percentile of the wage distribution where the minimum wage should hit.

that the minimum wage with optimal taxes is not necessarily smaller than in the case with no taxes, especially in the case where redistributive tastes are not too large ($\gamma = 0.5$).

3.2 Extension to a Model with Many Skills

In this section we consider a general model with I occupations (instead of 2) and a general production function, and any form for labor supply responses.[24]

• Model and Optimal Taxation

The model we use is the general occupation model described in the appendix of Saez (2002) and in Saez (2004). There are $I + 1$ occupations, paying wages $w_0 = 0, w_1, .., w_I$. Occupation 0 denotes unemployment. We assume that in equilibrium, occupations are ordered so that $0 < w_1 < .. < w_I$. For simplicity, we assume away income effects but we consider a fully general supply side model.[25] Each individual is therefore characterized by a cost parameter $\theta = (\theta_0 = 0, \theta_1, .., \theta_I)$ which describes the labor supply cost for the individual to work in each occupation $i = 0, 1, .., I$. By assumption, being out of work is costless. We assume that θ is distributed according to a measure $\nu(\theta)$ on Θ, with total population normalized to one.

The government can apply a general income tax and transfer system $T = (T_0, .., T_I)$. We denote by $c_i = w_i - T_i$ the disposable income (after taxes and transfers) in occupation i. An individual with cost θ picks the occupation i which maximizes $c_j - \theta_j$ for $j = 0, .., I$. Hence, the set Θ is partitioned into $I + 1$ subsets $\Theta_0, .., \Theta_I$ so that individuals with $\theta \in \Theta_i$ choose occupation i. We denote by $h_i = \nu(\Theta_i)$ the fraction of individuals in occupation i. Those supply functions are functions of $c = (c_0, .., c_I)$ and hence denoted by $h_i(c_0, .., c_I)$. We assume that θ is distributed smoothly across individuals so that the supply functions h_i are continuously differentiable. This is a fully general supply model with no income effects. Our no income effect assumption implies that for any R, $h_i(c_0 + R, .., c_I + R) = h_i(c_0, .., c_I)$ so that:

$$\sum_{j=0}^{I} \frac{\partial h_i}{\partial c_j} = 0.$$

The participation model from our previous section is a special case. Similarly, the intensive labor supply of Mirrlees (1971) can be represented in this discrete model by assuming that

---

[24]Introducing a capital input would also be possible as long as we assume that returns on capital can be taxed at a specific rate τ_K.

[25]The key results on the desirability of the minimum wage easily generalize to the case with income effects.

individuals of "type i" can work in job $i - 1$ at no cost or work in job i at cost $\theta_i > 0$ (see Saez 2002 for details).

Abstracting first from the minimum wage, the government chooses $c = (c_0, .., c_I)$ in order to maximize:

$$SW = \int_{\theta \in \Theta} G(c_i - \theta_i) d\nu(\theta), \tag{16}$$

subject to the budget constraint:

$$\sum_{j=0}^{I} (w_j - c_j) \cdot h_j(c) \geq 0. \tag{17}$$

where $G(.)$ in (16) is increasing and concave and where index i inside in integral in (16) denotes the utility maximizing job choice of individual θ. We denote again by λ the multiplier of the budget constraint (17).

Assuming first that the w_i are fixed, the first order condition with respect to c_i is simply:

$$(1 - g_i)h_i = \sum_{j=0}^{I} T_j \cdot \frac{\partial h_j}{\partial c_i}, \tag{18}$$

where g_i is the average social marginal welfare weight in occupation i, defined as:

$$g_i = \frac{1}{\lambda \cdot h_i} \int_{\theta \in \Theta_i} G'(c_i - \theta_i) d\nu(\theta). \tag{19}$$

The derivation is straightforward once one recognizes that the welfare effect of an small increase dc_i due to switching jobs behavioral responses is zero (because of a standard envelope theorem argument).

The no income effects assumption implies that

$$\sum_{j=0}^{I} g_i \cdot h_i = 1. \tag{20}$$

This can be obtained by increasing every c_i by dc uniformly. This generates no behavioral responses and hence the fiscal cost dc must be equal to the welfare gain $dc \cdot \sum_j h_j g_j$. Equation (20) implies that the average of g_i is one. It is natural to assume that $g_0 > g_1 > .. > g_I$. Thus low pay occupations will likely have $g_i > 1$: Absent behavioral responses, the government would like to redistribute toward those groups.

Let us assume now that w_i are endogenous and defined as the marginal product of a general production function $F(h_1, .., h_I)$. We assume that F has constant returns to scale so that there are no profits. It turns out that formula (18) remains valid in that case as well. This is a well known result in optimal taxation originally demonstrated by Diamond and Mirrlees (1971). The intuition can easily be seen as follows: Increasing c_i by dc_i generates changes in all wages $dw_1, .., dw_I$ through general equilibrium demand effects. Keeping each c_j (for $j \neq i$) constant, the fiscal cost of such changes $dw_1, .., dw_I$ is $h_1 dw_1 + .. + h_I dw_I$. This expression is zero because there are no profits in equilibrium: $0 = F(h_1, .., h_I) - (w_1 h_1 + .. + w_I h_I)$, implies that $h_1 dw_1 + .. + h_I dw_I = 0$ (as $\partial F / \partial h_i = w_i$).

Within the context of our occupation model, we note that for any i, j, we have the following symmetry equation:

$$\frac{\partial h_j}{\partial c_i} = \frac{\partial h_i}{\partial c_j}. \tag{21}$$

This is the equivalent of the symmetry of the Slutsky equation (in a context where there are no income effects and the h_i are both compensated and uncompensated supply functions). The proof is fairly simple. For a given i, increasing c_i by dc will lead $dh_{ij} = dc \cdot \partial h_j / \partial c_i$ workers to switch from job i to job j. Symmetrically, increasing c_j by dc will lead $dh_{ji} = dc \cdot \partial h_i / \partial c_j$ workers to switch from job j to job i. Suppose you now increase both c_i and c_j by dc, then the net flux of workers from i to j will be zero (as the relative pay between those two jobs remains constant). Hence, $dh_{ij} + dh_{ji} = 0$ and (21) is demonstrated.

This allows us to rewrite the optimal tax formula (18) as:

$$\frac{1}{h_i} \sum_{j=0}^{I} -T_j \cdot \frac{\partial h_i}{\partial c_j} = g_i - 1. \tag{22}$$

Following the tradition in optimal tax theory since Samuelson (1951), we can interpret the following equation as follows. The left-hand-side is called *index of encouragement* that the tax system imposes on good i.[26] Assuming linearity (constant partial derivatives of h_i), it measures the (percentage) effect of the tax system on the supply of work in occupation i: the tax system reduces c_j by T_j relative to the untaxed situation where $c_j = w_j$, inducing a change in h_i equal to $\Delta h_i \simeq -T_j \partial h_i / \partial c_j$.

---

[26]Optimal commodity tax theory uses the word "discouragement" because it refers in general to goods entering utility positively. We use word "encouragement" because labor supply enters utility negatively.

The right-hand-size $g_i - 1$ is positive for low paid occupations and negative for highly paid occupations. Hence, formula (22) can be interpreted as follows. The optimal tax system encourages the supply of low paid occupations and discourages the supply of highly paid occupations. This allows the government to redistribute from highly paid occupations toward low paid occupations (including of course those in the unemployed occupation).[27]

• Desirability of Minimum Wage Rationing

Before analyzing minimum wage policy, a useful first step is to study rationing in a fixed wage model, as the analysis can parallel closely Guesnerie and Roberts (1984). We will show subsequently that minimum wage policies are effectively generating rationing.

Suppose that the government can ration jobs in occupation $i \geq 1$ by imposing the additional constraint $h_i \leq \bar{h}_i$ where $\bar{h}_i$ is the constrained ration level. In case of rationing, there would be excess supply of labor in occupation i. Consistently with our previous analysis, we make the key *efficient rationing* assumption 1 that, with rationing, those with the highest surplus (relative to their second best job choice) are served first. Thus, those who would like to work in occupation i but cannot because of rationing are those with the smallest surplus (relative to their next best option). As discussed above, this assumption is the most favorable to quantity constraints policies.

