NBER WORKING PAPER SERIES

A MODEL OF AGGREGATE DEMAND AND UNEMPLOYMENT

Pascal Michaillat
Emmanuel Saez

Working Paper 18826
http://www.nber.org/papers/w18826

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 2013

This paper was previously circulated under the title "A Theory of Aggregate Supply and Aggregate Demand as Functions of Market Tightness with Prices as Parameters." We are deeply indebted to George Akerlof for his comments and suggestions. We thank Wouter den Haan, Peter Diamond, Emmanuel Farhi, Roger Farmer, Yuriy Gorodnichenko, Greg Kaplan, Phillip Kircher, Camille Landais, Etienne Lehmann, Kristof Madarasz, Alan Manning, Emi Nakamura, Nicolas Petrosky-Nadeau, Pontus Rendhal, Kevin Sheedy, Jon Steinsson, Etienne Wasmer, Danny Yagan, and numerous seminar and conference participants for helpful discussions and comments. Financial support from the Center for Equitable Growth at the University of California Berkeley and from the Fondation Banque de France is gratefully acknowledged. Michaillat is grateful to the W.E. Upjohn Institute for Employment Research for its financial support through Early Career Research Grant #12-137-09. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

A Model of Aggregate Demand and Unemployment
Pascal Michaillat and Emmanuel Saez
NBER Working Paper No. 18826
February 2013, Revised July 2013
JEL No. E12,E24,E32,E63

ABSTRACT

We present a static model of aggregate demand and unemployment. The economy has a nonproduced good, a produced good, and labor. Product and labor markets have matching frictions. A general equilibrium is a set of prices, market tightnesses, and quantities such that buyers and sellers optimize given prices and tightnesses, and actual tightnesses equal posted tightnesses. In each frictional market, there is one more variable than equilibrium condition. To close the model, we take all prices as parameters. We obtain the following results: (1) unemployment and unsold production prevail in equilibrium; (2) each market can be slack, efficient, or tight if the price is too high, efficient, or too low; (3) product market tightness and sales are positively correlated under aggregate demand shocks but negatively correlated under aggregate supply shocks; (4) transfers from savers to spenders stimulate aggregate demand, product market tightness, and employment; (5) the government-purchase multiplier is positive when the economy is slack, zero when the economy is efficient, and negative when the economy is tight; (6) with unequal distribution of profits and labor income, a wage increase may stimulate aggregate demand and reduce unemployment.

Pascal Michaillat
Department of Economics
London School of Economics
Houghton Street
London, WC2A 2AE
United Kingdom
p.michaillat@lse.ac.uk

Emmanuel Saez
Department of Economics
University of California, Berkeley
530 Evans Hall #3880
Berkeley, CA 94720
and NBER
saez@econ.berkeley.edu

1 Introduction

There is a view that fluctuations in aggregate demand explain some of the fluctuations in unemployment observed over the business cycle. However, the standard theory of unemployment—the matching theory developed by Diamond [1982], Mortensen [1982], and Pissarides [1985]—only accounts for fluctuations caused by changes in wages, matching process, production technology, or policy. In this paper we propose a model that builds on matching theory and that accounts for the influence of aggregate demand on unemployment.

The model is static. There are three goods in the economy: a nonproduced good, a produced good, and labor. The market for nonproduced good is perfectly competitive but the product and labor markets have matching frictions. Matching frictions have two components: a matching function, which governs the number of trades on the market, and a matching cost, incurred by each buyer and measured in terms of the traded good.[1] From a seller's perspective, a frictional market looks like a competitive market except that she must take into account not only the market price but also the probability to sell, which is less than one. From a buyer's perspective, a frictional market looks like a competitive market except that she must take into account the effective price of consumption, which is the market price times a wedge that captures the cost of matching. We define a market tightness for each frictional market. The market tightness determines the selling probability and the matching wedge. A general equilibrium is a set of prices, market tightnesses, and quantities such that buyers and sellers optimize given prices and tightnesses, and actual tightnesses equal posted tightnesses. In each frictional market, there is one more variable than equilibrium condition. This property implies that many combinations of prices and tightnesses are consistent with general equilibrium. To close the model, we take all prices as parameters. Nonetheless, the equilibrium is pairwise Pareto efficient in the sense that once a match is realized, there is no price change that could benefit both seller and buyer. Since prices are parameters, only tightnesses equilibrate the markets with matching frictions.

The model has three critical elements: nonproduced good, matching frictions, and parametric prices. The nonproduced good is necessary to obtain an interesting concept of aggregate because without it, consumers would mechanically spend all their income on produced good. In our model consumers allocate their income between consumptions of produced and nonproduced good, and

[1] By introducing these two components, we follow the common approach to modeling matching frictions. The matching function is a well-behaved function that summarizes the complicated exchange process between buyers an sellers. See Petrongolo and Pissarides [2001] and Rogerson, Shimer and Wright [2005] for discussions of the theoretical foundations and empirical characterization of the matching function. The matching cost is usually measured in terms of numéraire, but measuring it in terms of traded goods simplifies the exposition and welfare analysis. Farmer [2008, 2009] and Shimer [2010] make the same modeling choice.

aggregate demand is the desired consumption of produced good. Matching frictions generate unsold production in equilibrium to propagate aggregate demand shocks to the labor market, generate unemployment in equilibrium, and provide a theoretical justification for price and wage rigidity in equilibrium. Last, the parametric prices allow aggregate demand and other shocks to affect aggregate demand and labor demand and to have macroeconomic effects. The reason is that the prices do not respond to shocks, as in the Keynesian tradition. Modeling prices as parameters simplifies the mechanics of the model while describing the short-run reasonably well as long as prices adjust slowly.

Our model is parsimonious, even when we introduce heterogeneity in preferences, wealth, labor income, or profit income. The partial equilibrium on each market is represented by the intersection of a demand curve and supply curve in a (quantity, tightness) plane. It is easy to solve for the general equilibrium and obtain comparative statics since product market tightness and labor market tightness solve a simple system of two equations. It is also easy to analyse and represent welfare. We define a market as slack, efficient, or tight when market tightness is below, at, or above the efficient level of tightness. In a slack market, consumption and sales—the sum of consumption and aggregate matching costs—are too low. In a tight market, consumption is too low but sales are too high. When the market is tight, too much of the traded good is dissipated as matching cost. Each market is slack, efficient, or tight when the exogenous price is too high, efficient, or too low.

In Section 2, we present the simplest model connecting aggregate demand to unemployment. This model represents an economy of self-employed workers who sell and purchase services on a market with matching frictions. Labor and the produced good are a single good so that labor and product markets are a single market. Because of the matching frictions, workers cannot sell all their services and are idle part of the time. The rate of idleness, which we interpret as the unemployment rate, is a negative function of market tightness. We obtain a number of results. First, we show that market tightness and sales are positively correlated under aggregate demand shocks but negatively correlated under aggregate supply shocks. Second, when individuals differ in their marginal propensity to consume, transfers from savers to spenders stimulate aggregate demand and market tightness. Such transfers increase aggregate consumption when the economy is slack but reduce it when the economy is tight. Third, the government-purchase multiplier changes sign across economic regimes: it is positive when the economy is slack, zero when the economy is efficient, and negative when the economy is tight.

In Section 3, we introduce firms that mediate between workers and consumers by hiring workers on a labor market with matching frictions, employing these workers to produce goods, and selling the production on a product market with matching frictions. Following Michaillat [2012], we assume that firms are large, face a production function with diminishing marginal returns to labor, and maximize profits taking labor market tightness and real wage as given. This richer model allows us to study how aggregate demand shocks propagate from the product market to the labor market and how they affect unemployment, to study the effects of shocks to technology, labor force participation, and real wage, and to examine the impact of an unequal distribution of profits and labor income. For instance, we show that if profits are concentrated among savers while labor income is concentrated among spenders, a wage increase may reduce unemployment. This happens when the positive effect of the wage increase on aggregate demand dominates its negative effect on labor demand.

Section 4 argues that our matching framework is not restrictive. Using alternative assumptions about the functional forms of the utility, production, and matching functions, the value of matching costs, and the price and wage schedules, our model yields the same first-order conditions as a broad range of macroeconomic models. When we make assumptions to mimic a perfect-competition model, a matching model with Nash bargaining over price and wage, and a monopolistic-competition model, aggregate demand shocks have no effect because the price adjusts sufficiently to absorb them completely. In contrast in our model, price and wage are rigid and aggregate demand shocks therefore propagate to product and labor markets. When we make assumptions to mimic a matching model with rigid price and wage and with linear utility and production functions, many issues related to aggregate demand are trivial because aggregate demand and labor demand functions are perfectly elastic in tightness. In contrast in our model, utility function and production function are concave so the demand functions are decreasing in tightness. When we make assumptions to mimic a fixprice-fixwage model, we obtain aggregate demand effects, but the theory is much less tractable because it creates four different regimes, each with a different set of equilibrium conditions. The four regimes appear because the matching functions are equal to the minimum of their two arguments so all supply functions have kinks. In contrast in our model, the matching functions are smooth, the kinks disappear, and the general equilibrium is described by a unique set of equilibrium conditions.

Section 5 concludes by discussing some applications of the model. First, the model could be used to analyze the impact on unemployment of fiscal policies that affect simultaneously aggregate demand, labor demand, and labor supply (for example, unemployment insurance, employer and employee payroll taxes, or minimum wage). Second, empirical research could exploit the theoretical predictions of the model to identify the macroeconomic shocks driving business cycle fluctuations. A key result of the analysis is that product market tightness and sales are positively correlated under aggregate demand shocks but negatively correlated under other shocks. We explain how empirical research on sales and inventories could use this result to identify macroeconomic shocks. Third, the model could be a starting point to build a dynamic and stochastic macroeconomic model featuring unsold production and unemployment. To obtain a quantitatively realistic model, however, we would need to introduce a process for price adjustment instead of considering fixed prices only. For instance, prices might adjust toward the efficient level in the medium run through the competitive search mechanism of Moen [1997].

Our model is related to several other models that explore the link between aggregate demand and equilibrium unemployment.[2] Farmer [2008, 2009] exploits the indeterminacy arising in a matching model of the labor market to select an equilibrium using agents' beliefs about the level of economic activity. In Farmer's models, the wage satisfies an additional condition imposing that agents' beliefs are fulfilled in equilibrium. In contrast, in our model, price and wage are parameters. Lehmann and Van der Linden [2010] build a matching model of the product market and labor market. Money is required for transactions, giving rise to aggregate demand. In their model, the price is determined by competitive search and the wage by Nash bargaining so price and wage are completely flexible. In contrast, in our model, price and wage are parameters so they are rigid in response to macroeconomic shocks. Rendhal [2012] builds a New Keynesian model with matching frictions in the labor market, giving rise to unemployment, and a cash-in-advance constraint in the product market, giving rise to an aggregate demand. Kaplan and Menzio [2013] propose a model with matching frictions on the labor market and the search frictions of Burdett and Judd [1983] on the product market. Shopping externalities appear, generating multiple rational-expectation equilibria and self-fulfilling fluctuations in unemployment. The product markets are represented differently in the models of Rendhal [2012] and Kaplan and Menzio [2013] and in ours. Thus, aggregate demand shocks propagate differently in these models: the Euler equation and the zero lower bound play a key role in the model of Rendhal [2012]; price dispersion is critical in the model of Kaplan and Menzio [2013]; matching frictions and unsold production are central to our model.

2 A Basic Matching Model

This section presents the simplest model in which aggregate demand influences unemployment. All workers are self-employed and sell services on a market with matching frictions. Because of the matching frictions, workers may not be able to sell all their services. Thus, workers may be idle part

[2] Macroeconomists have recently shown renewed interest for aggregate demand. For example, see Mian and Sufi [2012] for an empirical investigation of the role of aggregate demand, Lorenzoni [2009] for a model in which news shocks act as aggregate demand shocks, Eggertsson and Krugman [2012] for a model in which debt deleveraging depresses aggregate demand, Heathcote and Perri [2012] for a model in which a housing market crash depresses aggregate demand, and Bai, Rios-Rull and Storesletten [2012] for a model of aggregate demand based on a matching model of the product market.

of the time in equilibrium. Idleness corresponds to unemployment in this simple model.

2.1 The Model

Market for Services. A measure 1 of identical self-employed workers sell services on a market with matching frictions. The capacity of each worker is y; that is, a worker would like to sell y units of services. Workers are also consumers of services, but they cannot consume their own services. Each consumer visits v workers to purchase their services. The number of trades between consumers and workers is given by a matching function with constant elasticity of substitution:

$$s = \left(y^{-\eta} + v^{-\eta}\right)^{-\frac{1}{\eta}},$$

where η determines the elasticity of substitution between inputs in the matching process.[3] We impose $\eta > 0$ such that the number of trades is less than aggregate capacity, y, and less than the total number of visits, v. In each trade, a consumer buys one unit of service from a worker at price $p > 0$.

We define market tightness as the ratio of visits to capacity: $x \equiv v/y$. Since the matching function displays constant returns to scale, market tightness determines the probabilities that one unit of service is sold and that one visit leads to a purchase. Workers sell each unit of service with probability

$$f(x) = \frac{s}{y} = \left(1 + x^{-\eta}\right)^{-\frac{1}{\eta}}, \tag{1}$$

and each visit leads to a purchase with probability

$$q(x) = \frac{s}{v} = \left(1 + x^{\eta}\right)^{-\frac{1}{\eta}}. \tag{2}$$

The matching probabilities, $f(x)$ and $q(x)$, are always between 0 and 1.[4] We abstract from randomness at the worker and consumer levels: a worker sells $f(x) \cdot y$ units of services for sure, and a consumer purchases $q(x) \cdot v$ units of services for sure. We define idleness as the fraction of services that are available but not sold in equilibrium: $u = 1 - f(x)$. Since $f(x) < 1$, idleness prevails in equilibrium. The function f is strictly increasing and the function q is strictly decreasing in x. In other words, when the market is slacker, a larger fraction of workers' capacity remains unsold and

[3] This functional form is borrowed from den Haan, Ramey and Watson [2000].

[4] With a conventional Cobb-Douglas matching function, $s = y^{\eta} \cdot v^{1-\eta}$, the trading probabilities $f(x)$ and $q(x)$ may be greater than 1, and the matching function may need to be truncated to obtain probabilities between 0 and 1. This truncation would complicate the analysis, which is why we use the matching function of den Haan, Ramey and Watson [2000].

idleness rises while a larger fraction of consumers' visits results in a purchase.

Market for Nonproduced Good. Consumers trade a nonproduced good on a perfectly competitive market. Each consumer has an endowment $\mu > 0$ of nonproduced good. We normalize the price of the nonproduced good to 1 and use the nonproduced good as numéraire. We introduce an nonproduced good in the model to avoid Say's law, the result that the supply of services automatically generates its own demand. With an nonproduced good that enters workers' utility function, workers allocate their income between consumption of services and consumption of nonproduced good as a function of the relative price of these goods. The optimal allocation then determines aggregate demand.[5]

Consumers. Consumers have a constant-elasticity-of-substitution (CES) utility function given by

$$\left[\chi \cdot c^{\frac{\epsilon-1}{\epsilon}} + (1-\chi) \cdot m^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{3}$$

where c is consumption of services, m is consumption of nonproduced good, $\chi \in (0,1)$ is a parameter measuring the marginal propensity to consume services out of income, and ϵ is a parameter measuring the elasticity of substitution between services and nonproduced good. To guarantee existence and unicity of the equilibrium, we impose $\epsilon > 1$.

The consumer visits v workers to purchase services. Mtching with workers requires other services: each visit costs $\rho \in (0,1)$ units of services. (For instance, one needs to purchase taxi services to get to the hair salon and purchase hairdressing services.) The $\rho \cdot v$ units of services for matching are purchased like the c units of services for consumption. Hence, the number of visits is related to consumption and market tightness by

$$q(x) \cdot v = c + \rho \cdot v.$$

The desired level of consumption directly determines the number of visits: $v = c/\left(q(x) - \rho\right)$. Because of the matching cost, consuming one unit of services requires to purchase $1+(\rho \cdot v/c) = 1+\tau(x)$ units of services, where

$$\tau(x) \equiv \frac{\rho}{q(x) - \rho}.$$

The function τ is positive and strictly increasing as long as $q(x) > \rho$, which holds in equilibrium.

[5]This modeling technique was common in the Keynesian literature of the 1970s and 1980s, when the nonproduced good was often money. See for instance Barro and Grossman [1971], Hart [1982], or Blanchard and Kiyotaki [1987].

In what follows, we focus on consumption decisions and relegate the matching process to the background. Consuming c requires purchasing $(1+\tau(x))\cdot c$ in the course of $(1+\tau(x))\cdot c/q(x)$ visits, which costs a total of $p\cdot(1+\tau(x))\cdot c$. The matching cost, ρ, imposes a wedge $\tau(x)$ on the price of services. At the limit where ρ is zero, the wedge disappears.

The consumer's income comes from the sales of μ units of nonproduced good at price 1 and $f(x)\cdot y$ units of services at price p. The consumer uses the income to purchase m units of nonproduced good at price 1 and c units of services at price $(1+\tau(x))\cdot p$. The consumer's budget constraint is

$$m+(1+\tau(x))\cdot p\cdot c=\mu+p\cdot f(x)\cdot y. \tag{4}$$

Given x and p, the consumer chooses m and c to maximize (3) subject to (4). The optimal choice of consumptions equalizes marginal utility of consumption for c and m given total prices and satisfies

$$(1-\chi)\cdot(1+\tau(x))\cdot p\cdot c^{\frac{1}{\epsilon}}=\chi\cdot m^{\frac{1}{\epsilon}}. \tag{5}$$

The nonproduced good market clears so $m=\mu$. Hence, the optimal consumption choice imposes

$$c=\left(\frac{\chi}{1-\chi}\right)^{\epsilon}\cdot\frac{\mu}{[(1+\tau(x))\cdot p]^{\epsilon}}. \tag{6}$$

At the limit where $\epsilon\to 1$, the utility function becomes Cobb-Douglas: $c^{\chi}\cdot m^{1-\chi}$. In that case, the optimal consumption choice (6) can be represented with a Keynesian cross in which the marginal propensity to consume is χ. The cross is displayed in Figure 1. The first condition in the cross is $E=I$: In general equilibrium expenditure on services, $E=(1+\tau(x))\cdot p\cdot c$, equal income from the sales of services, $I=p\cdot f(x)\cdot y$. The second condition in the cross is $E=\chi\cdot(\mu+I)$: Consumers spend on services a fraction χ of their wealth, which is the sum of their labor income, I, and the value of their endowment, μ. This second condition is obtained by combining (4) with (5) when $\epsilon=1$. Hence, in general equilibrium, $E=\mu\cdot\chi/(1-\chi)$, which is equivalent to (6) when $\epsilon=1$.

2.2 Equilibrium

We now define and characterize the equilibrium of the model.[6] We assume that a price, p, and a market tightness, x, are posted on the market for services, and that buyers and sellers of services take this price and tightness as given. We define an equilibrium as a triplet (p,x,c) such that the

[6]Appendix B contains a formal definition of the equilibrium, discusses the assumptions underlying the equilibrium, and draws a parallel between our equilibrium concept and the Walrasian equilibrium.

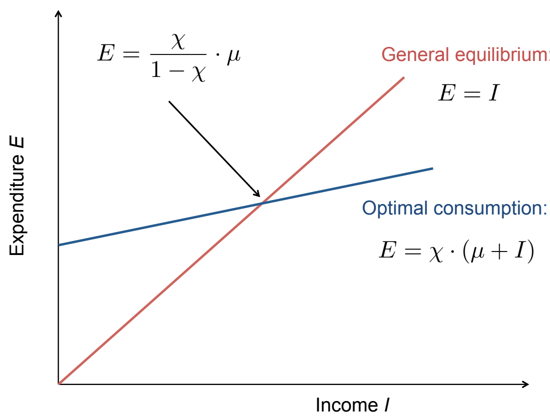


Figure 1: Keynesian cross with Cobb-Douglas utility function

following conditions hold: (1) taking as given x and p, the representative buyer chooses the number of visits $v(x, p)$ to maximize its utility subject to his budget constraint and to the constraint imposed by matching frictions: $c = v \cdot q(x)/(1 + \tau(x))$; (2) taking x and p as given, the representative seller chooses his capacity $y(x, p)$ to maximize its utility subject to the constraint imposed by matching frictions: $s = y \cdot f(x)$, where s are sales of the seller;[7] (3) the actual labor market tightness is x: $v(x, p)/y(x, p) = x$; and (4) the price p is pairwise Pareto efficient in all buyer-seller matches.[8] Under these equilibrium conditions, the market for nonproduced good necessarily clears.

Our equilibrium concept is a direct extension of the Walrasian equilibrium to a market with matching frictions. As in Walrasian theory, our assumption is that buyers and sellers are small relative to the size of the market so that they regard market tightness and price as unaffected by their own actions. In a Walrasian equilibrium, buyers and sellers behave optimally given the quoted price and the expectation that they will be able to trade with probability one. Here, Conditions (1) and (2) impose that buyers and sellers behave optimally given the quoted price and the quoted market tightness, which determines the trading probabilities of sellers and buyers. In a Walrasian equilibrium, the market clears. This condition can be reformulated as a consistency requirement: given that sellers and buyers expect to be able to trade with probability one, anybody desiring to trade at the quoted price must be able to trade in equilibrium. This condition can only be fulfilled if at the quoted price the quantity that buyers desire to buy equals the quantity that sellers desire to sell—that is, if the market clears. Condition (3) is the equivalent to this consistency requirement in presence of matching frictions. Last, Walrasian theory imposes that no mutually advantageous trades between two agents are available,

[7] y is exogenous in the single market model of this section but will be endogenous next section.

