OPTIMAL PUBLIC EXPENDITURE WITH INEFFICIENT UNEMPLOYMENT

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MAIN STABILIZATION POLICY: MONETARY POLICY

- Policymakers rely primarily on monetary policy for stabilization
  - Accordingly: extensive research on optimal monetary policy
- But monetary policy is sometimes constrained
  - Zero lower bound (Japan, USA, EU)
  - Monetary union (EU, USA)
  \[\Rightarrow\] High unemployment
- Then other stabilization policies are needed
  - But: very little is known about these alternative policies
public expenditure is commonly used for stabilization

- US: Great Depression (New Deal), Great Recession (ARRA)

framework: matching model from Michaillat & Saez (2015)

outcome: formula linking optimal stimulus spending to 3 sufficient statistics

1. unemployment gap
2. unemployment multiplier
3. elasticity of substitution between public consumption & private consumption
OPTIMAL PUBLIC EXPENDITURE: EXISTING RESULTS

• Samuelson (1954):
  – public goods financed by lump-sum taxation
  – efficient level of production
  – rule: spend until marginal utilities are equalized
  – but: what if production is inefficient?

• Keynes (1936):
  – no tradeoffs between public consumption & private consumption (multiplier > 1)
  – rule: spend to fill output gap
  – but: what if there is a tradeoff?

• our theory blends the theories of Samuelson & Keynes
INFORMAL DESCRIPTION OF THE MODEL
A SERVICE ECONOMY, WITHOUT FIRMS
A SERVICE ECONOMY, WITHOUT FIRMS
AN ASSET FOR SAVING
PRIVATE SERVICES (c) & PUBLIC SERVICES (g)
PRIVATE SERVICES (c) & PUBLIC SERVICES (g)
MATCHING: NOT ALL SERVICES ARE SOLD
MATCHING: NOT ALL SERVICES ARE SOLD
MATCHING: COSTLY TO PURCHASE SERVICES

Aunt Ann's In House Staffing Nanny, Housekeeper and Estate Agency of San Francisco, CA - the nationwide leader in Household Staffing.

Aunt Ann’s In House Staffing has been specializing in exclusive domestic staffing services for 50 years. We have worked with some of the world’s finest families and estates, representing highly trained and qualified domestic household personnel.

Aunt Ann's In House Staffing is a full-service private service placement firm that specializes in the staffing of key private service positions including nannies, housekeepers, baby nurses, doulas, executive housekeepers, private chefs, chauffeurs, butlers, household managers, domestic couples, estate managers, companions, personal assistants, executive assistants and more.

Aunt Ann's In House Staffing of San Francisco is staffed with seasoned, accomplished and professional placement consultants.

Our professional consultants are individuals who are trained to listen to your unique needs and are experienced in private service. We understand high standards and the art of service, absolute discretion and the special circumstances of our client’s lives.

Whether you are experienced in domestic service, or are a client searching to fill a position, we welcome the opportunity to offer our world class services to assist you.
**Fee Service Agreement:**

This Agreement confirms the terms and conditions whereby In-House Staffing @ Aunt Ann's, Inc. (dba Aunt Ann’s In-House Staffing) (the “Agency”) agrees to assist the undersigned (“Client”) whether singular or plural. Agency is a domestic referral agency and will make reasonable efforts according to applicable laws to refer individual(s) to provide in-home or office services to Client (“Candidate(s)”). However, Client understands and agrees that Client is responsible for making the final decision to hire or otherwise engage a Candidate. Client understands and agrees that signing up with or paying Agency does not guarantee that Agency will find a suitable Candidate for Client.

**Referral Fee:**
Upon Client’s hiring or other engagement of a Candidate referred by Agency, Client agrees to pay Agency its fee as stated in Agency’s Fee Schedule below. Client agrees to pay a fee for each Candidate Client employs or engages. Agency will bill the fee to Client upon Agency’s confirmation of Client’s hiring or engagement of a Candidate. Client agrees to pay Agency its fee in full after any applicable trial period and upon agency’s mailing or other transmission of the invoice to client within 14 days.

