Optimal Unemployment Insurance over the Business Cycle

Camille Landais, Pascal Michaillat, Emmanuel Saez

SIEPR, LSE, UC Berkeley

August 2011
Literature on Unemployment Insurance

- Optimal benefit level:
  - Baily ['78]
  - Chetty ['06]

- Optimal benefit levels over unemployment spell:
  - Shavell and Weiss ['79]
  - Hopenhayn and Nicolini ['97]
  - Shimer and Werning ['08]

- Optimal benefit levels over business cycle: –
Framework

- Model of equilibrium unemployment [Pissarides, ’00]
- Risk-averse workers, no self-insurance
- Unobservable job-search efforts [Baily, ’78]
- Recessions
  - real wage rigidity [Hall, ’05]
- Job rationing [Michaillat, forthcoming]
  - real wage rigidity & downward-sloping demand for labor
Framework

- Model of equilibrium unemployment [Pissarides, ’00]
- Risk-averse workers, no self-insurance
- Unobservable job-search efforts [Baily, ’78]

Recessions
- real wage rigidity [Hall, ’05]

Job rationing [Michaillat, forthcoming]
- real wage rigidity & downward-sloping demand for labor
Framework

- Model of equilibrium unemployment [Pissarides, ’00]
- Risk-averse workers, no self-insurance
- Unobservable job-search efforts [Baily, ’78]
- Recessions
  - real wage rigidity [Hall, ’05]
- Job rationing [Michaillat, forthcoming]
  - real wage rigidity & downward-sloping demand for labor
Why Job Rationing in Recessions?
Why Job Rationing in Recessions?
Why Job Rationing in Recessions?
Overview of Results

In recessions, unemployment insurance (UI) should be

- constant?
- more generous?
- less generous?
Overview of Results

In recessions, unemployment insurance (UI) should be

- constant
- more generous: \[
\frac{\text{Consumption of unemployed}}{\text{Consumption of employed}} \uparrow
\]
- less generous
What Happens in Recessions?

Diagram

Landais, Michaillat, and Saez (08/2011)  Optimal Unemployment Insurance

0.1
Unemployment rate

0.08
0.06
0.04
0.02
0


Rationing unemp.

Frictional unemp.

Diagram
What Happens in Recessions?

- Marginal benefits:
  - insurance
  - correction for negative *rat-race externality*

- Marginal cost:
  - increase of aggregate unemployment

UI
What Happens in Recessions?

- **Marginal benefits:**
  - insurance →
  - correction for negative *rat-race externality* ↑

- **Marginal cost:**
  - increase of aggregate unemployment ↓

- UI ↑
Outline of the Paper

1. Optimal UI formula: $\tau = \tau(\epsilon_m, \epsilon^M, \text{risk aversion})$
   - $\tau = c^u/c^e$: replacement rate
   - in generic model of equilibrium unemployment
   - formula in *sufficient statistics*

2. Optimal UI over the business cycle
   - model of recessions and job rationing [Michaillat, forthcoming]
   - characterize elasticities $\epsilon_m, \epsilon^M$ over business cycle
   - prove: optimal $\tau$ increases in recessions

3. Extension to an infinite-horizon model
   - verify robustness of theoretical results
   - extensions: (1) optimal UI with deficit spending; (2) optimal duration of benefits
1. Optimal UI Formula: $\tau = \tau(\epsilon^m, \epsilon^M, \text{risk aversion})$

2. Optimal UI over the Business Cycle

3. Extension to an Infinite-Horizon Model
UI Program

- Government gives $c^e$ to $n$ employed workers
- Government gives $c^u$ to $1 - n$ unemployed workers
- Budget constraint: $n \cdot w = n \cdot c^e + (1 - n) \cdot c^u$
One-Period Model with Matching Frictions

- Initial number of unemployed workers: $u$
- Job-search effort: $e$
- Job openings: $o$
- Number of matches: $h = m(e \cdot u, o)$
- Labor market tightness: $\theta \equiv o / (e \cdot u)$
- Vacancy-filling proba.: $q(\theta) = m(1/\theta, 1)$
- Job-finding proba.: $e \cdot f(\theta) = e \cdot m(1, \theta)$
One-Period Model with Matching Frictions

- Initial number of unemployed workers: $u$
- Job-search effort: $e$
- Job openings: $o$
- Number of matches: $h = m(e \cdot u, o)$
- Labor market tightness: $\theta \equiv o/(e \cdot u)$
- Vacancy-filling proba.: $q(\theta) = m(1/\theta, 1)$
- Job-finding proba.: $e \cdot f(\theta) = e \cdot m(1, \theta)$
One-Period Model with Matching Frictions

- Initial number of unemployed workers: $u$
- Job-search effort: $e$
- Job openings: $o$
- Number of matches: $h = m(e \cdot u, o)$
- Labor market tightness: $\theta \equiv o / (e \cdot u)$
- Vacancy-filling proba.: $q(\theta) = m(1/\theta, 1)$
- Job-finding proba.: $e \cdot f(\theta) = e \cdot m(1, \theta)$
One-Period Model with Matching Frictions

- Initial number of unemployed workers: $u$
- Job-search effort: $e$
- Job openings: $o$
- Number of matches: $h = m(e \cdot u, o)$
- Labor market tightness: $\theta \equiv o / (e \cdot u)$
- Vacancy-filling proba.: $q(\theta) = m(1/\theta, 1)$
- Job-finding proba.: $e \cdot f(\theta) = e \cdot m(1, \theta)$
Flows of Workers

Employed: 1-u

Unemployed: u
Flows of Workers

Job finding: \( e.f(\theta).u \)

Employed: \( n \)

