The Optimal Use of Government Purchases for Stabilization

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the policies for business-cycle stabilization

- extensive research on monetary policy
- but monetary policy is sometimes constrained
  - zero lower bound (Japan, USA, EU)
  - monetary union (EU, USA)
- leading to high unemployment
in this paper: government purchases

- framework: matching model of the business cycle
- outcome: a formula linking optimal budget-balanced government purchases to
  - unemployment gap
  - unemployment multiplier
  - social value of government consumption
relationship to other frameworks

- competitive model [Samuelson 1954]
  - ∼ matching with efficient unemployment
  - but matching also has inefficient unemployment

- disequilibrium model [Mankiw & Weinzierl 2011]
  - ∼ matching with no matching cost
  - but matching has supply in addition to demand

- New Keynesian model [Woodford 2010]
  - alternative representation of slack
  - but matching is a bit more tractable
a matching model
structure

- dynamic matching model
  - building on Michaillat & Saez [2015]
- identical self-employed households
- government
- 2 consumption goods traded on 1 matching market
  - public services + private services
- 1 asset for saving
matching market

- capacity of each household: \( k \) services
- household purchases: \( C(t) \) private services
- government purchases: \( G(t) \) public services
- output: \( Y(t) = C(t) + G(t) < k \)
- unemployment rate: \( u(t) = 1 - Y(t)/k \)
- price of services: \( p(t) \)
matching function and market tightness

- number of vacancies: $v(t)$
- matching function: $m(t) = \omega \cdot (k - Y(t))^\eta \cdot v(t)^{1-\eta}$
- **market tightness**: $x(t) = v(t)/(k - Y(t))$
- selling rate and buying rate:

  $$f(x(t)) = \frac{m(t)}{k - Y(t)} = \omega \cdot x(t)^{1-\eta}$$

  $$q(x(t)) = \frac{m(t)}{v(t)} = \omega \cdot x(t)^{-\eta}$$
market flows

- relationships separate at rate $s$

- given $x$, output and unemployment converge to

$$Y(x, k) = \frac{f(x)}{s + f(x)} \cdot k, \quad u(x) = \frac{s}{s + f(x)}$$

- convergence to steady state is extremely fast so we assume $Y(t) = Y(x(t), k)$ and $u(t) = u(x(t))$
matching cost: \( \rho \) services per vacancy

- output \((Y) = \) consumption \((y) + \) matching cost

\[
Y = y + \rho \cdot v = y + s \cdot Y \cdot \frac{\rho}{q(x)}
\]

- matching wedge: \( \tau(x) = \frac{s \cdot \rho}{q(x) - s \cdot \rho} \)

- total consumption: \( y = Y / [1 + \tau(x)] \)

- private consumption: \( c = C / [1 + \tau(x)] \)

- public consumption: \( g = G / [1 + \tau(x)] \)
supply structure: summary

capacity: $k$
supply structure: summary

output:

\[ Y(x, k) = (1 - u(x)) \cdot k \]

tightness \( x \)

capacity \( k \)

public + private services
supply structure: summary

output $Y(x,k)$  

capacity $k$

idle capacity:

$u(x) \cdot k$

public + private services
supply structure: summary

consumption:

\[ y(x, k) = \frac{Y(x, k)}{1 + \tau(x)} \]

output \( Y(x,k) \)  capacity \( k \)

public + private services
supply structure: summary

consumption $y(x,k)$

output $Y(x,k)$

capacity $k$

matching cost:

$y(x, k) \cdot \tau(x)$

tightness $x$

public + private services
demand structure: example

- asset: housing $h(t)$ in fixed supply $H$
  - traded on a competitive market
  - Iacoviello [2005] and Liu, Wang, Zha [2013]
- households choose $c(t)$ and $h(t)$ to maximize utility

$$
\int_{0}^{+\infty} e^{-\delta \cdot t} \cdot [U(c, g) + V(h)] \, dt
$$

- subject to flow budget constraint

$$
\frac{dh}{dt} = p \cdot [1 - u(x)] \cdot k - p \cdot [1 + \tau(x)] \cdot c - T
$$
aggregate demand in the example

- market clearing on housing market: \( h = H \)