**FEE SCHEDULE:**

### Fee for Long Term Services:

**CHILDCARE**
- **All Full Time** Nanny, Parent Helper, Family Assistant, Governess
  - 15% of annual Gross Compensation (minimum fee = $3000)
- **All Part Time** Nanny, Parent Helper, Family Assistant, Governess
  - 15% of annual Gross Compensation (minimum fee = $1500)

**HOUSEHOLD**
- **All Full Time** Housekeeper, Executive Housekeeper, Cook, Handyman, Companion
  - 15% of annual Gross Compensation (minimum fee = $3000)
- **All Part Time** Housekeeper, Executive Housekeeper, Cook, Handyman, Companion
  - 15% of annual Gross Compensation (minimum fee = $1500)

**ESTATE/ PRIVATE OFFICE**
- **All Full Time and Part Time** Estate Managers, Household Managers, Chefs, Valets, Butlers, Master Gardeners, Security Body Guards, Chauffeurs, Couples, Personal Assistants, Executive Assistant Candidates
  - 20% of annual Gross Compensation (minimum fee = $3000)

### Fee for On-Call & Temporary Services
- **All On-Call and Temporary Work Assignments except for Baby Nurses, Newborn Specialists and Doulas**
  - 35% of ongoing Gross Compensation (minimum fee = $35 a day)
- **All Baby Nurses, Newborn Specialists & Doulas**
  - 20% of ongoing Gross Compensation (minimum fee = $50 a day)
too much unemployment is bad
  – too many services are idle

too little unemployment is bad
  – too many services are devoted to recruiting

there is a socially efficient rate of unemployment ($u^*$)
  – number of services enjoyed ($y = g + c$) is maximized

when unemployment is efficient, Samuelson rule holds
FORMAL DESCRIPTION OF THE MODEL
STRUCTURE

- dynamic matching model
  - building on Michaillat & Saez (2015)
- identical, self-employed households
- government
- 2 consumption goods traded on a matching market
  - public services & private services
- 1 asset for saving
MATCHING MARKET

- capacity of each household: $k$ services
- household purchases: $C(t)$ private services
- government purchases: $G(t)$ public services
- output: $Y(t) = C(t) + G(t) < k$
- unemployment rate: $u(t) = 1 - Y(t)/k$
- price of services: $p(t)$
MATCHING FUNCTION

- number of vacancies: $v(t)$
- matching function: $h(t) = \omega \cdot [k - Y(t)]^\eta \cdot v(t)^{1-\eta}$
- market tightness: $x(t) = v(t)/(k - Y(t))$
- selling rate & buying rate:

$$f(x(t)) = \frac{h(t)}{k - Y(t)} = \omega \cdot x(t)^{1-\eta}$$
$$q(x(t)) = \frac{h(t)}{v(t)} = \omega \cdot x(t)^{-\eta}$$
• relationships separate at rate $s$

• given $x$, output and unemployment converge to

\[ Y(x, k) = \frac{f(x)}{s + f(x)} \cdot k, \quad u(x) = \frac{s}{s + f(x)} \]

• convergence to steady state is extremely fast, so we assume:

\[ Y(t) = Y(x(t), k) \]
\[ u(t) = u(x(t)) \]

• see Hall (2005)
MATCHING COST: $\rho$ SERVICES PER VACANCY

- output ($Y$) = consumption ($y$) + matching cost
  \[ Y = y + \rho \cdot v = y + s \cdot Y \cdot \frac{\rho}{q(x)} \]

- matching wedge: $\tau(x) = s \cdot \rho /[ q(x) - s \cdot \rho ]$

- total consumption: $y = Y/[1 + \tau(x)]$

- private consumption: $c = C/[1 + \tau(x)]$

- public consumption: $g = G/[1 + \tau(x)]$
SUPPLY STRUCTURE: SUMMARY

tightness $x$

capacity: $k$

public + private services
SUPPLY STRUCTURE: SUMMARY

output:

\[ Y(x, k) = (1 - u(x)) \cdot k \]

capacity \( k \)

tightness \( x \)

public + private services
idle capacity: $u(x) \cdot k$
output $Y(x,k)$
capacity $k$

tightness $x$

graph with axes labeled 'public + private services' and 'tightness $x$'.
SUPPLY STRUCTURE: SUMMARY

output $Y(x,k)$

capacity $k$

c Consumption:

$$y(x, k) = \frac{Y(x, k)}{1 + \tau(x)}$$

public + private services

tightness $x$
SUPPLY STRUCTURE: SUMMARY

consumption $y(x,k)$

output $Y(x,k)$
capacity $k$

matching cost:

