The Optimal Use of Government Purchases for Macroeconomic Stabilization

Pascal Michaillat (LSE) & Emmanuel Saez (Berkeley)

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“You are Barack Obama. It is early 2009. The unemployment rate has reached 9% and the federal funds rate is 0%. By how much should you increase government purchases?”

we propose an answer based on 2 sufficient statistics:

- balanced-budget multiplier
- elasticity of substitution between government and personal consumption
A model of unemployment and government purchases
Overview

- dynamic continuous-time model
- measure 1 of identical self-employed households
- benevolent government
- matching market where labor services are traded
  - not all services are sold in equilibrium → unemployment
  - labor and product markets are combined
  - modeling follows Michaillat and Saez [QJE, 2015]
Matching market

- households have productive capacity of 1
- households buy $C(t)$ services
- government buys $G(t)$ services
- households sell $Y(t) = C(t) + G(t) < 1$ services
- unemployment rate is $u(t) = 1 - Y(t)$
- services are traded through long-term relationships
Matching function and market tightness

- total number of vacancies: \( v(t) \)
- matching function: \( m(t) = \omega \cdot (1 - Y(t))^\eta \cdot v(t)^{1-\eta} \)
- **market tightness**: \( x(t) = \frac{v(t)}{1 - Y(t)} \)
- selling rate and buying rate:

\[
 f(x(t)) = \frac{m(t)}{1 - Y(t)} = \omega \cdot x(t)^{1-\eta}
\]

\[
 q(x(t)) = \frac{m(t)}{v(t)} = \omega \cdot x(t)^{-\eta}
\]
Market flows

- relationships separate at rate \( s \)
- output is a state variable: \( \dot{Y} = f(x) \cdot (1 - Y) - s \cdot Y \)
- assumption: flows are balanced, \( f(x) \cdot (1 - Y) = s \cdot Y \)
- output, unemployment become jump variables:

\[
Y(x) = \begin{cases} 
\frac{f(x)}{s + f(x)}, & \text{if } x > 0 \\
\frac{s}{s + f(x)}, & \text{if } x < 0 
\end{cases}
\]

\[
u(x) = \begin{cases} 
\frac{s}{s + f(x)}, & \text{if } x > 0 \\
\frac{s}{s + f(x)}, & \text{if } x < 0 
\end{cases}
\]
Matching cost: \( \rho \) services per vacancy

\[
\begin{align*}
Y \text{ (gross output)} &= y \text{ (net output)} + \rho \cdot v \\
\end{align*}
\]

Net output + matching cost

Market flows are balanced so \( s \cdot Y = v \cdot q(x) \) and

\[
Y = y + \rho \cdot \frac{s \cdot Y}{q(x)}
\]

\[
Y \cdot \left[ 1 - s \cdot \rho \frac{1}{q(x)} \right] = y
\]

\[
Y = \left[ 1 + \frac{s \cdot \rho}{q(x) - s \cdot \rho} \right] \cdot y
\]

\[
Y = [1 + \tau(x)] \cdot y
\]
Gross and net consumptions

Net consumptions enter utility function of households

- $C$: gross personal consumption
- $c = C/(1 + \tau(x))$: net personal consumption
- $G$: gross government consumption
- $g = G/(1 + \tau(x))$: net government consumption
Output and unemployment

market tightness $\chi$

labor services

capacity: 1
Output and unemployment

gross output:
\[ Y(x) = 1 - u(x) \]

unemployment rate:
\[ u(x) = \frac{s}{s + f(x)} \]

market tightness \( x \)
labor services
Output and unemployment

net output:

\[ y(x) = \frac{Y(x)}{1 + \tau(x)} \]

matching cost:

\[ y(x) \cdot \tau(x) \]
Feasible allocation and equilibrium

- assuming that the equilibrium system is a source, transitional dynamics are immediate because there are no state variables in the economy
- a feasible allocation \([c, g, y, x]\) satisfies \(y = y(x)\) and \(y = c + g\)
- an equilibrium function is \(g \mapsto [c, g, y, x]\)
- the equilibrium function reduces to \(g \mapsto x(g)\)
Efficient unemployment: \( x(g) = x^* \)
Inefficiently high unemployment: $x(g) < x^*$
Inefficiently low unemployment: $x(g) > x^*$
Sufficient-statistics formula for optimal government purchases
Value of government purchases