[8] In his search-theoretic model of interindustry wage differentials, Montgomery [1991] also defines an equilibrium concept in which agents take a posted market tightness as given and actual tightness equals posted tightness in equilibrium.

which in turn imposes that buyers and sellers expect to trade with probability one and not with any probability below one. This is because without a matching function, buyers who do not trade with anybody but would like to trade at the current price can come together on the market place. If excess supply or demand existed at the market price, buyers or sellers could initiate new trades at a different price until all opportunities for pairwise improvement are exhausted. Condition (4) is the equivalent of this condition in our theory. However, in presence of matching frictions, the condition that no mutually advantageous trades are available only applies to agents who are matched. Even though the equilibrium is pairwise Pareto efficient, the equilibrium might not be Pareto efficient overall in the sense that changing the price or wage could improve everybody's welfare.

To obtain a convenient representation of the equilibrium, we define the following functions:

DEFINITION 1. The *aggregate demand* is a function of market tightness and price defined by

$$c^d(x,p) = \left(\frac{\chi}{1-\chi}\right)^{\epsilon} \cdot \frac{\mu}{[(1+\tau(x)) \cdot p]^{\epsilon}} \tag{7}$$

for all $(x,p) \in [0,x^m] \times (0,+\infty)$, where $x^m > 0$ satisfies $\rho = q(x^m)$. The *aggregate supply* is a function of market tightness defined for all $x \in [0,x^m]$ by

$$c^s(x) = (f(x) - \rho \cdot x) \cdot y. \tag{8}$$

The aggregate demand gives the consumption of services that satisfies the consumer's optimal consumption choice given by (6). The aggregate supply gives the amount of services consumed after the matching process when workers offer y units of services for sale. Lemma 1 establishes a few properties of aggregate demand and aggregate supply:

LEMMA 1. *The function c^d is strictly decreasing in x and p, $c^d(0,p) = [\chi/(1-\chi)]^{\epsilon} \cdot [(1-\rho)/p]^{\epsilon} \cdot \mu$, and $c^d(x^m,p) = 0$. The function c^s is strictly increasing on $[0,x^*]$, strictly decreasing on $[x^*,x^m]$, $c^s(0) = 0$, $c^s(x^m) = 0$, and $c^s(x^*) = c^*$. x^* maximizes $c = [f(x) - \rho \cdot x] \cdot y$ so that $f'(x^*) = \rho$ and $c^* = [f(x^*) - \rho \cdot x^*] \cdot y$. The constants x^* and c^* depend solely on y, ρ, and η.*

The aggregate demand decreases with p and x because when either of them increases, the effective price of services, $(1+\tau(x)) \cdot p$, increases and the consumption of services is reduced relative to that of nonproduced good, fixed to μ. When x is low, the matching process is congested by the amount of services for sales. Consequently, the additional visits to sellers resulting from an increase in x lead to a large increase in sales and thus a large increase in $f(x)$. Conversely when x is high, the matching process is congested by visits, and the additional visits resulting from the increase in x only lead to small increases in sales and $f(x)$. It follows that $f(x) - \rho \cdot x$ and the aggregate supply increase for low x but decrease for high x. The value $x = x^*$ maximizes $f(x) - \rho \cdot x$.

Given the definition of aggregate demand, Condition (1) imposes that $v(x,p) = (1 + \tau(x)) \cdot c^d(x,p)/q(x)$. It also imposes that $c = c^d(x,p)$. Here there is no active decision from the seller: each worker's provision of services is exogenously set to y. Therefore, Condition (2) imposes that $y(x,p) = y$. Condition (3) imposes that

$$x = \frac{v(x,p)}{y(x,p)} = \frac{1 + \tau(x)}{q(x)} \cdot \frac{c^d(x,p)}{y}.$$

We rewrite this condition as

$$c^d(x,p) = \frac{x \cdot q(x)}{1 + \tau(x)} \cdot y = \frac{f(x)}{1 + \tau(x)} \cdot y = c^s(x).$$

Hence, Condition (3) imposes that aggregate supply equals aggregate demand. Condition (4) imposes that the price p is such that buyer and seller receive a positive surplus from their match. In our model, this condition is always respected when buyers and sellers behave optimally. The reason is that if buyers and sellers are willing to trade at price p before the matching process, when the search and production costs are not yet sunk, they are necessarily willing to trade at price p once they are matched and the search and production costs are sunk. Accordingly, an equilibrium consists of a triplet (p, x, c) such that $c^s(x) = c^d(x,p)$ and $c = c^s(x)$.

Since the equilibrium is composed of three variables that satisfy two conditions, there is one more variable than equilibrium conditions.[9] This property implies that infinitely many combinations of price and tightness are consistent with our equilibrium definition.[10] To close the model, we assume that the price is a parameter of the model and that only market tightness equilibrates the market.[11] In the definition of the equilibrium, the price becomes an exogenous variable; only tightness and quantity are endogenous variables such that the equilibrium is composed of two variables that satisfy two conditions. Furthermore, the price remains fixed in response to shocks in the comparative-statics anal-

[9] A Walrasian equilibrium is composed of as many variables as equations because Condition (4) imposes that x is such that the trading probabilities are one, thus adding one equation to the equilibrium system.

[10] The property that there is one more variable than equation in matching models was previously noted by Farmer [2008]. More broadly, the result that there is an indeterminacy in matching models is well known. For instance, Howitt and McAfee [1987] and Hall [2005] argue the price is indeterminate in matching models because each seller-buyer pair must share the positive surplus created by their pairing, thus deciding the price in a situation of bilateral monopoly. It has been known at least since Edgeworth [1881] that the solution to the bilateral monopoly problem is indeterminate.

[11] To close matching models of the labor market, several researchers assume that the wage is a parameter or a function of the parameters. See for instance Hall [2005], Blanchard and Galí [2010], and Michaillat [2012].

ysis. This assumption follows the Keynesian tradition. To explain persistent unemployment during the Great Depression, Keynes [1936] replaced the assumption that the wage clears the labor market by that of a wage floor under which nominal wages would not fall. This idea was formalized by Hicks [1965], who developed a model in which prices and wages are fixed and adjustments only take place through quantity rationing. Barro and Grossman [1971] then introduced the fixprice and fixwage assumptions in a general-equilibrium context. Prices and wages were assumed to be fixed to simplify the mechanics of the macroeconomy while describing the short-run reasonably well.

Our equilibrium concept does not require to close the model assuming that the price is a parameter. Any alternative assumption on the price could close the model. However, as showed in Section 4, readily available assumptions would not allow us to study the effects of aggregate demand shocks. For instance, the traditional way to close the model is to set prices using the Nash bargaining solution [Diamond, 1982; Mortensen, 1982]. In Section 4, we show that the Nash bargained price is so flexible that it completely eliminates the effects of aggregate demand shocks.

We now define and characterize a short-run equilibrium:

DEFINITION 2. Given a price p, a *short-run equilibrium* consists of a pair (x, c) of market tightness and consumption such that aggregate supply equals aggregate demand and consumption is given by the aggregate supply:

$$\begin{cases} c^s(x) & = & c^d(x,p) \\ c & = & c^d(x,p) \end{cases}$$

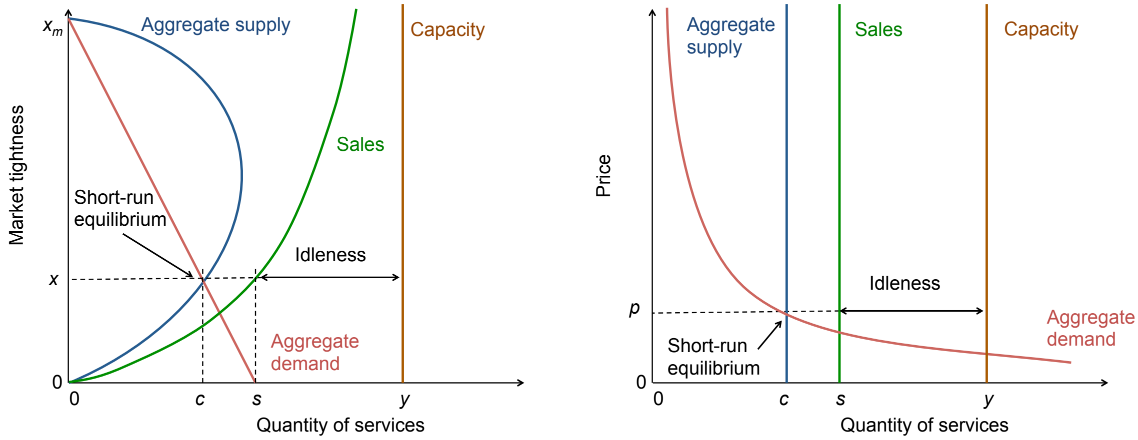
PROPOSITION 1. *For any price $p > 0$, there exits a unique short-run equilibrium with positive consumption. Equilibrium tightness, x, is the unique solution to*

$$(1+\tau(x))^{\epsilon-1} \cdot f(x) \cdot y = \left(\frac{\chi}{1-\chi}\right)^{\epsilon} \cdot \frac{\mu}{p^{\epsilon}}. \tag{9}$$

Equation (9) is obtained by manipulating the equilibrium condition $c^s(x) = c^d(x,p)$.

Figure 2(a) represents aggregate demand, aggregate supply, and the equilibrium in a (c, x) plane. The aggregate demand curve slopes downward. The aggregate supply curve slopes upward for $x \leq x^*$ and downward for $x \geq x^*$. The equilibrium corresponds to the intersection of the two curves with positive consumption.[12] The figure also shows capacity y and sales $s = f(x) \cdot y$. A fraction

[12]There is another equilibrium at the other intersection of the curves, but it has zero consumption. Appendix A extends Proposition 1 to characterize all the possible equilibria, with zero or positive consumption.



(a) Equilibrium in a (c, x) plane

(b) Equilibrium in a (c, p) plane

Figure 2: Short-run equilibrium in the model of Section 2

$c/s = 1/(1 + \tau(x))$ of sales are consumed, and a fraction $1 - (c/s) = \tau(x)/(1 + \tau(x))$ of sales are allocated to matching. The fraction $u = 1 - (s/y) = 1 - f(x)$ of services that are not sold is idleness. Figure 2(b) represents aggregate demand and aggregate supply in a (c, p) plane. Aggregate supply does not depend on the price so the aggregate supply curve is vertical. The aggregate demand curve is downward sloping. The equilibrium corresponds to the intersection of the aggregate supply and aggregate demand curves in this plane as well.

Even though the matching cost, ρ, plays an important role in the model, idleness would not necessarily disappear if the matching cost were arbitrarily small. What happens when the matching cost becomes arbitrarily small can be illustrated on Figure 2(a). The aggregate supply curve takes the shape of the sales curves and the aggregate demand curve becomes vertical and shits outwards. The equation of the aggregate demand curve when $\rho = 0$ is $c^d = [\chi/(1 - \chi)]^{\epsilon} \cdot \mu \cdot p^{-\epsilon}$. Hence, idleness remains positive in equilibrium if and only if the price is high enough: $p > [\chi/(1 - \chi)] \cdot (\mu/y)^{1/\epsilon}$.[13]

Our results do not rely on matching frictions in the product market. It is possible to obtain the same results through the same mechanism in a model with frictions in the labor market. Assume that firms hire workers at wage p on a labor market with matching frictions, that each employee produce one unit of service, and that firms sell services to consumers at price p^f on a competitive market. Consumers purchase any amount of services at price p^f from firms, without incurring matching costs. Firms bear the matching costs. They post v vacancies and fill each vacancy with probability $q(x)$. Posting a vacancy requires ρ workers so that $q(x) \cdot v = c + \rho \cdot v$. Hence selling one unit of good

[13] The labor market model of Michaillat [2012] exhibits the same property. In that model, when the wage is high enough, some unemployment remains even when the recruiting cost is arbitrarily small.

requires using $1+(\rho\cdot v/c)=1+\tau(x)$ workers. Firms' profits per sale are equal to $p^f-(1+\tau(x))\cdot p$, hence free entry of firms imposes $p^f=(1+\tau(x))\cdot p$. Workers find a job with probability $f(x)$ and consumption is $c=f(x)/(1+\tau(x))$. If the wage, p, is fixed and the price, p^f, adjusts, this model is isomorphic to our initial model, except that sales equal consumption in this new model.

2.3 Efficient Allocation

We now define and describe the efficient allocation, and we characterize the price that implements it:

DEFINITION 3. An *efficient allocation* is a pair (x,c) of market tightness and consumption that maximizes welfare, $\left[\chi\cdot c^{(\epsilon-1)/\epsilon}+(1-\chi)\cdot\mu^{(\epsilon-1)/\epsilon}\right]^{\epsilon/(\epsilon-1)}$, subject to the matching frictions, $c\leq(f(x)-\rho\cdot x)\cdot y$.

PROPOSITION 2. *The efficient allocation is (x^*,c^*), where x^* and c^* are defined by $f'(x^*)=\rho$ and $c^*=[f(x^*)-\rho\cdot x^*]\cdot y$. The price that implements the efficient allocation is*

$$p^*=\frac{\chi}{1-\chi}\cdot\left(\frac{\mu}{y}\right)^{\frac{1}{\epsilon}}\cdot\left(1-\rho^{\frac{\eta}{1+\eta}}\right)^{1-\frac{\eta+1}{\eta\cdot\epsilon}}.$$

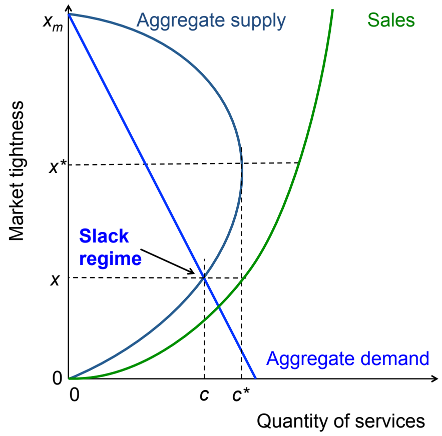
In Figure 2(b), the efficient allocation is the point that is furthest to the right on the aggregate supply curve. At this point, the aggregate supply function is maximized. The price p^* is such that the aggregate demand curve intersects the aggregate supply curve at the efficient allocation. This price necessarily exists because by increasing the price from 0 to $+\infty$, the aggregate demand curve rotates around the point $(0,x^m)$ from an horizontal position to a vertical position.

Depending on the value x of equilibrium market tightness, the economy can be in three regimes:

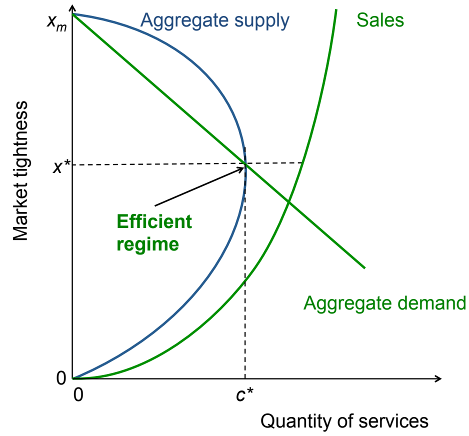
DEFINITION 4. The economy is *slack* if $x<x^*$, *tight* if $x>x^*$, and *efficient* if $x=x^*$.

PROPOSITION 3. *The economy is* slack *if and only if $p>p^*$,* tight *if and only if $p<p^*$, and* efficient *if and only if $p=p^*$.*

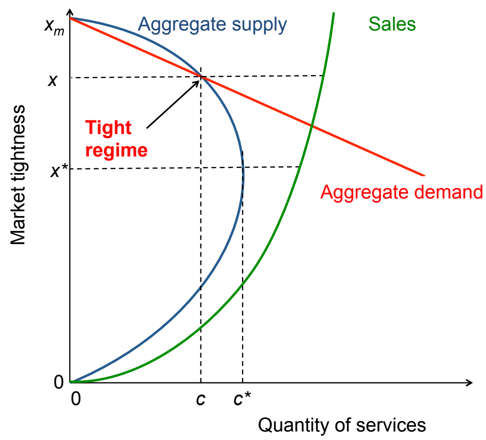
Figure 3 illustrates the regimes. In the slack regime, the price is above its efficient level so aggregate demand is too low and tightness is below its efficient level. Consumption and sales are below their efficient level. In the tight regime, the price is below its efficient level so aggregate demand is too high and tightness is above its efficient level. Consumption is again below its efficient level but sales are above their efficient level. In our model, higher consumption always implies higher welfare, which is not the case of higher sales. The economy behaves very differently in the three regimes



(a) An equilibrium in the slack regime



(b) Equilibrium in the efficient regime



(c) An equilibrium in the tight regime

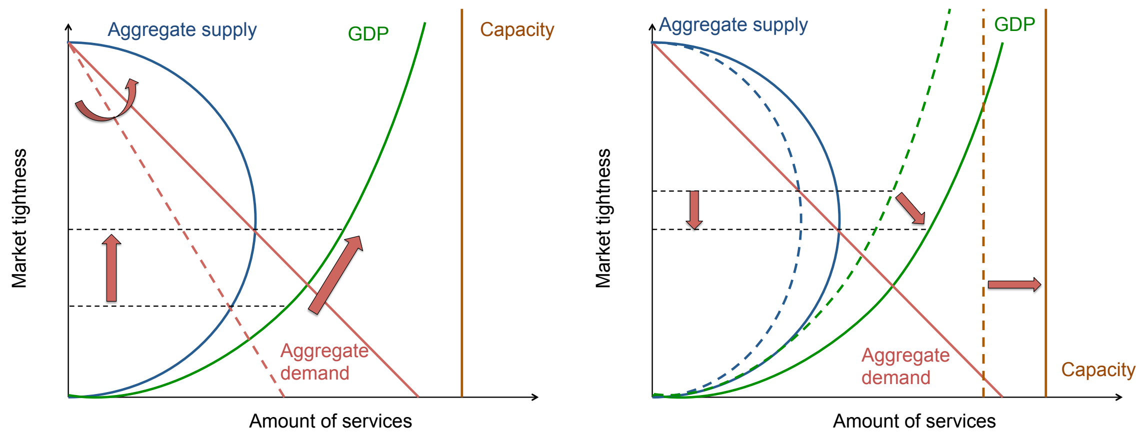
Figure 3: The three regimes of the model of Section 2

because the aggregate supply function has different slopes across regimes: $dc^s/dx > 0$ in the slack regime; $dc^s/dx < 0$ in the tight regime; and $dc^s/dx = 0$ in the efficient regime.

2.4 Aggregate Demand and Aggregate Supply Shocks

We use comparative statics to describe the response of consumption, market tightness, sales, and idleness to aggregate demand and aggregate supply shocks. Table 1 summarizes the results.

We parameterize an aggregate demand shock by a change in marginal propensity to consume, χ, in price, p, or in endowment, μ. Figure 4(a) illustrates a positive aggregate demand shock, corresponding to an increase in p or a decrease in χ or μ. The shock leads the aggregate demand curve to rotate outward and therefore increase market tightness and sales. Since tightness increases, idleness falls. The impact on consumption depends on the regime: in the slack regime, consumption increases; in



(a) Aggregate demand shock (b) Aggregate supply shock

Figure 4: Effects of aggregate demand and aggregate supply shocks in the model of Section 2

the efficient regime, consumption does not change; and in the tight regime, consumption falls. In the tight regime, a higher tightness reduces the sales devoted to consumption even though it increases total sales because it increases sharply the sales required for matching.

We parameterize an aggregate supply shock by a change in capacity, y. Figure 4(b) illustrates a positive aggregate supply shock, corresponding to an increase in y. The shock leads the aggregate supply curve to expand, raising consumption but reducing market tightness. Since tightness decreases, idleness increases. Since y increases but x falls, the impact on sales $s = f(x) \cdot y$ is not obvious. Equation (9) implies, however, that sales increase as x and therefore $(1 + \tau(x))^{\epsilon-1}$ fall when y increases. Interestingly, when the economy is in the efficient regime, shifts in aggregate supply do influence consumption whereas shifts in aggregate demand have no first-order effects on consumption.

Aggregate supply and aggregate demand shocks generate different correlations between variables. Market tightness and sales are positively correlated under aggregate demand shocks but negatively correlated under aggregate supply shocks. An implication is that idleness decreases after a positive aggregate demand shock but increases after a positive aggregate supply shock. The intuition is simple. After a positive aggregate demand shock consumers want to consume more services so workers sell a larger fraction of a fixed amount of services available. Hence, sales and market tightness are higher. On the other hand, after a positive aggregate supply shock workers offer more services for sale but consumers do not desire to consume more at a given price, so workers sell a smaller fraction of a larger amount of services available. Hence, market tightness is lower. Since tightness is lower in equilibrium, the effective price faced by consumers, $(1 + \tau(x)) \cdot p$, is lower, stimulating consumers to purchase more services and increasing sales.

Table 1: Comparative statics in the model of Section 2

	Effect on:			
Increase in:	Market tightness	Consumption	Sales	Idleness
Aggregate demand	> 0	> 0 if slack $= 0$ if efficient < 0 if tight	> 0	< 0
Aggregate supply	< 0	> 0	> 0	> 0

Notes: The comparative statics are derived in Section 2.4. An increase in aggregate demand results from an increase in endowment, μ, a decrease in price, p, or an increase in marginal propensity to consume, χ. An increase in aggregate supply results from an increase in capacity, y.

2.5 Transfers

We introduce heterogeneity of preferences and endowment across consumers. Aggregate demand admits a new expression. Yet, the equilibrium can be represented as in Figure 2. We show that a transfer of wealth from consumers with low taste for services to consumers with high taste for services creates a positive aggregate demand shock.