Unemployed: \( u \)
Unemployed Worker’s Problem

- Given $\theta, \Delta v = v(c^e) - v(c^u)$, choose $e$ to maximize

$$v(c^u) + e \cdot f(\theta) \cdot \Delta v - k(e)$$

- Utility-maximizing effort $e(\theta, \Delta v)$:

$$k'(e) = f(\theta) \cdot \Delta v$$

- Aggregate labor supply:

$$n^s(e(\theta, \Delta v), \theta) = (1 - u) + e(\theta, \Delta v) \cdot f(\theta) \cdot u$$
Unemployed Worker’s Problem

Given \( \theta, \Delta v = v(c^e) - v(c^u) \), choose \( e \) to maximize

\[
    v(c^u) + e \cdot f(\theta) \cdot \Delta v - k(e)
\]

Utility-maximizing effort \( e(\theta, \Delta v) \):

\[
    k'(e) = f(\theta) \cdot \Delta v
\]

Aggregate labor supply:

\[
    n^s(e(\theta, \Delta v), \theta) = (1 - u) + e(\theta, \Delta v) \cdot f(\theta) \cdot u
\]
Unemployed Worker’s Problem

- Given $\theta, \Delta v = v(c^e) - v(c^u)$, choose $e$ to maximize

$$v(c^u) + e \cdot f(\theta) \cdot \Delta v - k(e)$$

- Utility-maximizing effort $e(\theta, \Delta v)$:

$$k'(e) = f(\theta) \cdot \Delta v$$

- Aggregate labor supply:

$$n^s(e(\theta, \Delta v), \theta) = (1 - u) + e(\theta, \Delta v) \cdot f(\theta) \cdot u$$
Unemployed Worker’s Problem

- Given $\theta, \Delta v = v(c^e) - v(c^u)$, choose $e$ to maximize
  
  $$v(c^u) + e \cdot f(\theta) \cdot \Delta v - k(e)$$

- Utility-maximizing effort $e(\theta, \Delta v)$:
  
  $$k'(e) = f(\theta) \cdot \Delta v$$

- Aggregate labor supply:
  
  $$n^s(e(\theta, \Delta v), \theta) = (1 - u) + e(\theta, \Delta v) \cdot f(\theta) \cdot u$$
Unemployed Worker’s Problem

- Given $\theta, \Delta v = v(c^e) - v(c^u)$, choose $e$ to maximize

$$v(c^u) + e \cdot f(\theta) \cdot \Delta v - k(e)$$

- Utility-maximizing effort $e(\theta, \Delta v)$:

$$k'(e) = f(\theta) \cdot \Delta v$$

- Aggregate labor supply:

$$n^s(e(\theta, \Delta v), \theta) = (1 - u) + e(\theta, \Delta v) \cdot f(\theta) \cdot u$$
Unemployed Worker’s Problem

- Given $\theta, \Delta v = v(c^e) - v(c^u)$, choose $e$ to maximize
  $$v(c^u) + e \cdot f(\theta) \cdot \Delta v - k(e)$$

- Utility-maximizing effort $e(\theta, \Delta v)$:
  $$k'(e) = f(\theta) \cdot \Delta v$$

- Aggregate labor supply:
  $$n^s(e(\theta, \Delta v), \theta) = (1 - u) + e(\theta, \Delta v) \cdot f(\theta) \cdot u$$
Response of Labor Supply to Lower UI

![Graph showing the response of labor supply to lower unemployment insurance (UI). The x-axis represents employment n, and the y-axis represents labor market tightness θ. The graph illustrates how labor supply (high UI) changes as employment and labor market tightness vary.]

Landais, Michaillat, and Saez (08/2011)  
Optimal Unemployment Insurance
Response of Labor Supply to Lower UI

![Graph showing the response of labor supply to lower UI](image)

- **Labor supply (high UI)**
- **Labor supply (low UI)**

Landais, Michaillat, and Saez (08/2011) Optimal Unemployment Insurance
Government’s Problem

Choose \( c^e, c^u \) to maximize

\[
n \cdot v(c^e) + (1 - n) \cdot v(c^u) - u \cdot k(e)
\]

subject to:

- \( \Delta v = v(c^e) - v(c^u) \)
- budget: \( n \cdot c^e + (1 - n) \cdot c^u = n \cdot w \)
- labor market dynamics: \( n = (1 - u) + u \cdot e \cdot f(\theta) \)
- optimal job search: \( e = e(\theta, \Delta v) \)
- labor market clearing: \( n^d(\theta) = n^s(e(\theta, \Delta v), \theta) \)
Government’s Problem

Choose $c^e$, $c^u$ to maximize

$$n \cdot v(c^e) + (1 - n) \cdot v(c^u) - u \cdot k(e)$$

subject to:

- $\Delta v = v(c^e) - v(c^u)$
- budget: $n \cdot c^e + (1 - n) \cdot c^u = n \cdot w$
- labor market dynamics: $n = (1 - u) + u \cdot e \cdot f(\theta)$
- optimal job search: $e = e(\theta, \Delta v)$
- labor market clearing: $n^d(\theta) = n^s(e(\theta, \Delta v), \theta)$
Government’s Problem

Choose $c^e$, $c^u$ to maximize

$$n \cdot v(c^e) + (1 - n) \cdot v(c^u) - u \cdot k(e)$$

subject to:

- $\Delta v = v(c^e) - v(c^u)$
- budget: $n \cdot c^e + (1 - n) \cdot c^u = n \cdot w$
- labor market dynamics: $n = (1 - u) + u \cdot e \cdot f(\theta)$
- optimal job search: $e = e(\theta, \Delta v)$
- labor market clearing: $n^d(\theta) = n^s(e(\theta, \Delta v), \theta)$
Government’s Problem