It is also important to note that, in the general model, the second best option for those rationed is not necessarily involuntary unemployment so that rationing could possibly lead to the involuntary choice of a higher paying (but most costly) occupation.[28]

Starting from the tax/transfer optimum described above, suppose that the government introduces a small rationing on occupation i only by imposing the constraint $\bar{h}_i = h_i + dh_i$ where $dh_i < 0$ is exogenously set by the government. This constraint implies that individuals working in job i with surplus less than $d\theta_i$ (relative to their next best option) will have to switch to their next best option. The labor supply response to the ration is identical to the

---

[27]It is important to understand that this result is true in particular in the discrete Mirrlees (1971) model with intensive labor supply responses. Although a positive marginal tax rate at the bottom discourages low skilled workers from working in occupation 1, the tax system induces a larger number of slightly more skilled workers to work less and shift from occupation 2 to occupation 1.

[28]In practice, minimum wage policies are blamed for creating involuntary unemployment and not for creating involuntary work in higher paid but more strenuous occupations. This is consistent with the fact most of the labor supply responses at the bottom are concentrated along the participation margin as in our basic two-skill model.

behavioral response that would take place if c_i were decreased by $d\theta_i > 0$ (as those with surplus less than $d\theta_i$ would switch to their next best option). Thus, the ration dh_i induces behavioral responses such that:

$$dh_j = -\frac{\partial h_j}{\partial c_i} d\theta_i,$$

for $j = 0, .., I$. The case $j = i$ defines $d\theta_i = -dh_i/(\partial h_i/\partial c_i)$.[29]

The small ration has only a second order on welfare (as those who are constrained suffer from only a small welfare loss). However, the small ration has a first order fiscal effect dT equal to:

$$dT = \sum_{j=0}^{I} T_j dh_j = -d\theta_i \sum_{j=0}^{I} T_j \frac{\partial h_j}{\partial c_i} = (g_i - 1)h_i \cdot d\theta_i,$$

where we have used the optimal tax formula (18).

When wages are endogenous and determined through a general production function $F(h_1, .., h_I)$, rationing in occupation i can be obtained by imposing a minimum wage on occupation i. This minimum wage will effectively create excess supply in occupation i. Keeping our assumption that workers with maximum surplus (relative to next best job) are those who keep their job with rationing, the analysis exactly parallels our previous analysis with fixed wages[30] as long as the elasticity of demand for a given occupation is not infinite.[31] Hence, we immediately obtain the result that introducing a small minimum wage on occupation i is desirable if and only if $g_i > 1$. However, in practice the government cannot impose occupation specific minimum wages.[32] Thus, in practice the government is essentially constrained to impose the same minimum wage across all occupations. It is also important to note that the minimum wage cannot impose rationing on the unemployed in occupation 0 which would be desirable as $g_0 > 1$.[33] We can generalize proposition 3 as follows:

---

[29]Note that $d\theta_i$ is not well defined if the labor supply response is zero: $\partial h_i/\partial c_i = 0$. Hence, our analysis also requires assuming positive labor supply elasticities as in Section 2.

[30]Note that symmetric forced quotas can only be implemented with maximum wages and by forcing workers to keep working in those occupations subject to a quota. This is obviously not a feasible policy in practice as workers are always free to quit jobs at will (labor laws ban slavery).

[31]If the demand elasticity is infinite, then the wage rate is fixed and a minimum wage destroys all jobs in the occupation. Therefore, it is impossible to introduce a small ration as we described above in that context.

[32]Actual minimum wage laws in general impose a uniform minimum wage across all occupations with sometimes sub-minimum wages for small specific categories such as young workers, apprentices, or farm workers.

[33]This could justify policies aiming at helping the unemployed find jobs as well as force them to accept job offers they might obtain but would rather not take.

Proposition 6 *If the efficient rationing Assumption 1 holds, the labor demand elasticity on the lowest paid occupation is finite, and the labor supply elasticity is positive, then, starting from the optimal tax and transfer system with no minimum wage, introducing a binding minimum on occupation 1 (the lowest paid occupation) is desirable if and only if $g_1 > 1$.*

Barring extreme Ralwsian redistributive tastes or no redistributive tastes at all, the government values marginal redistribution toward the lowest paid workers so that $g_1 > 1$. Hence, this proposition constitutes a strong case for introducing a minimum wage even when the government has already implemented the optimal fully general and nonlinear tax and transfer system.

• Optimal Minimum Wage Rationing

Suppose that the optimum minimum wage $\bar{w}$ covers occupations $i = 1, .., i^*$ (we assumed that occupations were ordered). Then all those occupations pay the same wage $\bar{w}$. As a result, the government can no longer distinguish across those occupations and hence the government is forced to tax (or subsidize) them uniformly so that $c_1 = .. = c_{i^*} = \bar{c}$. We denote by $\bar{T} = \bar{w} - \bar{c}$ the net tax on minimum wage workers.

Again, increasing $\bar{c}$ does not produce any behavioral labor supply response (as occupations $1, .., i^*$ are rationed by the minimum wage). Hence, the government should increase $\bar{c}$ up to the point that $\bar{g} = 1$ where $\bar{g} = (h_1 g_1 + .. + h_{i^*} g_{i^*})/(h_1 + .. + h_{i^*})$ is the average social marginal welfare weight on minimum wage workers. This generalizes Proposition 4 that we obtained in the two-skill model.

Proposition 7 *If a minimum wage is desirable, then the optimal transfer to minimum wage workers is such that the average social marginal welfare weight across minimum wage workers equals one.*

Increasing the minimum wage $\bar{w}$ reduces labor demand for minimum wage occupations $i = 1, .., i^*$. This reduction in minimum wage labor demand will lead to a change in the price of other factors. Suppose the government increases $\bar{w}$ by $d\bar{w}$ while keeping $c = (c_0, .., c_I)$ constant. The direct fiscal effect due to the changes in factor prices $(w_1, .., w_I)$ is zero as $\sum_i h_i \cdot dw_i = 0$ (zero-profit condition).

The involuntary change in occupations due to the minimum wage increase will lead to a fiscal effect and a welfare effect. The fiscal effect is $\sum_i T_i dh_i$ and the welfare effect is $\sum_i \Delta G \cdot dh_i / \lambda$ where ΔG is the difference in social welfare for the marginal workers involuntary displaced from their minimum wage job because of $d\bar{w}$. The optimal minimum wage $\bar{w}$ is set so that those effects cancel out. Unfortunately, it does not seem possible to obtain a closed form formula for this optimum minimum wage in the general case. We explore the particular case of the participation model below.

- **Optimal Minimum Wage in the Participation Model with Many Skills**

In that case, we can use the concept of virtual consumption $\hat{c}$ to represent labor demand and constrained labor supply. For minimum wage occupations $i = 1, .., i^*$, $D_i(\bar{w}) = h_i^0 \cdot P_i(\hat{c}_i - c_0)$ where $\hat{c}_i < c_i = \bar{c}$ and $D_i(\bar{w})$ denotes skill i labor demand for a minimum wage $\bar{w}$. The optimum first order condition for $\bar{w}$ is:

$$\bar{T} - T_0 + \frac{\sum_{i=1}^{i^*}[G(\bar{c} - \hat{c}_i + c_0) - G(c_0)]D_i'(\bar{w})}{\lambda \cdot \sum_{i=1}^{i^*} D_i'(\bar{w})} = 0. \tag{23}$$

This formula is a generalization of the (14) in the multi-occupation case. The formula implies that $\bar{T} < T_0$ so that minimum wage work is subsidized (EITC type transfer). Although this model does not generate a closed form solution for $\bar{w}$, it is possible to obtain approximations using constant elasticity supply and demand equations. We can define the tax τ_i on occupation i by $c_i - c_0 = w_i(1 - \tau_i)$. Then, we can define the virtual wage rate $\underline{w}_i$ such that $\hat{c}_i - c_0 = \underline{w}_i(1 - \tau_i)$. We can denote by w_i^* the market wage absent the minimum wage defined as $D_i(w_i^*) = h_i^0 \cdot P_i(w_i^*(1 - \tau_i))$. Using a constant supply elasticity $e_i = \theta_i p_i / P_i$ and constant demand elasticity $\eta_i = -w_i D_i'/D_i$. We can rewrite (23) as:

$$\bar{\tau}\bar{w} + \frac{\sum_{i=1}^{i^*}[G(\bar{w}(1 - \bar{\tau})[1 - (w_i^*/\bar{w})^{1 + \frac{\eta_i}{e_i}}] + c_0) - G(c_0)]h_i \cdot \eta_i}{\lambda \cdot \sum_{i=1}^{i^*} h_i \cdot \eta_i} = 0. \tag{24}$$

It is straightforward to extend this model to the case with a continuum of skills as we did in Section 2. We will perform numerical simulations in that context (TO BE DONE).