Workers belong to one of G groups of measure $1/G$. Group g's per person utility is

$$c_g^{\chi_g} \cdot m_g^{1-\chi_g}, \tag{10}$$

where c_g is group g's per person consumption of services, m_g is group g's per person consumption of nonproduced good, and $\chi_g \in (0, 1)$ is group g's marginal propensity to consume services. We use a Cobb-Douglas utility function to simplify the exposition.[14] Group g's per person budget is

$$m_g + (1 + \tau(x)) \cdot p \cdot c_g = \mu_g + p \cdot f(x) \cdot y, \tag{11}$$

where $\mu_g \geq 0$ is group g's per person endowment of nonproduced good. Workers have the same labor income in all groups: any worker sells a fraction $f(x)$ of her capacity y at price p.

Given x and p, a consumer in group g chooses c_g and m_g to maximize (10) subject to (11). The

[14] Appendix C adapts Proposition 1 to a Cobb-Douglas utility function. An equilibrium with positive consumption exists if the price is high enough. When the equilibrium exists, equilibrium tightness is the unique solution to (9).

optimal consumption of services satisfies

$$(1+\tau(x)) \cdot p \cdot c_g = \chi_g \cdot [\mu_g + p \cdot f(x) \cdot y],$$

which is an application of the consumption cross ($E = \chi \cdot (\mu + I)$) to a model with heterogeneous preferences and endowments. We aggregate the demand for services of all the groups:

$$(1+\tau(x)) \cdot p \cdot \left(\sum_g c_g \right) = \left(\sum_g \mu_g \cdot \chi_g \right) + \left(\sum_g \chi_g \right) \cdot p \cdot f(x) \cdot y.$$

In general equilibrium, purchases and sales of services are equal: $(1+\tau(x)) \cdot \left(\sum_g c_g \right) / G = f(x) \cdot y$. Thus, aggregate demand is given by

$$c^d(x,p) \equiv \frac{1}{G} \cdot \sum_g c_g = \frac{1}{p \cdot (1+\tau(x))} \cdot \frac{\sum_g \mu_g \cdot \chi_g}{\sum_g (1-\chi_g)}.$$

The level of aggregate demand depends on the joint distribution of (μ_g, χ_g) so a transfer of endowment from one group to another affects aggregate demand, consumption, and idleness. Consider a transfer $\Delta\mu > 0$ of endowment from group g with low taste for consumption of services (low χ_g) to group g' with high taste for consumption of services (high $\chi_{g'}$). Aggregate demand becomes

$$c^d(x,p) = \frac{1}{p \cdot (1+\tau(x))} \cdot \frac{\Delta\mu \cdot (\chi_{g'} - \chi_g) + \sum_g \mu_g \cdot \chi_g}{\sum_g 1 - \chi_g}.$$

Since $\Delta\mu \cdot (\chi_{g'} - \chi_g) > 0$, the transfer stimulates aggregate demand. Hence, idleness falls and market tightness and sales increase. The response of aggregate consumption depends on the regime: aggregate consumption increases if the economy is slack but decreases if the economy is tight.

2.6 Government Purchases

We use comparative statics to describe the response of private and aggregate consumption to an increase in government purchases of services. First, we compute the responses when the government finances its purchases with an income tax. Next, we compute the responses when the government finances its purchases by selling part of its endowment of nonproduced good. The responses depend critically on the regime in which the economy is.

Government Purchases Financed by an Income Tax. The government consumes g units of services financed by an income tax at rate t. The consumer's budget constraint becomes $m + p \cdot (1 + \tau(x)) \cdot c = \mu + (1 - t) \cdot (p \cdot f(x) \cdot y)$. The government's budget constraint imposes that $p \cdot (1 + \tau(x)) \cdot g = t \cdot (p \cdot f(x) \cdot y)$. We assume that g enters separately into consumers' utility function such that g does not affect their consumption choice. In equilibrium, $m = \mu$. Thus, consumers' demand for services is given by (7) and the aggregate demand is $c^d(x, p) + g$. The aggregate supply is given by (8), and the short-run equilibrium (x, c) satisfies $c = c^d(x, p)$ and

$$c^s(x) = c^d(x, p) + g.$$

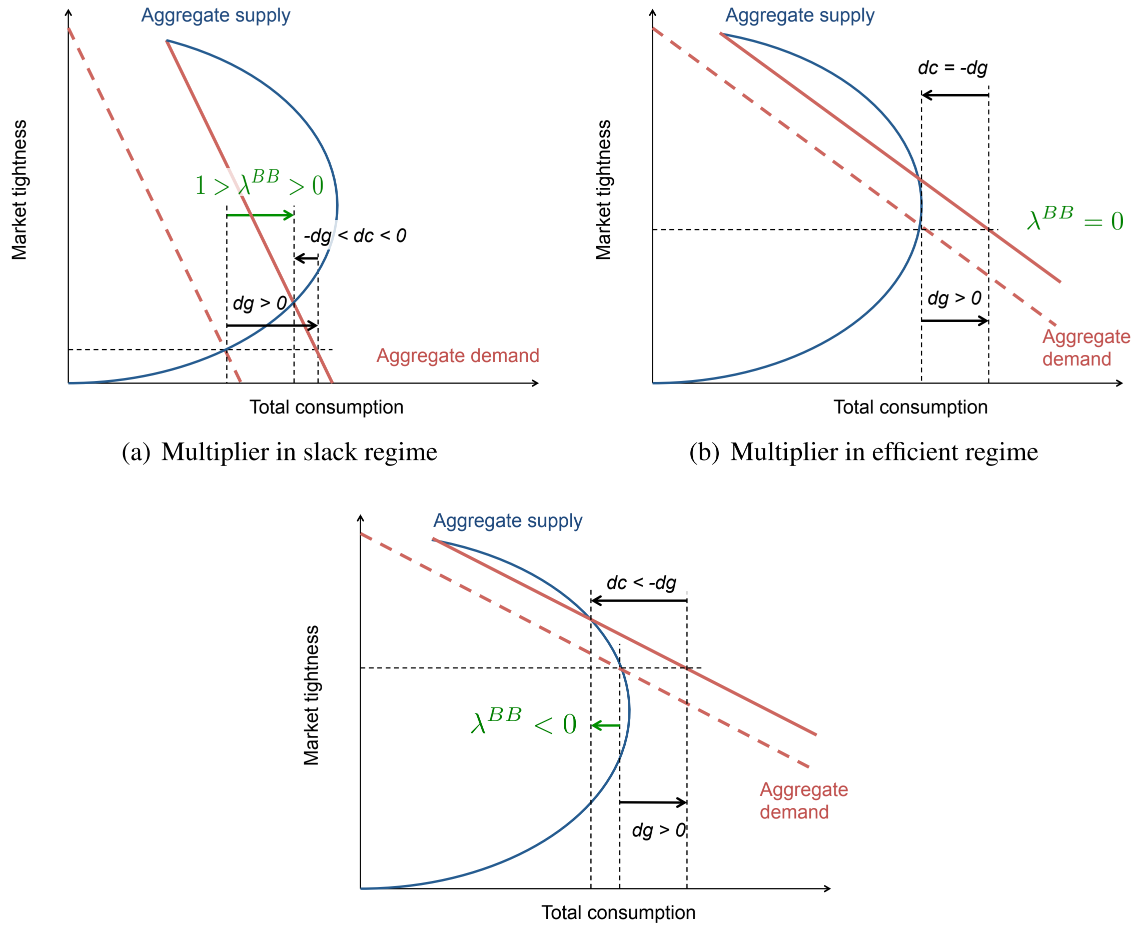
We study the effect of government consumption, g, on total consumption, $c + g$. We measure this effect with the balanced-budget multiplier, defined as $\lambda^{BB} \equiv 1 + dc/dg$. Differentiating the equilibrium condition with respect to g yields

$$\frac{\partial c^s}{\partial x} \cdot \frac{\partial x}{\partial g} = \frac{\partial c^d}{\partial x} \cdot \frac{\partial x}{\partial g} + 1.$$

Let $\epsilon^d \equiv -\partial c^d/\partial x > 0$ and $\epsilon^s \equiv \partial c^s/\partial x$. By normalization, $\epsilon^d > 0$. Furthermore, $\epsilon^s > 0$ in the slack regime, $\epsilon^s = 0$ in the efficient regime, and $\epsilon^s < 0$ in the tight regime. We obtain $\partial x/\partial g = 1/\left(\epsilon^d + \epsilon^s\right)$. Since $\lambda^{BB} = (\partial c^s/\partial x) \cdot (\partial x/\partial g)$, we obtain

$$\lambda^{BB} = \frac{1}{1 + (\epsilon^d/\epsilon^s)}.$$

As illustrated on Figure 5, the size of the multiplier depends on the slope of consumers' demand relative to the slope of aggregate supply. It follows that the sign and level of the balanced-budget multiplier depends on the regime in which the economy is. When the economy is slack, $\epsilon^s > 0$ and the balanced-budget multiplier is positive but necessarily less than 1. This means that government consumption increases total consumption but partially crowds out private consumption. Crowding out arises because after the increase in government purchases, the aggregate demand curve shifts outward, and market tightness increases to reach the new equilibrium. Therefore, it is more expensive for consumers to purchase goods: the effective price $(1 + \tau(x)) \cdot p$ increases. Consumers reduce consumption because of the increase in effective price. When the economy is efficient, $\epsilon^s = 0$ and the balanced-budget multiplier is 0. This means that government consumption crowds out private consumption one-for-one. When the economy is tight, $\epsilon^s < 0$ and $|\epsilon^s| < |\epsilon^d|$ and the balanced-budget multiplier is negative. This means that government consumption crowds out private consumption



(a) Multiplier in slack regime

(b) Multiplier in efficient regime

(c) Multiplier in tight regime

Figure 5: Balanced-budget multiplier in the three regimes of the model of Section 2

more than one-for-one such that it reduces total consumption.[15]

When the economy is slack, government purchases bring the economy closer from aggregate efficiency. This implies that providing more public good than the Samuelson rule—which requires that marginal utility of private consumption equals marginal utility of public-good consumption—is desirable when the economy is slack. Conversely, providing less public good than the Samuelson rule is desirable when the economy is tight. To see this, suppose that utility is given by $U = u(c, m) + v(g)$ with constraint $c + g = [f(x) - \rho \cdot x]y$. The Samuelson rule for optimal public good provision is that $u_c(c, \mu) = v'(g)$. Suppose the Samuelson rule holds and consider a small budget balanced increase

[15] We compute the multiplier λ^{BB} by following the methodology developed in Michaillat [forthcoming] to compute a public-employment multiplier. While our expression for λ^{BB} is quite similar to the expression of the public-employment multiplier of Michaillat [forthcoming], the multipliers have very different properties: λ^{BB} changes sign for different level of aggregate demand whereas only the amplitude of the public-employment multiplier varies when the level of labor demand varies (the public-employment multiplier is always positive). The difference arises because λ^{BB} is directly related to welfare whereas the public-employment multiplier is descriptive and not directly linked to welfare.

$dg > 0$. We have $dU = u_c(c,\mu)dc + v'(g)dg = v'(g)[dc + dg]$. Hence, increasing g above the Samuelson rule is desirable if and only if $dc + dg > 0$, i.e., $\lambda^{BB} > 0$, i.e., the economy is slack.[16]

Government Purchases Financed by Selling Nonproduced Good. The government consumes g units of services financed by the sale of a quantity t of nonproduced good that the government owns. The consumer's budget constraint remains given by (4). The government's budget constraint imposes that $p \cdot (1+\tau(x)) \cdot g = t$. The consumer's and government's budget constraints, together with equilibrium on the product market, impose that $m = \mu + t = \mu + p \cdot (1+\tau(x)) \cdot g$ in equilibrium. The consumer's holding of nonproduced good increases because the government depletes its endowment of nonproduced good. Furthermore, we assume that government consumption enters separately into consumers' utility function such that consumers' optimal consumption choice remains given by (5). Since consumers' income increases, consumers' demand for services is higher than in the budget-balanced case, and the aggregate demand becomes

$$c^d(x,p) + \left\{ \left(\frac{\chi}{1-\chi}\right)^{\epsilon} \cdot [p \cdot (1+\tau(x))]^{1-\epsilon} + 1 \right\} \cdot g.$$

The aggregate supply remains given by (8).

For the rest of the analysis, we assume a Cobb-Douglas utility function ($\epsilon = 1$) to simplify the analysis. Under this assumption, the short-run equilibrium (x, c) satisfies

$$c^s(x) = c^d(x,p) + \frac{1}{1-\chi} \cdot g,$$

and $c = c^d(x,p) + g \cdot \chi/(1-\chi)$. We define the deficit-financed multiplier as $\lambda^{DF} \equiv 1 + dc/dg$. Proceeding exactly as above, we obtain

$$\lambda^{DF} = \frac{1}{1-\chi} \cdot \frac{1}{1 + (\epsilon^d/\epsilon^s)} = \frac{1}{1-\chi} \cdot \lambda^{BB}.$$

The deficit-financed multiplier is equal to the budget balanced multiplier times $1/(1-\chi) > 1$; hence, the deficit-financed multiplier always has greater amplitude than the balanced-budget multiplier. The factor $1/(1-\chi) = 1/(1 - \text{marginal propensity to consume})$ appears under Cobb-Douglas utility following the same logic as in the textbook Keynesian-cross analysis of the multiplier. Thus, our model enriches the standard Keynesian-cross analysis of the multipliers with a well-defined concept

[16] Naturally, altering the Samuelson rule is second-best. A first-best solution to improve an inefficient allocation would be to change the price p. A complete optimal policy analysis in our model is left for future work.

of welfare and slack, efficient, and tight regimes.

The multipliers relevant for welfare are those defined in terms of consumption. In contrast, empirical studies typically estimate multipliers defined in terms of GDP, which is the empirical counterpart to sales.[17] These estimates may not be fully informative for welfare analysis because the consumption and sales multipliers have strikingly different behaviors. For instance with Cobb-Douglas utility function, the consumption multipliers are sharply countercyclical whereas the sales multipliers, denoted Λ^{BB} and Λ^{DF}, are acyclical: in any regime, $\Lambda^{BB} = 1$ and, according to the Keynesian-cross analysis of the multiplier, $\Lambda^{DF} = 1/(1-\chi)$.[18] An implication is that countercyclical government spending could be desirable even if multipliers in terms of GDP are estimated to be acyclical.

3 A Matching Model with Product Market and Labor Market

This section builds a model in which firms hire workers on a labor market with matching frictions, employ these workers to produce goods, and sell the production on a product market with matching frictions. The model allows us to study how aggregate demand shocks propagate from the product market to the labor market and how they affect unemployment. It also allows us to describe the effects of a number of supply-side shocks: technology shocks, labor force participation shocks, and real wage shocks. Finally, the model augmented with preferences and firm-ownership heterogeneity allows us to examine how wages influence unemployment through their effect on labor cost and aggregate demand.

The product market has the same structure as the market for services of Section 2 with firms' output being traded instead of workers' services. The only difference is that the amount of items for sale, y, is not exogenous but is determined endogenously from the production decision of firms. The labor market also has a very similar structure. The matching frictions on the labor market are isomorphic to those on the product market. Following Michaillat [2012], we assume that firms are large, face a production function with diminishing marginal returns to labor, and maximize profits taking labor market tightness and real wage as given. This section omits the description of the product market (it is identical to the market for services of Section 2) and focuses on the labor market.

[17] See Ramey [2011] for a recent survey of this literature.

[18] With CES utility function ($\epsilon > 1$), the sales multipliers are not acyclical but they are always positive whereas the consumption multipliers switch from positive to negative as tightness increases.

3.1 Labor Market

The economy has a measure 1 of identical firms and a measure 1 of identical households. Households own the firms and receive their profits. Household members pool their income before jointly deciding consumption. A number $h \in (0,1)$ of household members is in the labor force, and a number $1-h$ is out of the labor force. There are matching frictions on the labor market. All labor force participants are initially unemployed and search for a job. Each firm posts $\hat{v}$ vacancies to hire workers. The number l of workers who are hired is given by a matching function taking as argument unemployment and vacancy: $l = \left(h^{-\hat{\eta}} + \hat{v}^{-\hat{\eta}}\right)^{-\frac{1}{\hat{\eta}}}$. The parameter $\hat{\eta} > 0$ influences the curvature of the matching function. Labor market tightness is defined as the ratio of vacancy to unemployment: $\theta = \hat{v}/h$. Labor market tightness determines the probabilities that a jobseeker finds a job and a vacancy is filled. Jobseekers find a job with probability $\hat{f}(\theta) = l/h = \left(1 + \theta^{-\hat{\eta}}\right)^{-\frac{1}{\hat{\eta}}}$, and a vacancy is filled with probability $\hat{q}(\theta) = l/\hat{v} = \left(1 + \theta^{\hat{\eta}}\right)^{-\frac{1}{\hat{\eta}}}$. We assume away randomness at the firm and household level: a firm hires $\hat{v} \cdot \hat{q}(\theta)$ workers for sure, and $\hat{f}(\theta) \cdot h$ household members find a job for sure. The function $\hat{f}$ is increasing and the function $\hat{q}$ is decreasing in θ. That is, when the labor market is slacker, the probability to find a job is lower but the probability to fill a vacancy is higher.

3.2 Firms

The representative firm hires l workers. Some of the firm's workers are engaged in production while others are engaged in recruiting. More precisely, $n < l$ workers are producing output y according to the production function $y = a \cdot n^{\alpha}$. The parameter $a > 0$ measures the technology of the firm and the parameter $\alpha \in (0,1)$ captures decreasing marginal returns to labor. Because of matching frictions on the product market, the firm only sells a fraction $f(x)$ of its output.

Posting a vacancy requires a fraction $\hat{\rho} > 0$ of a worker's time. Thus, the firm devotes $l - n = \hat{\rho} \cdot \hat{v} = \hat{\rho} \cdot l/\hat{q}(\theta)$ workers to recruiting a total of l workers. The number n of production workers is therefore related to the number l of workers by $l = (1 + \hat{\tau}(\theta)) \cdot n$, where $\hat{\tau}(\theta) \equiv \hat{\rho}/\left(\hat{q}(\theta) - \hat{\rho}\right)$ measures the number of workers devoted to recruiting for each production worker. The function $\hat{\tau}$ is positive and strictly increasing as long as $\hat{q}(\theta) > \hat{\rho}$. The firm pays its l workers a *real wage* w, and the wage bill of the firm is $(1 + \hat{\tau}(\theta)) \cdot w \cdot n$. From this perspective, matching frictions in the labor market impose a wedge $\hat{\tau}(\theta)$ on the wage of production workers.

Given θ, x, p, and w, the firm chooses n to maximize profits

$$\Pi = p \cdot f(x) \cdot a \cdot n^{\alpha} - (1 + \hat{\tau}(\theta)) \cdot p \cdot w \cdot n.$$

The optimal number of production workers satisfies:

$$f(x) \cdot a \cdot \alpha \cdot n^{\alpha-1} = (1 + \hat{\tau}(\theta)) \cdot w. \tag{12}$$

This relationship says that at the optimum, the real marginal revenue of one production worker equals the real marginal cost of one production worker. The real marginal revenue is the marginal product of labor, $a \cdot \alpha \cdot n^{\alpha-1}$, times the selling probability, $f(x)$. The real marginal cost is the real wage, w, plus the marginal recruiting cost, $\hat{\tau}(\theta) \cdot w$.

3.3 Equilibrium

The equilibrium concept is the same in Section 2. To obtain a convenient representation of the equilibrium, we define aggregate demand, aggregate supply, labor demand, and labor supply functions. The aggregate demand is given by (7). The aggregate supply is given by $c^s(x, n) = (f(x) - \rho \cdot x) \cdot a \cdot n^{\alpha}$. Labor supply and labor demand are defined as follows:

DEFINITION 5. The *labor demand* is a function of labor market tightness, product market tightness, and real wage defined by

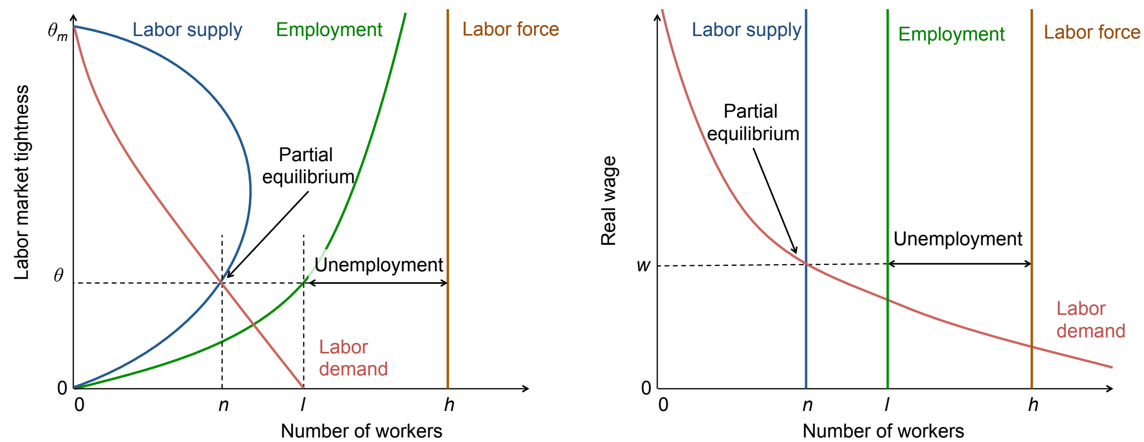
$$n^d(\theta, x, w) = \left[\frac{f(x) \cdot a \cdot \alpha}{(1 + \hat{\tau}(\theta)) \cdot w}\right]^{\frac{1}{1-\alpha}}$$

for all $(\theta, x, w) \in [0, \theta^m] \times (0, +\infty) \times (0, +\infty)$, where $\theta^m > 0$ satisfies $\hat{\rho} = \hat{q}(\theta^m)$. The *labor supply* is a function of labor market tightness defined for all for all $\theta \in [0, x^m]$ by

$$n^s(\theta) = \left(\hat{f}(\theta) - \hat{\rho} \cdot \theta\right) \cdot h.$$

The labor demand gives the number of production workers that satisfies the firm's optimal employment choice, given by (12). The labor supply gives the number of production workers employed after the matching process when a number h of household members are in the labor force. Lemma 2 establishes a few properties of labor demand and labor supply:

LEMMA 2. *The function n^d is strictly decreasing in θ, strictly increasing in x, strictly decreasing in w, $n^d(\theta = 0, x, w) = [f(x) \cdot a \cdot \alpha \cdot (1 - \hat{\rho})/w]^{\frac{1}{1-\alpha}}$, and $n^d(\theta^m, x, w) = 0$. The function n^s is strictly increasing on $[0, \theta^*]$, strictly decreasing on $[\theta^*, \theta^m]$, $n^s(\theta = 0) = 0$, $n^s(\theta^m) = 0$, and $n^s(\theta^*) = n^*$. θ^* maximizes $n = \left[\hat{f}(\theta) - \hat{\rho} \cdot \theta\right] \cdot h$ so that $\hat{f}'(\theta^*) = \hat{\rho}$ and $n^* = \left[\hat{f}(\theta^*) - \hat{\rho} \cdot \theta^*\right] \cdot h$. The constants θ^* and n^* depend on $\hat{\rho}$, $\hat{\eta}$, and h.*



(a) Equilibrium in a (n, θ) plane

(b) Equilibrium in a (n, w) plane

Figure 6: Short-run labor market equilibrium

The behavior of the labor supply is the same as that of the aggregate supply because the matching process is similar on labor and product markets. The labor demand decreases with w and θ because when either of them increases, the effective wage of production worker, $(1 + \hat{\tau}(\theta)) \cdot w$, increases and firms reduce hiring of production workers. The labor demand increases with x because when x increases, the probability $f(x)$ to sell output increases and firms increase hiring of production workers. Figure 6(a) represents labor demand and labor supply in a (n, θ) plane. The labor demand curve slopes downward. The labor supply curve slopes upward for $\theta \leq \theta^*$ and downward for $\theta \geq \theta^*$. The figure also shows the labor force, h, employment, $l = \hat{f}(\theta) \cdot h$, unemployment $u = h - l = (1 - \hat{f}(\theta)) \cdot h$, the number of production workers, $n = l/(1 + \hat{\tau}(\theta))$, and the number of recruiters, $l - n = [\hat{\tau}(\theta)/(1 + \hat{\tau}(\theta))] \cdot l$. Figure 6(b) represents labor demand and labor supply in a (n, w) plane.