- aggregate demand from Euler equation:

\[
\frac{\partial U}{\partial c}(c^d(x,g,p),g) = \frac{p \cdot (1 + \tau(x)) \cdot \psi'(H)}{\delta}
\]

- price of services relative to housing: \( p = p(g) \)
  - assumption required in matching model
  - here: general mechanism
equilibrium tightness is \( x = x(g) \)
efficient unemployment rate

\[ x(g) = x^* \]

\[ y \]

\[ Y \]

\[ u^* \]

\[ y^* \]

\[ Y^* \]

demand

supply

\[ x, y, Y \]
inefficiently high unemployment rate
inefficiently low unemployment rate
formulas for
optimal government purchases
government’s problem

- households’ instantaneous utility is \( U(c, g) \)
- government purchases are financed by a lump-sum tax to maintain a balanced budget
- given \( x(g) \), the government chooses \( g \) to maximize

\[
U \left( \frac{y(x(g), k) - g, g}{c} \right)
\]
correcting the Samuelson formula

- First-order condition of government’s problem is

\[ 0 = \frac{\partial U}{\partial g} - \frac{\partial U}{\partial c} + \frac{\partial U}{\partial c} \cdot \frac{\partial y}{\partial x} \cdot \frac{dx}{dg} \]

- Optimal government purchases satisfy

\[ 1 = MRS_{gc} + \frac{\partial y}{\partial x} \cdot \frac{dx}{dg} \]

\[ \text{Samuelson formula} \quad + \quad \text{correction} \]

- Correction: effect of \( g \) on welfare through \( x \)
introducing estimable statistics

- elasticity of substitution between $g$ and $c$:

$$1 - MRS_{gc} \approx \frac{1}{\varepsilon} \cdot \frac{g/c - (g/c)^*}{(g/c)^*}$$

- unemployment gap:

$$\frac{\partial y}{\partial x} \propto \frac{u - u^*}{u^*}$$

- unemployment multiplier:

$$\frac{dx}{dg} \propto m \equiv -\frac{y}{1 - u} \cdot \frac{du}{dg}$$
an implicit formula

\[
\frac{g/c - (g/c)^*}{(g/c)^*} \approx \frac{\varepsilon}{1 - \eta} \cdot m \cdot \frac{u - u^*}{u^*}
\]

- \((g/c)^*\): Samuelson level
- \(\varepsilon\): elasticity of substitution between \(g\) and \(c\)
- \(\eta\): elasticity of matching wrt unemployment
- \(u - u^*\): unemployment gap
- \(m\): unemployment multiplier
departure from the Samuelson level

<table>
<thead>
<tr>
<th>unemployed multiplier</th>
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<tbody>
<tr>
<td>$m &lt; 0$</td>
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<tr>
<td>$m = 0$</td>
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<tr>
<td>$m &gt; 0$</td>
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| $u > u^*$  | $g/c < (g/c)^*$ | $g/c = (g/c)^*$ | $g/c > (g/c)^*$ |
| $u = u^*$  | $g/c = (g/c)^*$ | $g/c = (g/c)^*$ | $g/c = (g/c)^*$ |
| $u < u^*$  | $g/c > (g/c)^*$ | $g/c = (g/c)^*$ | $g/c < (g/c)^*$ |
the marginal value of public services

- $\varepsilon = 0$: digging holes or building pyramids
  - $g/c = (g/c)^*$: no stabilization

- $\varepsilon \to +\infty$: perfect substitution
  - $u = u^*$: perfect stabilization
  - entirely fill unemployment gap

- $\varepsilon \in (0, +\infty)$: medium substitution
  - medium stabilization: $g/c \neq (g/c)^*$ but $u \neq u^*$
  - partially fill unemployment gap
making the formula explicit

- implicit formula: not useful for quantitative results because $u$ in RHS responds to $g/c$ in RHS

- we start from $(g/c)^*$ and $u_0 \neq u^*$:

$$\frac{g/c - (g/c)^*}{(g/c)^*} \approx \frac{\varepsilon \cdot m}{1 - \eta} \cdot \frac{u(g/c) - u^*}{u^*}$$

- first-order Taylor expansion of $u$ at $u((g/c)^*) = u_0$

$$\frac{u - u^*}{u^*} \approx \frac{u_0 - u^*}{u^*} - z \cdot m \cdot \frac{g/c - (g/c)^*}{(g/c)^*}$$
an explicit formula