Choose $c^e, c^u$ to maximize

$$n \cdot v(c^e) + (1 - n) \cdot v(c^u) - u \cdot k(e)$$

subject to:

- $\Delta v = v(c^e) - v(c^u)$
- budget: $n \cdot c^e + (1 - n) \cdot c^u = n \cdot w$
- labor market dynamics: $n = (1 - u) + u \cdot e \cdot f(\theta)$
- optimal job search: $e = e(\theta, \Delta v)$
- labor market clearing: $n^d(\theta) = n^s(e(\theta, \Delta v), \theta)$
Government’s Problem

Choose \( c^e, \ c^u \) to maximize

\[
n \cdot v(c^e) + (1 - n) \cdot v(c^u) - u \cdot k(e)
\]

subject to:

- \( \Delta v = v(c^e) - v(c^u) \)
- budget: \( n \cdot c^e + (1 - n) \cdot c^u = n \cdot w \)
- labor market dynamics: \( n = (1 - u) + u \cdot e \cdot f(\theta) \)
- optimal job search: \( e = e(\theta, \Delta v) \)
- labor market clearing: \( n^d(\theta) = n^s(e(\theta, \Delta v), \theta) \)
Micro-Elasticity $\epsilon^m$

$$\epsilon^m \equiv \frac{\Delta c}{1 - n} \cdot \frac{\partial n^s}{\partial e} \bigg|_{\theta} \cdot \frac{\partial e}{\partial \Delta c} \bigg|_{\theta}$$

- Response of individual job-search effort
- Elasticity used in the literature [Baily, '78]
- Interpretation: increase in probability of unemployment when individual UI increases
Macro-Elasticity $\epsilon^M$

\[ \epsilon^M \equiv \frac{\Delta c}{1 - n} \cdot \frac{dn}{d\Delta c} \]

- Response of aggregate unemployment
- Interpretation: increase in aggregate unemployment when aggregate UI increases
- Macro-elasticity = micro-elasticity + unemployment change due to equilibrium adjustment of $\theta$

\[ \epsilon^M = \epsilon^m + \frac{\Delta c}{1 - n} \cdot \frac{1 + \kappa}{\kappa} \cdot \left( \frac{\partial n^s}{\partial \theta} \bigg|_e \cdot \frac{d\theta}{d\Delta c} \right) \]
Macro-Elasticity $\epsilon^M$

$$\epsilon^M \equiv \frac{\Delta c}{1 - n} \cdot \frac{dn}{d\Delta c}$$

- Response of aggregate unemployment
- Interpretation: increase in aggregate unemployment when aggregate UI increases
- Macro-elasticity = micro-elasticity + unemployment change due to equilibrium adjustment of $\theta$

$$\epsilon^M = \epsilon^m + \frac{\Delta c}{1 - n} \cdot \frac{1 + \kappa}{\kappa} \cdot \left( \frac{\partial n^s}{\partial \theta} \bigg|_e \cdot \frac{d\theta}{d\Delta c} \right)$$
Macro-Elasticity $\epsilon^M$

$$\epsilon^M \equiv \frac{\Delta c}{1-n} \cdot \frac{dn}{d\Delta c}$$

- Response of aggregate unemployment
- Interpretation: increase in aggregate unemployment when aggregate UI increases
- Macro-elasticity = micro-elasticity + unemployment change due to equilibrium adjustment of $\theta$

$$\epsilon^M = \epsilon^m + \frac{\Delta c}{1-n} \cdot \frac{1+\kappa}{\kappa} \cdot \left( \frac{\partial n^s}{\partial \theta} \right)_{e} \cdot \frac{d\theta}{d\Delta c}$$
Macro-Elasticity $\epsilon^M$

$$\epsilon^M \equiv \frac{\Delta c}{1 - n} \cdot \frac{dn}{d\Delta c}$$

- Response of aggregate unemployment
- Interpretation: increase in aggregate unemployment when aggregate UI increases
- Macro-elasticity $= \text{micro-elasticity} + \text{unemployment change due to equilibrium adjustment of } \theta$

$$\epsilon^M = \epsilon^m + \frac{\Delta c}{1 - n} \cdot \frac{1 + \kappa}{\kappa} \cdot \left( \frac{\partial n^s}{\partial \theta} \bigg|_e \cdot \frac{d\theta}{d\Delta c} \right)$$
Exact Optimal UI Formula

\[
\frac{1}{n} \cdot \frac{\tau}{1 - \tau} = \left[ n + (1 - n) \cdot \frac{v'(c^u)}{v'(c^e)} \right]^{-1} \\
\cdot \left\{ \frac{n}{\epsilon^M} \cdot \left[ \frac{v'(c^u)}{v'(c^e)} - 1 \right] + \frac{\Delta v}{v'(c^e) \cdot \Delta c} \cdot \frac{\kappa}{\kappa + 1} \cdot \left[ \frac{\epsilon^m}{\epsilon^M} - 1 \right] \right\}
\]
Optimal UI Formula in Sufficient Statistics

\[
\frac{\tau}{1 - \tau} \approx \frac{\rho}{\epsilon^M} \cdot (1 - \tau) + \frac{\kappa}{1 + \kappa} \cdot \left[ \frac{\epsilon^m}{\epsilon^M} - 1 \right] \cdot \left[ 1 + \frac{\rho}{2} \cdot (1 - \tau) \right]
\]

- \( \tau \): replacement rate \( c^u/c^e \)
- \( \rho \): coefficient of relative risk aversion
- \( \kappa \): elasticity of marginal disutility of effort
- \( \epsilon^M \): macro-elasticity of unemployment
- \( \epsilon^m \): micro-elasticity of unemployment
Building on the Baily ['78] Formula

\[ \frac{\tau}{1 - \tau} \approx \frac{\rho}{\epsilon^m} (1 - \tau) \]

