From Labor to Intermediates: Firm Growth, Input Substitution, and Monopsony

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Abstract

We document and dissect a stylized fact about firm growth: the shift from labor to intermediate inputs. This shift occurs in input quantities, cost and output shares, and output elasticities. We establish this regularity in firm data for Germany and in firm (and industry) data for 11 (20) additional countries, and also in response to exogenous product demand shocks. We explain this regularity through a parsimonious model featuring an elasticity of substitution between intermediates and labor above one, and an increasing shadow price of labor (monopsony or adjustment costs). Our firm growth regressions identify a labor-intermediates substitution elasticity between 1.8 and 4.2. Labor-intermediates substitution also accounts for much of the labor share decline that we document accompanies firm and industry growth.

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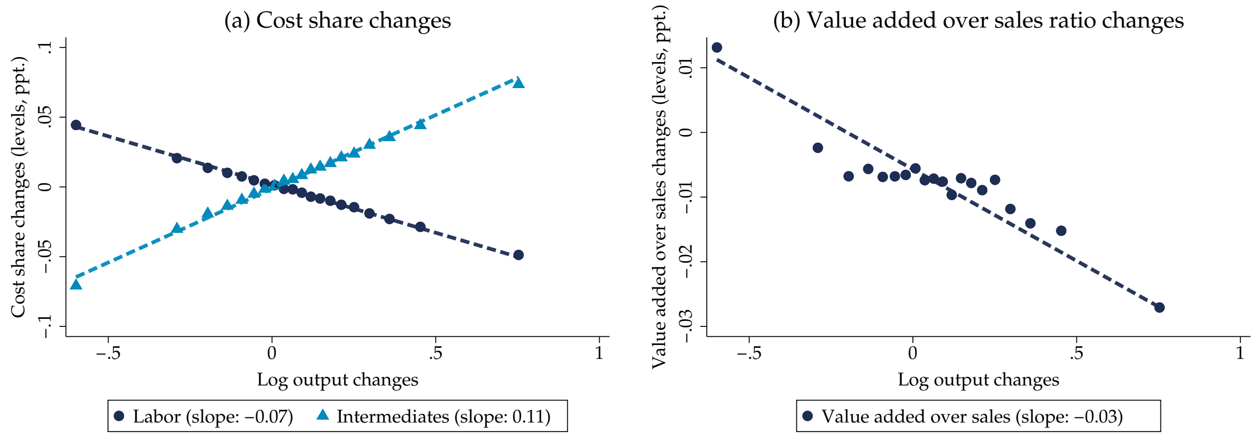
Figure 1: The shift from labor to intermediate inputs: within-firm 4-year change in cost shares and in and value added over sales ratios (levels) against output growth.



Notes: The figure reports binned scatter plots of the within-firm changes in labor and intermediate cost shares and value added over sales ratios in levels (i.e., percentage points) against log output changes (deflated sales). All panels depict 4-year differences and control for industry-year fixed effects. Panel (a) reports results for cost shares. Panel (b) reports results for value added over sales ratios. The data cover German manufacturing firms from 1995 to 2017; further details on the datasets, samples, and empirical specifications are discussed in the main part of the paper.

1 Introduction

How do firms grow? Standard Cobb-Douglas production and competitive input markets predict that firms simply scale up inputs proportionately, with constant cost shares. Our paper dissects and rationalizes systematic departures from this canonical benchmark. Figure 1 illustrates the stylized fact at the core of our analysis: as firms grow, they shift production inputs from labor to intermediate inputs, lowering their ratio of value added to sales. This shift from labor to intermediates holds across the board: in terms of cost shares (as depicted), as well as input quantities, output shares (i.e., the labor share), and output elasticities.[1] It holds across firm types, industries, and countries. To account for this set of facts, we offer a *parsimonious* model of firm growth.[2] The model leverages—and our analysis provides new estimates for—two features: (i) a labor-intermediates substitution elasticity above one, and (ii) an increasing shadow price of labor relative to intermediates, most likely due to monopsony. This paper is the first dedicated study of how these two forces jointly shape firm growth and how firm growth episodes, in turn, offer new insights into these two core parameters.

We primarily study firm growth in German manufacturing micro data. The data include firm-product-specific sales and real quantities, allowing us to construct firm-specific output prices and to address the price biases in output elasticities derived from production function estimation (De Loecker et al., 2016, Bond et al., 2021, De Ridder et al., 2024). We draw on OLS regressions as well as an IV strategy using export demand shocks as a firm growth shifter unrelated to factor-biased price or technological changes. We also establish the shift from labor to intermediates in nonparametric

[1]For manufacturing, intermediate inputs primarily consist of materials, energy, and product components. As we discuss below, temporary agency labor or services play a much smaller role given their small cost shares.

[2]Here, firm growth should be thought of as driven by input-neutral shifters, such as in TFP growth or product demand. Our model formalizes this through cost minimization, taking scale as given, and our empirical analyses include an instrument for growth that relies on product demand shifts.

scatter plots of raw firm-level data and, as a complementary robustness check, we also infer output elasticities from cost shares.

Additionally, we confirm our main results in administrative firm-level data from 11 additional countries as well as in industry-level data for 20 European countries using harmonized data from the Competitiveness Research Network (CompNet) for manufacturing *and* non-manufacturing industries—and for a subset of outcomes also for US manufacturing industries. We estimate our regressions at various horizons, from 1- to 10-year changes, and across different size and age classes of firms. As our evidence does not point toward a major role of production function non-homotheticities and/or fixed labor inputs in explaining our results, we rationalize our findings with a parsimonious production perspective on input substitutability as described below.

The shift from labor to intermediates in quantities and the reduction in output elasticities of labor accounts for *about one half to one third* of the negative effect of firm growth on the firm-level labor share. (In log changes, the labor share is equal to the output elasticity minus markups and markdowns.) Hence, our framework provides a novel, technological explanation for the negative association between the labor share and firm growth, complementing existing approaches that focus on large firms' product or labor market power. Importantly, unlike the existing literature on cross-sectional firm *size* gradients and concentration (e.g., Autor et al., 2020, De Loecker et al., 2020), we focus on *firm growth* in panel data.[3] Therefore, our additional results on labor shares in growing firms also resonate with the empirical study of Kehrig and Vincent (2021), who study firm growth dynamics in labor shares and highlight the role of demand-side factors (markups).