- **Many Consumption Goods and Production Efficiency**

It is also possible to extend the model to a situation with many goods. In that context, we can show that the standard theorems of public finance, namely the production efficiency

theorem of Diamond and Mirrlees (1971) and the no commodity taxation result of Atkinson and Stiglitz (1976) carry over to the model with optimal minimum wage with taxes and transfers.

The production efficiency theorem implies that, at the joint minimum and tax optimum, there should be production efficiency: producers should maximize profits using pre-tax prices for labor inputs and consumption outputs. This result is trivial to verify in the two skill model and remains true with many labor inputs and many consumption goods. As is well known, the production efficiency result implies that there should be no tariffs in the context of an open economy. This free trade result carries over when the government uses a minimum wage.

The Atkinson and Stiglitz (1976) implies that, if utility functions are separable between consumption goods and labor costs and the sub-utility of consumption is homogenous across all consumers, then the optimum tax/minimum wage system should tax labor only and not impose any differentiated taxes on consumption goods.

4 Conclusion

Our paper has proposed a theoretical analysis of optimal minimum wage policy for redistribution purposes in a perfectly competitive labor market, considering first the case with no taxes and transfers and then analyzing the case with optimal taxes and transfers. In light of the previous literature on this topic, we find that the standard competitive labor market model offers a surprisingly strong case for using the minimum wage when we make the efficient rationing assumption. A minimum wage is a useful tool if the government values redistribution toward low wage workers and, importantly, this result remains true in the presence of optimal nonlinear taxes and transfers. In that context, our model of occupational choice abstracting from hours of work allows us to overcome the informational inconsistency that has plagued previous work analyzing minimum wage policy with optimal income taxation. Our model fits into the general theory of rationing developed by Guesnerie (1981) and Guesnerie and Roberts (1984) as a minimum wage effectively rations low skilled labor. Such rationing is desirable because the optimal tax/transfer over-encourages the supply of low skilled labor.

We have also derived formulas for the optimal minimum wage in various settings, showing the key economic forces at play. The optimal minimum wage decreases with the demand elasticity for low skilled labor but increases with the supply elasticity of low skilled labor.

The optimal minimum wage follows an inverted U-shape as a function of the strength of the redistributive tastes of the government. When labor supply is along the extensive margin, a situation that might approximate well OECD low skilled labor markets, a minimum wage should always be associated with in-work subsidies: the co-existence of minimum wages with high participation tax rates for low skilled workers is (second-best) Pareto inefficient. In that situation, a cut in employer payroll taxes which decreases the gross minimum wage while keeping the net minimum wage constant, combined by an offsetting tax increase on higher skilled workers is Pareto improving.

There are a number of issues that we have abstracted from in our very stylized model that are worth pointing out as caveats and potential avenues for future research.

First, as mentioned, we abstract from the hours of choice decision which allows us to develop a model with no informational inconsistencies. However, the fact remains that, in practice, taxes and transfers are based on earnings while minimum wages are based on hourly rates. In reality, the government can observe both earnings and of hours of work of employees as this information is in general included in the payroll accounting of employers and is actually sometimes required to be reported to the government for administering payroll taxes or maximum hours laws.[34] Therefore, the puzzle remains for why taxes and transfers are based on earnings rather than wage rates. A possible explanation is that hours of work are not very elastic and that most of the labor supply response takes place along the occupation decision and in particular the participation decision. If hours were very elastic, taxes and transfers should be based (at least in part) on wage rates.[35] We conjecture that our results on the desirability of the minimum wage, would carry over to that case as well as long as the supply response along the occupation and participation margin is positive.[36]

Second, a minimum wage rationing mechanism operates very different from a tax and transfer which alters prices but lets markets clear freely. Our model has assumed that the efficiency costs of each tool should be evaluated using standard welfare theory. It is conceivable,

---

[34]For example, in France, hours of work are available in the administrative database DADS used by the social security administration.

[35]Some transfer programs are based partly on hours information. For example, the British Working Families Credit is given only to families where one earner works at least 16 hours a week. Similarly, the current US welfare program TANF imposes work requirements which is an indirect way of conditioning transfers on hours of work.

[36]If this occupational response is zero and the response is fully along the hours margin, then a tax based on wage rates only can achieve the first best.

however, that the rationing and hence involuntary unemployment creates additional psycho-logical costs that are not captured in our simple model, and that would make minimum wage policies less attractive in practice. Indeed, democratic states do put limits on quantity policies justified by individual rights that can sometimes conflict with the objective of the government. An extreme example is forced labor through serfdom which was an efficient way (from the perspective of the sovereign) to protect its fiscal base through quantity constraints.[37]

Third and related, in order to understand better the real costs of rationing, it could be valuable to consider labor market models with frictions. In particular, search frictions, would allow endogenize job matches and the degree of efficiency in the rationing created by the minimum wages. Therefore, it would be particularly interesting to analyze how our results would carry over in a model with frictions. Hungerbuhler and Lehmann (2007) have made an important step in analyzing optimal minimum wage policy with optimal tax in a search model and it would be interesting to understand more clearly how their results relate to ours.

Fourth, it is conceivable as well that minimum wage policies are favored by the public relative to taxes and transfers because the costs of higher wages at the bottom coming from other factors of productions are not directly visible and hence might be more acceptable to higher income earners than direct taxes to finance transfers toward low income families (this relates to political economy models imposing constraints on redistribution).

Finally, our numerical simulations have been purely illustrative and it would be worth trying to calibrate the simulations using empirically estimated parameters for the labor demand and supply elasticities. In particular, the demand elasticity is closely related to the extent of bunching at the minimum wage in the wage density distribution.

---

[37]The fiscal explanation for serfdom has been proposed by historians (see e.g., Ardant 1971, 1972) and also fits strikingly well into the theory of Guesnerie and Roberts (1987). A Leviathan government wants to extract as much revenue from its subjects and forbidding people from leaving the state is an effective quantity based policy in that context.

References

Abowd, John M., Francis Kramarz, David N. Margolis, and Thomas Philippon, "The Tail of Two Countries: Minimum Wages and Employment in France and the United States", mimeo September 2000 (revised version of NBER WP 6996, 1999).

Allen, Stephen (1987) "Taxes, Redistribution, and the Minimum Wage: A Theoretical Analysis" *Quarterly Journal of Economics* 102(3), 477-490.

Ardant, Gabriel (1971) *Histoire de l'Impôt: De l'Antiquité au XVIIe siècle* (Fayard: Paris, France).

Ardant, Gabriel (1972) *Histoire de l'Impôt: Du XVIIIe siècle au XXIe siècle* (Fayard: Paris, France).

Atkinson, Anthony B., Stiglitz, Joseph E., (1976) "The design of tax structure: Direct versus indirect taxation." *Journal of Public Economics* 6, 55-75.

Bacache, Maya and Etienne Lehmann (2005) "Minimum Wage or Negative Income Tax: Why Skilled Workers May Favor Wage Rigidities" , IZA discussion Paper No. 1570.

Blumkin, Tomer and Efraim Sadka (2005), "Income Taxation and Wage Policy: An Application to Minimum Wage", *International Tax and Public Finance* 12, 713–722.

Boadway, Robin, and Cuff, Katherine, (2001) "A minimum wage can be welfare-improving and employment enhancing" *European Economic Review* 45, 553-576.