- we follow Samuelson [REStat, 1954]
- households’ instantaneous utility is $U(c, g)$
- $U$ is homothetic so the marginal rate of substitution

$$MRS_{gc} = \frac{\partial U / \partial g}{\partial U / \partial c}$$

is a decreasing function of $G/C$ or $G/Y$
Government’s problem

- government purchases are financed by a lump-sum tax to maintain a balanced budget
- given an equilibrium function \( x(g) \), the government chooses \( g \) to maximize welfare

\[
U \left( \langle y(x(g)) - g, g \rangle \right)
\]
Formula in sufficient statistics

- first-order condition of government’s problem is

\[ 0 = \frac{\partial U}{\partial g} - \frac{\partial U}{\partial c} + \frac{\partial U}{\partial c} \cdot y'(x) \cdot x'(g) \]

- optimal government purchases satisfy

\[ 1 = MRS_{gc} + y'(x) \cdot x'(g) \]

- Samuelson formula

- correction term

- \((G/Y)^*\) is the Samuelson ratio: \(1 = MRS_{gc}((G/Y)^*)\)
\[ \frac{G}{Y} = \left( \frac{G}{Y} \right)^* \quad \text{if correction term} = 0 \]

\[ x'(g) > 0, \quad y'(x) = 0 \]
\( \frac{G}{Y} > (\frac{G}{Y})^* \) if correction term > 0

\[ x'(g) > 0, \quad y'(x) > 0 \]

\[ x^* \]

\[ u^* \]
$G/Y < (G/Y)^*$ if correction term $< 0$

$x'(g) > 0, \ y'(x) < 0$
Robustness

formula remains valid with:

- heterogeneity in preferences
- endogenous labor supply + distortionary taxes
- heterogeneity + endogenous labor supply

**benefit principle:** change in government purchases is financed by change in individual taxes designed to leave individual utilities unchanged
Summary

- Samuelson formula holds with unemployment as long as unemployment level is efficient.
- If $g$ stimulates tightness, $G/Y$ should be countercyclical.
- If $g$ depresses tightness, $G/Y$ should be procyclical.
- Formula has same structure as formula for optimal UI in Landais, Michaillat, Saez [2010].
Assessment of US government purchases for 1951–2014
Introducing estimable statistics

first-order Taylor approximation around efficient tightness $x^*$ and Samuelson ratio $(G/C)^*$:

$$1 - MRS_{gc} = y'(x) \cdot x'(g)$$

$$\frac{1}{\varepsilon} \cdot \frac{G/C - (G/C)^*}{(G/C)^*} \approx -\frac{x - x^*}{x^*} \cdot \frac{dY/dG}{1 - (G/Y) \cdot (dY/dG)}$$
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Implicit formula

\[ \frac{G/C - (G/C)^*}{(G/C)^*} \approx -\varepsilon \cdot \frac{dY/dG}{1 - (G/Y) \cdot (dY/dG)} \cdot \frac{x - x^*}{x^*} \]

- \( \varepsilon \): elasticity of substitution between \( g \) and \( c \)
  - \( \varepsilon = 0 \): Leontief preferences, “bridges to nowhere”
  - \( \varepsilon = 1 \): Cobb-Douglas preferences
  - \( \varepsilon \rightarrow +\infty \): linear preferences, perfect substitute

- \( dY/dG \): balanced-budget multiplier
  - \( = \): deficit-financed multiplier with Ricardian households
  - \( < \): deficit-financed multiplier otherwise
Estimates of $\varepsilon$ and $dY/dG$

- $\varepsilon = 1$: very little empirical evidence
  - also consider $\varepsilon = 0.5$ and $\varepsilon = 2$

- $dY/dG = 0.6$: large empirical literature
  - deficit-financed multiplier between 0.6 and 1.6
    $\rightarrow$ midpoint $= 1.1$
  - effect of lump-sum taxes between 0 [Ricardian households] and -1 [Romer & Bernstein, 2009]
    $\rightarrow$ midpoint $= -0.5$

- $dY/dG = 0.6$ implies $m = 0.7$
Labor market tightness in the US
Assessment of US government purchases

\[
\frac{G/C - (G/C)^*}{(G/C)^*} + \epsilon \cdot m \cdot \frac{x - x^*}{x^*}
\]

