A NEW KEYNESIAN MODEL
WITH WEALTH IN THE UTILITY FUNCTION

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Emmanuel Saez (Berkeley)
PATHOLOGIES OF NK MODEL AT ZLB

• explosive response of output & inflation to ZLB shock
  – Eggertsson, Woodford [2004], Cochrane [2017]
• forward-guidance puzzle
  – Del Negro, Giannoni, Patterson [2012]
• source of pathologies: ZLB steady state is a saddle of equilibrium system
REMEDY: MAKE ZLB STEADY STATE A SOURCE

• introduce real wealth (bonds) in the utility function
  – same derivations as in textbook NK model
  – but one extra parameter tilts Euler equation
  – thus changing dynamical properties of equilibrium system

• similar approaches to ZLB in NK models:
  – Gabaix [2016]: bounded rationality of households + firms
  – Eggertsson, Mehrotra [2014]: OLG + wage rigidity
  – Cochrane [2017]: fiscal theory of price level
  – Diba, Loisel [2017]: financial sector and bank reserves
PEOPLE MAY VALUE WEALTH IN ITSELF

• “following Irving Fisher, it has become customary to emphasize that it is the consumption stream that is the real aim of economic activity” [Dixit 1976]

• but Irving Fisher also said in “The Theory of Interest”:

\[ A \text{ man may include in the benefits of his wealth… the social standing he thinks it gives him, or political power and influence, or the mere miserly sense of possession, or the satisfaction in the mere process of further accumulation. } \]
PEOPLE VALUE CONVENIENCE OF GOVERNMENT BONDS

• convenience: liquidity, safety, legal status
• utility from bonds explains asset-pricing puzzles
  – Krishnamurthy, Vissing-Jorgensen [2012]
• shocks to marginal utility of bonds generate realistic fluctuations in DSGE models
  – Fisher [2015], Campbell et al [2017], Del Negro et al [2017]
• Heterogeneous agents models look in aggregate like representative agent with bond in the utility model
  – Auclert and Rognlie [2018], Hagedorn et al [2017], Kaplan and Violante [2018]
MODEL
SELF-EMPLOYED HOUSEHOLDS

• production function: \( y_j(t) = ah_j(t) \)

• consumption index: \( c_j(t) = \left[ \int_0^1 c_{jk}(t)^{(e-1)/e} \, dk \right]^{e/(e-1)} \)

• household \( j \) maximizes utility

\[
\int_0^\infty e^{-\delta t} \left[ \ln(c_j(t)) + u \left( \frac{b_j(t)}{p(t)} \right) - h_j(t) - \frac{\gamma}{2} \pi_j(t)^2 \right] \, dt
\]

• subject to the law of motion of bond holdings:

\[
\dot{b}_j(t) = i(t)b_j(t) + p_j(t)y_j(t) - \int_0^1 p_k(t)c_{jk}(t) \, dk
\]

• to the law of motion of price \( i \):

\[
\dot{p}_j(t)/p_j(t) = \pi_j(t)
\]

• and to the demand for good \( i \):

\[
y_j(t) = c(t) \left[ p_j(t)/p(t) \right]^{-\epsilon}
\]
SYMMETRIC GENERAL EQUILIBRIUM

• optimal pricing gives the **Phillips curve**:

\[ \dot{\pi} = \delta \pi - \frac{\epsilon}{\gamma a} (y - y^n) \quad \text{with} \quad y^n = \frac{\epsilon - 1}{\epsilon} a \]

• government bonds are in zero net supply: \( b = 0 \)

• optimal consumption/saving gives the **Euler equation**:

\[ \frac{\dot{y}}{y} = \underbrace{i - \pi}_{\text{real rate}} - \delta + \underbrace{u'(0)y}_{MRS_{\text{wealth,c}}} \]

• monetary policy is given by \( i = \max \{0, i^* + \phi \pi\} \)
TWO DIFFERENT MODELS: NK & WUNK

- New Keynesian (NK) model:
  \[ u'(0) = 0 \]