$y(x,k) \cdot \tau(x)$

tightness $x$

public + private services
DEMAND STRUCTURE: EXAMPLE

• asset: land \( l(t) \) in fixed supply \( l_0 \)
  - traded on a competitive market
  - Iacoviello (2005) and Liu, Wang, Zha (2013)
• households choose \( c(t) \) and \( l(t) \) to maximize utility
  \[
  \int_0^{+\infty} e^{-\delta t} \cdot [U(c, g) + V(l)] \, dt
  \]
• subject to flow budget constraint
  \[
  \dot{l} = p \cdot [1 - u(x)] \cdot k - p \cdot [1 + \tau(x)] \cdot c - T
  \]
market clearing on housing market: \( l = l_0 \)

private demand \( c^d(x, g, \rho) \) is solution to Euler equation:

\[
\frac{\partial U}{\partial c}(c, g) = \frac{\rho \cdot (1 + \tau(x)) \cdot V'(l_0)}{\delta}
\]

price of services relative to housing: \( \rho = \rho(x, g) \)

- general price mechanism
- (assumption required in matching model)
EQUILIBRIUM TIGHTNESS \( x(g) \)

\[
y(x,k) + c^d(x, g, p(g)) + g
\]
UNEMPLOYMENT MULTIPLIER $m$

c(x, p(x, g'), g') + g'  
c(x, p(x, g), g) + g

$y(x,k)$  
$Y(x,k)$

$m > 0$

$u(g')$

$u(g)$

$m > 0$
SOCIA LLY EFFICIENT UNEMPLOYMENT RATE $u^*$

$x(g) = x^*$

$u^*$

$x, y, Y$
INEFFICIENTLY LOW UNEMPLOYMENT RATE

The graph illustrates the relationship between supply and demand, with the supply curve labeled "supply" and the demand curve labeled "demand." The equation $x(g)$ is also shown, indicating a function of $g$. The graph highlights a situation where $u < u^*$, indicating an inefficiently low unemployment rate.
OPTIMAL PUBLIC EXPENDITURE
GOVERNMENT’S PROBLEM

• households’ flow utility is $U(c, g)$
• public expenditure is financed by a lump-sum tax to maintain a balanced budget
• given $x(g)$, the government chooses $g$ to maximize

$$U\left(\underbrace{y(x(g), k) - g, g}_{c}\right)$$
CORRECTING THE SAMUELSON FORMULA

- first-order condition of government’s problem is

\[ 0 = \frac{\partial U}{\partial g} - \frac{\partial U}{\partial c} + \frac{\partial U}{\partial c} \cdot \frac{\partial y}{\partial x} \cdot \frac{dx}{dg} \]

- optimal public expenditure satisfies

\[ 1 = MRS_{gc} + \underbrace{\frac{\partial y}{\partial x} \cdot \frac{dx}{dg}}_{\text{Samuelson formula}} \]

\[ \quad \underbrace{\text{correction}}_{\text{correction due to effect of public expenditure on welfare through tightness}} \]

- \[ MRS_{gc} = \frac{\partial U / \partial g}{\partial U / \partial c} \]
(g/c)*: Samuelson spending

elasticity of substitution between g and c:

\[ 1 - MRS_{gc} \approx \frac{1}{\epsilon} \cdot \frac{g/c - (g/c)^*}{(g/c)^*} \]

unemployment gap:

\[ \frac{\partial y}{\partial x} \propto u - u^* \]

unemployment multiplier:

\[ \frac{dx}{dg} \propto m = -\frac{y}{1 - u} \cdot \frac{du}{dg} \]
IMPLICIT FORMULA FOR OPTIMAL STIMULUS

\[
\frac{g/c - (g/c)^*}{(g/c)^*} \approx z_0 \epsilon m \cdot \frac{u - u^*}{u^*}
\]

- \(g/c - (g/c)^*\): stimulus spending
- \(\epsilon\): elasticity of substitution between \(g\) and \(c\)
  = marginal social value of public spending
- \(m\): unemployment multiplier
  - decrease in \(u\) when \(g\) increases by 1\% of \(y\)
- \(u - u^*\): unemployment gap
  = productive inefficiency
- \(z_0\): constant of the parameters \(\eta, u^*\)
### DEPARTURES FROM SAMUELSOON RULE

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<th>$m &lt; 0$</th>
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<td>$u &gt; u^*$</td>
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• $\epsilon = 0$: digging holes or building pyramids
  
  – $g/c = (g/c)^*$: Samuelson rule holds, no stimulus spending

• $\epsilon \to +\infty$: perfect substitution
  
  – $u = u^*$: entirely fill unemployment gap, as in Keynes

• $\epsilon \in (0, +\infty)$: medium substitution
  
  – medium stabilization: $g/c \neq (g/c)^*$ but $u \neq u^*$
  
  $\rightsquigarrow$ partially fill unemployment gap
MAKING THEFORMULAEXPLICIT

• implicit formula: not useful for quantitative results because $u$ in RHS responds to $g/c$ in LHS