Brecher, Richard A., 1974. "Minimum wage rates and the pure theory of international trade" *Quarterly Journal of Economics* 88, 98116.

Brown, Charles. 1999. "Minimum Wages, Employment, and the Distribution of Income." In Orley Ashenfelter and David Card, eds. *Handbook of Labor Economics* Vol. 3, 2101-2163. New York: Elsevier.

Brown, Charles, Curtis Gilroy and Andrew Kohen. (1982). "The Effect of MinimumWage on Employment and Unemployment," *Journal of Economic Literature* 20, 487528.

Burkhauser, R. V., Couch, K. A. and Glenn, J. A. (1996), "Public Policies for the Working Poor: Earned Income Tax Credit versus Minimum Wage", *Research in Labour Economics* 15, 65-109.

Cahuc, Pierre, Zylberberg, André and Saint-Martin, A. (2001), "The consequences of the minimum wage when other wages are bargained over", *European Economic Review* 45, 337-352.

Card, David and Alan Krueger (1995), Myth and measurement: the new economics of the minimum wage (Princeton University Press, Princeton, NJ).

Choné, Philippe and Laroque, Guy (2005)"Optimal Incentives for Labor Force Participation", *Journal of Public Economics* 89, 395-425.

Choné, Philippe and Laroque, Guy (2006), "Should Low Skilled Work be Subsidized?", Mimeo CREST-INSEE.

Crépon, Bruno and Rozenn Desplatz, "Evaluation of the effects of payroll tax subsidies for low wage workers" CREST Working Paper, (January 2002)

Danziger, Leif (2006), "The Elasticity of Labor Demand and the Optimal Minimum Wage", IZA Discussion Paper No. 2360.

Diamond, Peter (1980), "Income Taxation with Fixed Hours of Work," *Journal of Public Economics* 13, 101-110.

Diamond, Peter and James Mirrlees. (1971). "Optimal Taxation and Public Production I: Production Efficiency and II: Tax Rules," *American Economic Review* 61, 827 and 261278.

Dolado, Juan, Felgueroso, F. and Jimeno, J. (2000), "The Role of the Minimum Wage in the Welfare State: An Appraisal", IZA Discussion paper 152, *Swiss Journal of Economics and Statistics* 136, 1-33.

Dolado, Juan, Francis Kramarz; Stephen Machin; Alan Manning; David Margolis; Coen Teulings (1996), "The Economic Impact of Minimum Wages in Europe", *Economic Policy* 11(23), 317-372.

Drazen, Allan (1996) "Optimum Minimum Wage Legislation." *Economic Journal* 96 (1986): 774-784.

Drèze, Jacques H. and Christian Gollier (1993), "Risk Sharing on the Labour Market and Second Best wage Rigidities", European Economic Review, 37, 1457-1482.

Dube, Arindrajit, Suresh Naidu and Michael Reich 2007. "The Economic Effects of a Citywide

Minimum Wage." *Industrial and Labor Relations Review* 60(4), 522-43.

Flinn, C. (2006) "Minimum Wage E:ects on Labor Market Outcomes under Search, Bargaining, and Endogenous Contact Rates", *Econometrica* 74, 1013-1062.

Freeman, Richard (1996), "The Minimum Wage as a Redistributive Tool", *Economic Journal*, May, 639-649.

Glaeser, Edward L. and Erzo F. P. Luttmer (2003), "The Misallocation of Housing under Rent Control", *American Economic Review* 93(4), 1027-1046.

Gorostiaga, Arantza and Juan F. Rubio-Ramrez (2006). "Optimal Minimum Wage in a Competitive Economy: An Alternative Modelling Approach", unpublished Working Paper, October 2006.

Guesnerie, Roger (1981) "Could Giving out Freely a Private Good Be a Tool of Economic Policy?" *Canadian Journal of Economics* 14(2), 232-260.

Guesnerie, Roger, Roberts, Kevin, 1984. "Effective policy tools and quantity controls." *Econometrica* 52, 59-86.

Guesnerie, Roger, Roberts, Kevin, 1987. "Minimum wage Legislation as a Second Best Policy." *European Economic Review* 31, 490-498.

Hamermesh, Daniel S. (1996). *Labor Demand*, Princeton University Press: Princeton, New Jersey.

Hungerbuhler, M. and Lehmann, Etienne (2007), "On the Optimality of a Minimum Wage: New Insights from Optimal Tax Theory", mimeo CREST.

Immervoll, Herwig (2007), "Minimum Wages, Minimum Labour Costs and the Tax Treatment of Low-wage Employment", OECD Social, Employment and Migration Working Papers No.46.

Johnson, William R. and Edgar K. Browning (1983) "The Distributional and Efficiency Effects of Increasing the Minimum Wage: A Simulation", *American Economic Review* 73(1), 204-211.

Lang, Kevin (1987) "Pareto Improving Minimum Wage Laws." *Economic Inquiry* 25, 145-158.

Laroque, Guy (2005), "Income Maintenance and Labor Force Participation", *Econometrica* 73, 341-376.

Luttmer, Erzo F. P., 2007. "Does the Minimum Wage Cause Inefficient Rationing?" Harvard Unpublished Working Paper, March 2007.

Manning, Allan (2003), *Monopsony in Motion: Imperfect Competition in Labor Markets*, Princeton University Press: New Jersey.

Marceau, Nicolas, Boadway, Robin, 1994. "Minimum wage legislation and unemployment insurance." *Scandinavian Journal of Economics* 96, 67-81.

Mirrlees, James (1971). "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies* 38, 175208.

David Neumark William Wascher (2006) "Minimum Wages and Employment: A Review of Evidence from the New Minimum Wage Research" NBER Working Paper, No. 12663.

OECD (Organization for Economic Cooperation and Development), 1998. "Making the Most of the Minimum: Statutory Minimum Wages, Employment and Poverty", in *Employment outlook*, Chapter 2, OECD, Paris.

Piketty, Thomas (1997) "La Redistribution Fiscale face au Chômage." *Revue Française d'Economie* 12, 157-201.

Rebitzer, J., Taylor, L., 1995. The consequences of minimum wage laws: Some new theoretical ideas. Journal of Public Economics 56, 245-255.

Robinson, J., (1933), *The Economics of Imperfect Competition*, MacMillan, London.

Saez, Emmanuel (2002). "Optimal Income Transfer Programs: Intensive Versus Extensive Labor Supply Responses." *Quarterly Journal of Economics* 117, 1039-1073.

Saez, Emmanuel (2004). "Direct or indirect tax instruments for redistribution: short-run versus long-run," *Journal of Public Economics* 88, 503518.

Samuelson, Paul (1951). "Theory of optimal taxation", memorandum to the US Treasury, published in *Journal of Public Economics* 30(2) July 1986, 137-143.

Stigler, George, 1946. The economics of minimum wage legislation. *American Economic Review* 36, 358-365.

Stiglitz, Joseph E., 1982, "Self-Selection and Pareto E.cient Taxation", *Journal of Public Economics* 17, 213-240.

Swinnerton, K. (1996), "Minimum Wages in an Equilibrium Search Model with Diminishing Returns to Labor in Production." *Journal of Labor Economics* 14, 340-355.

Wijkander, Hans (1988). "Equity and Efficiency in Public Sector Pricing: A Case for Stochastic Rationing", *Econometrica* 56(6), 1455-1465.