We now define and characterize the general equilibrium:

DEFINITION 6. Given real wage $w > 0$ and product market tightness $x > 0$, a *short-run labor market equilibrium* consists of a pair (θ, n) of labor market tightness and employment such that labor supply equals labor demand and employment is given by the labor demand:

$$\begin{cases} n^s(\theta) & = & n^d(\theta, x, w) \\ n & = & n^d(\theta, x, w) \end{cases}$$

Given price $p > 0$ and production employment $n > 0$, a *short-run product market equilibrium* consists of a pair (x, c) of product market tightness and consumption such that aggregate supply equals aggregate demand and consumption is given by the aggregate demand:

$$\begin{cases} c^s(x,n) & = & c^d(x,p) \\ c & = & c^d(x,p) \end{cases}$$

Given prices (p,w), a *short-run general equilibrium* consists of a quadruplet (x,θ,c,n) of tightnesses and quantities such that (θ,n) is a short-run labor market equilibrium given (x,w) and (x,c) is a short-run product market equilibrium given (n,p).

The short-run labor market and product market equilibria are partial equilibria because they take as given the tightness and quantity in the other market. The product market equilibrium can be represented as in Figure 2 with $y = a \cdot n^{\alpha}$. Similarly, the labor market equilibrium is represented in Figure 6. In general equilibrium, the two partial-equilibrium systems hold simultaneously. The following proposition characterizes the general equilibrium:

PROPOSITION 4. *For any $p > 0$ and $w > 0$, there exists a unique short-run general equilibrium with positive consumption. The equilibrium tightnesses, (x,θ), are the unique solution to the system*

$$h^{1-\alpha} \cdot \hat{f}(\theta)^{1-\alpha} \cdot (1+\hat{\tau}(\theta))^{\alpha} = \frac{a \cdot \alpha}{w} \cdot f(x) \tag{13}$$

$$h \cdot \hat{f}(\theta) \cdot (1+\tau(x))^{\epsilon-1} = \frac{\alpha}{w} \cdot \left(\frac{\chi}{1-\chi}\right)^{\epsilon} \cdot \frac{\mu}{p^{\epsilon}}. \tag{14}$$

Equation (13) implicitly defines θ as a strictly increasing function of x while equation (14) implicitly defines θ as a strictly decreasing function of x. These two functions intersect exactly once.

Equation (13) arises from the partial-equilibrium condition on the labor market combining (12) with $n^s(\theta) = \left(\hat{f}(\theta) - \hat{\rho} \cdot \theta\right) \cdot h = \hat{f}(\theta) \cdot h / [1 + \hat{\tau}(\theta)]$. Equation (14) arises from a combination of the partial-equilibrium conditions on the labor and product markets[19] combining (9) with $y = f(x) \cdot a \cdot n^{\alpha} = \hat{f}(\theta) \cdot h \cdot w/\alpha$ obtained from (12). Figure 7 represents the general equilibrium as the intersection of an upward-sloping and a downward-sloping curve in a (x,θ) plane. The upward-sloping curve is the locus of points (x,θ) that solve (13), and the downward-sloping curve is the locus of points (x,θ) that solve (14).[20]

[19] In addition to the equilibrium with positive consumption, there exist two other equilibria with zero consumption. Appendix A extends Proposition 4 to characterize all the possible equilibria and to describe the domain and codomain of the functions implicitly defined by (13) and (14).

[20] On Figure 7, the domain of the function that solves equation (14) is $[0, x^m]$. If the price, p, is below some threshold, the function is only defined for x above some threshold (this threshold is necessarily below x^m). At the threshold, the function asymptotes to $+\infty$. See Appendix A for more details.

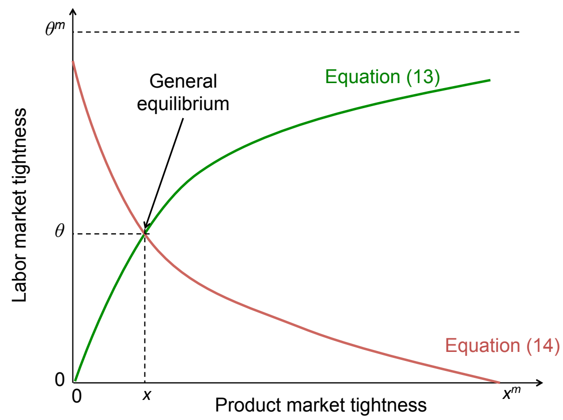


Figure 7: Short-run general equilibrium in the model of Section 3

In our model, firms may not want to hire all the workers in the labor force even if it is costless to hire and consumers may not want to purchase all the production even if it is costless to shop. If the matching costs are arbitrarily small ($\rho \to 0$ and $\hat{\rho} \to 0$), then $\tau(x) \to 0$ and $\hat{\tau}(\theta) \to 0$. Equations (13) and (14) indicate that in equilibrium, $f(x) < 1$ and $\hat{f}(\theta) < 1$ when w and p are large enough. In that case, some production remains unsold and some workers remain unemployed.

3.4 Efficient Allocation

DEFINITION 7. The *efficient allocation* is the quadruplet (x, θ, c, n) that maximizes welfare, $\left[\chi \cdot c^{\frac{\epsilon-1}{\epsilon}} + (1-\chi) \cdot \mu^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}$, subject to the matching frictions on the product market, $c \leq (f(x) - \rho \cdot x) \cdot a \cdot n^{\alpha}$, and to the matching frictions on the labor market, $n \leq \left(\hat{f}(\theta) - \hat{\rho} \cdot \theta\right) \cdot h$.

PROPOSITION 5. *The efficient allocation is $(x^*, \theta^*, c^*, n^*)$, where x^*, θ^*, c^* and n^* are defined by $f'(x^*) = \rho$, $c^* = [f(x^*) - \rho \cdot x^*] \cdot y$, $\hat{f}'(\theta^*) = \hat{\rho}$, and $n^* = [\hat{f}(\theta^*) - \hat{\rho} \cdot \theta^*] \cdot h$. The real wage w^* and price p^* that implement the efficient allocation are*

$$w^* = a \cdot \alpha \cdot h^{\alpha-1} \cdot \left(1 - \rho^{\frac{\eta}{1+\eta}}\right)^{\frac{1}{\eta}} \cdot \left(1 - \hat{\rho}^{\frac{\hat{\eta}}{1+\hat{\eta}}}\right)^{\alpha - \frac{1-\alpha}{\hat{\eta}}} \tag{15}$$

$$p^* = \frac{\chi}{1-\chi} \cdot \left(\frac{\mu}{a \cdot h^{\alpha}}\right)^{\frac{1}{\epsilon}} \cdot \left(1 - \rho^{\frac{\eta}{1+\eta}}\right)^{1 - \frac{1+\eta}{\epsilon \cdot \eta}} \cdot \left(1 - \hat{\rho}^{\frac{\hat{\eta}}{1+\hat{\eta}}}\right)^{-\frac{\alpha \cdot (1+\hat{\eta})}{\epsilon \cdot \hat{\eta}}}. \tag{16}$$

The economy can be in five different regimes:

DEFINITION 8. The economy is *efficient* if $\theta = \theta^*$ and $x = x^*$, *labor-slack and product-slack* if $\theta < \theta^*$ and $x < x^*$, *labor-slack and product-tight* if $\theta < \theta^*$ and $x > x^*$, *labor-tight and product-slack* if $\theta > \theta^*$ and $x < x^*$, *labor-tight and product-tight* if $\theta > \theta^*$ and $x > x^*$.

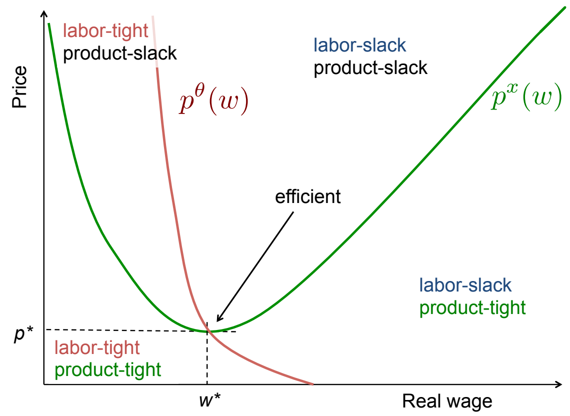


Figure 8: The five regimes of the model of Section 3

The economy behaves differently in the five regimes because the aggregate supply and labor supply functions have different properties across regimes. Proposition 6 establishes the boundaries of the regimes in a (w, p) plane:

PROPOSITION 6. *There exist a function $w \mapsto p^x(w)$ such that for any $w > 0$, the product market is slack ($x < x^*$) if and only if $p > p^x(w)$. There exist a function $w \mapsto p^\theta(w)$ such that for any $w > 0$, the labor market is slack ($\theta < \theta^*$) if and only if $p > p^\theta(w)$. The function p^x is strictly decreasing for $w \in (0, w^*)$ and strictly increasing for $w \in (w^*, +\infty)$. The function $p^\theta(w)$ is strictly decreasing for $w \in (0, w^L)$ and such that $p^\theta(w) = 0$ for all $w > w^L$, where $w^L > w^*$ is a constant of the parameters. Furthermore, $p^x(w^*) = p^\theta(w^*) = p^*$.*

Figure 8 displays the five regimes in a (w, p) plane. The labor market is slack above the curve $p = p^\theta(w)$ and tight below. The product market is slack above the curve $p = p^x(w)$ and tight below. Moreover, $\theta = \theta^*$ on the curve $p = p^\theta(w)$ and $x = x^*$ on the curve $p = p^x(w)$. As the price and wage implementing (x^*, θ^*) are unique, the curves $p = p^\theta(w)$ and $p = p^x(w)$ cross only once, at (w^*, p^*).

3.5 Aggregate Demand and Other Shocks

We use comparative statics to describe the response of the equilibrium to four types of shocks: aggregate demand shock, technology shock, real wage shock, and labor force participation shock. In particular, we study the correlations between product market tightness, labor market tightness, sales, and unemployment generated by these different shocks. The correlations are summarized in Table 2.

First, we parameterize a positive aggregate demand shock by an increase in marginal propensity to consume, χ, or in endowment, μ, or by a decrease in price, p. A transfer of wealth from consumers with low marginal propensity to consume to consumers with high marginal propensity to consume, as in Section 2.5, would have the same effects. A positive aggregate demand shock leads to an upward shift of the curve defined by equation (14) in Figure 7. After the shock, labor market tightness and product market tightness increase. Unemployment decreases because $u = h \cdot (1 - \hat{f}(\theta))$. Sales increase because equation (13) implies that $s = f(x) \cdot a \cdot n^{\alpha} = (w/\alpha) \cdot h \cdot \hat{f}(\theta)$. The response of consumption and employment of production workers depends on the regime. In the efficient regime, neither consumption nor employment respond to a marginal change in tightness. Employment of production workers decreases in a labor-tight regime but increases in a labor-slack regime. Consumption decreases in a product-tight and labor-tight regime but increases in a product-slack and labor-slack regime. In the two other regimes, n^{α} and $f(x) - \rho \cdot x$ move in opposite direction so it is not possible to determine the change in consumption. In the partial-equilibrium diagram of Figure 2, a positive aggregate demand shock leads to an upward rotation of the aggregate demand curve. This rotation raises product market tightness. In the partial-equilibrium diagram of Figure 6, the increase in product market tightness leads to an outward shift of labor demand because the probability to sell is higher. Labor market tightness increases as a result. Since the number of production workers changes, aggregate supply adjusts in Figure 2. Product market tightness adjusts again, which feedbacks on the labor demand. Feedbacks between labor demand and aggregate supply continue until convergence to the new general equilibrium with higher labor market and product market tightnesses.

Second, we consider an increase in technology, a. A positive technology shock leads to an upward shift of the curve defined by equation (14) in Figure 7. After the shock, labor market tightness increases but product market tightness decreases. For the same reasons as with a positive aggregate demand shock, unemployment decreases and sales increase. And as under an aggregate demand shock, the response of consumption and employment of production workers depends on the regime. In the partial-equilibrium diagram of Figure 2, an increase in technology leads to an expansion of the aggregate supply curve. In the partial-equilibrium diagram of Figure 6, an increase in technology leads to an outward shift of the labor demand curve. The resulting changes in product market tightness and labor market tightness influence the probability to sell and employment of production workers, which in turn feedback to the aggregate supply and labor demand curves. Feedbacks between labor demand and aggregate supply continue until convergence to the new general equilibrium with higher labor market tightness and lower product market tightness.

Third, we consider an increase in real wage, w. This increase leads to downward shifts of the curves defined by equations (13) and (14) in Figure 7. After the shock, labor market tightness decreases and unemployment increases. On the other hand, the response of product market tightness is ambiguous. We distinguish between labor-slack an labor-tight regimes. In a labor-slack regime, employment of production workers decreases when labor market tightness decreases. In the diagram of Figure 2, the aggregate supply curve contracts whereas the aggregate demand curve remains the same. Therefore, product market tightness, x, increases whereas consumption, given by $c = c^d(x, p)$, and sales, given by $s = (1+\tau(x)) \cdot c^d(x, p)$, decrease (both $x \mapsto c^d(x, p)$ and $x \mapsto (1+\tau(x)) \cdot c^d(x, p)$ are strictly decreasing). In a labor-tight regime, employment of production workers increases when labor market tightness decreases. In the diagram of Figure 2, the aggregate supply curve expands whereas the aggregate demand curve remains the same. Therefore, product market tightness decreases whereas consumption and sales increase.

Finally, we consider an increase in labor force participation, h. This increase leads to downward shifts of the curves defined by equations (13) and (14) in Figure 7. After the shock labor market tightness, θ, decreases. Unemployment, $u = h \cdot (1 - \hat{f}(\theta))$, increases because the unemployment rate, $1 - \hat{f}(\theta)$, increases and the number of workers in the labor force, h, increases. The response of product market tightness, x, is ambiguous on the general-equilibrium diagram of Figure 7; instead, we use the partial-equilibrium diagrams of Figures 2 and 6. Assume that x increases. Then the employment of production workers, $n = n^d(\theta, x, w)$, increases because the function n^d is strictly increasing in x and strictly decreasing in θ. Hence, the aggregate supply curve in Figure 2 expands and x falls in equilibrium. We reach a contradiction so x decreases. As a consequence, consumption, given by $c = c^d(x, p)$, and sales, given by $s = (1 + \tau(x)) \cdot c^d(x, p)$, increase. Finally, since $s = f(x) \cdot a \cdot n^\alpha$, s increases, and $f(x)$ decreases, it must be that n increases.

If we could map the variables of the model to macrodata, we could exploit the comparative-statics results summarized in Table 2 to separate between different types of macroeconomic shocks. An aggregate demand shock is the only shock under which product market tightness and sales are positively correlated. A labor force participation shock is the only shock under which sales and unemployment are positively correlated in a labor-slack regime (in a labor-tight regime, both labor force participation shock and real wage shock generate such a positive correlation). A technology shock is the only shock under which product market tightness and labor market tightness are negatively correlated in a labor-tight regime (in a labor-slack regime, both technology shock and real wage shock generate such a negative correlation).

Table 2: Comparative statics in the model of Section 3

	Effect on:			
Increase in:	Product market tightness	Labor market tightness	Sales	Unemployment
Aggregate demand	> 0	> 0	> 0	< 0
Technology	< 0	> 0	> 0	< 0
Real wage	> 0 if labor-slack	< 0	< 0 if labor-slack	> 0
	< 0 if labor-tight		> 0 if labor-tight	
Labor force	< 0	< 0	> 0	> 0

Notes: The comparative statics are derived in Section 3.5. An increase in aggregate demand results from an increase in endowment, μ, a decrease in price, p, or an increase in marginal propensity to consume, χ.

3.6 The Possibility of Reducing Unemployment by Increasing Wages

This section extends the model along two directions to improves its realism. First, we distinguish between buyers in a long-term relationship with a firm, which we call *customers*, and other buyers. This is equivalent to the usual assumption in labor market search models that some individuals are already employed. Second, we account for inequality in labor income, wealth, and share of profits received. With such extensions, a wage increase may reduce unemployment. With inequality and customer-firm relationships, it is difficult to describe the set of price-wage pairs for which the general equilibrium exists and to do comparative statics. To simplify, we assume Cobb-Douglas utility and no matching costs. We describe the results formally in Appendix D and informally here.

We begin by deriving the aggregate supply. Each firm has $\kappa < y$ customers, where $y = a \cdot n^{\alpha}$ is the firm's output. Each customer buys one good with certainty at price $p > 0$. The remaining $y - \kappa$ goods may be purchased by consumers who are not customers through the matching process. Each consumer purchases κ goods through customer relationships and visits v firms to purchase more goods through the matching process. The number of trades made in the matching process is $s - \kappa = \left[(y - \kappa)^{-\eta} + v^{-\eta}\right]^{-\frac{1}{\eta}}$. In each of these trades, a consumer buys one good at price p. The product market tightness is $x \equiv v / [y - \kappa]$. Let $f(x) \equiv (s - \kappa)/(y - \kappa)$ be the probability that a firm sells one good to a buyer who is not a customer. The probability $f(x)$ satisfies (1). The matching cost, ρ, is zero so consumption equals purchases. Accordingly, the aggregate supply is given by

$$c^s(x, n) = \kappa + f(x) \cdot (a \cdot n^{\alpha} - \kappa)\,. \tag{17}$$

Next, we derive the aggregate demand. Workers belong to one of G groups of measure $1/G$.

Group g's per person utility is given by (10). Group g's per person budget is

$$m_g + p \cdot c_g = \mu_g + \sigma_g \cdot \Pi + \varpi_g \cdot p \cdot w \cdot n,$$

where σ_g is group g's per person share of profits, Π are nominal profits, and ϖ_g is group g's per person share of labor income. By construction, $\left(\sum_g \sigma_g\right)/G = 1$ and $\left(\sum_g \varpi_g\right)/G = 1$. The optimal consumption of produced good, c_g, is related to the optimal consumption of nonproduced good, m_g, by a relationship similar to (5) with $\epsilon = 1$. Substituting out profits from group g's income using $\Pi = p \cdot [\kappa + f(x) \cdot (y - \kappa) - w \cdot n]$, we find that group g's optimal consumption satisfies

$$p \cdot c_g = \chi_g \cdot \{\mu_g + \sigma_g \cdot p \cdot [\kappa + f(x) \cdot (y - \kappa)] + (\varpi_g - \sigma_g) \cdot p \cdot w \cdot n\}$$

We define aggregate demand as the sum of each group's demand: $c^d \equiv \left(\sum_g c_g\right)/G$. In equilibrium, purchases equal sales: $\left(\sum_g c_g\right)/G = \kappa + f(x) \cdot (y - \kappa)$. Thus, aggregate demand is given by

$$c^d(n, p, w) = \frac{1}{p} \cdot \frac{\sum_g \mu_g \cdot \chi_g}{\sum_g (1 - \chi_g) \cdot \sigma_g} + \frac{\sum_g \chi_g \cdot (\varpi_g - \sigma_g)}{\sum_g (1 - \chi_g) \cdot \sigma_g} \cdot w \cdot n. \tag{18}$$

Aggregate demand does not depend on product market tightness because the matching wedge, $\tau(x)$, is zero. If profits and labor income are uniformly distributed across group such that $\sigma_g = \varpi_g = 1$ for all g, aggregate demand depends only on the price, p, groups' endowments, $\{\mu_g\}$, and groups' preferences, $\{\chi_g\}$. If profits or labor income are not uniformly distributed across group such that $\sum_g \chi_g \cdot (\varpi_g - \sigma_g) \neq 0$, aggregate demand also depends on the wage bill, $w \cdot n$. A wage increase stimulates aggregate demand if

$$\sum_g \chi_g \cdot \varpi_g > \sum_g \chi_g \cdot \sigma_g. \tag{19}$$

This condition says that across groups, the correlation between share of labor income and marginal propensity to consume is higher than the correlation between share of profits and marginal propensity to consume. Loosely speaking, wage earners have a higher marginal propensity to consume than firm owners; alternatively, profits are more concentrated among savers than labor income. A wage increase always redistributes income from firm owners to wage earners. If (19) holds, this redistribution stimulates aggregate demand.