• starting from $(g/c)^*$ and $u_0 \neq u^*$:

$$
\frac{g/c - (g/c)^*}{(g/c)^*} \approx z_0 \epsilon m \cdot \frac{u(g/c) - u^*}{u^*}
$$

• first-order Taylor expansion of $u$ at $u((g/c)^*) = u_0$:

$$
\frac{u - u^*}{u^*} \approx \frac{u_0 - u^*}{u^*} - z_1 m \cdot \frac{g/c - (g/c)^*}{(g/c)^*}
$$

• $z_1$: constant of the parameters $u^*, (g/c)^*$
optimal $g/c$ depends on fixed quantities:

$$\frac{g/c - (g/c)^*}{(g/c)^*} \approx \frac{z_0 \epsilon m}{1 + z_1 z_0 \epsilon m^2} \cdot \frac{u_0 - u^*}{u^*}$$

optimal $u$ depends on fixed quantities:

$$u \approx u^* + \frac{u_0 - u^*}{1 + z_1 z_0 \epsilon m^2}$$

approximations valid up to 2nd-order terms
RESULTS WITH DISTORTIONARY TAXATION

- endogenous capacity: \( U(c, g, k) \) with \( \partial U / \partial k < 0 \)
- linear income tax: \( T = \tau^L \cdot (1 - u(x)) \cdot k \)
- everything remains valid
  - but \((g/c)^*\) is lower because of tax distortions
- however: link between multipliers changes
  - no tax distortions: \( m = dY/dG \)
  - tax distortions: \( m > dY/dG \)
  - with taxes, we may have \( dY/dG < 0 \) but \( m > 0 \)
NUMERICAL ILLUSTRATION:

GREAT RECESSION IN THE US
STARTING POINT: WINTER 2008–2009

- unemployment = 6% and public spending = 16.5% of GDP
  - for illustration: we take these values as efficient
- unemployment is forecast to increase to 9%
  - initial unemployment gap = 9% – 6% = 3%
- we compute optimal stimulus for various elasticities of substitution and unemployment multipliers
OPTIMAL STIMULUS SPENDING (% OF GDP)

\[
\epsilon = 1
\]
OPTIMAL STIMULUS SPENDING (% OF GDP)
OPTIMAL STIMULUS SPENDING (% OF GDP)

$520 billion
OPTIMAL STIMULUS SPENDING (% OF GDP)
OPTIMAL STIMULUS SPENDING FOR VARIOUS $\epsilon$
UNEMPLOYMENT UNDER OPTIMAL STIMULUS

Unemployment multiplier
6%
7%
8%
9%

Resulting unemployment rate

\( \epsilon = 0.5 \)
\( \epsilon = 1 \)
\( \epsilon = 2 \)

Unemployment multiplier
SOME SIMULATIONS
Public expenditure (% of GDP)

Aggregate demand

Optimal $G$

$G/Y = 16.5\%$
UNEMPLOYMENT RATE IN CALIBRATED MODEL

Aggregate demand

4%
6%
8%
10%
12%

Unemployment rate

$G/Y = 16.5\%$

Optimal $G$
MUTLIPLIER IN CALIBRATED MODEL

Aggregate demand

Unemployment multiplier

$G/Y = 16.5\%$

Optimal $G$
QUALITY OF APPROXIMATIONS IN FORMULA

Aggregate demand

Public expenditure (% of GDP)

Optimal $G$: exact

Optimal $G$: sufficient-statistic formula
SUMMARY & DISCUSSION
1. $dY/dG > 1$ is not necessary for stimulus
   - stimulus requires unemployment multiplier $> 0$ (as in data)
2. bang-for-the-buck logic does not hold
   - strongest stimulus for $m = 0.4$
   - same stimulus for $m = 0.1$ and $m = 1.4$
3. completely filling the unemployment gap is not optimal
   - optimal to partially fill unemployment gap
   - except if public services = private services
4. low marginal social value of $g$ does not imply no stimulus
   - optimal to reduce unemployment gap
   - except if public services = digging holes
DISTORTIONARY TAXES $\Rightarrow$ SMALLER STIMULUS

- formula remains valid with distortionary taxation
  - but Samuelson spending is lower
- however, $dY/dG$ is not useful anymore because $dY/dG \neq m$
  - $dY/dG = m +$ labor-supply response to taxes
  - labor-supply distortion reduces $dY/dG$ but not $m$
  - so: $m > dY/dG$
  - possibly: $dY/dG < 0$ while $m > 0$
- distortionary taxation does not imply smaller stimulus
  - only average public spending is lower