- Public economics: Baily ['78], Chetty ['06]
- Government budget constraint in general equilibrium
- Correction for equilibrium adjustment of \( \theta \), with first-order welfare effect:

\[ \left. \frac{\partial n^s}{\partial \theta} \right|_e \cdot d\theta \propto [\epsilon^m - \epsilon^M] \]
Building on the Baily [’78] Formula

\[
\frac{\tau}{1 - \tau} \approx \frac{\rho}{\epsilon^M} (1 - \tau)
\]

- Public economics: Baily [’78], Chetty [’06]
- Government budget constraint in general equilibrium
- Correction for equilibrium adjustment of \( \theta \), with first-order welfare effect:

\[
\left. \frac{\partial n^s}{\partial \theta} \right|_e \cdot d\theta \propto [\epsilon^m - \epsilon^M]
\]
Building on the Baily ['78] Formula

\[
\frac{\tau}{1 - \tau} \approx \frac{\rho}{\epsilon^M} (1 - \tau) + \frac{\kappa}{1 + \kappa} \left[ \frac{\epsilon^m}{\epsilon^M} - 1 \right] \left[ 1 + \frac{\rho}{2}(1 - \tau) \right]
\]

- Public economics: Baily ['78], Chetty ['06]
- Government budget constraint in general equilibrium
- Correction for equilibrium adjustment of $\theta$, with first-order welfare effect:

\[
\left. \frac{\partial n^s}{\partial \theta} \right|_e \cdot d\theta \propto \left[ \epsilon^m - \epsilon^M \right]
\]
Optimal UI Formula: \( \tau = \tau(\epsilon^m, \epsilon^M, \text{risk aversion}) \)

Optimal UI over the Business Cycle

Extension to an Infinite-Horizon Model
A Model of Recessions and Job Rationing

- Given \((\theta, a)\), firm chooses \(n \geq 1 - u\) to maximize

\[
 a \cdot n^\alpha - \omega \cdot a^\gamma \cdot n - \frac{r \cdot a}{q(\theta)} \cdot [n - (1 - u)]
\]

- Profit-maximizing employment \(n^d(\theta, a)\):

\[
 \alpha \cdot n^{\alpha - 1} = \omega \cdot a^{\gamma - 1} + \frac{r}{q(\theta)}
\]

- Wage rigidity: \(\gamma < 1\)

- Diminishing marginal returns to labor: \(\alpha < 1\)
A Model of Recessions and Job Rationing

Given \((\theta, a)\), firm chooses \(n \geq 1 - u\) to maximize

\[
\underbrace{a \cdot n^\alpha}_{\text{production}} - \underbrace{\omega \cdot a^\gamma \cdot n}_{\text{wage}} - \underbrace{\frac{r \cdot a}{q(\theta)} \cdot [n - (1 - u)]}_{\text{hiring cost}}
\]

Profit-maximizing employment \(n^d(\theta, a)\):

\[
\alpha \cdot n^{\alpha - 1} = \omega \cdot a^{\gamma - 1} + \frac{r}{q(\theta)}
\]

Wage rigidity: \(\gamma < 1\)

Diminishing marginal returns to labor: \(\alpha < 1\)
A Model of Recessions and Job Rationing

- Given \((\theta, a)\), firm chooses \(n \geq 1 - u\) to maximize

\[
\underbrace{a \cdot n^\alpha}_{\text{production}} - \underbrace{\omega \cdot a^\gamma \cdot n}_{\text{wage}} - \underbrace{\frac{r \cdot a}{q(\theta)} \cdot [n - (1 - u)]}_{\text{hiring cost}}
\]

- Profit-maximizing employment \(n^d(\theta, a)\):

\[
\alpha \cdot n^\alpha - 1 = \omega \cdot a^\gamma - 1 + \frac{r}{q(\theta)}
\]

- Wage rigidity: \(\gamma < 1\)

- Diminishing marginal returns to labor: \(\alpha < 1\)
A Model of Recessions and Job Rationing

- Given \((\theta, a)\), firm chooses \(n \geq 1 - u\) to maximize

\[
\underbrace{a \cdot n^\alpha}_{\text{production}} - \underbrace{\omega \cdot a^\gamma \cdot n - \frac{r \cdot a}{q(\theta)}}_{\text{wage} \quad \text{cost}} \cdot [n - (1 - u)]
\]

- Profit-maximizing employment \(n^d(\theta, a)\):

\[
\alpha \cdot n^{\alpha - 1} = \omega \cdot a^{\gamma - 1} + \frac{r}{q(\theta)}
\]

- Wage rigidity: \(\gamma < 1\)

- Diminishing marginal returns to labor: \(\alpha < 1\)
A Model of Recessions and Job Rationing

- Given \((\theta, a)\), firm chooses \(n \geq 1 - u\) to maximize

\[
a \cdot n^\alpha - \omega \cdot a^\gamma \cdot n - \frac{r \cdot a}{q(\theta)} \cdot [n - (1 - u)]
\]

- Profit-maximizing employment \(n^d(\theta, a)\):

\[
\alpha \cdot n^{\alpha-1} = \omega \cdot a^{\gamma-1} + \frac{r}{q(\theta)}
\]

- Wage rigidity: \(\gamma < 1\)

- Diminishing marginal returns to labor: \(\alpha < 1\)
A Model of Recessions and Job Rationing