Figure 1: Desirability of Small Minimum Wage

Wage w_1

Transfer $h_1 d\bar{w}$ from other factors to min wage workers

Supply $S(w_1)$

$\bar{w}=w^*_1+d\bar{w}$
w^*_1

Loss due to unemployment: 2nd order

Demand $D(w_1)$

h^*_1

Labor h_1
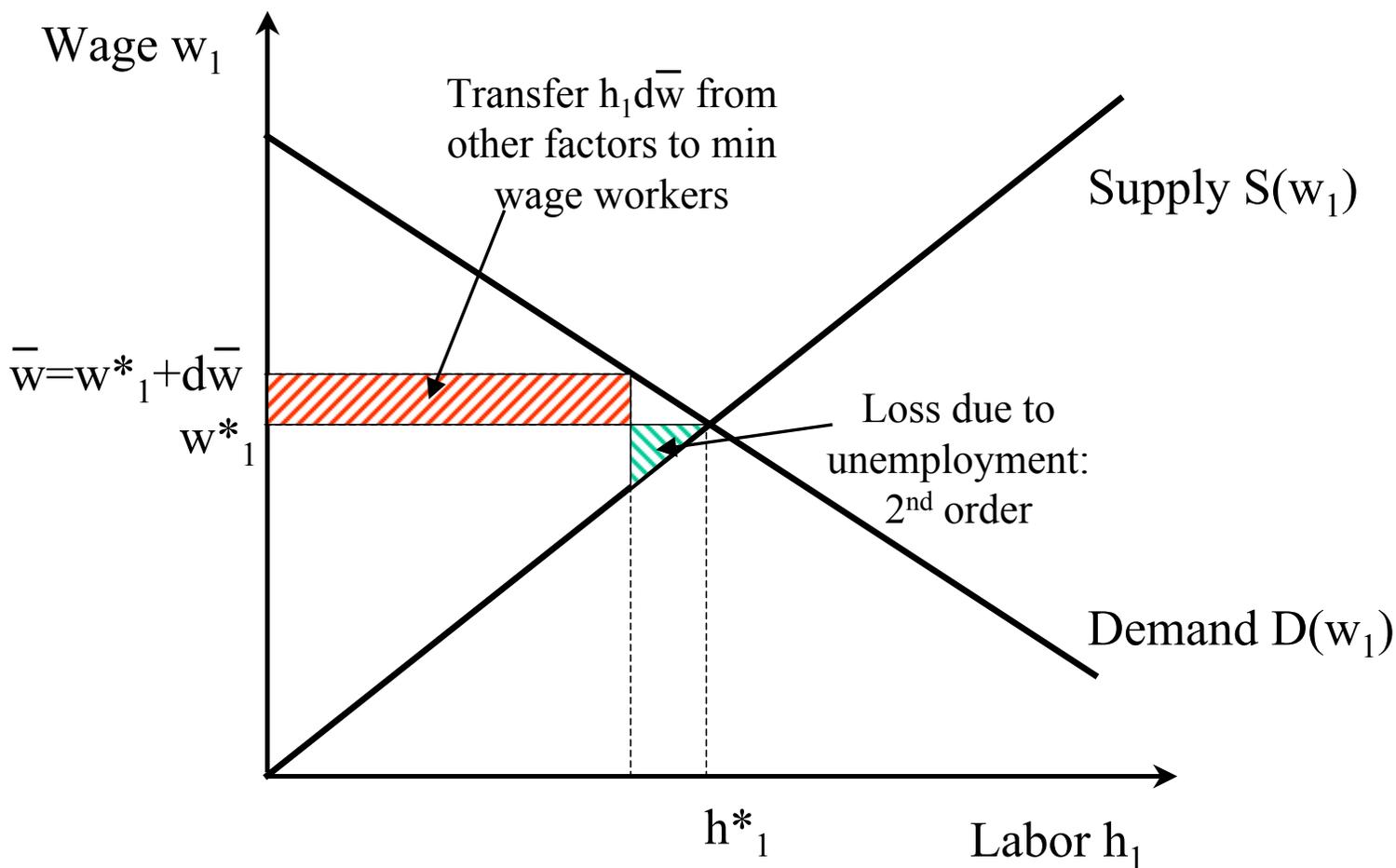
Figure 2: Deriving the Optimal Minimum Wage

Wage w_1

Transfer from other factors to
min wage workers: $h_1 d\bar{w}$
Weight $g_1 - g_2$

Supply $S(w_1)$

$\bar{w} + d\bar{w}$

min wage $\bar{w}$

$\underline{w}$

Unemployment loss:
$(\bar{w} - \underline{w}) dh_1$
Weight g_0^a

Demand $D(w_1)$

$dh_1 < 0$

Labor h_1
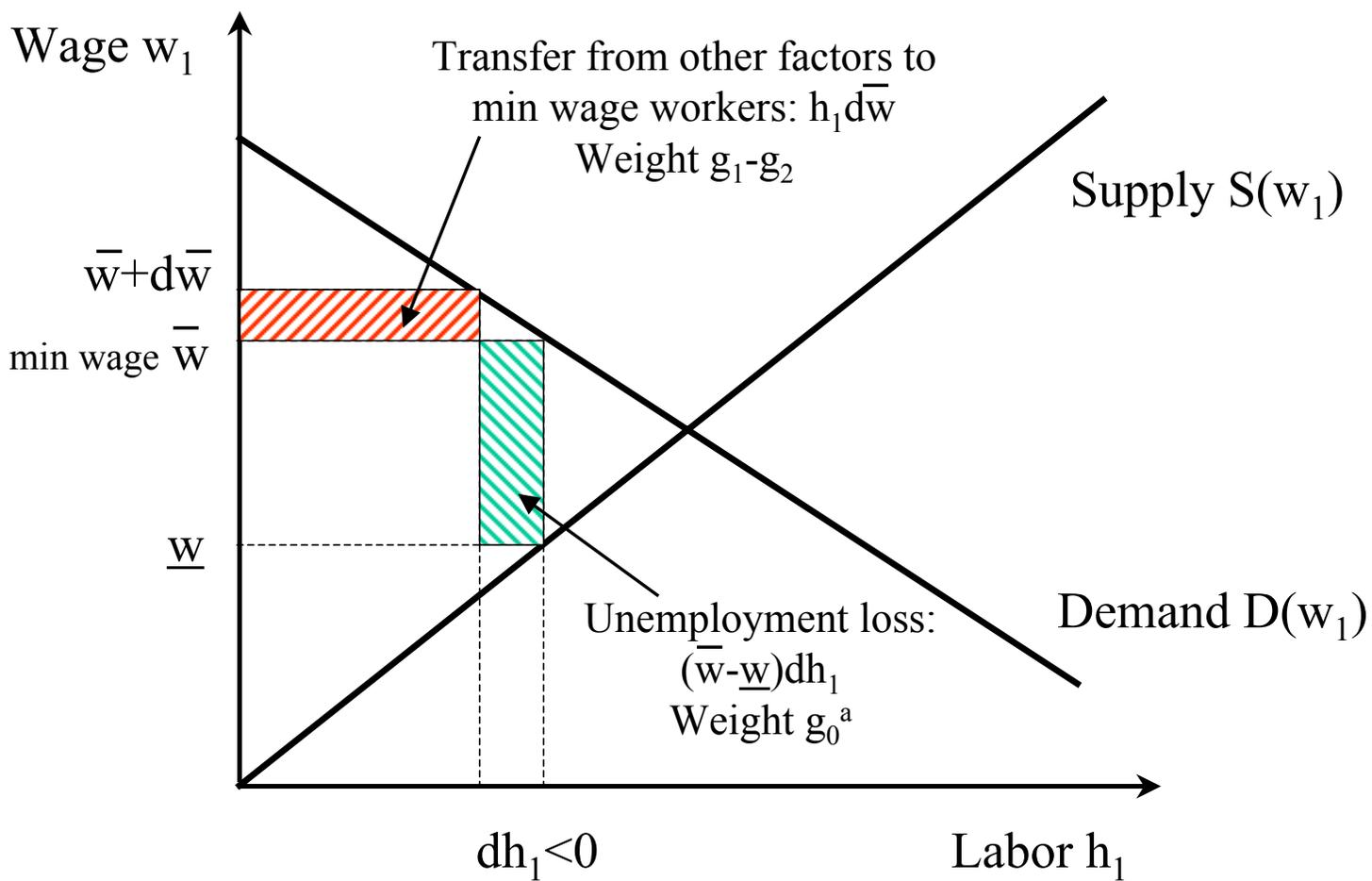
Figure 3: Min wage with continuous wage distribution