To rationalize the *full* set of facts, we propose a deliberately *parsimonious* model of firm growth. It leverages the combination of two features typically studied in isolation: substitutability between intermediates and labor, and an increasing relative shadow price of labor. In a nutshell, under an increasing relative shadow price of labor, as firms grow, they lower their demand for labor relative to intermediate inputs. If labor and intermediates are substitutes, this shift toward intermediates translates into a lower output elasticity of labor relative to intermediates. This, in turn, lowers the labor cost share and the labor share in output.

Substitution elasticities above one imply that as the intermediate-labor input quantity ratio increases, output elasticities shift from labor to intermediates—consistent with our firm growth facts. In fact, we formally show that this comovement—expressed as the ratio of our reduced-form regression coefficients on firm growth—identifies the substitution elasticity in a constant elasticity of substitution (CES) production function (or, more generally and independently of the production function, the implied elasticity of relative input quantity changes to relative shadow price changes). Our approach to estimating substitution elasticities is new in that it relies on growth-induced (including IV-based) panel comovement in input quantities and output elasticities, rather than relying on variation in input prices.[4]

[3]We find much smaller (but qualitatively similar) cross-sectional differences in output elasticities by firm *size* than by firm *growth*, perhaps due to firm-specific permanent factors shaping input intensities and output elasticities.

[4]While we intentionally focus on firm-growth variation (including an IV strategy relying on product demand shocks), a complementary approach to identify substitution elasticities would rely on input price variation. Existing studies relying on input price variation are difficult to compare to our approach, as they typically estimate substitution elasticities between intermediates and a capital-labor bundle rather than labor. Three studies that use price variation and directly estimate a labor-intermediate substitution elasticity are Doraszelski and Jaumandreu (2018), Huneeus et al. (2022), and Chan (2023),

Quantitatively, our firm growth regressions identify substitution elasticities well above one, ranging from 1.8 to 2.7 (OLS) and from 3.8 to 4.2 (IV). Our paper situates these values in a systematic meta-analysis of existing estimates. We also apply our (OLS) identification of substitution elasticities to firm micro data for additional 11 countries (and for 20 in industry data). For all those countries, we provide new, harmonized estimates of labor-intermediate substitution elasticities, which consistently exceed unity.

To account for *why* growing firms choose to shift their input mix from labor to intermediates, our model features an increasing relative shadow price of labor. A natural source is a finitely elastic firm-specific labor supply curve, i.e., firms holding monopsony power over labor. (Alternatively, adjustment costs may play a role, at least in the short run.) In fact, for a given substitution elasticity and under the assumption of firm cost minimization, our input mix estimates identify the elasticity of the labor (shadow) price to the firm (i.e., the inverse labor supply elasticity). We offer a range of implied elasticities across our specifications and wage measurement approaches, and compare them to existing estimates using data from the review of Sokolova and Sorensen (2021). Our results also imply that supply elasticities appear higher in the long run (i.e., labor markets are more competitive), for our IV estimates, and when we net out markdown shifts or use direct (average) wage estimates rather than implied ones.

Our paper focuses on *firm*-level growth. Studying aggregate implications would necessitate an input-output network analysis or open economy perspective. However, we confirm that, qualitatively, all our firm-level patterns transfer to the industry level, drawing on CompNet data for 20 countries and the United States. Specifically, industry-level inputs, cost shares, and output elasticities shift from labor to intermediates, and these inputs are substitutes also at the industry level. Strikingly, at the industry level, the negative association between output growth and labor shares is fully accounted for by labor output elasticities, with no role for markups or wage markdowns. Hence, reductions in the output elasticity of labor may act as a new, production-function-based factor in aggregate labor share declines—but we caveat that potential aggregation is beyond the scope of our paper's focus on micro growth dynamics of individual firms.[5]

Additional related literature. Broadly, our study complements work showing that estimates of the substitution elasticity between production factors are inconsistent with a Cobb-Douglas production model (Chirinko et al., 2011, Raval, 2019).

Our paper also adds to existing studies that document factor substitution in response to firm-specific shocks and trends—though the existing literature has largely focused on capital-labor substitution. For example, Acemoglu and Restrepo (2019), Acemoglu and Restrepo (2020), Dauth et al. (2021), and Deng et al. (2023) study substitution of labor with robots. Lashkari et al. (2024) analyze how non-homotheticities in the production function cause firms to shift toward higher IT-capital in-

who report estimates ranging from 0.12 to 1.8, 1.05 to 1.62, and 1.6 to 9.6, respectively. The findings in the latter two studies are qualitatively consistent with our results in the context of firm growth (i.e., substitution elasticities exceed unity). Huneeus et al. (2022) also show that accounting for input heterogeneity when constructing intermediate and labor input prices leads to higher estimated substitution elasticities. We provide further details on these and other studies in our meta-analysis.

[5]Consequently, our paper generalizes and provides a micro-founded explanation for the aggregate time series results for German manufacturing in Mertens (2022), who shows that output elasticities of labor and labor shares declined over the last decades. Similarly, Elsby et al. (2013) argue that China's accession to the WTO and the resulting offshoring of labor-intensive tasks have contributed to aggregate labor share declines in many advanced countries.

tensities.[6] Hubmer and Restrepo (2021) use Compustat data and show that capital-output elasticities have increased in the largest Compustat firms in the most recent years, consistent with automation. Karabarbounis and Neiman (2014) discuss the role of declining capital prices in the global labor share decline. Huneeus et al. (2022) estimate that labor and intermediates are substitutes and show that firms with access to cheaper suppliers have lower labor shares. Dhyne et al. (2022) show that labor adjustments to demand shocks are weaker compared to intermediate inputs and emphasize the role of fixed cost in labor. Our paper also rationalizes these empirical patterns *and* additional ones specific to our mechanism, offering a complementary but distinct mechanism, based on labor-intermediates substitution.[7] Castro-Vincenzi and Kleinman (2024) use *aggregate* data to study how rising material prices may lower labor shares if labor and materials are *complements*. We focus on firm-level mechanisms and firm growth, and find empirical evidence showing that intermediates and labor are substitutes and that, as firms and industries grow, shadow prices of labor relative to intermediates increase.

In parallel work, Chan et al. (2024) document that larger firms have (cross-sectionally) higher returns to scale due to higher intermediate input output elasticities and study the resulting implications for efficiency losses under financial frictions. Our two papers complement each other. We focus on shifts from labor to intermediates within firms due to firm growth, analyze implications for firm and industry labor shares, and provide a parsimonious micro-foundation for our findings based on substitution elasticities and imperfect input markets.