1960 1975 1990 2005
Assessment of US government purchases
Assessment of US government purchases

\[ \frac{G/C - (G/C)^*}{(G/C)^*} + \epsilon \cdot m \cdot x - x^* \]

1960 1975 1990 2005

excessive government purchases

\[ \epsilon \cdot m = 0.7 \]

insufficient government purchases
Assessment of US government purchases

\[
\frac{G/C}{G/C^*} + \epsilon \cdot m \cdot \frac{x - x^*}{x^*}
\]

excessive government purchases

insufficient government purchases

\[\epsilon \cdot m = 0.04\]
How accurate is first-order approximation?

even for $x$ very far from $x^*$, a good approximation for optimal government purchases is

$$\frac{G/C - (G/C)^*}{(G/C)^*} \approx -\varepsilon \cdot \frac{dY/dG}{1 - (x/x^*) \cdot (G/Y) \cdot (dY/dG) \cdot \frac{x - x^*}{x^*}}$$

the implicit formula is accurate if

- $(G/Y) \cdot (dY/dG)$ is small $\rightarrow$ about 0.1 in US data
- $\varepsilon$ and $dY/dG$ are stable when $x$ varies $\rightarrow$ simulations
Summary

- **US government purchases are optimal for** $\varepsilon \cdot m = 0.04$, which implies
  - either a tiny multiplier: $dY/dG = 0.04$ and $\varepsilon = 1$
  - or bridges to nowhere: $dY/dG = 0.6$ and $\varepsilon = 0.06$

- **If** $\varepsilon \cdot m > 0.04$, **US government purchases are not countercyclical enough**
Optimal response to an increase in unemployment from 5.9% to 9%
An explicit formula

- Assume that government purchases are at Samuelson level: \((G/C)_0 = (G/C)^*\)
- Assume that tightness is inefficient: \(x_0 \neq x^*\)
- Optimal response of government purchases is

\[
\frac{G/C - (G/C)^*}{(G/C)^*} \approx -\varepsilon \cdot m \cdot \frac{x(G/C) - x^*}{x^*}
\]
An explicit formula

- assume that government purchases are at Samuelson level: \( (G/C)_0 = (G/C)^* \)
- assume that tightness is inefficient: \( x_0 \neq x^* \)
- optimal response of government purchases is

\[
\frac{G/C - (G/C)^*}{(G/C)^*} \approx \frac{-\varepsilon \cdot m}{1 + \left[ \frac{(G/Y)^* \cdot (1 - (G/Y)^*)}{(1 - \eta) \cdot u^*} \right] \cdot \varepsilon \cdot m^2} \cdot \frac{x_0 - x^*}{x^*}
\]
\[
\frac{u - u^*}{u^*} \approx -(1 - \eta) \cdot \frac{x - x^*}{x^*}
\]
Optimal increase in $G/Y$ when unemployment rises from 5.9% to 9%
Optimal increase in $G/Y$ when unemployment rises from 5.9% to 9%
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Optimal increase in $G/Y$ when unemployment rises from 5.9\% to 9\%
Application to 2009 stimulus package

Billions of USD vs.Multiplier

For different values of $\epsilon$:

- $\epsilon = 2$ (red dashed line)
- $\epsilon = 1$ (blue solid line)
- $\epsilon = 0.5$ (green dotted line)
Reduction in unemployment with optimal policy

\[ \epsilon = 0.5 \]

\[ \epsilon = 1 \]

\[ \epsilon = 2 \]
Reduction in unemployment with optimal policy

Multiplier

0 0.5 1 1.5 2

5%
6%
7%
8%
9%

perfect stabilization
Summary (I)

- $dY/dG$ determines optimal government purchases
  - huge macro literature estimates $dY/dG$
- elasticity of substitution between government and personal consumption is as important as $dY/dG$ to determine optimal government purchases
  - empirical effort would be valuable to measure marginal value of government purchases
cutoff value justifying an increase in government purchases in slumps is $dY/dG > 0$, not $dY/dG > 1$

even for $dY/dG = 0.2$, optimal government purchases respond strongly to unemployment

relation between the deviation of optimal $G/Y$ from $(G/Y)^\ast$ and $dY/dG$ is not increasing but hump-shaped, with a peak at $dY/dG = 0.4$
The equilibrium function $x(g)$ in the model of Michaillat and Saez [2014]
Representative household