Density

Wage density
above $\bar{w}$ shifts
down (slightly)
$\Delta w < 0$

Some workers with $w < \bar{w}$
become unemployed

Some workers with
$w < \bar{w}$ are bunched at $\bar{w}$

$\bar{w}$

Wage w

Figure 4a: Optimal Tax/Transfer Derivation

Consumption c

c_2

Net Welfare effect: $h_1 dc_1(g_1-1) > 0$

Labor Supply: $dh_1 w_1 \tau_1 < 0$

$c_1 + dc_1$

c_1

At the optimum:
$dh_1 w_1 \tau_1 + h_1 dc_1(g_1-1) = 0$
implies
$\tau_1/(1-\tau_1) = (1-g_1)/e_1 < 0$

c_0

45°

0

w_1

w_2

Wage w

Figure 4b: Optimal Tax/Transfer Derivation

Consumption c

Net Welfare effect: $h_1 dc_1(g_1-1)>0$

c_2

Labor Supply: $dh_1 w_1 \tau_1 < 0$

$c_1 + dc_1$
c_1

c_0

Endogenous wages do not affect optimal formula as $h_1 dw_1 + h_2 dw_2 = 0$ (no profits) and tax$= h_1(w_1-c_1)+h_2(w_2-c_2)$

45°

0

$dw_1<0$

$dw_2>0$

Wage w

Figure 5: Desirability of Min Wage with Optimal Taxes

Consumption
c

Net Welfare effect: $h_1 dc_1(g_1-1)>0$

c_2

Labor Supply:
$dh_1=0$

$c_1 +dc_1$

c_1

With min wage set at w_1,
$dc_1>0$ does not affect labor
supply because w_1 cannot go
down: $dh_1=0$

c_0

45°

0

$\bar{w}=w_1$

w_2

Wage w

Figure 6: Optimal c_1 with Min Wage such that $g_1=1$

Consumption c

Net Marginal welfare effect:
$h_1dc_1(g_1-1)=0$

c_2

$c_1 +\Delta c_1$

c_1

With min wage set at w_1, c_1 is a
pure lumpsum transfer and
hence adjusted up till $g_1=1$
(creates invol.unemployment)

c_0

45°

0

$\overline{w}=w_1$

w_2
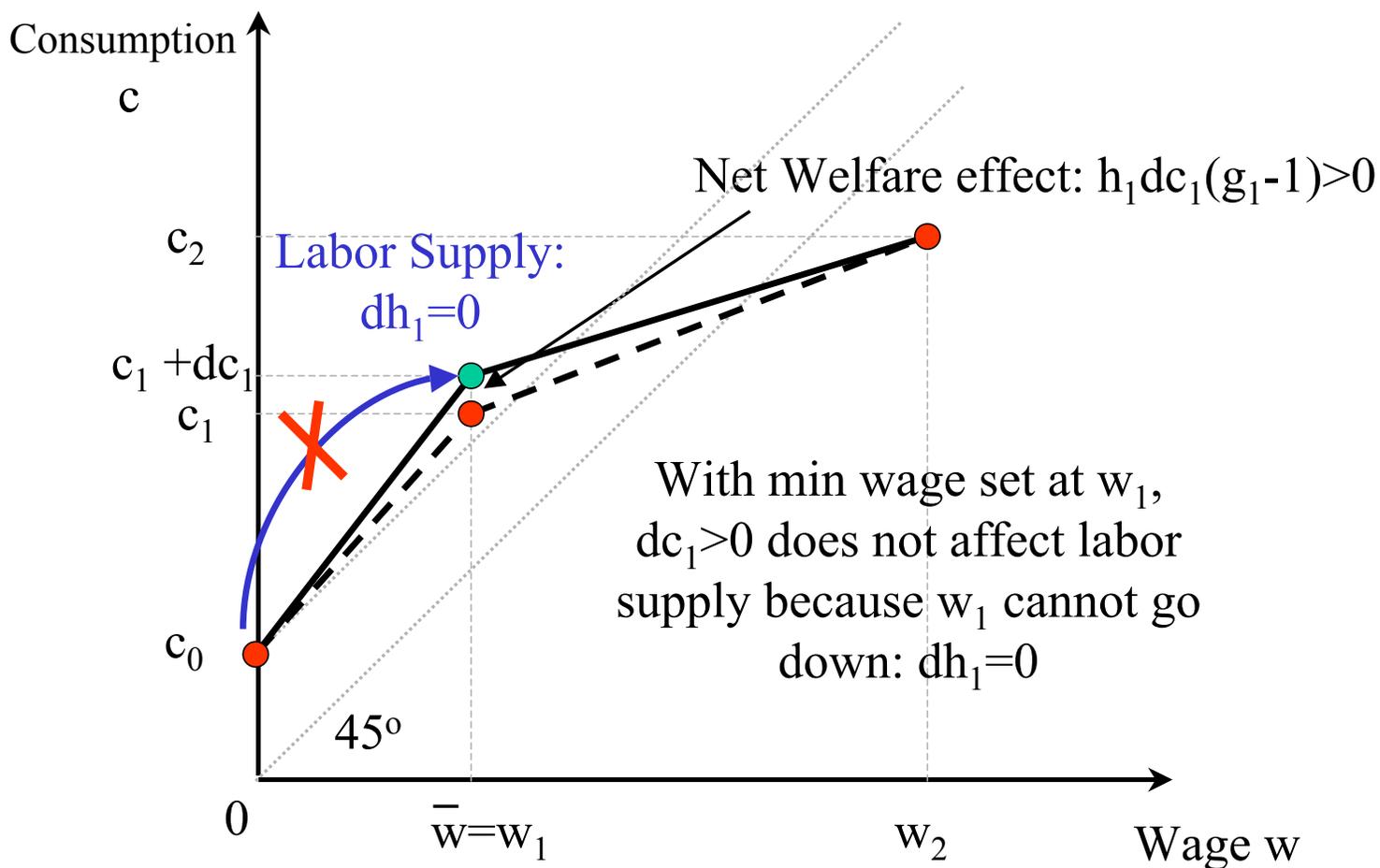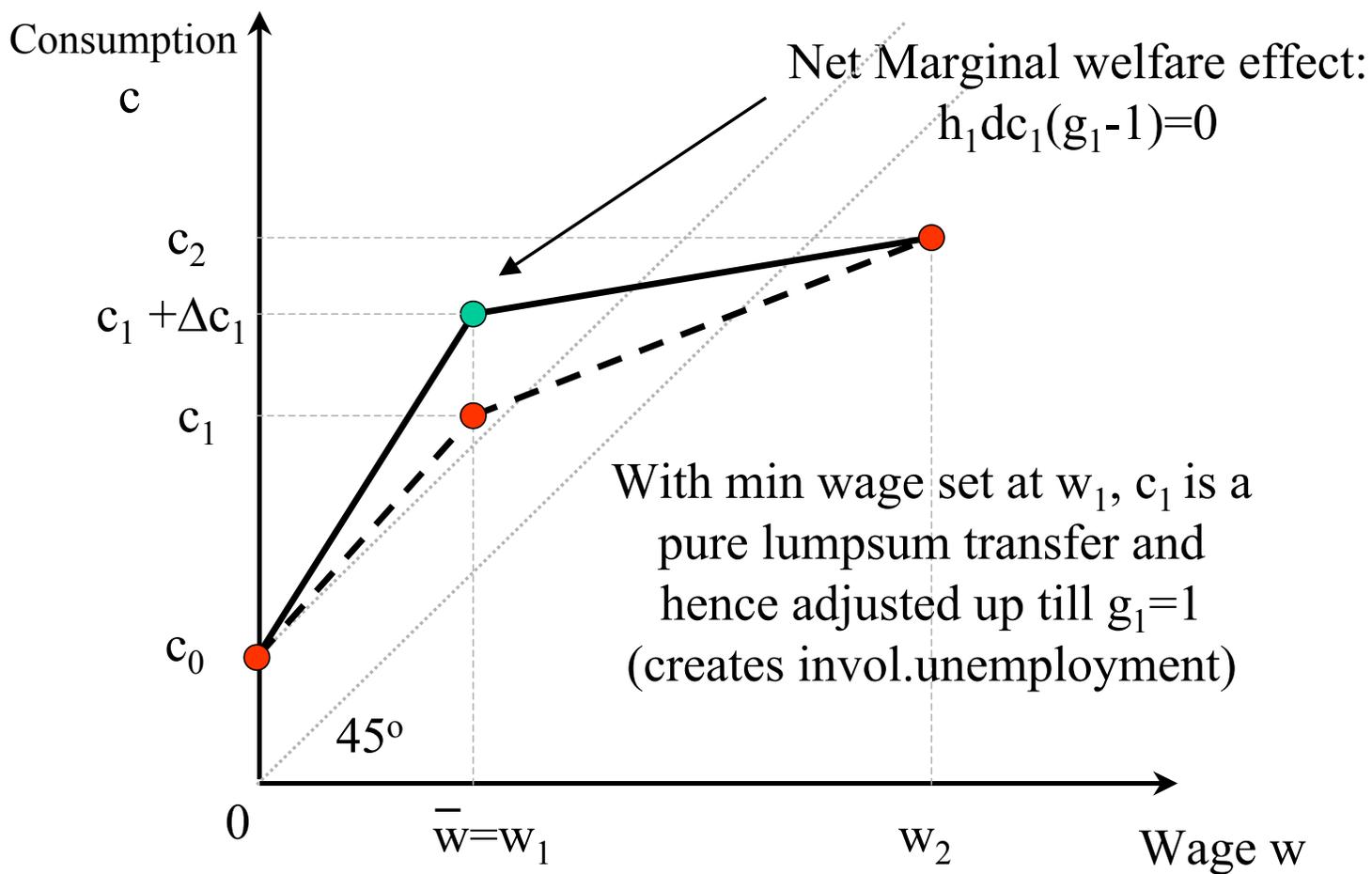
Wage w