- Given \((\theta, a)\), firm chooses \(n \geq 1 - u\) to maximize

\[
\text{production: } a \cdot n^\alpha - \text{wage: } \omega \cdot a^\gamma \cdot n - \frac{r \cdot a}{q(\theta)} \cdot [n - (1 - u)]
\]

- Profit-maximizing employment \(n^d(\theta, a)\):

\[
\alpha \cdot n^{\alpha - 1} = \omega \cdot a^{\gamma - 1} + \frac{r}{q(\theta)}
\]

- Wage rigidity: \(\gamma < 1\)

- Diminishing marginal returns to labor: \(\alpha < 1\)
A Model of Recessions and Job Rationing

- Given \((\theta, a)\), firm chooses \(n \geq 1 - u\) to maximize

\[
\underbrace{a \cdot n^\alpha}_{\text{production}} - \underbrace{\omega \cdot a^\gamma \cdot n}_{\text{wage}} - \underbrace{\frac{r \cdot a}{q(\theta)} \cdot [n - (1 - u)]}_{\text{hiring cost}}
\]

- Profit-maximizing employment \(n^d(\theta, a):\)

\[
\alpha \cdot n^{\alpha - 1} = \omega \cdot a^{\gamma - 1} + \frac{r}{q(\theta)}
\]

- Wage rigidity: \(\gamma < 1\)

- Diminishing marginal returns to labor: \(\alpha < 1\)
Labor Demand over the Business Cycle

Landais, Michaillat, and Saez (08/2011)
Labor Demand over the Business Cycle

- Labor demand (boom)
- Labor demand (recession)

Landais, Michaillat, and Saez (08/2011) Optimal Unemployment Insurance
Baily [’78] Partial-Equilibrium Model

![Graph showing Labor market tightness (θ) vs. Employment (n). The graph illustrates the labor supply (high UI) and highlights a fixed θ value.](image-url)
Baily ['78] Partial-Equilibrium Model

![Graph showing the relationship between employment n and labor market tightness \( \theta \).]

- Labor supply (high UI)
- Labor supply (low UI)

- Fixed \( \theta \)

Landais, Michaillat, and Saez (08/2011) Optimal Unemployment Insurance
Baily [’78] Partial-Equilibrium Model

![Graph showing labor market tightness and employment](image)

- Labor supply (high UI)
- Labor supply (low UI)

Labor market tightness \( \theta \)

Employment \( n \)

\( \varepsilon_m \)
Our General-Equilibrium Model

\begin{figure}
\centering
\includegraphics[width=\textwidth]{equilibrium.png}
\caption{Equilibrium \((\theta, n)\) with Labor supply (high UI) and Labor demand (boom).}
\end{figure}
Our General-Equilibrium Model

Landais, Michaillat, and Saez (08/2011)  Optimal Unemployment Insurance
Our General-Equilibrium Model

Landais, Michaillat, and Saez (08/2011) Optimal Unemployment Insurance
Our General-Equilibrium Model
Our General-Equilibrium Model

![Graph showing labor market tightness and employment](image)

Landais, Michaillat, and Saez (08/2011)
Micro-Elasticity $\epsilon^m >$ Macro-Elasticity $\epsilon^M$

- Positive wedge between $\epsilon^m$ and $\epsilon^M$:
  \[ \epsilon^m > \epsilon^M \]

- Estimable statistic:
  \[ \left[ \epsilon^m - \epsilon^M \right] \propto \frac{\Delta c}{\theta} \cdot \frac{d\theta}{d\Delta c} \]

- Testable implication:
  - model with Nash bargaining [Pissarides, ’00]:
    \[ \epsilon^m < \epsilon^M \]
  - model with rigid wages [Hall, ’05]:
    \[ \epsilon^m = \epsilon^M \]
Micro-Elasticity $\epsilon^m >$ Macro-Elasticity $\epsilon^M$

- Positive wedge between $\epsilon^m$ and $\epsilon^M$:
  $$\epsilon^m > \epsilon^M$$

- Estimable statistic:
  $$\left[ \epsilon^m - \epsilon^M \right] \propto \frac{\Delta c}{\theta} \cdot \frac{d\theta}{d\Delta c}$$

- Testable implication:
  - model with Nash bargaining [Pissarides, ’00]:
    $$\epsilon^m < \epsilon^M$$
  - model with rigid wages [Hall, ’05]:
    $$\epsilon^m = \epsilon^M$$
**Micro-Elasticity $\epsilon^m >$ Macro-Elasticity $\epsilon^M$**

- Positive wedge between $\epsilon^m$ and $\epsilon^M$:
  \[ \epsilon^m > \epsilon^M \]

- Estimable statistic:
  \[ [\epsilon^m - \epsilon^M] \propto \frac{\Delta c}{\theta} \cdot \frac{d\theta}{d\Delta c} \]

- Testable implication:
  - model with Nash bargaining [Pissarides, '00]:
    \[ \epsilon^m < \epsilon^M \]
  - model with rigid wages [Hall, '05]:
    \[ \epsilon^m = \epsilon^M \]

Landais, Michaillat, and Saez (08/2011)
Micro-Elasticity $\epsilon^m >$ Macro-Elasticity $\epsilon^M$

- Positive wedge between $\epsilon^m$ and $\epsilon^M$:

$$\epsilon^m > \epsilon^M$$

- Estimable statistic:

$$\left[ \epsilon^m - \epsilon^M \right] \propto \frac{\Delta c}{\theta} \cdot \frac{d\theta}{d\Delta c}$$

- Testable implication:
  - model with Nash bargaining [Pissarides, ’00]:

$$\epsilon^m < \epsilon^M$$

  - model with rigid wages [Hall, ’05]:

$$\epsilon^m = \epsilon^M$$
Effect of Lower UI in Expansion

Landais, Michaillat, and Saez (08/2011)
Effect of Lower UI in Expansion

Landais, Michaillat, and Saez (08/2011) Optimal Unemployment Insurance
Effect of Lower UI in Expansion