- wealth-in-the-utility New Keynesian (WUNK) model:
  \[ u'(0) > \frac{\epsilon}{\delta \gamma a} \]
EQUILIBRIUM DYNAMICS
NK | STEADY-STATE PHILLIPS CURVE

\[ y^n = \frac{\epsilon - 1}{\epsilon} a \]

\[ y = y^n + \frac{\delta \gamma a}{\epsilon} \pi \]
EE: $i = \delta + \pi$

MP: $i = \max\{0, \delta + \phi\pi\}$
NK | SOURCE WITH ACTIVE POLICY ($\phi > 1$)
$i^* = r^n \equiv \delta - u'(0)y^n$

**AD:**

$$y = y^n - \frac{\phi - 1}{u'(0)}\pi$$
\[ \text{AD: } y = y^n - \frac{\phi - 1}{u'(0)} \pi \]
WUNK | SOURCE WITH $\phi = 1$
WUNK | SOURCE WITH PASSIVE POLICY ($\phi < 1$)
WUNK | SOURCE WITH INTEREST-RATE PEG ($\phi = 0$)
AD: \[ y = \frac{\delta}{u'(0)} + \frac{1}{u'(0)} \pi \]
INFLATION & OUTPUT AT ZLB
<table>
<thead>
<tr>
<th>at intended steady state</th>
<th>ZLB</th>
<th>back to intended steady state</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta &gt; 0 ) ( i^* = \delta )</td>
<td>( \delta &lt; 0 ) ( i = 0 )</td>
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</tr>
</tbody>
</table>

\[ t = 0 \quad t = T \]
NK | ZLB SCENARIO

\[
\pi
\]

\[
y^n
\]

\[
y \geq T
\]
NK | ZLB SCENARIO

\[ \pi \]

\[ y \]

\[ y^n \]

\[ t \geq T \]

\[ t = 0 \]
NK | LONGER ZLB

\[ \pi \]

[Diagram showing a graph with a line labeled PC and AD, a point labeled \( y^n \), and arrows indicating a path from \( t = 0 \) to \( t \geq T \).]
## WUNK | ZLB SCENARIO

<table>
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<tr>
<th>at intended steady state</th>
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<tbody>
<tr>
<td>$r^n &gt; 0$</td>
<td>higher $u'(0)$</td>
<td>lower $\delta$ : $r^n = \delta - u'(0)y^n &lt; 0$</td>
</tr>
<tr>
<td>$i^* = r^n$</td>
<td>$i = 0$</td>
<td>$r^n &gt; 0$</td>
</tr>
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$t = 0$  \hspace{1cm}  $t = T$
WUNK | ZLB SCENARIO
WUNK | ZLB SCENARIO

The diagram illustrates a scenario where the economy is constrained by the zero lower bound (ZLB) on interest rates. The graph shows the relationship between inflation ($\pi$) and output ($y$), with the AD (aggregate demand) and PC (policy curve) lines.

At time $t = 0$, the economy is at point $y^0$. Between $t = 0$ and $t \geq T$, the economy moves along the AD line, indicating a positive response to policy actions. The ZLB constraint is represented by the horizontal line at the zero interest rate level, limiting the ability of monetary policy to further stimulate the economy.

Key points:
- $y^0$: Initial output level at $t = 0$.
- $t \geq T$: Time period beyond which policy actions are applied.
- AD: Aggregate demand curve.
- PC: Policy curve.
ZLB steady state: \( r^n < 0 \)

\[
\pi = \frac{r^n}{\frac{u'(0)\delta\gamma a}{\epsilon} - 1}
\]

\[
y - y^n = \frac{r^n}{\frac{\epsilon}{\delta\gamma a}}
\]
FORWARD GUIDANCE
### NK | Forward-Guidance Scenario

<table>
<thead>
<tr>
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<td>( t = T_1 )</td>
</tr>
<tr>
<td>( t = T_2 )</td>
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NK | LONG FORWARD GUIDANCE

\[ y^n \]

\[ t = T_2 \]
NK | LONG FORWARD GUIDANCE

![Diagram showing NK with long forward guidance](image-url)
NK | LONG FORWARD GUIDANCE

\[ \pi \]

\[ -\delta \]