Finally, by focusing on input mix and production function dynamics accompanying firm growth, our study complements existing studies of firm growth using one-input (labor) models and measures of firm size (e.g., Sterk et al., 2021), and we leave firm life cycles for future research because we lack information on firm age in our main dataset. However, using additional data from other countries, we show that our results hold for young and mature growing firms.

Outline. The paper is organized as follows. Section 2 provides a formal model and derives predictions. Section 3 describes the firm-level data, sample, and production function estimation. Section 4 uses German firm-level data to empirically establish the shift from labor to intermediates. Section 5 interprets the results quantitatively, identifies parameters of interest, and discusses alternative explanations for our findings. Section 6 draws implications of our findings for firm-level labor shares. Section 7 uses additional firm- and industry-level data to transfer our analysis to other European countries, sectors outside of manufacturing, and US manufacturing industries. Section 8 concludes.

2 Theory

This section provides a parsimonious framework for firm growth and its effects on input intensities, output elasticities, and cost and output shares. Section 2.1 presents our production model, which we use in Section 2.2 to formulate testable predictions about firm growth.

[6] Among others, Zeira (1998), Acemoglu (2002), and Rubens (2022) also study how relative factors prices induce technological change by directing firms' decision to innovate.

[7] While we do not aim to adjudicate between fixed costs and the substitution mechanism, our paper shows that several firm-growth patterns our paper primarily focuses on cannot be rationalized by at least basic versions of a fixed-cost model. In this context, our study also provides a new and alternative micro-foundation for why larger/growing firms outsource a higher share of their production tasks as documented in Fort (2017).

2.1 Firm Optimization

Production function. We consider a constant returns to scale (CRS) constant-elasticity-of-substitution (CES) production function of firm i in period t that transforms labor (L_{it}), intermediates (M_{it}), and capital (K_{it}) into output (Q_{it}):

$$Q_{it} = \Omega_{it}\Lambda_i^K K_{it}^{1-\kappa}\left(\Lambda_i^{LM}\alpha_i^L L_{it}^{\frac{\sigma-1}{\sigma}} + \Lambda_i^{LM}\alpha_i^M M_{it}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\kappa}. \tag{1}$$

Ω_{it} represents firm productivity. α_i^L, α_i^M, Λ_i^{LM}, and Λ_i^K are distribution parameters. Capital enters multiplicatively with a Cobb-Douglas exponent $(1-\kappa)$. Labor and intermediates enter as a CES nest with substitution elasticity σ, forming a labor-intermediate bundle with Cobb-Douglas exponent κ.

We choose this homothetic CES specification for its analytical simplicity, and as it can fully explain our main empirical finding of a shift from labor to intermediates in production through the substitution elasticity σ (Section 5.3 discusses non-homotheticities).

The specification foreshadows our main result, that the substitution from labor to intermediates is the primary pattern accompanying firm growth, with capital intensity shifts being comparatively unimportant. In fact, in the production function in Equation (1), the capital output elasticity is constant. In our empirical analysis that builds on more flexible production function specifications, we will allow output elasticities (and cost shares) of all inputs (including capital) to vary. As the capital output elasticity is small (see below), its level changes in response to output growth are also small, even with sizable percent effects, and one could indeed think of $(1-\kappa)$ as being approximately constant in our context.

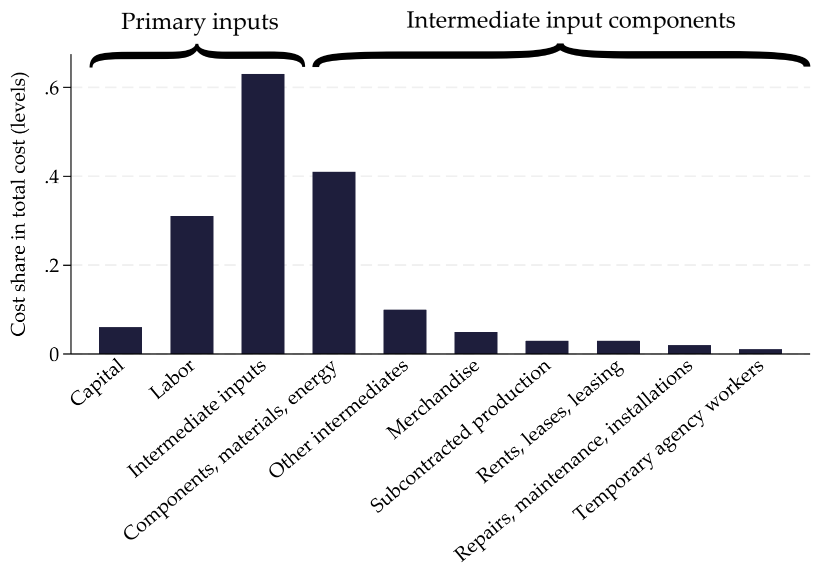
Cost shares. Figure 2 details input cost shares for German manufacturing firms (data described in Section 3). The average capital cost share is just 6% (which equals $1-\kappa$ under perfect input markets).[8] The average capital output elasticity that we will later estimate is 0.12. The low average capital share, along with our empirical findings (showing relatively small capital cost share and output elasticity changes in levels compared to labor and intermediates), further motivates our focus on labor-intermediate substitution. The figure also decomposes the intermediate input cost share. Two thirds of intermediate inputs consist of materials, energy, and external product components (e.g., car tires). Of the remainder, half are classified as "other intermediate inputs," which include services like transport, postage, insurance, and legal services. The other half comprises merchandise, rents, and subcontracted production steps (i.e., when a firm contracts another firm to perform specific production tasks). Temporary agency labor represents only about 1% of total costs.

Cost minimization. We rely on cost minimization at a given output level to study the input and production function dynamics accompanying firm growth. Cost minimization conveniently introduces firm size and growth as quasi-parameters and permits us to cast our reduced-form empirical regression equations as structural equations. This section focuses on key equations. Appendix C.1 details all derivations.

We allow for imperfect product market competition and input market frictions, such as monopsony

[8] In our firm-level data, labor's average share of value added is 80%.

Figure 2: Cost shares of production inputs.