The labor market is the same as above and the marginal decisions of firms are not affected by the presence of customers; hence, labor demand and labor supply satisfy Definition 5. Aggregate supply and aggregate demand satisfy (17) and (18). A short-run general equilibrium satisfies Definition 6. A major difference with the results from Proposition 4 is that a general equilibrium does not exist for all parameter values and all price-wage pairs. But when a general equilibrium with positive consumption and employment exists, it is unique and we can perform comparative statics.

We identify conditions on parameter values such that a wage increase reduces unemployment. Condition (19) is necessary because it ensures that a wage increase stimulates aggregate demand. Another necessary condition is $\left[\sum_g \mu_g \cdot \chi_g\right] / \left[\sum_g (1-\chi_g) \cdot \sigma_g\right] < \kappa$, which requires the customer base κ to be positive. Under this condition the labor-income component of aggregate demand tends to be large relative to the endowment component, and a wage increase tends to have a large positive effect on aggregate demand.

In fact, a wage increase has two opposite effects. First, a wage increase raises the marginal cost of labor and depresses labor demand. This is the conventional effect, which is found in all matching models of the labor market. Second, a wage increase redistributes income from firm owners to wage earners. This redistribution stimulates aggregate demand if wage earners have a higher marginal propensity to consumer than firm owners. An increase in aggregate demand leads to higher product market tightness, which stimulates labor demand. This is an unconventional effect, which arises in our model only in presence of a frictional product market and inequality. A wage increase lowers unemployment when the unconventional positive response of aggregate demand to the wage increase is strong enough to dominate the conventional negative response of labor demand.

The result that a wage increase may decrease unemployment has a number of implications. One implication is that the effect of the minimum wage on employment may be different in partial equilibrium and in general equilibrium. In partial equilibrium the influence of the minimum wage on aggregate demand is omitted. In general equilibrium the influence of the minimum wage on aggregate demand is accounted for. Increasing the minimum wage necessarily depresses employment in partial equilibrium, but when (19) holds, the disemployment effect of the minimum wage is not as strong in general equilibrium as in partial equilibrium. Under stronger conditions, the minimum wage may have a negative employment effect in partial equilibrium but a positive employment effect in general equilibrium.[21] One possible reason why empirical studies estimating the employment effect of the minimum wage reach conflicting conclusions is that these studies do not distinguish between

[21] This discussion parallels the comparison of the microelasticity and macroelasticity of unemployment with respect to unemployment insurance in Landais, Michaillat and Saez [2010].

partial- and general-equilibrium effects.[22] On the one hand, studies measuring employment effects within a labor market, for example by comparing workers eligible for the minimum wage to workers ineligible, estimate partial-equilibrium effects. On the other hand, studies measuring employment effects across labor markets estimate general-equilibrium effects. If (19) held, the within-state studies would systematically estimate larger disemployment effect than the cross-state studies.

4 Relation to Other Macroeconomic Models

In this section, we argue that our matching framework is not restrictive. With alternative assumptions about the functional forms of the utility, production, and matching functions, the value of matching costs, and the price and wage schedules, our framework can replicate the key first-order conditions of a broad range of macroeconomic models—perfect-competition model, existing matching models, fixprice-fixwage model, and monopolistic-competition model. We then explain why making these alternative assumptions would eliminate aggregate demand effects captured by our model.

4.1 Perfect-Competition Model

Assume that the matching costs, ρ and $\hat{\rho}$, are zero. Assume that the real wage equals the marginal product of the last worker in the labor force and that the price equals the marginal rate of substitution between the produced good and the nonproduced good at full employment:

$$w = a \cdot \alpha \cdot h^{\alpha-1} \tag{20}$$

$$p = \frac{\chi}{1-\chi} \cdot \left(\frac{\mu}{a \cdot h^{\alpha}}\right)^{\frac{1}{\epsilon}}. \tag{21}$$

Under these assumptions, equation (14) implies that

$$\hat{f}(\theta) = \frac{1}{h} \cdot \frac{\alpha}{a \cdot \alpha \cdot h^{\alpha-1}} \cdot \left(\frac{\chi}{1-\chi}\right)^{\epsilon} \cdot \mu \cdot \left(\frac{\chi}{1-\chi}\right)^{-\epsilon} \cdot \left(\frac{a \cdot h^{\alpha}}{\mu}\right) = 1.$$

Thus, $\theta \to +\infty$ and $n = \hat{f}(\theta) \cdot h = h$. Equation (13) implies that

$$f(x) = \frac{a \cdot \alpha \cdot h^{\alpha-1}}{a \cdot \alpha} \cdot h^{1-\alpha} = 1.$$

[22] For a survey of the vast empirical literature on the minimum wage, see for instance Card and Krueger [1995].

Thus, $x \to +\infty$ and $c = f(x) \cdot a \cdot h^{\alpha} = a \cdot h^{\alpha}$. Labor and product markets behave as if they were perfectly competitive. The matching wedges are zero: $\tau(x) = \hat{\tau}(\theta) = 0$. Firms sell all of their production at the going price: $f(x) = 1$. All labor force participants find a job at the going wage: $\hat{f}(\theta) = 1$. There is no unemployment: $n = h$. There is no unsold production: $c = a \cdot h^{\alpha}$.

These assumptions are inadequate to study aggregate demand because they eliminate all the effects from aggregate demand shocks. Indeed, employment and consumption do not depend on aggregate demand: $n = h$ and $c = a \cdot h^{\alpha}$ for any value μ of endowment and any value χ of the marginal propensity to consume. Aggregate demand shocks have no effect because any shock to the marginal propensity to consume, χ, or the endowment, μ, is absorbed by a corresponding change in price, p, such that the aggregate demand, $c^d = [\chi/(1-\chi)]^{\epsilon} \cdot (\mu/p^{\epsilon})$, does not change.

4.2 Other Matching Models

The following assumptions capture the main features of existing matching models with a product market and a labor market. We assume that consumers have a linear utility function, which is the special case of the CES utility function when $\epsilon \to +\infty$, and that firms have a linear production function, which is the special case of our production function when $\alpha = 1$. The optimal consumption choice of consumers, given by (5), and the optimal employment choice of firms, given by (12), yield

$$(1 + \tau(x)) \cdot p = \frac{\chi}{1 - \chi} \tag{22}$$

$$(1 + \hat{\tau}(\theta)) \cdot w = a \cdot f(x). \tag{23}$$

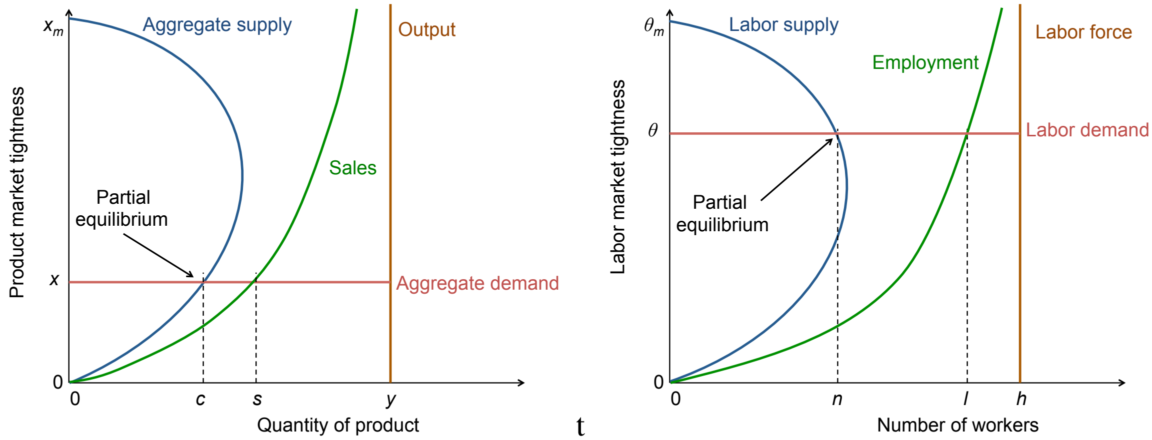
Equations (22) and (23) show that tightnesses are pinned down independently of quantities so that both aggregate demand and labor demand are perfectly elastic with respect to x and θ respectively. The diagrams of Figure 9 represent the product market in a (c, x) plane and the labor market in a (n, θ) plane. Both aggregate demand and labor demand are represented by horizontal curves.

Existing matching models assume either that price and wage are the outcome of bargaining, or that price and wage are rigid.[23] In a bargaining model, real wage and price are given by

$$w = \hat{\beta} \cdot a \cdot f(x) \tag{24}$$

$$p = \beta \cdot \frac{\chi}{1 - \chi}, \tag{25}$$

[23] See Wasmer [2011] for a model assuming bargaining and Hall [2008] for a model assuming rigid price and wage.



(a) Partial equilibrium on the product market (b) Partial equilibrium on the labor market

Figure 9: The matching model with linear utility function and linear production function

where $\beta \in (0,1)$ and $\hat{\beta} \in (0,1)$ are parameters. The real wage is the generalized Nash solution of the bargaining problem between a worker and a firm when the worker has bargaining power $\hat{\beta}$. The surplus to the firm of hiring one worker is $\mathcal{F}(w) = a \cdot f(x) - w$. The surplus to the worker of being hired is $\mathcal{W}(w) = w$. The Nash solution maximizes $\mathcal{F}(w)^{1-\hat{\beta}} \cdot \mathcal{W}(w)^{\hat{\beta}}$, so $\mathcal{W}(w) = \hat{\beta} \cdot [\mathcal{W}(w) + \mathcal{F}(w)] = \hat{\beta} \cdot a \cdot f(x)$ and w satisfies (24). The price is the generalized Nash solution to the bargaining problem between a consumer and a firm when the firm has bargaining power β. The surplus to the consumer of buying one unit of produced good is $\mathcal{C}(p) = \chi - p \cdot (1-\chi)$. The surplus to the firm of selling one unit of produced good is $\mathcal{F}(p) = p \cdot (1-\chi)$. Firms are owned by consumers so firms' marginal income is valued at consumers' marginal utility of consumption of nonproduced good. The Nash solution maximizes $\mathcal{C}(p)^{1-\beta} \cdot \mathcal{F}(p)^{\beta}$, so $\mathcal{F}(p) = \beta \cdot [\mathcal{F}(p) + \mathcal{C}(p)] = \beta \cdot \chi$ and p satisfies (25). Combining (22), (23), (24) and (25), we determine the equilibrium tightnesses:

$$\beta \cdot (1 + \tau(x)) = 1 \quad \text{and} \quad \hat{\beta} \cdot (1 + \hat{\tau}(\theta)) = 1.$$

Hence both x and θ are pinned down independently of quantities hereby eliminating all the effects from aggregate demand shocks. Indeed, employment and consumption are given by the supply equations $n = n^s(\theta)$ and $c = c^s(x, n)$ and are independent of the aggregate demand parameters, μ and χ. Aggregate demand shocks are eliminated because the bargained price is proportional to the marginal rate of substitution between produced good and nonproduced good, $\chi/(1-\chi)$.

Instead of assuming price and wage bargaining, assume instead that price and real wage are parameters of the model as in Hall [2008]. For the equilibrium to exist, it is necessary that $p < \chi/(1-\chi)$

and $w < a$. The equilibrium tightnesses are given by (22) and (23). Since price and wage are rigid, aggregate demand shocks (shocks to χ) affect equilibrium tightnesses and quantities. However, many elements of our analysis disappear in this model because the demand curves are perfectly elastic: the transfers of Section 2.5 have no effect because wealth plays no role; the government-purchase multiplier of Section 2.6 is always zero; unemployment and unsold production would disappear in absence of matching costs, irrespective of the level of aggregate demand or labor demand.

4.3 Monopolistic-Competition Model

We begin by presenting a model with monopolistic competition on the product market and labor market. The model is a variant of the model of Blanchard and Kiyotaki [1987, Section 2]. Since this model is standard, we omit derivations and only report equilibrium conditions. The economy is composed of a continuum of firms indexed by $i \in [0, 1]$ and a continuum of households indexed by $j \in [0, 1]$. The goods produced by firms are imperfect substitutes and the types of labor supplied by households are also imperfect substitutes, so firms and households have some monopoly power.

The utility function of household j is given by

$$\chi \cdot \ln(c_j) + (1 - \chi) \cdot \ln(m_j) - \nu \cdot \frac{\xi}{1 + \xi} \cdot n_j^{\frac{1+\xi}{\xi}}. \tag{26}$$

Utility depends on consumption of nonproduced good, m_j, number of hours worked, $n_j \in (0, 1)$, and a consumption index, $c_j \equiv \left(\int_0^1 c_{ij}^{\frac{\zeta-1}{\zeta}} di\right)^{\frac{\zeta}{\zeta-1}}$, where c_{ij} is consumption of good i. The parameters $\nu > 0$, $\xi > 0$, and $\zeta > 1$, measure the disutility from labor, the curvature of the disutility for labor, and the elasticity of substitution between goods in utility. The budget constraint of household j is

$$m_j + \int_0^1 p_i \cdot c_{ij} di = \mu_j + W_j \cdot n_j + \Pi_j, \tag{27}$$

where p_i is the price of good i, W_j is the nominal wage for labor of type j, Π_j is the share of aggregate profits distributed to the household, and $\mu_j > 0$ is the endowment received by the household. The aggregate endowment of nonproduced good is $\mu = \int_0^1 \mu_j dj$. Given $\{p_i\}$, household j chooses $\{c_{ij}\}$, m_j, n_j, and W_j to maximize (26) subject to (27) and to a demand schedule for its labor, a decreasing function of W_j arising from firms' profit maximization.

Firm i hires labor to produce output. Its production function is

$$c_i = a \cdot \left(\int_0^1 n_{ij}^{\frac{\gamma-1}{\gamma}} \, dj \right)^{\alpha \cdot \frac{\gamma}{\gamma-1}}, \tag{28}$$

where c_i is output of good i, a is the technology level, n_{ij} is the number of workers of type j hired, $\gamma > 1$ is the elasticity of substitution between types of labor in production, and $\alpha < 1$ indicates decreasing returns to scale. Given $\{W_j\}$, firm i chooses $\{n_{ij}\}$, c_i and p_i to maximize profits

$$p_i \cdot c_i - \int_0^1 W_j \cdot n_{ij} dj.$$

subject to (28) and to a demand schedule for its good, a decreasing function of p_i arising from households' utility maximization.

In the symmetric general equilibrium, all households and firms are identical, they set the same prices and wages, and they produce and work the same amounts. In this equilibrium, we have

$$\frac{W}{p} = \frac{\gamma}{\gamma - 1} \cdot \nu \cdot n^{\frac{1}{\xi}} \cdot \frac{c}{\chi} \tag{29}$$

$$\frac{p}{W} = \frac{\zeta}{\zeta - 1} \cdot \frac{1}{\alpha \cdot a} \cdot n^{1-\alpha}, \tag{30}$$

where W is the nominal wage paid by all firms, p is the price charged by all firms, and n and c measure the aggregate number of hours worked and aggregate quantity of goods produced. Equation (29) says that households set the real wage at a markup $\gamma/(\gamma - 1) > 1$ over its marginal rate of substitution between leisure and consumption. Equation (30) says that firms set the price at a markup $\zeta/(\zeta-1) > 1$ over the marginal cost of producing one item.

Our matching model generates the same markups as the model with monopolistic competition under appropriate assumptions. Assume that matching costs are zero and that consumers have a utility function given by $\chi \cdot \ln(c) + (1 - \chi) \cdot \ln(m) - \nu \cdot [\xi/(1+\xi)] \cdot h^{\frac{1+\xi}{\xi}}$, where h is the number of workers in the labor force searching for a job. Given that $h = n/\hat{f}(\theta)$, the optimal choice of labor force participation imposes

$$\frac{W}{p} = \frac{1}{\hat{f}(\theta)^{\frac{1+\xi}{\xi}}} \cdot \nu \cdot n^{\frac{1}{\xi}} \cdot \frac{c}{\chi},$$

where $W \equiv w \cdot p$ is the nominal wage. With $\hat{\rho} = 0$ and hence $\hat{\tau}(\theta) = 0$, equation (12) becomes

$$\frac{p}{W} = \frac{1}{f(x)} \cdot \frac{1}{\alpha \cdot a} \cdot n^{1-\alpha}.$$

We make two assumptions on tightnesses to close the model:

$$f(x) = \frac{\zeta - 1}{\zeta} \tag{31}$$

$$\hat{f}(\theta) = \left(\frac{\gamma - 1}{\gamma}\right)^{\frac{\xi}{1+\xi}}. \tag{32}$$

These assumptions impose that the job-finding probability is the inverse of the labor market markup and the selling probability is the inverse of the product market markup to the power of $\xi/(1+\xi)$. Under these conditions, the quadruplet (c, n, W, p) satisfies equations (29) and (30) both in the model with matching frictions and in the model with monopolistic competition. The ratio between the marginal product of labor and the marginal rate of substitution, sometimes called the *labor wedge*, is therefore identical in the two models and equal to $\gamma \cdot \xi / [(\gamma - 1) \cdot (\xi - 1)] > 1$.

These assumptions are inadequate to study aggregate demand because they eliminate all the effects from aggregate demand shocks. Indeed, unemployment rate and consumption do not depend on aggregate demand: $u = \left(1 - \hat{f}(\theta)\right) \cdot h$ and $c = f(x) \cdot a \cdot \hat{f}(\theta)^{\alpha} \cdot h^{\alpha}$ where $f(x)$ and $\hat{f}(\theta)$ depend only on the parameters ζ, γ, and ξ and not on the parameters capturing aggregate demand, μ and χ. Aggregate demand shocks have no effect because the price always adjusts such that (31) holds. Given that $f(x)$ and the real wage remain the same, the labor demand does not respond to aggregate demand shocks. The labor supply does not respond either so unemployment is unaffected. In fact, the price response is such that the aggregate demand curve does not shift in response to aggregate demand shocks. Aggregate demand shocks impact unemployment in this model only if (31) and (32) do not always hold. One way to eliminate (31) and (32) is to assume instead that price and real wage are somewhat rigid, which brings us back to the model of Section 3.[24]

4.4 Fixprice-Fixwage Model

The standard fixprice-fixwage model was developed by Barro and Grossman [1971].[25] Our model cannot replicate the allocation of that model because firms' sales are sometimes determined by a

[24] This is only a reinterpretation of the insight of Blanchard and Kiyotaki [1987]. They showed that aggregate demand shocks have effects in the monopolistic-competition model only if prices are rigid.

[25] See Bénassy [1993] for an overview of this class of models and their properties.

demand constraint in that model whereas they are always determined by a marginal decision in our model.[26] Our model can, however, replicate the allocation of a fixprice-fixwage model with a proportional rationing rule. With this rule, all agents on the rationed side of the market trade with the same probability. This alternative model predicts slightly different allocations from the standard fixprice-fixwage model, but the allocations can still be sorted into its four traditional regimes.

To replicate the allocation of the fixprice-fixwage model with proportional rationing, we assume that the matching costs are zero and that the matching functions are given by $s = \min\{v, y\}$ and $l = \min\{\hat{v}, h\}$. These matching functions are the special case of those considered in our model when $\eta \to +\infty$ and $\hat{\eta} \to +\infty$. Since the matching functions have a kink, the matching probabilities have a kink as well: $f(x) = \min\{x, 1\}$ and $\hat{f}(\theta) = \min\{\theta, 1\}$. Accordingly, the supply functions are defined piecewise, with one piece when the relevant tightness is below 1 and one piece when it is above 1: $n^s(\theta) = \min\{\theta, 1\} \cdot h$ and $c^s(x, n) = \min\{x, 1\} \cdot a \cdot n^\alpha$. Since the matching wedges, $\tau(x)$ and $\hat{\tau}(\theta)$, are zero, the demand functions do not depend on the tightness in their market: $n^d(\theta, x, w) = (\min\{x, 1\} \cdot a \cdot \alpha/w)^{\frac{1}{1-\alpha}}$ and $c^d(p) = [\chi/(1-\chi)]^\epsilon \cdot \mu/p^\epsilon$. In particular, equilibrium consumption is $c = [\chi/(1-\chi)]^\epsilon \cdot \mu/p^\epsilon$. Supply and demand curves are represented in Figure 10 in a (c, x) plane and in a (n, θ) plane. The aggregate supply and labor supply are piecewise linear with a kink at $x = 1$ and $\theta = 1$, respectively; the aggregate demand is vertical; and the labor demand is vertical. Given price and real wage, tightnesses and quantities are determined by the intersection of supply and demand curves. By introducing matching functions that govern the number of trades on each market, we rewrite the fixprice-fixwage model, which traditionally is a disequilibrium model, as an equilibrium model. Before describing the four regimes, we define two functions to help delimitate them: $p \mapsto w^\theta(p) = (\alpha/h) \cdot [\chi/(1-\chi)]^\epsilon \cdot \mu/p^\epsilon$ and $p \mapsto w^x(p) = \alpha \cdot a^{\frac{1}{\alpha}} \cdot \{[(1-\chi)/\chi]^\epsilon \cdot p^\epsilon/\mu\}^{\frac{1-\alpha}{\alpha}}$. The function w^θ decreases from $+\infty$ to 0 for $p \in (0, +\infty)$. The function w^x increases from 0 to $+\infty$ for $p \in (0, +\infty)$. Accordingly, the equation $w^\theta(p) = w^x(p)$ admits a unique solution on $(0, +\infty)$, which we denote p^*. We denote $w^* = w^\theta(p^*)$. We can show that p^* is given by (21) and w^* is given by (20). Figure 10(c) delimitates the four regimes in a (p, w) plane.

