ANOMALIES IN NK MODEL AT ZLB

1. collapse of output & inflation
   - Eggertsson, Woodford [2004]
   - Werning [2011]

2. implausibly large effects of forward guidance
   - del Negro, Giannoni, Patterson [2015]
   - Cochrane [2017]

3. implausibly large effects of government spending
   - Christiano, Eichenbaum, Rebello [2011]
   - Cochrane [2017]
EXISTING REMEDIES TO ZLB ANOMALIES

• Cochrane [2018]: fiscal theory of price level
• Bilbiie [2018] & Acharya, Dogra [2020]: heterogeneous agents
• Gabaix [2020]: bounded rationality
• Diba, Loisel [2021]: interest on bank reserves
• but these remedies complicate the textbook model
  – sometimes equilibrium system becomes 3-dimensional
  – sometimes derivations are complicated by heterogeneity or bounded rationality
• New Keynesian model with relative wealth in the utility function
  • only one additional parameter
    – marginal utility of wealth in Euler equation
  ⇝ equilibrium system remains 2-dimensional
    – 2 variables: output & inflation
    – 2 differential equations: Euler equation & Phillips curve
  ⇝ derivations remain exactly the same
WHY WOULD PEOPLE VALUE WEALTH IN ITSELF?

• Keynes [1919]: “The duty of saving became nine-tenths of virtue and the growth of the cake the object of true religion…. Saving was for old age or for your children; but this was only in theory—the virtue of the cake was that it was never to be consumed, neither by you nor by your children after you.”

• Irving Fisher [1930]: “A man may include in the benefits of his wealth… the social standing he thinks it gives him, or political power and influence, or the mere miserly sense of possession, or the satisfaction in the mere process of further accumulation.”
Camerer, Loewenstein, Prelec [2005]: “brain-scans conducted while people win or lose money suggest that money activates similar reward areas as do other primary reinforcers like food and drugs, which implies that money confers direct utility, rather than simply being valued only for what it can buy.”

evidence from economics, social psychology, sociology, social neuroscience: wealth is a marker of social status, and people value high social status
NK MODEL WITH WEALTH IN THE UTILITY
• self-employed household \( j \in [0, 1] \) maximizes utility

\[
\int_0^\infty e^{-\delta t} \left[ \ln(c_j(t)) + u\left(\frac{b_j(t)}{p(t)} - \frac{b(t)}{p(t)}\right) - \kappa h_j(t) - \frac{\gamma}{2} \pi_j(t)^2 \right] dt
\]

- consumption index: \( c_j(t) = \left[ \int_0^1 c_{jk}(t) (e^{-1})/\epsilon \, dk \right]^{\epsilon/(\epsilon-1)} \)
- aggregate wealth: \( b(t) = \int_0^1 b_j(t) \, dj \)
- inflation: \( \pi_j(t) = \dot{p}_j(t)/p_j(t) \)

• subject to budget constraint:

\[
\dot{b}_j(t) = i(t) b_j(t) + p_j(t) y_j(t) - \int_0^1 p_k(t) c_{jk}(t) \, dk
\]

• to production function: \( y_j(t) = a h_j(t) \)

• to demand for good \( i \): \( y_j(t) = \left[ p_j(t)/p(t) \right]^{-\epsilon} c(t) \)
EQUILIBRIUM: EULER-PHILLIPS SYSTEM

- Phillips curve: standard

\[ \dot{\pi} = \delta \pi - \frac{\epsilon \kappa}{\gamma a} (y - y^n) \quad \text{with} \quad y^n = \frac{\epsilon - 1}{\epsilon} \cdot \frac{a}{\kappa} \]

- Euler equation: “discounted”

\[ \frac{\dot{y}}{y} = r(\pi) + u'(0)y - \delta \]