Figure 7: Optimal Minimum Wage Derivation

Consumption c

Unemployment $dh_1 < 0$
Welfare effect < 0
Fiscal effect: $\tau_1 w_1 \, dh_1 > 0$

c_2

c_1

At the optimum:
Fiscal effect + welfare effect = 0
→ with min wage, we need $\tau_1 < 0$

c_0

45°

0

$d\bar{w} > 0$

$dw_2 < 0$
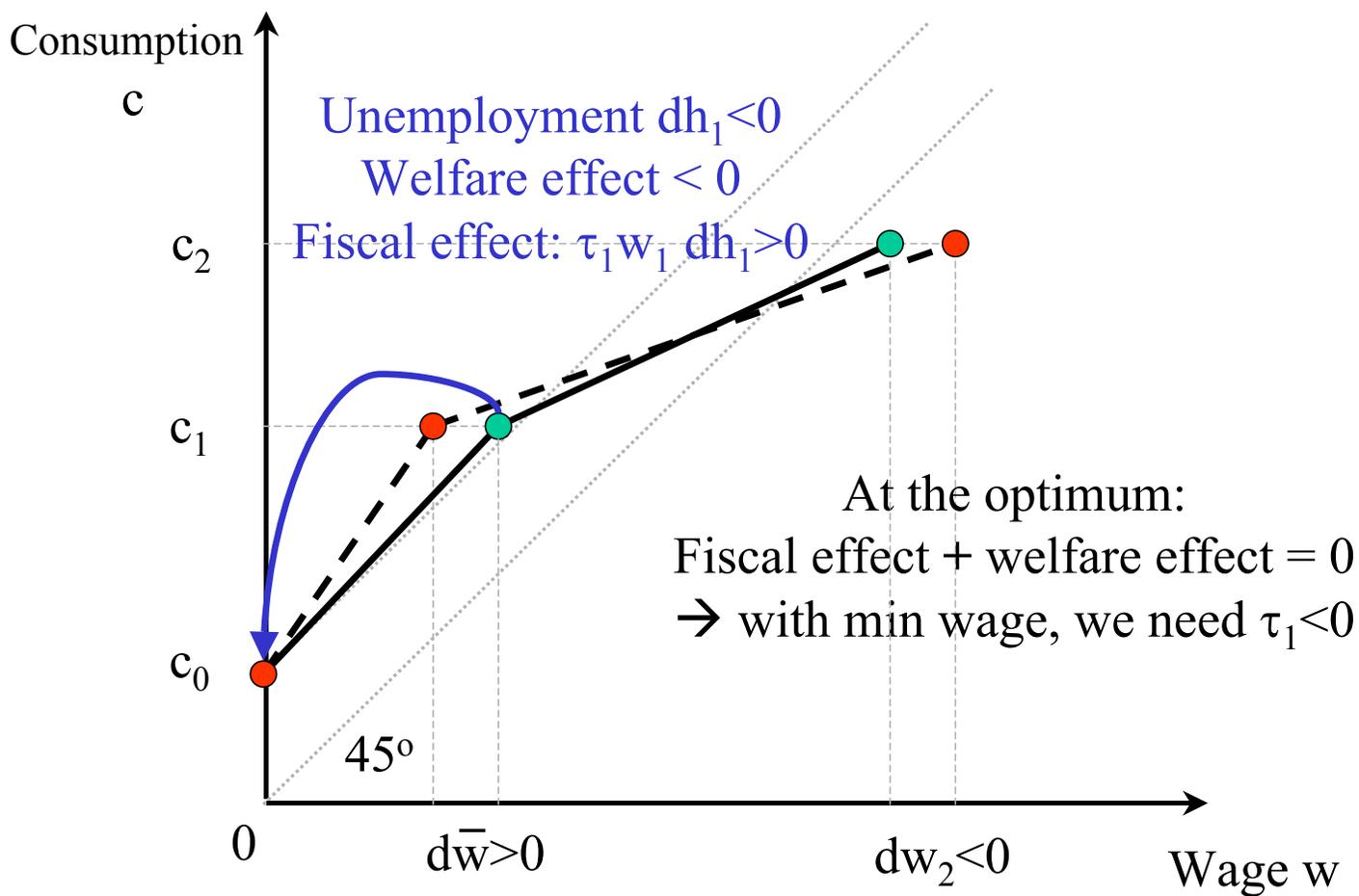
Wage w

Fig. 8: Double Dividend Policy when $\tau_1 > 0$ and min wage binds

Consumption c

Unemployment decreases: $dh_1 > 0$
Welfare effect > 0
Fiscal effect: $\tau_1 w_1\, dh_1 > 0$

c_2

c_1

Reduce $\bar{w}$ while keeping c_1, c_2 constant:
Fiscal effect > 0 and welfare effect > 0
Situation was (2nd best) Pareto inefficient
$\rightarrow$ Cut employer payroll taxes at bottom