Notes: The figure reports average firm-level cost shares for capital, labor, intermediate inputs, and the components of intermediate inputs. Capital costs are approximated as 8% of the capital stock following Dhyne et al. (2024) and the approach used in the CompNet data (CompNet, 2023), which we use in Section 7. Separate information for temporary agency worker cost shares is available from 1999. All other variables are available from 1995. The data cover German manufacturing firms and is described in Section 3.

power and adjustment costs, creating wedges between the marginal costs of production inputs and their unit costs. This feature aligns with our empirical analysis that accommodates firm- and time-specific markups and input wedges. We will revisit these assumptions when formulating growth predictions in Section 2.2. Cost minimization implies a FOC for each input, labor, capital, and intermediates, $X = \{L, K, M\}$,

$$P_{it}^X \underbrace{\left(1 + \frac{\partial P_{it}^X}{\partial X_{it}} \frac{X_{it}}{P_{it}^X} + \frac{\partial \chi^X}{\partial X_{it}}\right)}_{\gamma_{it}^X} = \lambda_{it} \frac{\partial Q_{it}}{\partial X_{it}}, \tag{2}$$

where P^X is the price for input X (e.g., wage for labor L), χ^X is an adjustment cost function, λ_{it} is marginal cost, and γ_{it}^X is the input price wedge, such that $P_{it}^X \gamma_{it}^X$ is the input *shadow price*. We express the adjustment cost function in flexible "quasi-static" terms (as in Bond et al., 2021, see also Appendix C.1) without formulating the dynamic problem (hence, sidestepping continuation-related terms) to highlight the key take-away from Equation (2): monopsony power and adjustment costs raise marginal input costs (the overall shadow price) beyond an input's price, P_{it}^X.[9]

Using the production function and cost minimization, we derive two key equations. First, we show how the substitution elasticity is related to and can be identified by the co-movement of relative

[9]Under profit maximization, we could also write γ_{it}^X as the wedge between marginal revenue products and input costs.

output elasticity and input ratio changes:

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{-1}{\sigma}} \quad \Leftrightarrow \quad \frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}} \quad \Rightarrow \quad \frac{\sigma-1}{\sigma} = \frac{\Delta\ln(\theta_{it}^L) - \Delta\ln(\theta_{it}^M)}{\Delta\ln(L_{it}) - \Delta\ln(M_{it})}, \tag{3}$$

where, in the last step, we have taken changes within firms (and used the assumption of constant α_i levels within the firm, departures from which we discuss below). $\theta^X = \frac{\partial Q}{\partial X}\frac{X}{Q}$ is the output elasticity of input $X = \{L, K, M\}$.

Equation (3) is reminiscent of the substitution elasticity in Hicks (1932). The difference is that we connect changes in input quantities to changes in output elasticities rather than marginal products. If labor and intermediates are substitutes ($\sigma > 1$, consistent with our evidence below), the two equations imply that a decrease in the labor-intermediates ratio will decrease the ratio of the labor output elasticity to the intermediate output elasticity. With Cobb-Douglas ($\sigma = 1$), no such change would occur, and complements ($\sigma < 1$) would imply the opposite. Moreover, in Appendix C.3 and Section 3, we show and discuss that Equation (3), more generally and regardless of the functional form of the production function, describes the elasticity of relative input quantity changes to relative shadow prices changes (holding capital inputs and prices fixed).

Second, inserting Equation (2) into Equation (3) recovers the implied shadow price ratio change that rationalizes a given shift in input mix at the firm level for a given level of σ:

$$\Delta\ln\left(P_{it}^L\gamma_{it}^L\right) - \Delta\ln\left(P_{it}^M\gamma_{it}^M\right) = \frac{\Delta\ln(L_{it}) - \Delta\ln(M_{it})}{-\sigma}. \tag{4}$$

Hence, for any $\sigma > 0$, an increase in the shadow price of labor relative to intermediates leads to a decrease in the labor-to-intermediate ratio, with σ scaling this relationship (absent input-biased shifts). Analogously, σ guides the size of the implied input price ratio shift that must have rationalized a given shift in the input mix.

We will use Equations (3) and (4) in our empirical analysis to estimate substitution elasticities and the implied shadow price ratios that accompany the production function dynamics of firm growth.

Cost and output shares. Changing output elasticities have important implications. Appendix C.1 shows that using Equation (2) for all inputs determines an input X's cost share as follows:

$$CS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it}^X L_{it} + P_{it}^M M_{it} + P_{it}^K K_{it}} = \frac{\frac{\theta_{it}^X}{\gamma_{it}^X}}{\frac{\theta_{it}^L}{\gamma_{it}^L} + \frac{\theta_{it}^M}{\gamma_{it}^M} + \frac{\theta_{it}^K}{\gamma_{it}^K}}. \tag{5}$$

Similarly, reformulating Equation (2) defines an input's output share (e.g., the labor share in output) as a function of markups, input cost markdowns (i.e., γ_{it}^X, the wedge between the input's marginal cost and its price), and output elasticities (Appendix C.1):

$$OS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it} Q_{it}} = \frac{\theta_{it}^X}{\mu_{it}\gamma_{it}^X}, \tag{6}$$

where $\mu_{it} = \frac{P_{it}}{\lambda_{it}}$ is the output price over (total) marginal cost markup. Thus, a decrease in an input's output elasticity leads to a reduction in both the cost share and output share of an input.[10] Equation (3) highlights that such a decline in the labor output elasticity can result from changes in the labor-intermediate input mix (holding fixed returns to scale and if $\sigma \geq 1$).

2.2 Predictions for Firm Growth

To make predictions about firm growth, we require additional structure and model *firm-specific* labor and intermediate supply as isoelastic, $P_{it}^X = a_{it}^X X_{it}^{\varepsilon^X}$ for $X = \{L, M\}$, where a_{it}^X is a baseline input price normalization and ε^X is the inverse *firm-specific* input supply elasticity (such that a finite supply elasticity denotes a case of monopsony in input markets).[11] For simplicity, we also assume constant markups, that a firm's capital shadow prices do not depend on its own capital demand, and that γ_{it}^L and γ_{it}^M are determined by the supply functions, which is sufficient to make growth predictions consistent with our empirical results. Importantly, in our empirical analysis, we relax these assumptions and allow for firm- and time-specific markups and wage markdowns (resulting from monopsony/finitely elastic firm-specific labor supply curves, or adjustment costs).