- financial returns: real interest rate = \( r(\pi) = i(\pi) - \pi \)
- hedonic returns: MRS(wealth, consumption) = \( u'(0)y^n \)

so

\[ \frac{\dot{y}}{y} = r(\pi) - r^n + u'(0)(y - y^n) \quad \text{with} \quad r^n = \delta - u'(0)y^n \]
TWO MODELS

- NK: standard New Keynesian model
  \[ u'(0) = 0 \]

- WUNK: wealth-in-the-utility New Keynesian model
  \[ u'(0) > \frac{\epsilon \kappa}{\delta \gamma a} \]
OUTPUT & INFLATION COLLAPSE
SCENARIO: ZLB

- $r^n < 0$
- $i(\pi) = 0$
- $r^n > 0$
- $i(\pi) = r^n + \phi \pi$
  \[\phi > 1\]
\[ \pi = 0 \]

\[ y = y^n \]

\[ \dot{\pi} = \delta \pi - \frac{\epsilon \kappa}{\gamma a} (y - y^n) \]

Phillis \( \dot{\pi} = 0 \)
\[ \dot{\pi} = \delta \pi - \frac{\varepsilon \kappa}{\gamma a} (y - y^n) \]
\[ \frac{\dot{y}}{y} = (\phi - 1)\pi \]
\[ \frac{\dot{y}}{y} = (\phi - 1)\pi \]
NK | PHASE DIAGRAM IN NORMAL TIMES: SOURCE

\[ \pi = 0 \]

\[ y = y^n \]

Phillips
\[
\frac{\dot{y}}{y} = (\phi - 1)\pi
\]
\[ \frac{\dot{y}}{y} = -\pi - r^n \]
NK | PHASE DIAGRAM AT ZLB: SADDLE

\[ \pi = 0 \]

Euler

Phillips

\[ y = y^n \]
\[ y = y^n \]

\[ t = T \]

\[ \pi = 0 \]

Euler

Phillips
\pi = 0 \quad y = y^n 

\text{Euler} 

\text{Phillips} 

\pi \quad y 

\text{t} = 0 \quad \text{t} = T
\[
\frac{\dot{y}}{y} = (\phi - 1)\pi
\]
\[
\frac{\dot{y}}{y} = (\phi - 1)\pi + u'(0)(y - y^n)
\]
WUNK | PHASE DIAGRAM IN NORMAL TIMES: SOURCE

\[ \pi = 0 \]

Euler

Phillips

\[ y = y^n \]
\[ \frac{\dot{y}}{y} = -\pi - r^n \]
WUNK | PHASE DIAGRAM AT ZLB

\[ y = y^n \]

\[ \frac{\dot{y}}{y} = -\pi - r^n + u'(0)(y - y^n) \]
\[
\frac{\dot{y}}{y} = -\pi - r^n + u'(0)(y - y^n)
\]
WUNK | PHASE DIAGRAM AT ZLB: SOURCE

\[ \pi = 0 \]

\[ y = y^n \]

Euler

Phillips
LONGER ZLB CONVERGES TO STEADY STATE

WUNK

\[ \pi = 0 \]

\[ y = y^n \]

\[ t = 0 \]

\[ t = T \]
FORWARD GUIDANCE
# Scenario: ZLB + Forward Guidance

<table>
<thead>
<tr>
<th>ZLB</th>
<th>Forward Guidance</th>
<th>Back to Natural Steady State</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r^n &lt; 0 )</td>
<td>( r^n &gt; 0 )</td>
<td>( r^n &gt; 0 )</td>
</tr>
<tr>
<td>( i(\pi) = 0 )</td>
<td>( i(\pi) = 0 )</td>
<td>( i(\pi) = r^n + \phi \pi )</td>
</tr>
</tbody>
</table>

\[ t = 0 \quad t = T \quad t = T + \Delta \]
\[ y = y^n \]

\[ t = T + \Delta \]

Euler

\[ \pi = 0 \]

Phillips
NK | LONGER GUIDANCE: BOOM AT ZLB

\[ y = y^n \]

