A NEW KEYNESIAN MODEL

WITH WEALTH IN THE UTILITY FUNCTION

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REMEDY TO ZLB PATHOLOGIES: DETERMINE ZLB

STEADY STATE

1. modification of IS & Phillips curves
   - Eggertsson, Mehrotra [2014]: OLG + wage rigidity
   - Gabaix [2016]: bounded rationality

2. addition of one state variable & one equation
   - Diba, Loisel [2017]: stock of bank reserves
   - Cochrane [2018]: fiscal theory of price level

3. this paper: modification of IS curve, no extra equation
Why would people value wealth in itself?

Irving Fisher answers in “The Theory of Interest”:

A man may include in the benefits of his wealth… the social standing he thinks it gives him, or political power and influence, or the mere miserly sense of possession, or the satisfaction in the mere process of further accumulation.
NK MODEL WITH WEALTH IN THE UTILITY

• production function: \( y_j(t) = ah_j(t) \)

• consumption index: \( c_j(t) = \left[ \int_0^1 c_{jk}(t)^{(e-1)/e} \, dk \right]^{e/(e-1)} \)

• household \( j \) maximizes utility

\[
\int_0^\infty e^{-\delta t} \left[ \ln(c_j(t)) + u \left( \frac{b_j(t)}{p(t)} \right) - h_j(t) - \frac{\gamma}{2} \pi_j(t)^2 \right] \, dt
\]

• subject to the law of motion of bond holdings:

\[
\dot{b}_j(t) = i(t)b_j(t) + p_j(t)y_j(t) - \int_0^1 p_k(t)c_{jk}(t) \, dk
\]

• to the law of motion of price \( i \):

\[
\dot{p}_j(t)/p_j(t) = \pi_j(t)
\]

• and to the demand for good \( i \):

\[
y_j(t) = c(t) \left[ p_j(t)/p(t) \right]^{-\epsilon}
\]
DYNAMICAL SYSTEM REPRESENTING GE

• Phillips curve describes optimal pricing:

\[ \dot{\pi} = \delta \pi - \frac{\epsilon}{\gamma a} (y - y^n) \quad \text{with} \quad y^n = \frac{\epsilon - 1}{\epsilon} a \]

• Euler equation described optimal consumption/saving:

\[ \frac{\dot{y}}{y} = i - \pi + u'(0)y - \delta \]

- real rate
- MRS_{wealth,c}

- bonds are in zero net supply: \( b = 0 \)
- monetary policy follows \( i = \max \{0, i^* + \phi \pi\} \)
TWO MODELS: NK & WUNK

• New Keynesian (NK) model:

\[ u'(0) = 0 \]

• wealth-in-the-utility New Keynesian (WUNK) model:

\[ u'(0) > \frac{\epsilon}{\delta \gamma a} \]
INFLATION & OUTPUT AT ZLB
<table>
<thead>
<tr>
<th>at intended steady state</th>
<th>ZLB</th>
<th>back to intended steady state</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta &gt; 0$</td>
<td>$\delta &lt; 0$</td>
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</tr>
<tr>
<td>$i^* = \delta$</td>
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</tr>
<tr>
<td>$\phi &gt; 1$</td>
<td></td>
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</table>

$t = 0$  $t = T$
\[ \dot{\pi} = 0 \quad \implies \quad \pi = \frac{\varepsilon}{\delta \gamma a} (y - y^n) \]
\[ \dot{\pi} = \delta \pi - \frac{\epsilon}{\gamma a} (y - y^n) \]
\[ \pi \]

\[ y_n \]

\[ \dot{y} = 0 \implies \pi = \frac{\delta - i^*}{\phi - 1} \]

\[ i^* = \delta \implies \pi = 0 \]
NK | BACK TO INTENDED STEADY STATE

$\frac{\dot{y}}{y} = (\phi - 1)\pi$

steady state = source
NK | BACK TO INTENDED STEADY STATE

\[ \pi \]

\[ y^n \]

\[ t \geq T \]
\[ \dot{y} = 0 \implies \pi = -\delta \]
steady state = saddle

\[ \frac{\dot{y}}{y} = -\pi - \delta \]
# WUNK | ZLB SCENARIO

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<th>at intended steady state</th>
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<tr>
<td>$r^n \equiv \delta - u'(0)y^n$</td>
<td>higher $u'(0)$:</td>
<td>$r^n &gt; 0$</td>
</tr>
<tr>
<td>$r^n &gt; 0$</td>
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$t = 0$  \hspace{0.5cm}  $t = T$
\[ \dot{y} = 0 \iff \pi = -\frac{u'(0)}{\phi - 1} (y - y^n) \]
steady state = source