In the *Keynesian unemployment regime*, there is excess supply in the labor and product market:

[26] Indeed when demand is low in the standard fixprice-fixwage model, firms sell all of their output up to a fixed number of items and none above that number. In our model, firms sell all their output with the same probability; they choose output and thus sales to equalize marginal cost with marginal revenue.

$1 > \hat{f}(\theta) = \theta$ and $1 > f(x) = x$. The equilibrium tightnesses satisfy

$$\theta = \frac{\alpha}{h \cdot w} \cdot \left(\frac{\chi}{1-\chi}\right)^{\epsilon} \cdot \frac{\mu}{p^{\epsilon}}$$
$$x = \frac{w^{\alpha}}{a \cdot \alpha^{\alpha}} \cdot \left[\left(\frac{\chi}{1-\chi}\right)^{\epsilon} \cdot \frac{\mu}{p^{\epsilon}}\right]^{1-\alpha}.$$

Employment $n = \theta \cdot h$ is determined partly by aggregate demand and partly by the real wage. In the standard fixprice-fixwage model, employment only depends on demand in the Keynesian unemployment regime. In our model, employment also depends on the wage because firms have a positive probability to sell any amount of production so employment is determined by the marginal cost of labor and not by a quantity constraint. This regime prevails if $w > w^{\theta}(p)$ and $w < w^{x}(p)$. In the *classical unemployment regime*, there is excess supply in the labor market and excess demand in the product market: $1 > \hat{f}(\theta) = \theta$ and $1 = f(x)$. Product market tightness $x \geq 1$ is irrelevant. Labor market tightness satisfies

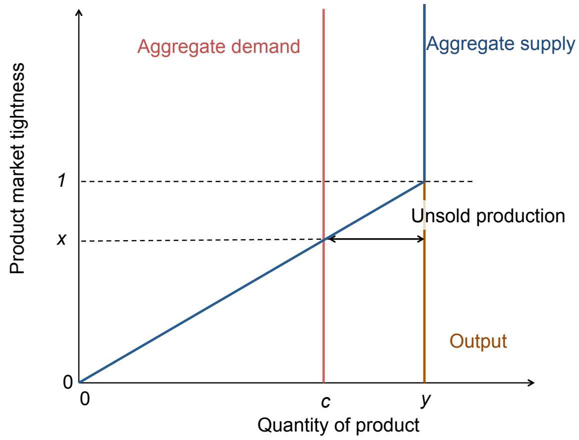
$$\theta = \frac{1}{h} \cdot \left(\frac{\alpha \cdot a}{w}\right)^{\frac{1}{1-\alpha}}.$$

Employment $n = \theta \cdot h$ is completely determined by the real wage. Aggregate demand does not matter because firms sell all their production for sure on the product market. This regime prevails if $w > w^{*}$ and $w = w^{x}(p)$. If $w > w^{x}(p)$, there is no equilibrium. In the *underconsumption regime*, there is excess demand in the labor market and excess supply in the product market: $\hat{f}(\theta) = 1$ and $1 > f(x) = x$. Labor market tightness $\theta \geq 1$ is irrelevant. Product market tightness satisfies

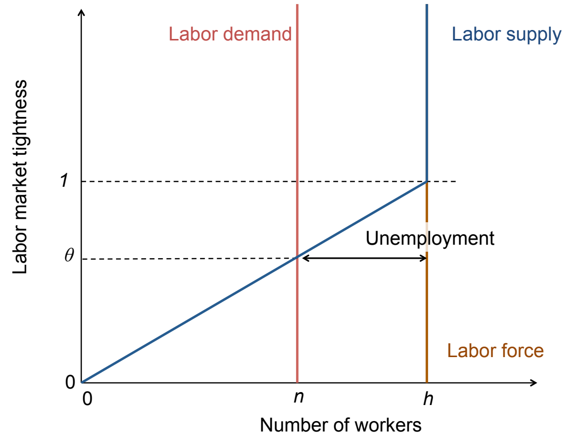
$$x = \frac{w}{a \cdot \alpha \cdot h^{\alpha-1}}.$$

There is no unemployment: $n = h$. This regime prevails if $w < w^{*}$ and $w = w^{\theta}(p)$. If $w < w^{\theta}(p)$, there is no equilibrium. In the *repressed inflation regime*, there is excess demand in the labor market and excess demand in the product market: $\hat{f}(\theta) = 1$ and $f(x) = 1$. The tightnesses $x \geq 1$ and $\theta \geq 1$ are irrelevant. There is no unemployment: $n = h$. This regime prevails for $w = w^{*}$ and $p = p^{*}$. Price and real wage are exactly the same as in the perfectly competitive case. Thus, the repressed inflation regime is identical to the Walrasian equilibrium.

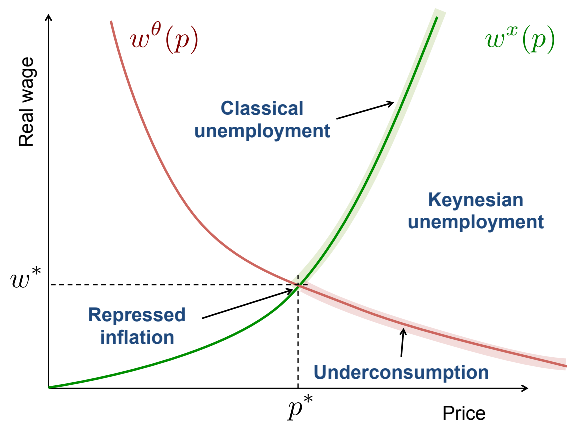
The assumptions made in this section could be used to study the effect of aggregate demand on unemployment. However, these assumptions limit the theory without clear advantages. First, the equilibrium is not well defined for all positive prices and wages. Second, the theory is complex



(a) Excess demand in the product market



(b) Excess demand in the labor market



(c) The four regimes of the fixprice-fixwage model

Figure 10: The fixprice-fixwage model of Section 4.4

because it requires to determine in which of the four regimes the economy operates in order to know which equilibrium conditions hold. Third, aggregate demand plays an interesting role only in the Keynesian unemployment regime, and not in the other regimes.

5 Conclusion

This paper proposes a parsimonious model that links unemployment to aggregate demand. We envision three applications for the model. First, the model could be used to analyse the impact on unemployment of a broad range of fiscal policies that affect simultaneously aggregate demand, labor demand, and labor supply. The model could address the following questions: Should unemployment insurance be more generous in recessions to stimulate aggregate demand, or should it be less generous to incentivize jobseekers to search more? Should payroll tax shift from employees to employers in recessions to stimulate aggregate demand, or should it shift from employers to employees to reduce labor cost and stimulate hiring? Should the minimum wage rise in recessions to increase the income of poorer workers with a high marginal propensity to consume and thus stimulate aggregate demand, or should it fall to stimulate hiring of low-income workers? Should income tax be more progressive in recessions to stimulate aggregate demand, or should it be more regressive to encourage work?

Second, empirical research could exploit the theoretical predictions of the model to identify the macroeconomic shocks driving business cycle fluctuations. A key result of our analysis is that product market tightness and sales are positively correlated under aggregate demand shocks, whereas they are negatively correlated under other shocks. This result implies that the probability to sell goods is procyclical under aggregate demand shocks and countercyclical under other shocks. This probability can be measured by the ratio of sales to stock for sales in a given period. Hence, empirical research on the cyclical behavior of sales and inventories could be useful to separate between different types of macroeconomic shocks. For instance, Bils and Kahn [2000] study empirically the cyclicality of the ratio of sales to stock for sales in the manufacturing sector in the US. They find that this ratio is strongly procyclical, suggesting that aggregate demand shocks play an important role in their data.

Third, the model could be a starting point to build a more sophisticated dynamic stochastic general equilibrium model featuring unsold production and unemployment. An important restriction of the theory, however, needs to be addressed to obtain a quantitatively realistic macroeconomic model. The restriction is that prices are completely rigid in the short-run in response to a shock and that all adjustments take place through tightnesses. Naturally in the long-run, market forces could push prices to adjust if their level is inefficient. For instance if prices are too high, new markets may be created with lower prices but higher tightness.[27] Sellers and buyers have incentives to move to these new markets because they are more efficient so there is a larger surplus to share. Sellers are compensated for the lower price with a higher probability to sell. Buyers are compensated for the higher matching wedge by a lower price. A mechanism that could lead to the creation of these new markets and dynamic price adjustments in the medium-run is the competitive search mechanism of Moen [1997]. Modeling these market forces would provide a microfoundation for the aggregate supply curve of the traditional AS-AD model; indeed, the AS curve represents the price increase arising when the economy is overheating and the price decrease arising when the economy is slack.

[27] Lazear [2010] finds evidence of such behavior for price and tightness in the housing market in the US.

References

Bai, Yan, Jose-Victor Rios-Rull, and Kjetil Storesletten. 2012. "Demand Shocks as Productivity Shocks." http://www.econ.umn.edu/~vr0j/papers/brs_Sept_27_2012.pdf.

Barro, Robert J., and Herschel I. Grossman. 1971. "A General Disequilibrium Model of Income and Employment." *American Economic Review*, 61(1): 82–93.

Bénassy, Jean-Pascal. 1993. "Nonclearing Markets: Microeconomic Concepts and Macroeconomic Applications." *Journal of Economic Literature*, 31(2): 732–761.

Bils, Mark J., and James A. Kahn. 2000. "What Inventory Behavior Tells Us about Business Cycles." *American Economic Review*, 90(3): 458–481.

Blanchard, Olivier J., and Jordi Galí. 2010. "Labor Markets and Monetary Policy: A New-Keynesian Model with Unemployment." *American Economic Journal: Macroeconomics*, 2(2): 1–30.

Blanchard, Olivier J., and Nobuhiro Kiyotaki. 1987. "Monopolistic Competition and the Effects of Aggregate Demand." *American Economic Review*, 77(4): 647–666.

Burdett, Kenneth, and Kenneth L. Judd. 1983. "Equilibrium Price Dispersion." *Econometrica*, 51(4): 955–969.

Card, David, and Alan B. Krueger. 1995. *Myth and Measurement: The New Economics of the Minimum Wage.* Princeton University Press.

den Haan, Wouter J., Garey Ramey, and Joel Watson. 2000. "Job Destruction and Propagation of Shocks." *American Economic Review*, 90(3): 482–498.

Diamond, Peter A. 1982. "Wage Determination and Efficiency in Search Equilibrium." *Review of Economic Studies*, 49(2): 217–227.

Edgeworth, Francis Y. 1881. *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences.* London:Kegan Paul.

Eggertsson, Gauti B., and Paul Krugman. 2012. "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo approach." *Quarterly Journal of Economics*, 127(3): 1469–1513.

Farmer, Roger E. A. 2008. "Aggregate Demand and Supply." *International Journal of Economic Theory*, 4(1): 77–93.

Farmer, Roger E.A. 2009. "Confidence, Crashes and Animal Spirits." National Bureau of Economic Research Working Paper 14846.

Hall, Robert E. 2005. " Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review*, 95(1): 50–65.

Hall, Robert E. 2008. "General Equilibrium with Customer Relationships: A Dynamic Analysis of Rent-Seeking." http://www.stanford.edu/~rehall/GECR_no_derivs.pdf.

Hart, Oliver. 1982. "A Model of Imperfect Competition with Keynesian Features." *Quarterly Journal of Economics*, 97(1): 109–138.

Heathcote, Jonathan, and Fabrizio Perri. 2012. "Wealth and Volatility."

Hicks, John. 1965. *Capital and Growth.* Oxford University Press.

Howitt, Peter, and R. Preston McAfee. 1987. "Costly Search and Recruiting." *International Economic Review*, 28(1): 89–107.

Kaplan, Greg, and Guido Menzio. 2013. "Shopping Externalities and Self-Fulfilling Unemployment Fluctuations." National Bureau of Economic Research Working Paper 18777.

Keynes, John Maynard. 1936. *The General Theory of Employment, Interest and Money.* Macmillan.

Landais, Camille, Pascal Michaillat, and Emmanuel Saez. 2010. "Optimal Unemployment Insurance over the Business Cycle." National Bureau of Economic Research Working Paper 16526.

Lazear, Edward P. 2010. "Why Do Inventories Rise When Demand Falls in Housing and Other Markets?" National Bureau of Economic Research Working Paper 15878.

Lehmann, Etienne, and Bruno Van der Linden. 2010. "Search Frictions on Product and Labor Markets: Money in the Matching Function." *Macroeconomic Dynamics*, 14(01): 56–92.

Lorenzoni, Guido. 2009. "A Theory of Demand Shocks." *American Economic Review*, 99(5): 2050–2084.

Mian, Atif, and Amir Sufi. 2012. "What Explains High Unemployment? The Aggregate Demand Channel." Chicago Booth Working Paper 13-43.

Michaillat, Pascal. 2012. "Do Matching Frictions Explain Unemployment? Not in Bad Times." *American Economic Review*, 102(4): 1721–1750.

Michaillat, Pascal. forthcoming. "A Theory of Countercyclical Government Multiplier." *American Economic Journal: Macroeconomics*.

Moen, Espen R. 1997. "Competitive Search Equilibrium." *Journal of Political Economy*, 105(2): 385–411.

Montgomery, James D. 1991. "Equilibrium Wage Dispersion and Interindustry Wage Differentials." *Quarterly Journal of Economics*, 106(1): 163–179.

Mortensen, Dale T. 1982. "Property Rights and Efficiency in Mating, Racing, and Related Games." *American Economic Review*, 72(5): 968–979.

Petrongolo, Barbara, and Christopher A. Pissarides. 2001. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature*, 39(2): 390–431.

Pissarides, Christopher A. 1985. "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages." *American Economic Review*, 75(4): 676–690.

Ramey, Valerie A. 2011. "Can Government Purchases Stimulate the Economy?" *Journal of Economic Literature*, 49: 673–685.

Rendhal, Pontus. 2012. "Fiscal Policy in an Unemployment Crisis." sites.google.com/site/pontusrendahl/stimulus.pdf.

Rogerson, Richard, Robert Shimer, and Randall Wright. 2005. "Search-Theoretic Models of the Labor Market: A Survey." *Journal of Economic Literature*, 43(4): 959–988.

Shimer, Robert. 2010. *Labor Markets and Business Cycles.* Princeton, NJ:Princeton University Press.

Wasmer, Etienne. 2011. "A Steady-State Model of Non-Walrasian Economy with Three Imperfect Markets." http://econ.sciences-po.fr/sites/default/files/file/ewasmer/WasmerNequal3-3June2011.pdf.

Appendix

A Proofs

We start by proving a lemma that we use repeatedly in the proofs. The lemma characterizes the matching probabilities, defined by $q(x) = (1 + x^{\eta})^{-\frac{1}{\eta}}$ and $f(x) = (1 + x^{-\eta})^{-\frac{1}{\eta}}$, the matching wedge, defined by $\tau(x) = \rho/(q(x) - \rho)$, and the tightnesses x^m and x^*, defined by $f'(x^*) = \rho$ and $q(x^m) = \rho$.

LEMMA A1. *The functions f, q, and τ and the values x^m and x^* satisfy the following properties:*

- $x^m = (\rho^{-\eta} - 1)^{\frac{1}{\eta}}$
- $f(x^m) = (1 - \rho^{\eta})^{\frac{1}{\eta}}$.
- $\tau(x^m) = +\infty$ *and* $1/(1 + \tau(x^m)) = 0$.
- $x^* = \left(\rho^{-\frac{\eta}{1+\eta}} - 1\right)^{\frac{1}{\eta}}$
- $q(x^*) = \rho^{\frac{1}{1+\eta}}$.
- $f(x^*) = \left(1 - \rho^{\frac{\eta}{1+\eta}}\right)^{\frac{1}{\eta}}$.
- $\tau(x^*) = 1/\left(\rho^{-\frac{\eta}{1+\eta}} - 1\right)$ *and* $1/(1 + \tau(x^*)) = 1 - \rho^{\frac{\eta}{1+\eta}}$.
- $f(0) = 0$ *and* $\lim_{x \to +\infty} f(x) = 1$.
- f *is positive, smooth, and strictly increasing on* $[0, +\infty)$.
- $q(0) = 1$ *and* $\lim_{x \to +\infty} q(x) = 0$.
- q *is positive, smooth, and strictly decreasing on* $[0, +\infty)$.
- $\tau(0) = \rho/(1 - \rho)$ *and* $\lim_{x \to x^m} \tau(x) = +\infty$.
- τ *is positive, smooth, and strictly increasing on* $[0, x^m)$.
- $f(x) = q(x) \cdot x$.
- $q'(x) = -q(x)^{1+\eta} \cdot x^{\eta - 1}$.
- $f'(x) = q(x)^{1+\eta}$.

Proof. The results follow from simple algebra. □

A.1 Proof of Lemma 1

The proof follows directly from the definitions of x^* and c^*, and from Lemma A1.

A.2 Proof of Proposition 1

In this section we propose an extension of Proposition 1 and prove it.

PROPOSITION A1. *For any $p > 0$, there are two short-run equilibria: $(x^m, 0)$ and (x, c) where x is implicitly defined by*

$$(1+\tau(x))^{\epsilon-1} \cdot f(x) \cdot y = \left(\frac{\chi}{1-\chi}\right)^{\epsilon} \cdot \frac{\mu}{p^{\epsilon}} \tag{A1}$$

and $c = c^s(x)$. In particular, $x \in (0, x^m)$ and $c > 0$.

Proof. We are looking for a tightness $x \in [0, x^m]$ that satisfies $c^s(x) = c^d(x, p)$ or

$$\frac{1}{1+\tau(x)} \cdot \left[f(x) \cdot y - \left(\frac{\chi}{1-\chi}\right)^{\epsilon} \cdot \frac{\mu}{p^{\epsilon}} \cdot \frac{1}{(1+\tau(x))^{\epsilon-1}}\right] = 0.$$

$1/\left(1+\tau(x^m)\right) = 0$ by definition of x^m; hence, $(x^m, c^s(x^m)) = (x^m, 0)$ is always an equilibrium. Furthermore, $1/\left(1+\tau(x)\right) > 0$ for $x < x^m$ so any equilibrium tightness $x < x^m$ must satisfy (A1). Since $\epsilon > 1$, Lemma A1 implies that $x \mapsto (1+\tau(x))^{\epsilon-1} \cdot f(x)$ is strictly increasing and $(1+\tau(0))^{\epsilon-1} \cdot f(0) = 0$ and $\lim_{x \to x^m} (1+\tau(x))^{\epsilon-1} \cdot f(x) = +\infty$. Thus, there is a unique $x \in (0, x^m)$ that satisfies (A1). □

A.3 Proof of Proposition 2

An efficient allocation (x, c) is such that $c = c^s(x)$ and x maximizes $c^s(x)$. Lemma 1 implies that $x = x^*$ and $c = c^*$. There is a unique price p^* that implements the efficient allocation. This price satisfies

$$(p^*)^{\epsilon} = \left(\frac{\chi}{1-\chi}\right)^{\epsilon} \cdot \frac{\mu}{y} \cdot \frac{1}{f(x^*) \cdot (1+\tau(x^*))^{\epsilon-1}}.$$

The reason is that with $p = p^*$, x^* satisfies (A1). Lemma A1 implies that $f(x^*) = \left(1 - \rho^{\frac{\eta}{1+\eta}}\right)^{\frac{1}{\eta}}$ and $(1+\tau(x^*))^{\epsilon-1} = \left(1 - \rho^{\frac{\eta}{1+\eta}}\right)^{1-\epsilon}$. Thus,

$$p^* = \frac{\chi}{1-\chi} \cdot \left(\frac{\mu}{y}\right)^{\frac{1}{\epsilon}} \cdot \left(1 - \rho^{\frac{\eta}{1+\eta}}\right)^{1-\frac{\eta+1}{\eta \cdot \epsilon}}.$$

A.4 Proof of Proposition 3

Obvious using the results of Proposition 1 and Proposition 2.

A.5 Proof of Lemma 2

This proof is similar to the proof of Lemma 1.

A.6 Proof of Proposition 4

In this section we propose an extension of Proposition 4 and prove it.

PROPOSITION A2. *For any $p > 0$ and $w > 0$, there are three short-run general equilibria: (x, θ, n, c) where $x < x^m$ and $\theta < \theta^m$ solve the system*

$$h^{1-\alpha} \cdot \hat{f}(\theta)^{1-\alpha} \cdot (1 + \hat{\tau}(\theta))^{\alpha} = f(x) \cdot \frac{a \cdot \alpha}{w} \tag{A2}$$

$$h \cdot \hat{f}(\theta) \cdot (1 + \tau(x))^{\epsilon - 1} = \frac{\alpha}{w} \cdot \left(\frac{\chi}{1-\chi}\right)^{\epsilon} \cdot \frac{\mu}{p^{\epsilon}}. \tag{A3}$$

and $n = n^s(\theta) > 0$ and $c = c^s(x, n) > 0$; $(x^m, \theta^m, 0, 0)$; and $(x^m, \theta, 0, n)$ where $\theta < \theta^m$ solves (A2) for $x = x^m$ and $n = n^s(\theta) > 0$.