AD

\[ y \]

\[ y^n \]

\[ t = T_2 \]

\[ t = T_1^- \]
NK | LONG FORWARD GUIDANCE

π

y

PC

AD

t = 0

y^n

t = T_1

t = T_2

−δ

0
NK | MEDIUM FORWARD GUIDANCE

\[ \pi \]

\[ -\delta \]

AD

\[ y^n \]

\[ t = T_1 \]

\[ t = T_2 \]

\[ t = 0 \]
NK | SHORT ZLB + SHORT FORWARD GUIDANCE

\[ \pi \]

\[ y \]

\[ \delta \]

\[ AD \]

\[ t = 0 \]

\[ y^n \]

\[ t = T_1 \]

\[ t = T_2 \]
NK | LONG ZLB + SHORT FORWARD GUIDANCE
### WUNK | FORWARD-GUIDANCE SCENARIO

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$t = 0$ \hspace{1cm} $t = T_1$ \hspace{1cm} $t = T_2$
WUNK | FORWARD GUIDANCE

\[ \pi, 0, y, PC, AD \]

\[ y^n, t = T_2 \]
WUNK | FORWARD GUIDANCE

Diagram showing the relationship between inflation (π) and output (y) with time (t) at T1 and T2.
WUNK | MEDIUM ZLB + FORWARD GUIDANCE
WUNK | LONG ZLB + FORWARD GUIDANCE

\[
\begin{aligned}
\pi & \quad AD \\
y & \quad PC \\
0 & \quad t = 0 \\
\end{aligned}
\]

- \( t = 0 \)
- \( y^* \)
- \( t = T_1 \)
- \( t = T_2 \)
WUNK | POSSIBLE FORWARD-GUIDANCE OUTCOMES

\[ \pi \]

\[ y \]

\[ \text{AD (ZLB)} \]

\[ \text{PC} \]

\[ t = T_1 \]

\[ t = T_2 \]

\[ t = 0 \]

\[ y^n \]
EMPIRICAL SUPPORT FOR THE WUNK MODEL

• WUNK assumption:

\[ u'(0) > \frac{\epsilon}{\delta \gamma a} \]

• equivalent to: \( \delta \times (\delta - r^n) > \) output gap coeff. in Phillips curve
  - macro evidence: coefficient in Phillips curve \( \approx 3\% \)
  - experimental evidence: wide range with average \( \delta \approx 30\% \)
  - macro evidence: annual \( r^n \approx 2\% \)

• WUNK world: high discount rate, but utility for wealth makes people save, so \( r \) is small
WUNK BRIDGES NK AND OLD IS-MP

• IS-MP model uses fixed prices and steady-state analysis

• NK | flat steady-state IS curve: \( r = \delta \)
  
  – monetary policy cannot affect \( r \) in steady state
  
  – NK model cannot handle fixprice limit

• WUNK | downward-sloping steady-state IS curve: \( r = \delta - u'(0)y \)
  
  – monetary policy affects \( r \) and \( y \) in steady state
  
  – at fixprice limit, WUNK boils down to IS-MP
NK | MONETARY POLICY $i^*$ SHIFTS AD KEEPING $r = \delta$
NK | MONETARY POLICY $i^*$ SHIFTS AD KEEPING $r = \delta$
NK | PC BECOMES FLAT WITH FIXED PRICES = PROBLEM
WUNK | MONETARY POLICY $i^*$ SHIFTS AD $\Rightarrow$ CHANGES $r$
WUNK | MONETARY POLICY $i^*$ SHIFTS AD $\Rightarrow$ CHANGES $r$

$r = \delta - u'(0)y$
WUNK | PC BECOMES FLAT WITH FIXED PRICES = FINE

\[ \pi \]

\[ 0 \]

\[ y^n \]

\[ PC \]

\[ AD \]