- optimal \( g/c \) depends on fixed quantities:
  \[
  \frac{g/c - (g/c)^*}{(g/c)^*} \approx \frac{\varepsilon}{1 - \eta} \cdot \frac{m}{1 + z \cdot \frac{\varepsilon}{1 - \eta} \cdot m^2} \cdot \frac{u_0 - u^*}{u^*}
  \]

- optimal \( u \) depends on fixed quantities:
  \[
  u \approx u^* + \frac{1}{1 + z \cdot \frac{\varepsilon}{1 - \eta} \cdot m^2} \cdot (u_0 - u^*)
  \]

- \( z = (g/y)^* \cdot [1 - (g/y)^*] \cdot (1 - u_0)/u^* \)
results with distortionary taxation

- endogenous capacity: \( \mathcal{U}(c, g, k) \) with \( \partial \mathcal{U} / \partial k < 0 \)
- linear income tax: \( T = \tau^L \cdot (1 - u(x)) \cdot k \)
- everything remains valid but \( (g/c)^* \) lower because of tax distortions
- however: link between multipliers changes
  - no tax distortions: \( m = dY / dG \)
  - tax distortions: \( m > dY / dG \)
  - with taxes, we may have \( dY / dG < 0 \) but \( m > 0 \)
numerical application:

Great Recession in the US
calibration

- starting point: winter 2008–2009
  - $u = 6\%$, $u_0 = 9\%$, and $G/Y = 16.5\%$
  - assumption: $u^* = 6\%$ and $(G/Y)^* = 16.5\%$

- $\varepsilon$: input into policy problem
- $\eta \in [0.5, 0.7]$: Pissarides & Petrongolo [2001]
- $m \in [0.2, 1]$ on average in the US [Ramey 2013]
  - in addition, $m$ may be higher in recessions
  - see Auerbach & Gorodnichenko [2012]
optimal stimulus spending

Increase in G (% GDP)

Unemployment multiplier

$\epsilon = 1$
optimal stimulus spending

![Diagram showing the relationship between unemployment multiplier and increase in G (% GDP). The graph depicts a curve where the maximum increase in G occurs at a multiplier of 0.1, with a peak of 2%. Below the maximum, the increase in G decreases as the multiplier increases.]
optimal stimulus spending

Unemployment multiplier

Increase in G (% GDP)

$520$ billion

0.4

0 0.5 1 1.5 2
optimal stimulus spending

[Graph showing the relationship between unemployment multiplier and increase in G (% GDP). The graph peaks at 2% increase in G at a unemployment multiplier of 1.4.]
optimal stimulus spending

![Graph showing the optimal stimulus spending with different values of \( \epsilon \). The graph plots the increase in G (\% GDP) against the unemployment multiplier. There are three lines representing different values of \( \epsilon \): 0.5, 1, and 2. Each line peaks at a different point, indicating the optimal stimulus spending at various unemployment multipliers.](image-url)
unemployment with optimal stimulus
quality of approximations in formula

Unemployment multiplier

Aggregate demand

Constant $G/Y$
quality of approximations in formula

Govt. purchases/output vs. Aggregate demand

- Constant $G/Y$
- Optimal $G/Y$
- Explicit formula

16.5%
summary: implications for policy
1. $\frac{dY}{dG} > 1$ is not necessary for stimulus spending
   - sufficient: $\frac{du}{dG} > 0$
   - available evidence suggests that $\frac{du}{dG} > 0$

2. tax distortions $\nRightarrow$ smaller stimulus
   - tax distortions only reduce the average $G$

3. tax distortions $\nRightarrow$ $\frac{du}{dG} < 0$
   - tax distortions do reduce $\frac{dY}{dG}$
   - but they raise $\frac{du}{dG}$
4. low marginal value of $g \nRightarrow$ no stimulus
   - optimal: use $g$ to reduce unemployment gap
   - except if $g =$ bridges to nowhere

5. positive value of $g \nRightarrow$ filling the unemployment gap
   - optimal: partially filling unemployment gap
   - except if $g$ and $c$ are perfect substitutes

6. bang-for-the-buck logic does not hold
   - strongest stimulus for $m = 0.4$
   - same stimulus for $m = 0.1$ and $m = 1.4$