c_0

45°

0 $d\bar{w} < 0$ $dw_2 > 0$ **Wage w**
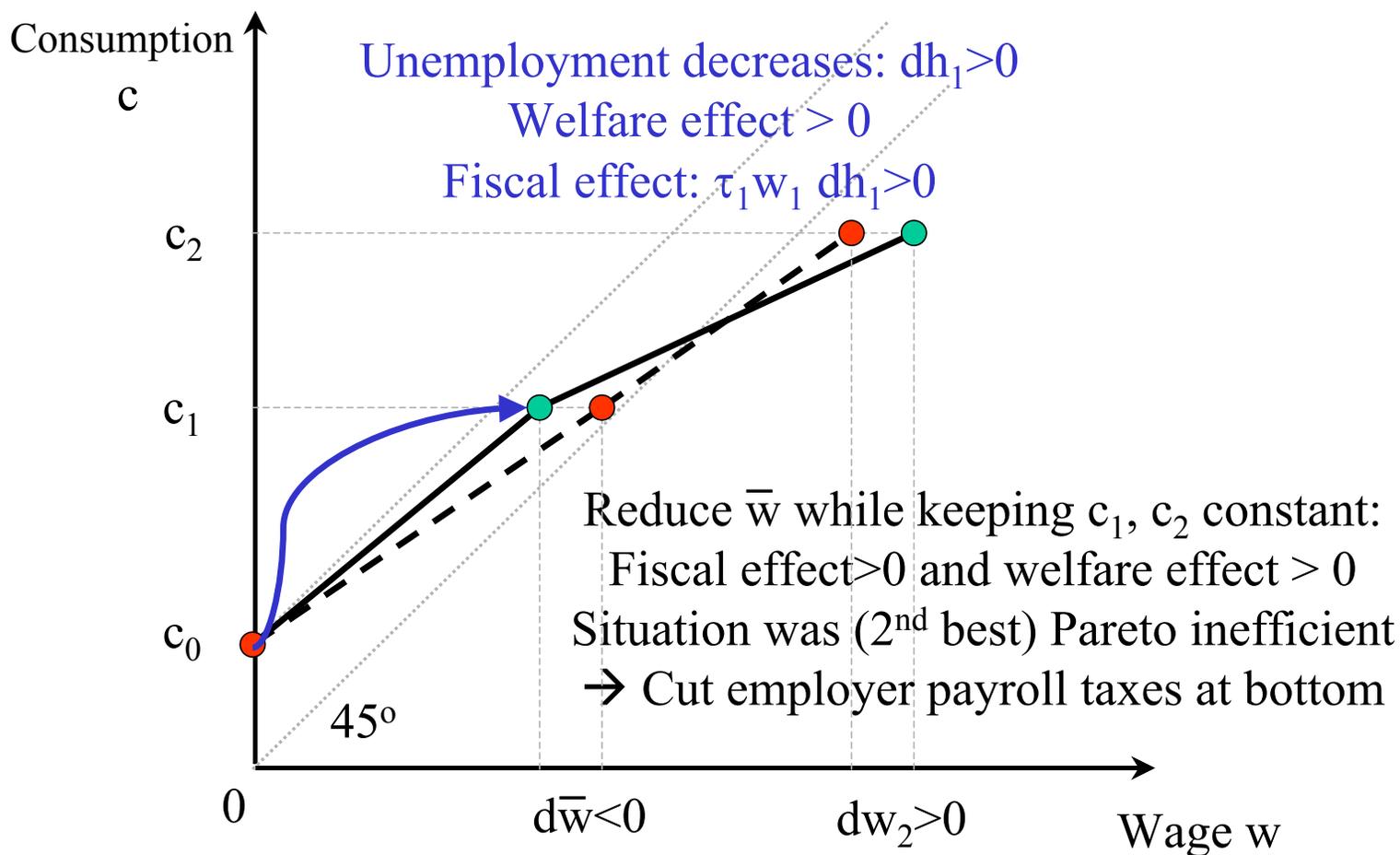
Table 1: Optimal Minimum Wage with No Taxes or Fixed Taxes

|	$\sigma=0.5$ $e_1=0.25$ (1)	$\sigma=0.5$ $e_1=0.5$ (2)	$\sigma=0.5$ $e_1=1$ (3)	$\sigma=0.25$ $e_1=0.5$ (4)	$\sigma=0.5$ $e_1=0.5$ (5)	$\sigma=1$ $e_1=0.5$ (6)
A. Optimum Minimum Wage with no taxes and transfers						
A1. Case $\gamma=1$						
Minimum Wage / Market Wage	1.12	1.21	1.34	1.44	1.21	1.07
Unemployment Rate	7.6%	16.8%	39.8%	24.8%	16.8%	9.2%
A2. Case $\gamma=3$						
Minimum Wage / Market Wage	1.03	1.08	1.17	1.18	1.08	1.03
Unemployment Rate	2.3%	6.9%	20.1%	10.9%	6.9%	3.8%
A3. Case $\gamma=0.5$						
Minimum Wage / Market Wage	1.11	1.19	1.29	1.41	1.19	1.06
Unemployment Rate	6.9%	15.0%	34.9%	23.1%	15.0%	8.1%
B. Optimum Minimum Wage with exogenous taxes (uniform tax rate $\tau=0.35$)						
B1. Case $\gamma=1$						
Minimum Wage / Market Wage	1.00	1.01	1.04	1.13	1.01	1.00
Unemployment Rate	0.0%	0.5%	5.5%	8.0%	0.5%	0.0%

Notes: The table reports the minimum wage (relative to market wage rate w^*_1) and the induced unemployment rate among the low skilled as a function of the elasticity of substitution σ between low and high skilled labor in production, the elasticity of labor supply of low skilled workers e_1 (the high skilled labor supply elasticity $e_2=0.25$ in all cases), and the risk aversion γ of the social welfare function.
The production function is CES with elasticity of substitution γ, calibrated so that market equilibrium with no minimum wage is $(w^*_1,w^*_2)=(1,3)$. The supply functions are calibrated so that $(h^*_0,h^*_1,h^*_2)=(.2,.2,.6)$
The social welfare function is such that $G(u)=(u+0.5)^{1-\gamma}/(1-\gamma)$.

Table 2: Optimal Minimum Wage with Optimal Taxes

	$\sigma=0.5$ $e_1=0.25$	$\sigma=0.5$ $e_1=0.5$	$\sigma=0.5$ $e_1=1$	$\sigma=0.25$ $e_1=0.5$	$\sigma=0.5$ $e_1=0.5$	$\sigma=1$ $e_1=0.5$
	(1)	(2)	(3)	(4)	(5)	(6)

A. Case $\gamma=1$

A1. Optimal Tax Rates with no Minimum Wage

Tax rate on low skilled workers τ_1	-9.0%	-20.7%	-20.1%	-25.7%	-20.7%	-16.6%
Tax rate on high skilled workers τ_2	45.4%	46.2%	47.6%	47.0%	46.2%	45.4%

A2. Optimal Tax Rates and optimal Minimum Wage

Tax rate on low skilled workers τ_1	-13.1%	-45.0%	-98.4%	-56.4%	-45.0%	-35.6%
Tax rate on high skilled workers τ_2	45.2%	44.9%	44.2%	45.5%	44.9%	44.3%
Minimum Wage / Market Wage	1.02	1.11	1.31	1.20	1.11	1.06
Unemployment Rate	1.6%	11.2%	52.1%	13.2%	11.2%	9.3%

B. Case $\gamma=3$

B1. Optimal Tax Rates with no Minimum Wage

Tax rate on low skilled workers τ_1	28.6%	10.2%	-7.6%	5.3%	10.2%	14.1%
Tax rate on high skilled workers τ_2	64.0%	64.2%	64.7%	64.6%	64.2%	63.8%

B2. Optimal Tax Rates and optimal Minimum Wage

Tax rate on low skilled workers τ_1	28.6%	10.2%	-15.5%	5.3%	10.2%	14.1%
Tax rate on high skilled workers τ_2	64.0%	64.2%	64.4%	64.6%	64.2%	63.8%
Minimum Wage / Market Wage	1.00	1.00	1.05	1.00	1.00	1.00
Unemployment Rate	0.0%	0.0%	6.8%	0.0%	0.0%	0.0%

C. Case $\gamma=0.5$

C1. Optimal Tax Rates with no Minimum Wage

Tax rate on low skilled workers τ_1	-21.9%	-23.3%	-16.6%	-25.9%	-23.3%	-17.8%
Tax rate on high skilled workers τ_2	32.4%	33.6%	35.2%	34.4%	33.6%	32.0%

C2. Optimal Tax Rates and optimal Minimum Wage

Tax rate on low skilled workers τ_1	-41.0%	-81.3%	-153.3%	-93.2%	-81.3%	-34.1%
Tax rate on high skilled workers τ_2	31.3%	30.7%	29.1%	31.5%	30.7%	31.1%
Minimum Wage / Market Wage	1.08	1.21	1.49	1.35	1.21	1.04
Unemployment Rate	5.5%	21.9%	91.5%	23.8%	21.9%	4.7%

Notes: The table reports optimal tax rates (on low and high skilled) with no minimum wage and
the joint optimal tax rates and minimum wage (relative to market wage rate w^*_1) and the induced unemployment rate
among the low skilled as a function of the elasticity of substitution σ between low and high skilled labor
in production, the elasticity of labor supply of low skilled workers e_1 (the high skilled labor supply elasticity
$e_2=0.25$ in all cases), and the risk aversion γ of the social welfare function.
The production function is CES with elasticity of substitution γ, calibrated so that market equilibrium with
no minimum wage is $(w^*_1,w^*_2)=(1,3)$. The supply functions are calibrated so that $(h^*_0,h^*_1,h^*_2)=(.2,.2,.6)$
The social welfare function is such that $G(u)=(u+0.5)^{1-\gamma}/(1-\gamma)$.