Proof. In general equilibrium, (x, θ) satisfies the following system of two equations:

$$n^s(\theta) = n^d(\theta, x, w) \tag{A4}$$

$$c^s(x, n^s(\theta)) = c^d(x, p). \tag{A5}$$

First case: $\theta < \theta^m$ and $x < x^m$. $1/(1 + \tau(x)) \in (0, +\infty)$ and $1/(1 + \hat{\tau}(\theta)) \in (0, +\infty)$ so we can rewrite the system (A4)–(A5) as (A2)–(A3). Equation (A4) is equivalent to

$$\left(\frac{1}{1 + \hat{\tau}(\theta)}\right)^{1-\alpha} \cdot \left[\hat{f}(\theta)^{1-\alpha} \cdot h^{1-\alpha} - \frac{a \cdot \alpha}{w} \cdot f(x) \cdot \frac{1}{(1 + \hat{\tau}(\theta))^{\alpha}}\right] = 0. \tag{A6}$$

Since the first factor is positive, this equation implies that the second factor must be zero. Multiplying the second factor by $(1 + \hat{\tau}(\theta))^{\alpha}$ yields (A2). Following the proof of Proposition A1, we modify (A5) to obtain

$$f(x) \cdot a \cdot \left(\frac{\hat{f}(\theta)}{1 + \hat{\tau}(\theta)} \cdot h\right)^{\alpha} \cdot (1 + \tau(x))^{\epsilon - 1} = \left(\frac{\chi}{1-\chi}\right)^{\epsilon} \cdot \frac{\mu}{p^{\epsilon}}; \tag{A7}$$

Multiplying both sides of the equation by α/w and substituting (A2) into this equation yields (A3).

We now show that for any $p > 0$ and $w > 0$, the system (A2)–(A3) admits a unique solution. Since $\alpha < 1$, Lemma A1 implies that $\theta \mapsto \hat{f}(\theta)^{1-\alpha} \cdot (1 + \hat{\tau}(\theta))^{\alpha}$ is strictly increasing from 0 to $+\infty$ for $\theta \in [0, \theta^m)$. Hence, equation (A2) implicitly defines θ as a function of $x \in [0, +\infty)$: $\theta = \Theta^L(x)$. Lemma A1 shows that f is strictly increasing from 0 to 1 on $(0, +\infty)$; thus, Θ^L is strictly increasing on $(0, +\infty)$, $\Theta^L(0) = 0$, and $\lim_{x \to +\infty} \Theta^L(x) = \theta^L > 0$ where $\theta^L \in (0, \theta^m)$ is implicitly defined by $h^{1-\alpha} \cdot \hat{f}(\theta^L)^{1-\alpha} \cdot \left(1 + \hat{\tau}(\theta^L)\right)^{\alpha} = a \cdot \alpha / w$.

If $[\alpha/(w \cdot h)] \cdot [\chi/(1 - \chi)]^{\epsilon} \cdot (\mu/p^{\epsilon}) \geq 1$, define $x^P(p, w)$ by

$$\left(1 + \tau(x^P)\right)^{\epsilon - 1} = \frac{\alpha}{w \cdot h} \cdot \left(\frac{\chi}{1-\chi}\right)^{\epsilon} \cdot \frac{\mu}{p^{\epsilon}}.$$

If $[\alpha/(w \cdot h)] \cdot [\chi/(1 - \chi)]^{\epsilon} \cdot (\mu/p^{\epsilon}) < 1$, $x^P(p, w) \equiv 0$. Since $\epsilon > 1$, Lemma A1 implies that $x \mapsto (1 + \tau(x))^{\epsilon - 1}$ is strictly increasing from 1 to $+\infty$ for $x \in [0, x^m)$; therefore, x^P is well defined and $x^P(p, w) \in (0, x^m)$. Lemma A1 shows that $\hat{f}$ is strictly increasing from 0 to 1 on $(0, +\infty)$, which implies that equation (A3) implicitly defines θ as a function of $x \in (x^P(p,w), x^m)$: $\theta = \Theta^P(x)$. Moreover, Θ^P is strictly decreasing on $(x^P(p,w), x^m)$, $\lim_{x \to x^P(p,w)} \Theta^P(x) = +\infty$, and $\lim_{x \to x^m} \Theta^P(x) = 0$.

The system (A2)–(A3) is equivalent to

$$\begin{cases} \Theta^L(x) & = & \Theta^P(x) \\ \theta & = & \Theta^P(x) \end{cases}$$

Given the properties of the functions Θ^L and Θ^P, we conclude that this system admits a unique solution (x, θ) with $x \in (x^P(p,w), x^m)$ and $\theta \in (0, \theta^L)$.

Second case: $x = x^m$. $c^s(x^m, n^s(\theta)) = 0 = c^d(x^m, p)$ so (A5) is necessarily satisfied. When (A4) is rewritten as (A6), it is clear that it admits exactly two solutions for $x = x^m$: θ^m and $\Theta^L(x^m) < \theta^m$. To summarize, there are exactly two general equilibria when $x = x^m$: $(x^m, \theta^m, 0, 0)$ and $(x^m, \theta, 0, n)$ where $\theta < \theta^m$ solves (A2) for $x = x^m$ and $n = n^s(\theta) > 0$.

Third case: $\theta = \theta^m$. $n^s(\theta^m) = 0 = n^d(\theta^m, x, w)$ so (A4) is necessarily satisfied. Then, $n^s(\theta^m) = 0$ so (A5) becomes $c^s(x, 0) = c^d(x, p)$. Since $c^s(x, 0) = 0$, x solves $c^d(x, p) = 0$, which imposes $x = x^m$. Thus, we are back to the second case. □

A.7 Proof of Proposition 5

An efficient allocation (x, θ, c, n) is such that $n = n^s(\theta)$ and θ maximizes $n^s(\theta)$. Lemma 2 implies that $\theta = \theta^*$ and $n = n^*$. An efficient allocation is also such that $c = c^s(x, n^*)$ and x maximizes $c^s(x, n^*)$. Lemma 1 implies that $x = x^*$ and $c = c^* = (f(x^*) - \rho \cdot x^*) \cdot a \cdot (n^*)^\alpha$.

There exists a unique pair (p^*, w^*) that implements the efficient allocation. The pair satisfies

$$\begin{aligned} w^* &= a \cdot \alpha \cdot f(x^*) \cdot h^{\alpha-1} \cdot \hat{f}(\theta^*)^{\alpha-1} \cdot (1 + \hat{\tau}(\theta^*))^{-\alpha} \\ (p^*)^\epsilon &= \frac{\alpha}{w^*} \cdot \left(\frac{\chi}{1-\chi}\right)^\epsilon \cdot \mu \cdot \frac{1}{h \cdot \hat{f}(\theta^*)} \cdot (1 + \tau(x^*))^{1-\epsilon}. \end{aligned}$$

The reason is that with $p = p^*$ and $w = w^*$, (x^*, θ^*) satisfies the system (A2)–(A3). Lemma A1 implies that $f(x^*) = \left(1 - \rho^{\frac{\eta}{1+\eta}}\right)^{\frac{1}{\eta}}$, $\hat{f}(\theta^*) = \left(1 - \hat{\rho}^{\frac{\hat{\eta}}{1+\hat{\eta}}}\right)^{\frac{1}{\hat{\eta}}}$, $(1 + \tau(x^*)) = \left(1 - \rho^{\frac{\eta}{1+\eta}}\right)^{-1}$, and $(1 + \hat{\tau}(\theta^*)) = \left(1 - \hat{\rho}^{\frac{\hat{\eta}}{1+\hat{\eta}}}\right)^{-1}$. After some algebra, we conclude that

$$\begin{aligned} w^* &= a \cdot \alpha \cdot h^{\alpha-1} \cdot \left(1 - \rho^{\frac{\eta}{1+\eta}}\right)^{\frac{1}{\eta}} \cdot \left(1 - \hat{\rho}^{\frac{\hat{\eta}}{1+\hat{\eta}}}\right)^{\alpha - \frac{1-\alpha}{\hat{\eta}}} \\ p^* &= \frac{\chi}{1-\chi} \cdot \left(\frac{\mu}{a \cdot h^\alpha}\right)^{\frac{1}{\epsilon}} \cdot \left(1 - \rho^{\frac{\eta}{1+\eta}}\right)^{1 - \frac{1+\eta}{\epsilon \cdot \eta}} \cdot \left(1 - \hat{\rho}^{\frac{\hat{\eta}}{1+\hat{\eta}}}\right)^{-\frac{\alpha \cdot (1+\hat{\eta})}{\epsilon \cdot \hat{\eta}}}. \end{aligned}$$

A.8 Proof of Proposition 6

We build on the proof of Proposition 4 and use the same notations. The function $\Theta^L : [0, +\infty) \times (0, +\infty) \to (0, +\infty)$ is defined such that $\theta = \Theta^L(x, w)$ solves (A2). The function Θ^L is strictly increasing in x and strictly decreasing in w. The function Θ^P: $\{(x,p,w)|p>0,\ w>0,\ x^P(p,w)<x<x^m\}\rightarrow(0,+\infty)$ is defined such that $\theta=\Theta^P(x,p,w)$ solves (A3). The function Θ^P is strictly decreasing in x, strictly decreasing in p, and strictly decreasing in w. The first part of the proof is illustrated in Figure 1(a). The second part of the proof is illustrated in Figure 1(b).

First part: condition such that $\theta<\theta^*$. Let w^L be defined by $\Theta^L(x^m,w^L)=\theta^*$. For all $w>w^L$ and for all $x\in[0,x^m]$, $\Theta^L(x,w)<\theta^*$. For all $w\leq w^L$, there exists a unique $x\in[0,x^m]$ such that $\Theta^L(x,w)=\theta^*$. We implicitly define the function $x^L:(0,w^L]\rightarrow[0,x^m]$ by $\Theta^L(x^L(w),w)=\theta^*$. In particular, $\lim_{w\rightarrow 0}x^L(w)=0$, $x^L(w^*)=x^*$ and $x^L(w^L)=x^m$ and x^L is strictly increasing. We define the function $p^\theta:(0,w^L)\rightarrow(0,+\infty)$ by

$$p^\theta(w)=\frac{\chi}{1-\chi}\cdot\left[\frac{\left(1+\tau(x^L(w))\right)^{1-\epsilon}}{h\cdot\hat{f}(\theta^*)}\cdot\frac{\alpha\cdot\mu}{w}\right]^{\frac{1}{\epsilon}}.$$

The function p^θ is strictly decreasing from $+\infty$ to 0 for $w\in(0,w^L)$ and $p^\theta(w^*)=p^*$. By definition, $\Theta^P(x^L(w),p^\theta(w),w)=\theta^*$.

Let θ denote equilibrium labor market tightness and x denote equilibrium product market tightness. For any $w>w^L$, $\theta<\theta^*$ because $\Theta^L(x,w)<\theta^*$ for all $x\in[0,x^m]$ and because $\theta=\Theta^L(x,w)$. Consider $w\leq w^L$. For any $p>p^\theta(w)$, $\Theta^P(x^L(w),p,w)<\Theta^P(x^L(w),p^\theta(w),w)=\theta^*=\Theta^L(x^L(w),w)$. Given that Θ^L is strictly increasing in x and Θ^P is strictly decreasing in x and Θ^P and Θ^L cross only once in a (x,θ) plane, we conclude that $x<x^L(w)$ for any $p>p^\theta(w)$. Thus, $\theta=\Theta^L(x,w)<\Theta^L(x^L(w),w)=\theta^*$ for any $p>p^\theta(w)$. To simplify the exposition, we extend the definition of p^θ by $p^\theta(w)=0$ for all $w\geq w^L$. To summarize, $\theta<\theta^*$ if and only if $p>p^\theta(w)$ for any $w>0$.

Second part: condition such that $x<x^*$. We define the function $p^x:(0,+\infty)\rightarrow(0,+\infty)$ by

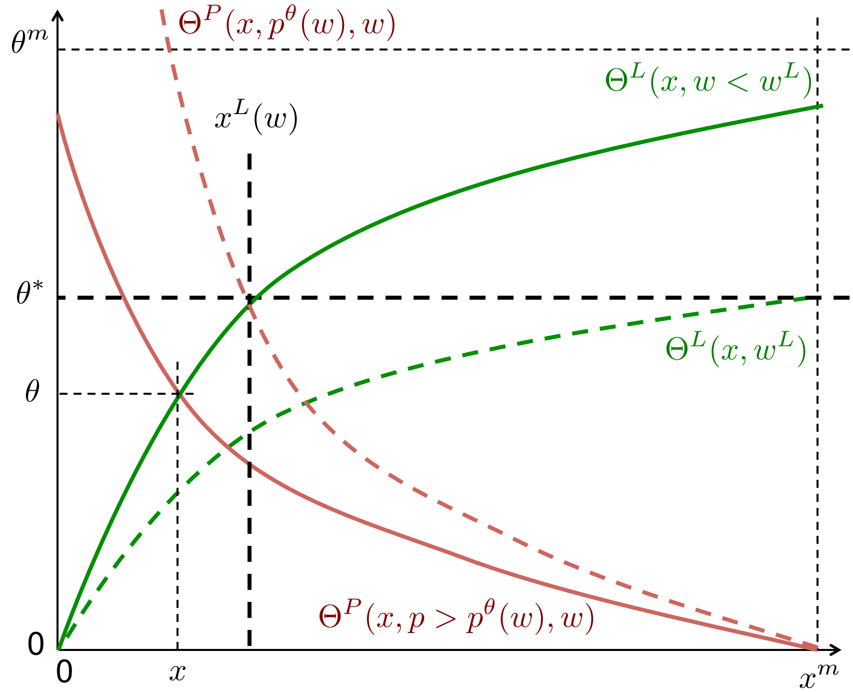
$$p^x(w)=\frac{\chi}{1-\chi}\cdot\left[\frac{(1+\tau(x^*))^{1-\epsilon}}{h\cdot\hat{f}(\Theta^L(x^*,w))}\cdot\frac{\alpha\cdot\mu}{w}\right]^{\frac{1}{\epsilon}}.$$

By definition, $\Theta^P(x^*,p^x(w),w)=\Theta^L(x^*,w)$; thus, x^* is the equilibrium product market tightness when the value of the real wage is w and the value of the price is $p^x(w)$. We define the auxiliary function $Z:(0,+\infty)\rightarrow(0,+\infty)$ by

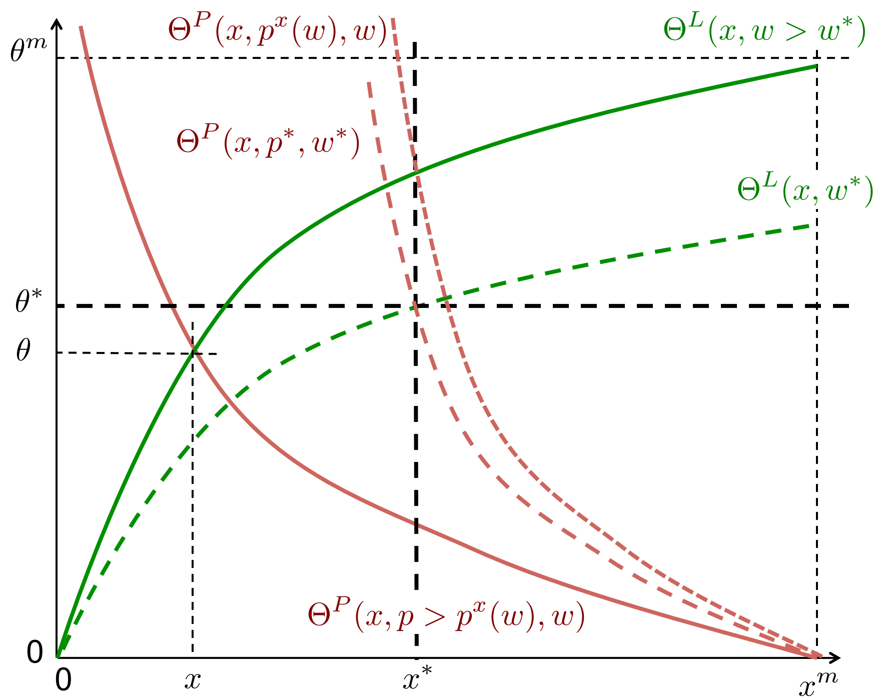
$$Z(w)=f(x^*)\cdot a\cdot\alpha\cdot n^s(\Theta^L(x^*,w)).$$

Given that $\Theta^L(x^*,w^*)=\theta^*$ and Θ^L is strictly increasing in w, $\Theta^L(x^*,w)>\theta^*$ if and only if $w>w^*$. Using Lemma 2 and the fact that Θ^L is strictly increasing in w, we infer that Z is strictly increasing for $w\in(0,w^*)$ and strictly decreasing for $w\in(w^*,+\infty)$. Given the definition of Θ^L, we infer that $Z(w)=h\cdot w\cdot\hat{f}(\Theta^L(x^*,w))$ and therefore that

$$p^x(w)=\frac{\chi}{1-\chi}\cdot\left[\frac{(1+\tau(x^*))^{1-\epsilon}}{Z(w)}\cdot\alpha\cdot\mu\right]^{\frac{1}{\epsilon}}.$$



(a) Condition such that $\theta < \theta^*$



(b) Condition such that $x < x^*$

Figure A1: Illustration of the proof of Proposition 6

The properties of Z imply that the function p^x strictly decreasing for $w \in (0, w^*)$ and strictly increasing for $w \in (w^*, +\infty)$ and $p^x(w^*) = p^*$.

Let θ denote equilibrium labor market tightness and x denote equilibrium product market tightness. Consider $w \in (0, +\infty)$. For any $p > p^x(w)$, $\Theta^P(x^*, p, w) < \Theta^P(x^*, p^x(w), w) = \Theta^L(x^*, w)$. Given that Θ^L is strictly increasing in x and Θ^P is strictly decreasing in x and Θ^P and Θ^L cross only once in a (x, θ) plane, we conclude that $x < x^*$ for any $p > p^x(w)$. To summarize, $x < x^*$ if and only if $p > p^x(w)$ for any $w > 0$.

B Equilibrium Concept

This appendix provides more details about our equilibrium concept. We draw a parallel between our concept and the Walrasian equilibrium. Following Walrasian theory, we make the institutional assumption that a price, p, and a market tightness, x, are posted on the market for services, and we make the behavioral assumption that buyers and sellers of services take price and tightness as given. The assumption is that buyers and sellers are small relative to the size of the market so that they regard market tightness and price as unaffected by their own actions. Market tightness is the ratio of aggregate buying effort to aggregate selling effort; thus, it seems reasonable for buyers and sellers to take it as given if they are small relative to the size of the market. The issue is more complicated for the price since buyer and seller could bargain over the price once they are matched, implying that they have some control over the price. However, the actual transaction price has no influence on the production and search decisions once a match is realized; what matters is the price that buyers and sellers expect to trade at. Since the transaction price depends on the other party and possibly other factors (for instance, custom, social norms, other buyers, and other sellers), we assume that each party takes the expected transaction price as given.

We define a general equilibrium as follows:

DEFINITION A1. A general equilibrium is a price p, market tightness x, aggregate consumption of services c, aggregate sales of services s, a collection of visits $\{v(i),\ i \in [0, 1]\}$, and a collection of capacities $\{y(j),\ j \in [0, 1]\}$ such that

(1) Taking x and p as given, buyer $i \in [0, 1]$ chooses the number of visits $v(i)$ to maximize her utility subject to her budget constraint and to the constraint imposed by matching frictions: $c(i) = v(i) \cdot q(x)/(1 + \tau(x))$, where $c(i)$ is consumption of buyer i.

(2) Taking x and p as given, seller $j \in [0, 1]$ chooses the capacity $y(j)$ to maximize her utility subject to the constraint imposed by matching frictions: $s(j) = y(j) \cdot f(x)$, where $s(j)$ is sales of seller j.

(3) The actual labor market tightness is x:

$$x = \frac{\int_0^1 v(i)di}{\int_0^1 y(j)dj}.$$

(4) The price p is pairwise Pareto efficient in all buyer-seller matches.

(5) Aggregate consumption satisfies $c = \int_0^1 c(i)di$ and aggregate sales satisfy $s = \int_0^1 s(j)dj$.

As in a Walrasian equilibrium, our equilibrium concept imposes that buyers and sellers behave optimally given the quoted price and tightness. A key difference between the two equilibrium concepts is that in a Walrasian equilibrium, buyers and sellers decide the quantity that they desire to buy or sell whereas in our equilibrium, buyers and sellers decide the buying effort and selling effort that they desire to exert, and these efforts lead to a trade with a probability determined by the market tightness. Consumers decide how many sellers of services to visit, knowing that each visits lead to a purchase with probability $q(x)$; workers decide how many units of services to offer for sale, knowing that each unit is sold with probability $f(x)$.

In a Walrasian equilibrium, the condition that sellers and buyers behave optimally is complemented by a market-clearing condition: at the quoted price, the quantity that buyers desire to buy equals the quantity that sellers desire to sell. It is possible to reformulate this condition as a consistency requirement: given that sellers and buyers expect to be able to trade with probability one, it must be that anybody desiring to trade is able to trade in equilibrium; this condition can only be fulfilled if the market clears. Condition (3) is the equivalent to this consistency requirement in presence of matching frictions. Once buyers and sellers have chosen $\{v(i),\ i \in [0,1]\}$ and $\{y(j),\ j \in [0,1]\}$, the number of trades is given by

$$\left[\left(\int v(i)di\right)^{-\eta} + \left(\int y(j)dj\right)^{-\eta}\right]^{\frac{-1}{\eta}} = \left(\int y(j)dj\right) \cdot f\left(\frac{\int v(i)di}{\int y(j)dj}\right) = \left(\int v(i)di\right) \cdot q\left(\frac{\int v(i)di}{\int y(j)dj}\right).$$

These equalities imply that the selling probability faced by sellers is $f\left(\int v(i)di / \int y(j)dj\right)$ and the buying probability faced by buyers is $q\left(\int v(i)di / \int y(j)dj\right)$. Both probabilities do not have to be equal to the probabilities on which sellers and buyers based their calculations, $f(x)$ and $q(x)$. In equilibrium, we impose the consistency requirement that these probabilities match, or equivalently, that the posted tightness equals the actual tightness, $\int v(i)di / \int y(j)dj$.