Inserting the input supply functions into Equation (2) determines labor and intermediate demand as functions of marginal products and parameters (see Appendix C.2):

$$X_{it} = \left(\frac{\lambda_{it} \alpha_i^X}{(1+\varepsilon^X) a_{it}^X} \right)^{\frac{1}{\varepsilon^X}} \left(\frac{\partial Q_{it}}{\partial X_{it}} \right)^{\frac{1}{\varepsilon^X}} \quad \text{for} \quad X = \{L, M\}. \tag{7}$$

Inserting the production function and expressing the resulting equation in terms of the labor-intermediates ratio yields:

$$\frac{L_{it}}{M_{it}} = \varrho_{it} \lambda_{it}^{\frac{\sigma+\kappa-1}{\kappa}\left(\frac{1}{\sigma\varepsilon^L+1} - \frac{1}{\sigma\varepsilon^M+1}\right)} Q_{it}^{\left(\frac{1}{\sigma\varepsilon^L+1} - \frac{1}{\sigma\varepsilon^M+1}\right)}, \tag{8}$$

where ϱ_{it} (expression in Appendix C.2) captures effects that are unrelated to firm growth (i.e., parameters, baseline prices, and TFP). $\lambda_{it}^{\frac{\sigma+\kappa-1}{\kappa}\left(\frac{1}{\sigma\varepsilon^L+1} - \frac{1}{\sigma\varepsilon^M+1}\right)}$ captures the effect of marginal costs, which increase with quantities produced due to increasing supply curves. The key insight from Equation (8) is that the response of *input ratios* (and thus *output elasticities* and, in turn, cost and output shares) to an increase in output (i.e., firm growth) depends on $\frac{1}{\sigma\varepsilon^L+1} - \frac{1}{\sigma\varepsilon^M+1}$.

Growth predictions. Table 1 summarizes our predictions for various assumptions about labor-intermediate substitution elasticities and relative input supply elasticities.[12] This section's discussion remains qualitative. Our quantitative interpretation is presented in Section 5, where we back out the

[10] We can express θ_{it}^X in terms of returns to scale ($RTS_{it} = \theta_{it}^L + \theta_{it}^M + \theta_{it}^K$) and the input output elasticity relative to other output elasticities, $\theta_{it}^X = \frac{\theta_{it}^X}{RTS_{it}} RTS_{it}$, which separates returns to scale from the relative technological importance of inputs vis-à-vis other production factors.

[11] Strictly speaking, our analysis does not require firms to exploit upward-sloping firm-specific labor supply; what matters is that shadow input prices increase as output grows. In our empirical analysis we study within-firm changes and include industry-year fixed effects.

[12] We focus on the empirically relevant case of $\sigma + \kappa > 1$, where κ is approximately the sum of labor and intermediate output elasticities or, under constant returns to scale and perfect input markets, the sum of labor and intermediate cost shares. κ is therefore close to unity (see Figure 2 and Appendix Table A.1). Our estimates of σ are well above unity (see Table 4).

Table 1: Growth predictions for different substitution and supply elasticities (ceteris paribus).

	$\sigma < 1$	$\sigma = 1$	$\sigma > 1$	
L **less** elastic than M $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$	↓	↓	↓	
	↑	=	↓	
	↑	=	↓	
	↑	=	↓	
L **as** elastic as M $(\varepsilon^L)^{-1} = (\varepsilon^M)^{-1}$	=	=	=	$\Delta \ln L_{it}/M_{it}$
	=	=	=	$\Delta \ln \theta^L_{it}/\theta^M_{it}$
	=	=	=	$\Delta \ln CS^L_{it}/CS^M_{it}$
	=	=	=	$\Delta \ln OS^L_{it}/OS^M_{it}$
L **more** elastic than M $(\varepsilon^L)^{-1} > (\varepsilon^M)^{-1}$	↑	↑	↑	
	↓	=	↑	
	↓	=	↑	
	↓	=	↑	

Notes: Ceteris paribus refers to constant returns to scale and non-changing market imperfections with output growth ($\Delta \ln Q_{it}$) (but our empirical treatment will relax those expositional assumptions). The shaded area represents the region of the parameter space consistent with our empirical findings. These assumptions are relaxed in the empirical analysis.

implied values for σ and labor supply elasticities, $\epsilon^L = (\varepsilon^L)^{-1}$, identified by our empirical estimates.

Potential cases. We use a CRS Cobb-Douglas production function with perfect markets as a benchmark to fix ideas (CES with $\sigma = 1$), and consider departures under input complementarity ($\sigma < 1$) and substitutability ($\sigma > 1$). We consider three cases for firm-specific input supply elasticities: labor is more or less elastic than intermediates ($(\varepsilon^L)^{-1} > (\varepsilon^M)^{-1}$ and $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$), and the inputs are equally elastic ($(\varepsilon^L)^{-1} = (\varepsilon^M)^{-1}$).

The shaded, top-right area in Table 1 highlights the combination of parameters implied by our empirical evidence: substitutability and labor being less elastically supplied to the firm than intermediates.

Rejected by evidence: equally elastic supply. We first consider the benchmark of equal supply elasticities, which nests the competitive input prices case (the middle row of Table 1). In this case, input ratios, output elasticities, cost shares, and output shares are constant as firms grow for any value of the substitution elasticity.

Rejected by evidence: Cobb-Douglas and less elastic labor supply. Second, consider the case of an increasing relative shadow price of labor (the top row). With Cobb-Douglas ($\sigma = 1$), firms substitute from labor to intermediates, but output elasticities and thus cost and output shares remain unchanged. That is, even though labor becomes relatively more expensive as firms grow, the quantity substitution from labor exactly offsets the price increase, leaving labor cost and output shares constant.

Rejected by evidence: complements and less elastic labor supply. Now, consider that labor and intermediates are complements ($\sigma < 1$). In this case, a relative reduction in labor quantities increases the output elasticity of labor relative to intermediates. Intuitively, firms reduce their labor-intermediate ratio less than one-to-one with the input price ratio increase, causing cost and output shares of labor to *increase*.

Case supported by evidence: substitutes and monopsony. Input substitutability ($\sigma > 1$) implies the opposite: the reduction in the labor-intermediate input quantity ratio translates into declines in output elasticity ratios and cost and output shares of labor. As labor becomes expensive, firms substitute away from it by more than one-to-one. This scenario aligns with our empirical evidence discussed below (and Figure 1).