- given tightness $x(t)$, real interest rate $r(t)$, government consumption $g(t)$, and taxes $T(t)$
- chooses consumption $c(t)$ and real bonds $b(t)$
- to maximize utility

$$\int_{0}^{+\infty} e^{-\delta \cdot t} \cdot [\mathcal{U}(c(t), g(t)) + \mathcal{V}(b(t))] \, dt$$

- subject to law of motion of real wealth

$$\frac{db}{dt} = [1 - u(x(t))] - [1 + \tau(x(t))]c(t) + r(t) \cdot b(t) - T(t)$$
Aggregate demand and real interest rate

- The aggregate demand \( c^d(x, g, r) \) is implicitly defined by a modified Euler equation:

\[
\frac{\partial U}{\partial c}(c, g) = \frac{(1 + \tau(x)) \cdot \gamma'(0)}{\delta - r}
\]

- On a matching market, we need to specify a price mechanism (bargaining, efficient, or rigid)

\[\rightarrow\] we specify a schedule for the real interest rate:

\[
\delta - r(g) = \mu \cdot \frac{\gamma'(0)^{1-\alpha}}{\frac{\partial U}{\partial c}(y^* - g, g)^{1-\beta}}
\]
General equilibrium

\[ y(x) = \frac{44}{53} \]

\[ c^d(x, g, r(g)) + g \]
Numerical simulations with countercyclical multiplier
Calibration

- we set $\varepsilon = 1$
- with efficient tightness and Samuelson formula:
  \[
  \frac{dY}{dG} = \frac{\beta}{(G/Y)^* + \varepsilon \cdot [1 - (G/Y)^*]}
  \]
- we set $\beta = 0.6$ to match $dY/dG = 0.6$
- the multiplier is countercyclical because of slack as in Michaillat [AEJMacro, 2014]
Unemployment rate without stabilization

Marginal utility of wealth

<table>
<thead>
<tr>
<th>G/Y</th>
<th>0.98</th>
<th>1</th>
<th>1.02</th>
</tr>
</thead>
<tbody>
<tr>
<td>3%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
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<td>7%</td>
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<td>9%</td>
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<td></td>
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<tr>
<td>11%</td>
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</tbody>
</table>

Constant G/Y
Multiplier without stabilization

![Graph showing the marginal utility of wealth with a constant G/Y. The graph demonstrates a linear increase as the marginal utility of wealth increases from 0.98 to 1.02.]
Optimal government purchases-output ratio

Marginal utility of wealth

<table>
<thead>
<tr>
<th>G/Y</th>
<th>Marginal Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.98</td>
</tr>
<tr>
<td>Optimal</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Graph showing the relationship between marginal utility of wealth and the optimal government purchases-output ratio.
Unemployment rate with stabilization

Marginal utility of wealth

Constant G/Y
Optimal G/Y

Marginal utility of wealth

0.98  1  1.02

3%
5%
7%
9%
11%
Accuracy of explicit formula

Marginal utility of wealth

<table>
<thead>
<tr>
<th>Govt. purchases/output</th>
<th>13%</th>
<th>15%</th>
<th>17%</th>
<th>19%</th>
<th>21%</th>
<th>23%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant $G/Y$</td>
<td>17%</td>
<td>17%</td>
<td>17%</td>
<td>17%</td>
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<tr>
<td>Exact formula</td>
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<tr>
<td>Explicit formula</td>
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</tr>
</tbody>
</table>

Marginal utility of wealth
Welfare gains from government purchases

Marginal utility of wealth

Social welfare

Constant $G/Y$

Exact formula

Explicit formula

Marginal utility of wealth

Social welfare

Marginal utility of wealth
Conclusion: balanced-budget government purchases are a key tool for stabilization

- with $\varepsilon > 0$ and $dY/dG > 0$, government purchases should be adjusted when unemployment is inefficient
- key when monetary and debt policy are unavailable
  1. zero lower bound + debt ceiling (USA, 2011–13)
  2. monetary union + balanced budget (US States)
  3. monetary union + restricted credit (eurozone)