Euler

\[ \pi = 0 \]

\[ t = 0 \]

\[ t = T + \Delta \]

\[ t = T \]

Phillips

unstable line
NK | LONGER GUIDANCE: BOOM AT ZLB

Euler

$\pi = 0$

$y = y^n$

$\pi$

Phillips

$t = 0$

$t = T + \Delta$

$t = T$

unstable line
LONGER GUIDANCE: BOOM AT ZLB

\[ \pi = 0 \]

\[ y = y^n \]

Euler

Phillips

unstable line

\[ t = 0 \]

\[ t = T \]

\[ t = T + \Delta \]
$t = T + \Delta$

$\pi = 0$

$y = y^n$
\[ t = T + \Delta \]

\[ y = y^n \]

Euler

Phillips

\[ \pi = 0 \]
$y = y^n$
WUNK | LONGER GUIDANCE: LIMITED EFFECT

\begin{align*}
\pi &= 0 \\
y &= y^n \\
t &= 0 \\
t &= T \\
t &= T + \Delta
\end{align*}
GOVERNMENT SPENDING
### SCENARIO: ZLB + GOVERNMENT SPENDING $g$

<table>
<thead>
<tr>
<th>ZLB</th>
<th>back to natural steady state</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^n &lt; 0$</td>
<td>$r^n &gt; 0$</td>
</tr>
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<td>$i(\pi) = 0$</td>
<td>$i(\pi) = r^n + \phi \pi$</td>
</tr>
<tr>
<td>$g &gt; 0$</td>
<td>$\phi &gt; 1$</td>
</tr>
<tr>
<td>$t = 0$</td>
<td>$g = 0$</td>
</tr>
</tbody>
</table>
NK | ZLB + SMALL SPENDING

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A diagram showing the Phillips curve and Euler equation. The graph plots the relationship between inflation ($\pi$) and consumption ($c$) over time ($t$). The curve $c = c^n$ represents a constrained consumption line, and the unstable line indicates the path towards a stable state at $t = T$. The diagram illustrates the concept of small spending at various points in time.
NK □ ZLB + LARGE SPENDING: BOOM AT ZLB

π

Euler

π = 0

Phillips

t = 0

t = T

unstable line

c

\( c = c^n \)
WUNK | ZLB + NO SPENDING

\[ \pi = 0 \]

Euler

Phillips

\[ t = 0 \]

\[ t = T \]

\[ c = c^n \]

unstable line
WUNK | ZLB + LARGE SPENDING: LIMITED EFFECT

\[ t = 0 \]

\[ \pi = 0 \]

\[ c = c^n \]

unstable line
OTHER ZLB PROPERTIES IN WUNK
People want to save more...
People want to save more…
but they end up saving the same and consuming less.
PARADOX OF TOIL: LOWER DISUTILITY OF LABOR

People want to work more…
People want to work more…
but they end up working less.
PARADOX OF FLEXIBILITY: LOWER PRICE-ADJUSTMENT COST

Prices are more flexible…
Prices are more flexible… yet the slump worsens.
When public consumption $g$ increases...

$\pi = 0$

$c = c^n$
When public consumption $g$ increases... private consumption $c$ increases.
ASSESSMENT OF WUNK ASSUMPTION
• WUNK assumption in measurable statistics:

\[ \delta - r^n > \frac{\lambda}{\delta} \]

• \( \delta \) = annual time discount rate \( \approx 43\% \)
  - Frederick, Loewenstein, O’Donoghue [2002]
  - Andersen, Harrison, Lau, Rutstrom [2014]

• \( r^n \) = natural rate of interest \( \approx 2\% \)

• \( \lambda \) = output-gap coefficient in Phillips curve \( \approx 1.6\% \)
  - Mavroeidis, Plagborg-Moller, Stock [2014]

• assumption holds: \( 43\% - 2\% = 0.41 > 0.037 = 1.6\% / 43\% \)
  - lowest acceptable household discount rate: 27%
  - lowest acceptable firm discount rate: 16%