Rejected by evidence and implausible: more elastic labor supply. For completeness, we also consider the case where intermediate inputs are less elastically supplied than labor. In this scenario, the opposite sign for quantities emerges across the board, irrespective of σ.

Discussion: non-constant input wedges/supply elasticities and markups. As noted, the previous predictions hold ceteris paribus with respect to input wedges (γ^L/γ^M) and markups. This simplified framework can fully explain our findings. However, in our empirical analysis, we will directly measure both markups and relative input wedges and explore their dynamics in response to firm growth in Section 6.

Markups tend to rise with firm growth, which dampens input responsiveness to growth relatively more for the more elastically supplied input (this can be seen in Equation (8) as $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$). However, empirically, this attenuation is small and does not overturn the predicted reduction in the labor-intermediate input ratio, because markups only increase moderately as firms grow.

Regarding changing input wedges, we find that they *amplify* the shift from labor to intermediates as the relative labor wedge increases with firm growth (intuitively, the difference in supply elasticities in Equation (8) gets larger as firms grow). One implication is that, in this case, even under Cobb-Douglas, we may observe declines in the labor share of costs and output with *constant* output elasticities. We formally discuss this case in Section 6, but note that we will have direct estimates of output elasticities in our baseline analysis using the production function methods outlined below.

Discussion: biased technological change. In Equation (1), α_i^L and α_i^M are assumed to be constant within the firm. If these parameters changed at different rates within firms, the estimation of the substitution elasticity would need to account for such changes (i.e., they would enter Equations (3) and (4)).[13] As we discuss in our empirical section, we address potential biases arising from firm-specific biased technological change through our IV regressions—where causality goes from output to output elasticities (we also discuss non-homotheticities).

3 Firm-level Data

We now describe the German firm-level data, the sample, and the estimation of output elasticities.

Production data. The firm-product-level panel data for Germany's manufacturing sector cover the period of 1995-2017. The data are collected and supplied by the German Statistical Offices.[14] The unit of observation is firms (not establishments or plants, although 90% of firms are single-plant firms).[15] The data include employment, investment (includes tangible and intangible assets), intermediate input costs, wage bills, depreciation, and total gross output. Importantly, as a rare feature, the data

[13]For instance, if α_{it}^L and α_{it}^M entered as time-varying factor-specific technologies, Equation 3 would change to: $\frac{\sigma-1}{\sigma} = \frac{\Delta\ln(\theta_{it}^L)-\Delta\ln(\theta_{it}^M)}{\Delta\ln(\alpha_{it}^L)-\ln(\alpha_{it}^M)+\Delta\ln(L_{it})-\Delta\ln(M_{it})}$.

[14]Data source: RDC of the Federal Statistical Office and Statistical Offices of the Federal States, DOI: 10.21242/42131.2017.00.03.1.1.0, 10.21242/42221.2018.00.01.1.1.0, and 10.21242/42111.2018.00.01.1.1.0.

[15]In this dataset, firms are defined as legal units, referring to the smallest legally independent unit that keeps accounts for commercial or tax purposes.

also include *real product quantities and sales, both at a ten-digit product classification, allowing us to compute firm-product-level prices (as in Mertens, 2022).*[16] The data cover 40% of firms with at least 20 employees and consist of a rotating panel that is redrawn every 4-5 years. Labor is defined as the number of employees on September 30th. All other variables pertain to the full calendar year.[17] We clean and prepare the data following Mertens (2022) and provide further details on data preparation, capital stock construction, variable definitions, and summary statistics in Appendix D.[18]

Supplementary trade data. To provide *causal* evidence on the relationship between output growth (i.e., growth in response to an input -neutral shifter) and our variables of interest, we merge bilateral trade flows from the United Nations Comtrade Database at the firm-product-year level to the German micro data (1995 to 2017). We map product codes in both datasets into the PRODCOM2002 classification using official concordance tables following the code of Bräuer et al. (2023). As described below, we will use the IV approach by Hummels et al. (2014) to instrument output changes with foreign export demand to study how output elasticities respond to exogenous output changes (more precisely, output shifts in response to input-neutral product demand shifters).

Production function estimation. We use two complementary approaches that allow for time-varying and firm-specific output elasticities. Our baseline specification estimates the following flexible translog production function. While we ultimately find that our evidence can be well explained by our simple CES production function from Section 2.1, it is useful to start with this general specification, which permits non-constant returns to scale and a direct estimation of more flexible output elasticities (e.g., with varying capital output elasticities).[19] This general translog specification is given by (lower case letters denote logs):

$$\begin{aligned}\tilde{q}_{it} = \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \beta_{l2} l_{it}^2 + \beta_{k2} k_{it}^2 + \beta_{m2} m_{it}^2 + \\ \beta_{lk} l_{it} k_{it} + \beta_{lm} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \beta_{lkm} l_{it} k_{it} m_{it} + \omega_{it} + \nu_{it},\end{aligned} \tag{9}$$

where ν_{it} is an i.i.d. error term. $\tilde{q}_{it}$ denotes log firm sales deflated by a *firm-specific* price index that we derive from firm-product-level price information. ω_{it} is log total factor productivity. Labor enters Equation (9) in quantities, intermediates and capital enter as expenditures deflated by 2-digit-industry-year-specific deflators. *The production function is estimated for each industry separately* (NACE Rev. 1.1 two-digit), although we omit industry indices in Equation (9).

We detail our production function estimation approach in Appendix E, where we also discuss how we address several recently highlighted biases to production function estimation. Most importantly, we apply a *correction for price biases (De Loecker et al., 2016) by constructing a firm-specific output price index from our firm-product-level price data* as in Eslava et al. (2004) and by additionally including an input price control function, which proxies for unobserved input price variation using information on firms' output prices and market shares. The latter follows the firm-level adaption of the control function

[16]Examples of products are "Tin sheets and tapes, thicker than 0.2mm" or "Workwear: long trousers for men, cotton".

[17]We will estimate results for 1-, 4-, and 10-year changes—assuaging concerns about timing differences between labor and other variables.

[18]The dataset has been used in various studies, e.g., Mertens (2020, 2022, 2023), Mertens and Müller (2022), Haelbig et al. (2023), Mertens et al. (2022), Bräuer et al. (2023), and Bighelli (2023).