![Graph showing the effect of lower UI in expansion]

- Labor supply (high UI)
- Labor supply (low UI)
- Labor demand (boom)

Equation: \( \varepsilon^m - \varepsilon^M \)

Landais, Michaillat, and Saez (08/2011)
Effect of Lower UI in Recession

Landais, Michaillat, and Saez (08/2011) Optimal Unemployment Insurance
Effect of Lower UI in Recession

Landais, Michaillat, and Saez (08/2011)
Effect of Lower UI in Recession

![Graph showing the effect of lower UI in recession. The graph plots labor market tightness against employment. Different curves represent labor supply (high and low UI) and labor demand (recession). The graph includes labels for labor supply and demand, as well as the employment level.](image-url)
Effect of Lower UI in Recession

![Graph showing the effect of lower UI in recession. The graph plots employment (n) against labor market tightness (θ). The labor supply for high UI and low UI is represented by dotted and solid lines, respectively. The labor demand during recession is shown with a dashed line. The graph illustrates the movement of the economy from a higher UI to a lower UI environment, indicating a decrease in labor supply and an increase in labor demand.](image-url)
Effect of Lower UI in Recession

![Graph showing the effect of lower UI in recession](image)

- Labor supply (high UI)
- Labor supply (low UI)
- Labor demand (recession)

Landais, Michaillat, and Saez (08/2011)
Cyclicality of Elasticities

- Assume: isoelastic utility functions, Cobb-Douglas matching function

- Wedge $\frac{\epsilon^m}{\epsilon^M} > 1$ is countercyclical:

$$\frac{\partial}{\partial a} \left( \frac{\epsilon^m}{\epsilon^M} \right) \bigg|_\tau < 0$$

- Macro-elasticity $\epsilon^M$ is procyclical:

$$\frac{\partial \epsilon^M}{\partial a} \bigg|_\tau > 0$$
Intuition for Optimal UI in Recession

\[
\frac{\tau}{1 - \tau} \approx \frac{\rho}{\epsilon M} \cdot (1 - \tau) + \frac{\kappa}{1 + \kappa} \cdot \left[ \frac{\epsilon^m}{\epsilon M} - 1 \right] \cdot \left[ 1 + \frac{\rho}{2} \cdot (1 - \tau) \right]
\]

- Small impact of UI on unemployment: \( \epsilon^M \downarrow \)
- Strong rat-race externality: \( \epsilon^m/\epsilon^M \uparrow \)
- \( \tau \uparrow \): UI should be more generous
Intuition for Optimal UI in Recession

\[
\frac{\tau}{1 - \tau} \approx \frac{\rho}{\epsilon^M} \cdot (1 - \tau) + \frac{\kappa}{1 + \kappa} \cdot \left[ \frac{\epsilon^m}{\epsilon^M} - 1 \right] \cdot \left[ 1 + \frac{\rho}{2} \cdot (1 - \tau) \right]
\]

- Small impact of UI on unemployment: \( \epsilon^M \downarrow \)
- Strong rat-race externality: \( \epsilon^m / \epsilon^M \uparrow \)
- \( \tau \uparrow \): UI should be more generous
Intuition for Optimal UI in Recession

\[
\frac{\tau}{1 - \tau} \approx \frac{\rho}{\epsilon^M} \cdot (1 - \tau) + \frac{\kappa}{1 + \kappa} \cdot \left[ \frac{\epsilon^m}{\epsilon^M} - 1 \right] \cdot \left[ 1 + \frac{\rho}{2} \cdot (1 - \tau) \right]
\]

- Small impact of UI on unemployment: $\epsilon^M \downarrow$
- Strong rat-race externality: $\epsilon^m / \epsilon^M \uparrow$
- $\tau \uparrow$: UI should be more generous
Intuition for Optimal UI in Recession

\[ \frac{\tau}{1 - \tau} \approx \frac{\rho}{\epsilon M} \cdot (1 - \tau) + \frac{\kappa}{1 + \kappa} \cdot \left[ \frac{\epsilon^m}{\epsilon M} - 1 \right] \cdot \left[ 1 + \frac{\rho}{2} \cdot (1 - \tau) \right] \]

- Small impact of UI on unemployment: \( \epsilon^M \downarrow \)
- Strong rat-race externality: \( \epsilon^m / \epsilon^M \uparrow \)
- \( \tau \uparrow: \) UI should be more generous
Optimal Replacement Rate $\tau$ is Countercyclical

- $\tau = c^u/c^e$ captures generosity of UI
- Use exact optimal UI formula
- Prove: optimal UI is more generous in recessions:
  $$ \frac{d\tau}{da} < 0 $$
1 Optimal UI Formula: $\tau = \tau(\epsilon^m, \epsilon^M, \text{risk aversion})$

2 Optimal UI over the Business Cycle

3 Extension to an Infinite-Horizon Model
Flows of Workers

Job finding: $e.f(\theta).u$

Employed: $n$

Unemployed: $u$
Flows of Workers

Job finding: $e.f(\theta).u$

Employed: $n$

Unemployed: $u$

Job destruction: $s.n$
Stochastic Environment

- Fluctuations are driven by technology $\{a_t\}_{t=0}^{\infty}$.
- All workers receive the same $c_t^e$ (if employed) and $c_t^u$ (if unemployed) at time $t$.
- Firm’s, worker’s, and government’s decisions at time $t$ are measurable wrt $a^t = (a_0, a_1, \ldots, a_t)$.
- Government can commit to policy.
Worker’s Problem (Labor Supply)