\[
\frac{\dot{y}}{y} = (\phi - 1)\pi + u'(0)(y - y^n)
\]
\[ \dot{y} = 0 \implies \pi = -r^n + u'(0)(y - y^n) \]
steady state = source

\[
\frac{\dot{y}}{y} = -\pi - r^n + u'(0)(y - y^n)
\]
FORWARD GUIDANCE
### NK | FORWARD-GUIDANCE SCENARIO

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$t = 0$  \hspace{1cm} $t = T_1$  \hspace{1cm} $t = T_2$
NK | FORWARD GUIDANCE

\[ \pi \]

\[ y \]

\[ y^n \]

\[ t = T_2 \]

PC

EE
NK | FORWARD GUIDANCE

\[ \pi \]

\[ 0 \]

\[ y \]

\[ y^n \]

\[ t = T_1 \]

\[ t = T_2 \]

PC

EE
NK | FORWARD GUIDANCE
NK | ZLB + FORWARD GUIDANCE

\[ \pi \]

\[ y \]

EE

PC

\[ t = 0 \]

\[ y^n \]

\[ t = T_1 \]

\[ t = T_2 \]
NK | ZLB + FORWARD GUIDANCE

π

EE

0

t = 0

π

y

t = T_1

y^n

0

t = T_2

PC
NK | DISCONTINUITY OF FORWARD GUIDANCE

![Diagram showing the discontinuity of forward guidance with a graph and annotations indicating points and times.]

- EE: End Effector
- PC: Pose Capture
- $t = 0$: Initial Time
- $t = T_1$: Time at $y^n$
- $t = T_2$: End Time
NK | DISCONTINUITY OF FORWARD GUIDANCE

The diagram illustrates the concept of discontinuity in forward guidance, with key points labeled:

- $\pi$ (pi) axis
- $y$ axis
- EE (Equilibrium Envelope)
- Unstable arm

Key points and times:

- $t = 0$?
- $y^n$ (point $y$ at time $n$)
- $t = T_1$
- $t = T_2 + 2\varepsilon$
NK | DISCONTINUITY OF FORWARD GUIDANCE

\[ \pi \]

\[ 0 \quad y \]

\[ \text{EE} \]

\[ y^n \]

\[ t = T_2 + 2\varepsilon \]

\[ t = T_1 \]

\[ t = 0 \]

unstable arm

PC
## WUNK | FORWARD-GUIDANCE SCENARIO

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\( t = 0 \) \hspace{1cm} \( t = T_1 \) \hspace{1cm} \( t = T_2 \)
WUNK | FORWARD GUIDANCE

\[ \pi \]

\[ 0 \]

\[ y \]

\[ y^n \]

\[ t = T_2 \]

PC

EE
WUNK | FORWARD GUIDANCE

\[ y^n \]

\( t = T_1 \)

\( t = T_2 \)

\[ \pi \]

PC

EE
WUNK | ZLB + FORWARD GUIDANCE

\[ \pi \]

\[ y \]

\[ EE \]

\[ PC \]

\[ t = T_1 \]

\[ t = T_2 \]
WUNK | ZLB + FORWARD GUIDANCE

\[ \pi \]

\[ 0 \]

\[ t = 0 \]

\[ y^n \]

\[ t = T_1 \]

\[ t = T_2 \]
WUNK | CONTINUITY OF FORWARD GUIDANCE

\[ \pi \]

EE (ZLB)

PC

\[ t = T_1 \]

\[ t = T_2 \]

\[ y^n \]

\[ t = 0 \]

\[ 0 \]

\[ y \]
IS THE WUNK ASSUMPTION REALISTIC?

• WUNK assumption:

\[ u'(0) > \frac{\epsilon}{\delta \gamma a} \]

• equivalent to: \( \delta \times (\delta - r^n) > \) output gap coeff. in Phillips curve
  
  – macro evidence: coeff. in Phillips curve \( \approx 3\% \)
  
  – experimental evidence: annual \( \delta \approx 40\% \)
  
  – macro evidence: annual \( r^n \approx 2\% \)

• WUNK assumption holds: \( 0.4 \times (0.4 - 0.02) = 0.15 > 0.03 \)