[19]Moreover, the production function estimation in the European firm and industry data in Section 7 draws on a translog production function as well.

method in De Loecker et al. (2016) by Mertens (2022).[20] As described in more detail in Appendix E, including the input price control function captures, among others, quality differences in inputs that create (unobserved) input price variation across firms. Importantly, we explicitly include labor quantities rather than expenditures in Equation (9) as labor markets are imperfect (see also Footnote 20).

Using the estimated coefficients from Equation (9) (varying across industries) and the information on input levels (varying across firms), we compute output elasticities for each firm-year observation as $\theta_{it}^X = \frac{\partial \tilde{q}_{it}}{\partial x_{it}}$ for input $X = \{L, M, K\}$. As production function coefficients are industry-specific, output elasticities vary across firms within an industry due to heterogeneity in input quantities between firms and across time and vary across industries additionally due to different coefficients (and we allow for more flexibility using a cost-share approach below).

Our production function estimation allows for imperfect product market competition, labor adjustment costs, imperfect input markets, and non-constant returns to scale. However, the control function approach relies on Hicks-neutrality to formulate a control function for productivity based on flexible production inputs (raw materials, product components, and energy expenditures; see Appendix E).[21]

CES vs. translog. Another potential concern is the lack of consistency with our initial model. Specifically, one might worry that the substitution elasticity derived from the CES function is invalid under a translog specification. However, this concern is unwarrented for two reasons. First, we can still estimate an average substitution elasticity from the translog output elasticities using Equation (3), which locally approximates the nested two-input CES-Hicks elasticity under certain parameter restrictions (Berndt and Christensen, 1973). Second, and more importantly, we can still apply Equation (3) in the translog framework and interpret the result as an elasticity of relative input quantity changes with respect to relative input shadow price changes, holding capital inputs and prices fixed. This result follows directly from cost minimization (see Appendix C.3) and is independent of the production function. The elasticity measures how two inputs adjust in response to changes in their relative *shadow* prices, which is exactly what we are interested in.[22]

Alternative approach: cost shares. Nevertheless, to complement our analysis, we also measure output elasticities using cost shares, which avoids relying on Hicks-neutral productivity, does not require parametric assumptions, and can be directly applied within the CES model from Section 2. Under constant returns to scale and perfect input markets, cost shares equal output elasticities, $\theta_{it}^X = \frac{P^X X}{\sum_{X'} P^{X'} X'}$ (see Equation (5)). While these are typical assumptions in cross-sectional settings, for our

[20] This approach relies on the positive correlation between output and input prices (e.g., due to high quality outputs requiring high quality inputs). To account for the dependence of input decisions on productivity, we utilize a control function approach similar to Wooldridge (2009) and proxy productivity with information on expenditures on raw materials, product components, and energy. The control function for productivity also includes (lagged) wages (in addition to the input price control function), which helps address some of the identification concerns raised by Doraszelski and Jaumandreu (2021), particularly relevant in our context, where labor markets are imperfect.

[21] Recent studies (e.g., Demirer, 2025, Raval, 2023) estimate production functions allowing for labor-augmenting productivity, though at the cost of imposing competitive labor markets. Demirer (2025) also shows that biases from ignoring non-Hicks-neutral productivity are significantly smaller under a flexible translog specification compared to Cobb-Douglas—in some cases, differences in estimated output elasticities between Hicks-neutral and non-Hicks-neutral production functions become statistically insignificant when considering the translog specification.

[22] This elasticity fits our setting, where firm growth induces a movement along the labor and intermediate input supply curves, possibly shifting two input prices at the same time (though not necessarily if, e.g., intermediates are supplied competitively).

within-firm analyses of changes, it is sufficient to assume *non-changing* returns to scale and input market imperfections in the form of input wedges (e.g., from iso-elastic labor supply)—i.e., our changes-based specification (including Equation (10)) considerably relaxes the conventional assumptions when using cost share approaches that assumes away such features. We will find that both approaches to identifying output elasticities yield qualitatively similar results (substitution elasticities above one); in fact, the cost share approach will yield even larger elasticities.[23] Hence, our preferred baseline method is the translog approach given its flexibility and conservative results.

Summary statistics and sample. Appendix Table A.1 provides summary statistics for our sample. Our sample sizes vary between 180,000 and 50,000 observations, depending on the time horizon of the analysis (1-10 years).

4 Firm-level Evidence: Reduced Form Analysis

We now study firm growth and associated dynamics of input use, output elasticities, and cost and output shares in the micro data. This section presents our empirical results. Section 5 will interpret these reduced form moments structurally and argue how they identify the implied substitution elasticities and input supply elasticities.

4.1 OLS Regressions

Strategy. For each firm i in year t and firm-level outcome $\mathbb{O}_{it}$ (input quantities, cost shares, output shares, output elasticities, and, later, markups and markdowns), we estimate OLS regressions in within-firm log differences across h years (i.e., $\Delta^h x_{it} = x_{it+h} - x_{it}$) of the form:

$$\Delta^h \ln(\mathbb{O}_{it}) = \beta_Q^h \Delta^h \ln Q_{it} + \upsilon_{jt} + \nu_{it}. \tag{10}$$

Q_{it} is deflated sales, and υ_{jt} captures 4-digit NACE rev. 1.1 industry (j) times year (t) fixed effects.[24] The difference specification accounts for unobserved constant firm characteristics. That is, we focus on within-firm and within-industry idiosyncratic firm growth, rather than industry growth or industry response to shocks. We will study industry-aggregated analogs of this specification in Section 7.[25] ν_{it} is an error term. We cluster standard errors at the firm level. The regression is unweighted (i.e., each firm has the same weight). We conduct robustness checks by size (sales) quintiles below, finding similar results for all key outcomes. The coefficient of interest is β_Q^h. It captures the percent effect

[23]This is, as mentioned above, because in our changes-based specification, the cost-share approach does accommodate labor wedges but only in the form of a constant wedge—so that any changes in input wedges load on the substitution elasticity in the cost share approach. Empirically, we will find that input wedges increase with firm growth in the translog approach.

[24]For this regression, we use the 2-digit industry output price deflator provided by the Statistical Office (but also include industry-year fixed effects). Using our own firm-specific output price index yields, however, similar results. The former may be preferable if there is measurement error in the firm-level output price index or under correlation between input and output prices.