- Given \( \{ a_t, \theta_t, c_t^e, c_t^u \}_{t=0}^{\infty} \),

- Choose job-search effort \( \{ e_t \}_{t=0}^{\infty} \)

- To maximize expected utility:

\[
E_0 \sum_{t=0}^{+\infty} \delta^t \{ (1 - n_t^s) v(c_t^u) + n_t^s v(c_t^e) - [1 - (1 - s)n_{t-1}^s] k(e_t) \}
\]

- Subject to

\[
n_t^s = [1 - (1 - s) \cdot n_{t-1}^s] \cdot e_t \cdot f(\theta_t) + (1 - s) \cdot n_{t-1}^s.
\]
Firm’s Problem (Labor Demand)

- Given \( \{a_t, \theta_t, w_t\}_{t=0}^{\infty} \)
- Choose hiring \( \{h_t\}_{t=0}^{\infty} \)
- To maximize expected profits:

\[
\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \left\{ a_t \cdot (n_t^d)^\alpha - w_t \cdot n_t^d - \frac{r \cdot a_t}{q(\theta_t)} \cdot h_t \right\}
\]

- Subject to:

\[
n_t^d = (1 - s) \cdot n_{t-1}^d + h_t.
\]
Equilibrium on the Labor Market

- Wage is indeterminate
  \[ w_t = \omega \cdot a_t^\gamma, \quad \gamma < 1 \]

- Tightness \( \theta \) equalizes labor supply and labor demand
  \[ n_t \equiv n_t^s = n_t^d \]
Government’s Problem

- Given \( \{ a_t \}_{t=0}^{\infty} \)
- Choose consumptions \( \{ c_t^e, c_t^u \}_{t=0}^{\infty} \)
- To maximize worker’s expected utility

\[
\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \left\{ (1 - n_t) \nu(c_t^u) + n_t \nu(c_t^e) - [1 - (1 - s)n_{t-1}] k(e_t) \right\}
\]

- Subject to worker’s and firm’s optimality conditions, equilibrium conditions, and budget constraints

\[
n_t \cdot w_t = n_t \cdot c_t^e + (1 - n_t) \cdot c_t^u.
\]
# Calibration: US, weekly frequency

<table>
<thead>
<tr>
<th>Interpretation</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ Relative risk aversion</td>
<td>1</td>
<td>Chetty ['06]</td>
</tr>
<tr>
<td>$\gamma$ Real wage rigidity</td>
<td>0.5</td>
<td>Haefke et al. ['08], Pissarides ['09]</td>
</tr>
<tr>
<td>$\eta$ Effort-elasticity of matching</td>
<td>0.7</td>
<td>Petrongolo &amp; Pissarides ['01]</td>
</tr>
<tr>
<td>$s$ Separation rate</td>
<td>0.95%</td>
<td>JOLTS, 2000–2010</td>
</tr>
<tr>
<td>$\omega_m$ Effectiveness of matching</td>
<td>0.23</td>
<td>JOLTS, 2000–2010</td>
</tr>
<tr>
<td>$r$ Recruiting cost</td>
<td>0.21</td>
<td>Barron et al. ['97], Silva &amp; Toledo ['09]</td>
</tr>
<tr>
<td>$\alpha$ Marginal returns to labor</td>
<td>0.67</td>
<td>Matches labor share= 0.66</td>
</tr>
<tr>
<td>$\omega$ Steady-state real wage</td>
<td>0.67</td>
<td>Matches unemployment= 5.9%</td>
</tr>
<tr>
<td>$\kappa$ Curvature of disutility of effort</td>
<td>2.1</td>
<td>Matches Meyer ['90]</td>
</tr>
<tr>
<td>$\omega_k$ Disutility of effort</td>
<td>0.58</td>
<td>Matches effort = 1 for $t = 7.65%$, $b = 60%$</td>
</tr>
</tbody>
</table>
Steady State: Optimal Replacement Rate

\[
\text{Replacement rate } \tau \quad \text{vs. Unemployment rate}
\]

Landais, Michaillat, and Saez (08/2011)
Steady State: Optimal Benefit Rate

Unemployment rate vs. Benefit rate graph.
Steady State: Optimal Labor Tax Rate

Landais, Michaillat, and Saez (08/2011) Optimal Unemployment Insurance
Comparison with Baily ['78] Formula

![Graph showing elasticity of (1-n) with respect to ΔC against unemployment rate]

- Elasticity of (1-n) wrt $\Delta C$
- Unemployment rate

Microsimulation

Optimal UI formula in infinite-horizon model
Comparison with Baily ['78] Formula

![Graph showing the comparison between Baily's formula and the micro-elasticity model.](image)

- Baily with micro-elasticity

Landais, Michaillat, and Saez (08/2011)  |  Optimal Unemployment Insurance
Comparison with Baily [’78] Formula

Optimal UI formula in infinite-horizon model

Landais, Michaillat, and Saez (08/2011) Optimal Unemployment Insurance
Comparison with Baily ['78] Formula

![Graph showing comparison between Baily's formula with micro-elasticity and macro-elasticity against unemployment rate and replacement rate.](image)

- **Baily with micro-elasticity**
- **Baily with macro-elasticity**

**Optimal UI formula in infinite-horizon model**

Landais, Michaillat, and Saez (08/2011)
Comparison with Baily [’78] Formula

\[
\epsilon_m / \epsilon_M - 1 > 0
\]

Elasticity of \((1-n)\) wrt \(\Delta C\)

- **Micro**
- **Macro**

Optimal UI formula in infinite-horizon model

Landais, Michaillat, and Saez (08/2011)  Optimal Unemployment Insurance
Comparison with Baily [’78] Formula