The last substantial equilibrium condition is that the price is pairwise Pareto efficient in all buyer-seller matches. In Walrasian theory, the condition that no mutually advantageous trades between two agents are available impose that the probability to trade is one. This is because without a matching function, buyers who do not trade with anybody but would like to trade can come together on the market place. If excess supply or demand existed at the market price, buyers or sellers could initiate new trades at a different price until all opportunities for pairwise improvement are exhausted. For example, if there is excess demand for a good, a buyer who is not receiving as much of the good as she desires could offer a slightly higher price and get sellers to sell the good to her first, making both buyer and seller better off. In our theory, this condition only constrains the price to allocate to buyer and seller a positive share of the surplus arising from the seller-buyer match. This surplus arises because workers remain idle and do not sell one unit of service if the match is broken, and also because buyer's matching costs are sunk at the time of matching.

Under these equilibrium conditions, the market for nonproduced good necessarily clears: $\int_0^1 m(i)di = \mu$, where $m(i)$ is consumption of nonproduced good by buyer i. The reason is that the budget constraints of all consumers are satisfied, and that sales equal purchases through the matching process.

C Equilibrium Under Cobb-Douglas Utility Function

We determine conditions such that an equilibrium with positive consumption exists when consumers have Cobb-Douglas utility function (which is a CES utility function with $\epsilon \to 1$). We state and prove a proposition for the model of Section 2, and another one for the model of Section 3.

PROPOSITION A3. *Consider the model of Section 2. Assume that consumers have a Cobb-Douglas utility function, $c^{\chi} \cdot m^{1-\chi}$. Define*

$$p^m \equiv \frac{\chi}{1-\chi} \cdot \frac{\mu}{y} \cdot (1-\rho^{\eta})^{-\frac{1}{\eta}}.$$

There are two cases:

(i) If $p \leq p^m$, the only short-run equilibrium is $(x^m, 0)$.

(ii) If $p > p^m$, one short-run equilibrium is $(x^m, 0)$ and one short-run equilibrium is (x, c) where x satisfies

$$f(x) \cdot y = \frac{\chi}{1-\chi} \cdot \frac{\mu}{p} \tag{A8}$$

and $c = c^s(x)$. In particular, $x \in (0, x^m)$ and $c > 0$.

Proof. We are looking for a tightness $x \in [0, x^m]$ that satisfies $c^s(x) = c^d(x, p)$ or

$$\frac{1}{1+\tau(x)} \cdot \left(f(x) \cdot y - \frac{\chi}{1-\chi} \cdot \frac{\mu}{p} \right) = 0.$$

$1/\left(1+\tau(x^m)\right) = 0$ by definition of x^m. Hence, $(x, c) = (x^m, c^s(x^m)) = (x^m, 0)$ is always an equilibrium. Furthermore, $1/\left(1+\tau(x)\right) > 0$ for $x < x^m$ so any other equilibrium tightness must satisfy (A8). Since f is strictly increasing and $f(0) = 0$, there is a unique $x \in (0, x^m)$ that satisfies this condition if and only if

$$f(x^m) = (1-\rho^{\eta})^{\frac{1}{\eta}} > \frac{\chi}{1-\chi} \cdot \frac{\mu}{p \cdot y}. \tag{A9}$$

Let us summarize the results. If (A9) is not satisfied, $(x^m, 0)$ is the unique equilibrium. If (A9) is satisfied, there are two equilibria: $(x^m, 0)$ and (x, c) where x satisfies (A8) and $c = c^s(x)$.

In the interior equilibrium (x, c), demand is higher than supply when tightness is below x (and conversely). Hence, if tightness is below equilibrium, demand is above supply, leading to an increase in tightness (and conversely). In contrast, in the corner equilibrium $(x^m, 0)$, demand is lower than supply when tightness is below x^m. Loosely speaking, the interior equilibrium is stable while the corner equilibrium is unstable. Hence, the main text focuses solely on the interior equilibrium. □

PROPOSITION A4. *Consider the model of Section 3. Assume that consumers have a Cobb-Douglas utility function, $c^{\chi} \cdot m^{1-\chi}$. Define*

$$W^m \equiv \alpha \cdot \frac{\chi}{1-\chi} \cdot \mu \cdot \left(1-\hat{\rho}^{\hat{\eta}}\right)^{\frac{1}{\hat{\eta}}}.$$

Define $\Theta : [W^m, +\infty) \to [0, \theta^m)$ *by* $\Theta(W) = \hat{f}^{-1}(\alpha \cdot [\chi/(1-\chi)] \cdot (\mu/W))$. *Finally, define* $p^m : [W^m, +\infty) \to [0, +\infty)$ *by*

$$p^m(W) = \frac{\chi}{1-\chi} \cdot \frac{\mu}{a} \cdot (1-\rho^\eta)^{-\frac{1}{\eta}} \cdot [n^s(\Theta(W))]^{-\alpha}.$$

There are two cases:

(i) If $p \cdot w \leq W^m$ *or* $p \leq p^m(p \cdot w)$*, there are two short-run general equilibria: the two equilibria with zero consumption of Proposition A2.*

(ii) If $p \cdot w > W^m$ *and* $p > p^m(p \cdot w)$*, there are three short-run general equilibria: the two equilibria with zero consumption of Proposition A2 and* (x, θ, n, c) *where* (θ, x) *solve*

$$\alpha \cdot \frac{\chi}{1-\chi} \cdot \mu = p \cdot w \cdot \hat{f}(\theta) \quad \text{(A10)}$$

$$\frac{\chi}{1-\chi} \cdot \mu = p \cdot f(x) \cdot a \cdot [n^s(\theta)]^\alpha \quad \text{(A11)}$$

and $n = n^s(\theta)$ *and* $c = c^s(x, n)$*.*

Proof. In general equilibrium, (x, θ) satisfies the system (A4)–(A5).

First case: $\theta = \theta^m$ **or** $x = x^m$**.** As explained in the proof of Proposition A2, there are exactly two general equilibria in that case: the two equilibria with zero consumption of Proposition A2. These equilibria exist for any $p > 0$ and any $w > 0$.

Second case: $\theta < \theta^m$ **and** $x < x^m$**.** $1/(1+\tau(x)) > 0$ and $1/(1+\hat{\tau}(\theta)) > 0$ so we can rewrite (A4) and (A5) as

$$w \cdot \hat{f}(\theta) = f(x) \cdot a \cdot \alpha \cdot [n^s(\theta)]^\alpha$$

$$f(x) \cdot a \cdot [n^s(\theta)]^\alpha = \frac{\chi}{1-\chi} \cdot \frac{\mu}{p}.$$

The first equation is just (A2) and the second equation is just (A7) when $\epsilon = 1$, substituting $n^s(\theta) = h \cdot \hat{f}(\theta)/(1+\hat{\tau}(\theta))$. We can recombine this system as (A10)–(A11). The system (A10)–(A11) does not admit solutions for any p and w. As we are looking for $\theta < \theta^m$, we need $\hat{f}(\theta) < \hat{f}(\theta^m) = \left(1 - \hat{\rho}^{\hat{\eta}}\right)^{\frac{1}{\hat{\eta}}}$. Equation (A10) therefore requires

$$p \cdot w > \alpha \cdot \frac{\chi}{1-\chi} \cdot \mu \cdot \left(1 - \hat{\rho}^{\hat{\eta}}\right)^{-\frac{1}{\hat{\eta}}} \equiv W^m.$$

Since $\hat{f}$ is strictly increasing from 0 to $\left(1 - \hat{\rho}^{\hat{\eta}}\right)^{\frac{1}{\hat{\eta}}}$ for $\theta \in [0, \theta^m)$, equation (A10) admits a unique solution in $[0, \theta^m)$ if $p \cdot w > W^m$. The function Θ defined in the proposition is the unique solution to (A10) when $p \cdot w \in (W^m, +\infty)$.

We turn to (A11). As we are looking for $x < x^m$, we also need $f(x) < f(x^m) = (1-\rho^\eta)^{\frac{1}{\eta}}$.

Equation (A11) therefore requires

$$p > \frac{\chi}{1-\chi} \cdot \frac{\mu}{a} \cdot (1-\rho^\eta)^{-\frac{1}{\eta}} \cdot [n^s(\Theta(p \cdot w))]^{-\alpha} \equiv p^m(p \cdot w).$$

We replaced θ by $\Theta(p \cdot w)$ in (A11) since we have already solved (A10) for θ. Since f is strictly increasing from 0 to $(1-\rho^\eta)^{\frac{1}{\eta}}$ for $x \in [0, x^m)$, equation (A11) admits a unique solution in $[0, x^m)$ if $p > p^m(p \cdot w)$. □

D Reducing Unemployment by Increasing Wages

To formalize the discussion of Section 3.6, we state and prove two propositions that apply to the model with customer relationships and inequality. The first proposition describes the set of parameters such that a general equilibrium exists. The second proposition establishes conditions such that a wage increase leads to a reduction in unemployment.

PROPOSITION A5. *Assume that consumers have Cobb-Douglas utility function, $c^\chi \cdot m^{1-\chi}$. Assume that there are no matching costs: $\rho = \hat{\rho} = 0$. Assume that the customer base is $\kappa > 0$. Normalize technology to $a = 1$ and price to $p = 1$. Then equilibrium employment solves*

$$\frac{\alpha}{w} \cdot [K_2 - \kappa] = K_1 \cdot n - \kappa \cdot n^{1-\alpha}$$

where the constants K_1 and K_2 are defined by

$$K_1 \equiv \frac{\sum_g (1-\chi_g) \cdot [\alpha \cdot \varpi_g + (1-\alpha) \cdot \sigma_g]}{\sum_g (1-\chi_g) \cdot \sigma_g} \tag{A12}$$

$$K_2 \equiv \frac{\sum_g \mu_g \cdot \chi_g}{\sum_g (1-\chi_g) \cdot \sigma_g}. \tag{A13}$$

In addition, define the following constants: $B_1 \equiv \kappa^{-1/\alpha} \cdot (\kappa - K_2)/(1-K_1)$, $B_2 \equiv (\kappa - K_2)/(\kappa - K_1)$, and $B_3 \equiv (K_1/K_2)^{\frac{1-\alpha}{\alpha}}$. There are several possible cases in which a general equilibrium with positive tightnesses exists:

(i) if $K_1 < \kappa$, $K_2 < \kappa$, and $K_1 < K_2$, for any wage such that $(w/\alpha) \in [B_1, B_2]$;

(ii) if $K_1 < 1$, $K_2 < \kappa$, and $K_1 > K_2$, for any wage such that $(w/\alpha) \in [B_1, B_3]$;

(iii) if $K_1 < 1$, $\kappa < K_2$, and $K_1 > K_2$, for any wage such that $(w/\alpha) \in [B_2, B_3]$;

(iv) if $1 < K_1$, $\kappa < K_2$, and $K_1 > K_2$, for any wage such that $(w/\alpha) \in [B_2, \min\{B_1, B_3\}]$;

In all these cases, the general equilibrium with positive tightnesses is unique.

Proof. Assume that $\rho = \hat{\rho} = 0$, $a = 1$, and $p = 1$. In this case, $\tau(x) = \hat{\tau}(\theta) = 0$. Also assume that $\kappa > 0$. A key general-equilibrium condition is that $c^d(n, w) = c^s(x, n)$. Using the expressions for $c^s(x, n)$ and $c^d(n, w)$ obtained in Section 3.6, we obtain

$$\frac{\sum_g \mu_g \cdot \chi_g}{\sum_g (1-\chi_g) \cdot \sigma_g} + \frac{\sum_g \chi_g \cdot (\varpi_g - \sigma_g)}{\sum_g (1-\chi_g) \cdot \sigma_g} \cdot w \cdot n = \kappa \cdot (1 - f(x)) + f(x) \cdot n^\alpha.$$

Another general equilibrium condition is that $n = n^d(\theta, x, w)$, which implies $\alpha \cdot f(x) = w \cdot n^{1-\alpha}$. Combining these two conditions, we obtain

$$\alpha \cdot \frac{\sum_g \mu_g \cdot \chi_g}{\sum_g (1-\chi_g) \cdot \sigma_g} = \alpha \cdot \kappa - \kappa \cdot w \cdot n^{1-\alpha} + \left[1 - \alpha \cdot \frac{\sum_g \chi_g \cdot (\varpi_g - \sigma_g)}{\sum_g (1-\chi_g) \cdot \sigma_g}\right] \cdot w \cdot n.$$

Using the constants defined by (A12) and (A13), this equation simplifies to

$$\frac{\alpha}{w} \cdot (K_2 - \kappa) = K_1 \cdot n - \kappa \cdot n^{1-\alpha}. \tag{A14}$$

Define the polynomial

$$P(z) = K_1 \cdot z^{\frac{1}{1-\alpha}} - \kappa \cdot z + \frac{\alpha}{w} \cdot (\kappa - K_2)$$

Solving the general equilibrium of the model is equivalent to solving $P(z) = 0$ and imposing that the root z^* of P satisfies $z^* \in \left[\kappa^{\frac{1-\alpha}{\alpha}}, \min\{1, \alpha/w\}\right]$. Once we have found z^*, the general equilibrium (n, x, θ, c) is given by $n = (z^*)^{\frac{1}{1-\alpha}}$, $x = f^{-1}(z^* \cdot w/\alpha)$, $\theta = \hat{f}^{-1}\left((z^*)^{\frac{1}{1-\alpha}}\right)$, and $c = \kappa + (z^* \cdot w/\alpha) \cdot \left((z^*)^{\frac{\alpha}{1-\alpha}} - \kappa\right)$. The three conditions on z^* ensure that we can construct x, θ, and c:

- $z^* \geq \kappa^{\frac{1-\alpha}{\alpha}}$ ensures that $c \geq \kappa$;
- $z^* < 1$ ensures that we can construct θ given that $\hat{f}(\theta) \in [0, 1]$;
- $z^* < \alpha/w$ ensures that we can construct x given that $f(x) \in [0, 1]$.

We first show that $P(z)$ is strictly increasing for $z \in [\kappa^{\frac{1-\alpha}{\alpha}}, +\infty)$.

$$P'(z) = \frac{K_1}{1-\alpha} \cdot z^{\frac{\alpha}{1-\alpha}} - \kappa.$$

Hence,

$$P'(\kappa^{\frac{1-\alpha}{\alpha}}) = \left[\frac{K_1}{1-\alpha} - 1\right] \cdot \kappa.$$

As

$$K_1 = \frac{\sum_g (1-\chi_g) \cdot [\alpha \cdot \varpi_g + (1-\alpha) \cdot \sigma_g]}{\sum_g (1-\chi_g) \cdot \sigma_g} \geq \frac{\sum_g (1-\chi_g) \cdot [(1-\alpha) \cdot \sigma_g]}{\sum_g (1-\chi_g) \cdot \sigma_g} = 1 - \alpha,$$

$K_1/(1-\alpha) \geq 1$ and $P'(\kappa^{\frac{1-\alpha}{\alpha}}) \geq 0$. Since P' is strictly increasing for $z \geq 0$, we infer that $P'(z) > 0$ for $z > \kappa^{\frac{1-\alpha}{\alpha}}$. As a conclusion, P is strictly increasing for $z \in [\kappa^{\frac{1-\alpha}{\alpha}}, +\infty)$ and P has at most one root on $[\kappa^{\frac{1-\alpha}{\alpha}}, +\infty)$.

The system of equations has a solution if and only if P has a root in the interval $\left[\kappa^{\frac{1-\alpha}{\alpha}}, \min\{1, \alpha/w\}\right]$. As a function of the parameters K_1, K_2, and κ, we determine for which wages such a root exists. For such a root to exist, it is necessary and sufficient that $P(\kappa^{\frac{1-\alpha}{\alpha}}) < 0$ and $\min\{P(1), P(\alpha/w)\} > 0$.

The first condition is that $P(\kappa^{\frac{1-\alpha}{\alpha}}) < 0$, which is equivalent to

$$\frac{w}{\alpha} \cdot (K_1 - 1) < (K_2 - \kappa) \cdot \kappa^{-\frac{1}{\alpha}}.$$

Define $B_1 \equiv \kappa^{-1/\alpha} \cdot (\kappa - K_2)/(1 - K_1)$. There are three cases:

- if $K_1 < 1$, then we need $w/\alpha > B_1$;
- if $K_1 = 1$, then any wage works as long as $K_2 > \kappa$;
- if $K_1 > 1$, then we need $w/\alpha < B_1$.

The next necessary condition is that $P(1) > 0$, which is equivalent to

$$\frac{w}{\alpha} \cdot (K_1 - \kappa) > (K_2 - \kappa).$$

Define $B_2 \equiv (\kappa - K_2)/(\kappa - K_1)$. There are three cases:

- if $K_1 < \kappa$, then we need $w/\alpha < B_2$;
- if $K_1 = \kappa$, then any wage works as long as $K_2 < \kappa$;
- if $K_1 > \kappa$, then we need $w/\alpha > B_2$.

The last necessary condition is that $P(\alpha/w) > 0$, which is equivalent to

$$K_1 \cdot \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} > K_2.$$

Define $B_3 \equiv (K_1/K_2)^{\frac{1-\alpha}{\alpha}}$. Then we need $w/\alpha < B_3$ for any $K_1 > 0$ and $K_2 > 0$.

We now need to consider all the possible combinations of (K_1, K_2) to determine whether there exists a wage satisfying these three conditions.

1. $K_2 < K_1 < \kappa$: We need $w/\alpha > B_1$, $w/\alpha < B_2$, and $w/\alpha < B_3$. $K_2 < K_1$ so $B_3 > 1$ and $B_2 > 1$. However, when $w/\alpha > 1$, $P(1) > P(\alpha/w)$ so the constraint $w/\alpha < B_2$ is not binding. Hence, we only need to impose $B_1 < w/\alpha < B_3$.

2. $K_1 < K_2 < \kappa$: We need $w/\alpha > B_1$, $w/\alpha < B_2$, and $w/\alpha < B_3$. $K_1 < K_2$ so $B_3 < 1$ so $w/\alpha < 1$ so $P(1) < P(\alpha/w)$ so the constraint $w/\alpha < B_3$ is not binding. Hence, we only need to impose $B_1 < w/\alpha < B_2$.

3. $K_1 < \kappa$ and $K_2 > \kappa$: We need $w/\alpha > B_1$, $w/\alpha < B_2$, and $w/\alpha < B_3$. But $B_2 < 0$. Therefore, no such w/α exists.

4. $K_1 \in (\kappa, 1)$ and $K_2 < \kappa$: We need $w/\alpha > B_1$, $w/\alpha > B_2$, and $w/\alpha < B_3$. But $B_2 < 0$ so the constraint $w/\alpha > B_2$ is always satisfied. Hence, we only need to impose $B_1 < w/\alpha < B_3$.

5. $K_1 \in (\kappa, 1)$ and $K_2 \in (\kappa, 1)$ and $K_2 < K_1$: We need $w/\alpha > B_1$, $w/\alpha > B_2$, and $w/\alpha < B_3$. But $B_1 < 0$ so the constraint $w/\alpha > B_1$ is always satisfied. Hence, we only need to impose $B_2 < w/\alpha < B_3$.

6. $K_1 \in (\kappa, 1)$ and $K_2 > K_1$: We need $w/\alpha > B_1$, $w/\alpha > B_2$, and $w/\alpha < B_3$. But $B_2 > 1$ and $B_3 < 1$. Therefore, no such w/α exists.

7. $K_1 > 1$ and $K_2 < \kappa$: We need $w/\alpha < B_1$, $w/\alpha > B_2$, and $w/\alpha < B_3$. But $B_1 < 0$. Therefore, no such w/α exists.

8. $K_1 > 1$ and $K_2 \in (\kappa, K_1)$: We need $w/\alpha < B_1$, $w/\alpha > B_2$, and $w/\alpha < B_3$.

9. $K_1 > 1$ and $K_2 > K_1$: We need $w/\alpha < B_1$, $w/\alpha > B_2$, and $w/\alpha < B_3$. But $B_2 > 1$ and $B_3 < 1$. Therefore, no such w/α exists.

Only Cases 1, 2, 4, 5, and 8 offer a solution. □

PROPOSITION A6. *In all general equilibria such that $K_2 < \kappa$, a wage increase leads to a reduction in unemployment, as well as an increase in employment, consumption, product market tightness, and labor market tightness.*

Proof. Consider a general equilibrium with $K_2 < \kappa$. Using the notations of the proof of Proposition 6, a general equilibrium characterized by z^* satisfies $P(z^*) = 0$. We perform a comparative-statics exercise with respect to w:

$$\frac{\partial P}{\partial w} + \frac{\partial P}{\partial z} \cdot \frac{dz^*}{dw} = 0.$$

Since $K_2 < \kappa$,

$$\frac{\partial P}{\partial w} = \alpha \cdot (K_2 - \kappa) \cdot \frac{1}{w^2} < 0.$$

Furthermore, since $z^* > \kappa^{\frac{1-\alpha}{\alpha}}$ and $K_1 \geq 1 - \alpha$,

$$\frac{\partial P}{\partial z} = \frac{K_1}{1-\alpha} \cdot z^{\frac{\alpha}{1-\alpha}} - \kappa \geq z^{\frac{\alpha}{1-\alpha}} - \kappa > \left(\kappa^{\frac{1-\alpha}{\alpha}}\right)^{\frac{\alpha}{1-\alpha}} - \kappa = 0.$$

We conclude that $dz^*/dw > 0$. All the results listed in the proposition follow given the relationship between z^* and the equilibrium variables described in the proof of Proposition 6. □