- Baily with micro-elasticity
- Baily with macro-elasticity
- Approximated formula

Optimal UI formula in infinite-horizon model
Comparison with Baily ['78] Formula

Unemployment rate

Optimal UI formula in infinite-horizon model

Landais, Michaillat, and Saez (08/2011)  Optimal Unemployment Insurance

38 / 42
Dynamics: Government Cannot Borrow

- Technology
- Unemployment
- Replacement rate $\tau$
- Deficit
- Consumption (employed)
- Consumption (unemployed)

Weeks after shock: 0 50 100 150 200 250 300

-0.5% 0% 0.5% 1%
Dynamics: Government Can Borrow

Technology

No deficit spending
Deficit spending

Unemployment

Replacement rate $\tau$

Deficit

Consumption (employed)

Consumption (unemployed)

Weeks after shock

Landais, Michaillat, and Saez (08/2011) Optimal Unemployment Insurance 39 / 42
Flows of Workers: Finite-Duration Model

Job finding: $e^u f(\theta) x^u$

Employed: $n$

Eligible Unemployed: $x^u$

Job destruction: $s \cdot n$
Flows of Workers: Finite-Duration Model

- Employed: \( n \)
  - Job finding: \( e^u.f(\theta).x^u \)
  - Job destruction: \( s.n \)

- Eligible Unemployed: \( x^u \)
- Ineligible Unemployed: \( x^a \)
  - Ineligibility: \( \lambda.x^u \)
Flows of Workers: Finite-Duration Model

Job finding: $e^a f(\theta) x^a$

Employed: $n$

Ineligible Unemployed: $x^a$

Ineligibility: $\lambda x^u$

Job finding: $e^u f(\theta) x^u$

Eligible Unemployed: $x^u$

Job destruction: $s n$
Optimal Composition of Unemployment

Landais, Michaillat, and Saez (08/2011)
Optimal Job-Search Effort

![Graph showing the relationship between unemployment rate and search effort. The graph compares eligible and ineligible search efforts.](image-url)
Optimal Weekly Arrival Rate of Ineligibility

Landais, Michaillat, and Saez (08/2011)  Optimal Unemployment Insurance
Optimal Expected Duration of Benefits

![Graph showing the relationship between unemployment rate and benefit duration in weeks. The x-axis represents the unemployment rate ranging from 0.04 to 0.09, and the y-axis represents the benefit duration in weeks, ranging from 0 to 500. The graph illustrates an upward trend, indicating that as the unemployment rate increases, the optimal expected duration of benefits also increases.]
Next Step: Estimation of $\epsilon^m$ and $\epsilon^M$

- Estimation of $\epsilon^m$:
  - evidence for Germany: Schmieder et al. [’11]
  - our data: Continuous Wage & Benefit History (CWBH)
  - regression kink design: use kink in schedule of UI benefits
  - details: Micro-elasticity

- Direct estimation of $\epsilon^M$:
  - preliminary evidence: Notowidigdo & Kroft [’11]
  - but, very difficult

- Indirect estimation of $\epsilon^M$: estimation of $\epsilon^m/\epsilon^M$
  - our data: Regional Extended Benefit Program (REBP)
  - Austria, 1988–1995
  - difference-in-difference: compare job-finding probability of non-treated in treated vs. non-treated regions
  - details: Macro-elasticity
BACK-UP SLIDES
Equilibrium: Pissarides [’00] Model

![Graph showing the relationship between employment and gross marginal profit and marginal recruiting expenses. The graph illustrates how these variables change with varying levels of employment.](image-url)
Equilibrium: Pissarides ['00] Model

- Gross marginal profit
- Marginal recruiting expenses

Employment

Canonical model

Gross marginal profit
Marginal recruiting expenses
Equilibrium: Hall ['05] Model

Model with wage rigidity

Gross marginal profit
Marginal recruiting expenses

Employment

Landais, Michaillat, and Saez (08/2011)  Optimal Unemployment Insurance
Equilibrium: with Job Rationing

Model with job rationing

Gross marginal profit
Marginal recruiting expenses

Rationing unemp.
Frictional unemp.

Employment

Return
Equilibrium: with Job Rationing

Model with job rationing

Gross marginal profit
Marginal recruiting expenses

Frictional unemp.
Rationing unemp.
Optimal UI Formula in Infinite-Horizon Model

\[
\frac{\tau}{1 - \tau} \approx \frac{\rho}{\epsilon^M} \cdot (1 - \tau) + \frac{1 + \kappa}{\kappa} \cdot \left[ \frac{\epsilon^m}{\epsilon^M} - 1 \right] \cdot \left[ 1 + \frac{\rho}{2} \cdot (1 - \tau) \right]
\]
Unemployment Rates in CWBH

The graph illustrates the unemployment rates in various states (GA, ID, LA, MO, NM, PA, WA) from 1978m1 to 1984m1. The x-axis represents the years, and the y-axis represents the unemployment rate (CPS). The states are differentiated by line types and markers, allowing for a clear comparison of unemployment trends over time.
Weekly UI Benefits Schedule: Missouri

Return

Landais, Michaillat, and Saez (08/2011)  Optimal Unemployment Insurance  47 / 42
MO 1980–84

Highest Quarter Earnings

duration wba

duration

wba

0 2500 5000

0 75 150
Micro-Elasticity over the Business Cycle

Landais, Michaillat, and Saez (08/2011)  Optimal Unemployment Insurance  49 / 42
Regional Distribution of REBP

With Extended Benefits = Shaded
Without Extended Benefits = White
Difference in Non-Employment Duration: Age 50-53

Return
Difference in Non-Employment Duration: Age 46-49