[25]In unreported robustness checks, we also ran specifications with firm effects to take out firm-specific trends. This specification leads to slightly higher coefficient estimates, particularly at longer horizons, likely due to its remaining variation capturing more transitory fluctuations around the firm-specific trends. This consistency of results suggests that our findings are not merely responses to transitory shocks.

(co-movement) of a one percent change in firms' output on firm-level outcome $\mathbb{O}_{it}$ (compared to the industry-year mean growth). We examine time differences h ranging from one year to ten years.

Results. Table 2 presents OLS regression results for various outcomes (1- to 10-year changes across panels). As a companion exhibit, Figure 3 visualizes firm-level relationships from Table 2 using binned scatter plots and residualizing variables by industry-year fixed effects as in Equation (10)—revealing linear relationships that support the linear regression specification. As OLS coefficients are precisely estimated, we focus our discussion on point estimates (regression tables contain standard errors).

Input quantities. Table 2 Columns (1)-(3) and Figure 3 Panels (a)-(c) report effects on input use. We measure labor in head counts. Capital and intermediates are measured as industry-deflated input expenditures (Appendix D describes the capital stock calculation).[26] We discuss hours worked, capacity utilization, and input quality below. (Moreover, we omit TFP as another factor that accounts for firm growth with inputs scaling (weakly) less than one to one with output, and below, test and find no evidence for increasing returns to scale.)

Intermediate inputs exhibit approximately a unit elasticity with output growth, while labor and capital increase by much less. These relative slopes result in a declining ratio of labor (and capital) to intermediate inputs. The small standard errors allow us to reject the hypothesis of proportionate input growth, which would be expected in a Cobb-Douglas model with constant or uniformly shifting (shadow) input prices.

Cost shares. One possible explanation for the shift from labor to intermediates is divergence of input prices, while production remains consistent with a Cobb-Douglas model. In this scenario, cost shares would remain stable, as firms adjust input quantities inversely proportionately to rising input prices. (Input wedges are not captured by monetary costs and hence cost shares—we explore their role in Section 6.)

Table 2 Columns (4)-(6) and Figure 3 Panels (d)-(f) report effects on cost shares, i.e., input expenditures divided by total costs.[27] The data reveal a striking shift towards intermediate costs, away from labor (and capital), indicating that firms are increasingly outsourcing production. This is reflected in the declining ratio of value added to total shipments (output) shown in Figure 1.

Complementing the cost share analysis in logs, we also run the specification in level (ppt.) changes

[26] Not accounting for firm-specific intermediate input prices may slightly attenuate our results on real intermediate input quantities. This bias is likely small, and we can ballpark it as follows. Percent changes can be expressed as $\frac{dM_{it}}{M_{it}} + \frac{d\widetilde{P}_{it}^M}{\widetilde{P}_{it}^M} = \frac{d\widetilde{M}_{it}}{\widetilde{M}_{it}}$, where M_{it} is real quantities (our object of interest), $\widetilde{M}_{it}$ is the *firm-specific* intermediate price deviation from the industry intermediate input price index, and $\widetilde{P}^M$ is the measured real intermediate input quantity adjusted for the industry index. Assuming that firm-specific price movements are linked to firm-specific real quantities through firm-specific elasticity ϵ', such that $\frac{d\widetilde{P}_{it}^M}{\widetilde{P}_{it}^M} = \epsilon' \cdot \frac{dM_{it}}{M_{it}}$, we can write: $\frac{dM_{it}}{M_{it}} = \frac{d\widetilde{M}_{it}/\widetilde{M}_{it}}{1+\epsilon'}$. Thus, the estimated regression coefficient on the basis of our industry-deflated intermediate input measure is related to the object of interest by $\rho_{M,Q} = \frac{\rho_{\widetilde{M},Q}}{1+\epsilon'}$. The attenuation is small; for instance, even if we assumed that intermediate inputs were as inelastically supplied as labor, with $\epsilon' \approx 0.1$, then latent real intermediate inputs would only move slightly less elasticically with output than suggested by our coefficient estimate—so that our overall results are unlikely to be affected by this approach. Intermediate inputs are likely more elastically supplied than this benchmark suggests, implying negligible bias in our results, although our paper highlights the need for better evidence on firm-specific supply elasticities of intermediates.

[27] We approximate capital costs as 8 percent of the capital stock, following Dhyne et al. (2024) and the approach used in the CompNet data (CompNet, 2023), which we use in Section 7. For logged specifications, this homogeneous multiplicative factor does not identify the coefficient due to the (industry-)year fixed effects. Similarly, an industry-specific capital deflator would drop out. For that reason, we also do not attempt to provide more granular heterogeneous cost of capital measures.

(as in Figure 1), showing that intermediate cost shares absorb the decrease in the labor cost shares and, with a quantitatively much smaller role, the shift in capital cost shares (see Appendix Tables A.3 and A.4). (We also study the labor income share in Section 6.)

Output elasticities and returns to scale. The prediction of constant input cost shares in a Cobb-Douglas model relies on the assumption of fixed input wedges and stable output elasticities. Indeed, Equation (5) shows that with constant input wedges and returns to scale, shifts in cost shares correspond directly to changes in output elasticities. To account for the possibility of varying input wedges, Table 2 Columns (7)-(9) and Figure 3 Panels (g)-(i) analyze output elasticities based on our production function estimates.

We find a strong negative relationship between output growth and labor output elasticities. Table 2 Column (7) reveals that an additional 10 percentage point output growth reduces the (growth of) labor output elasticity by 3 percentage points, with a coefficient of -0.30 (SE 0.004) at a one-year horizon. Figure 3 Panel (g) visualizes the underlying relationship. This shift away from labor output elasticities is accompanied by a significant increase in intermediate output elasticities, as shown in Column (9) and Panel (i). Notably, capital output elasticities decline as well (Column (8) and Panel (h)).

For additional clarity, Appendix Tables A.3 and A.4 reproduce our findings in levels rather than logs for the dependent variables. The level changes in labor and capital output elasticities offset those in intermediate output elasticities, with labor and intermediates driving most of the variation. This result is consistent with the much smaller capital cost shares and output elasticities (Figure 2 and Table A.1).

Notably, we normalize the output elasticities in our regressions by the returns-to-scale parameter to account for any changes in scale, which is consistent with our constant returns-to-scale production function in Section 2. However, the results remain consistent without this adjustment (Appendix Tables A.3 and A.4). Column (10) shows that returns to scale are relatively stable in the short term and only change slightly over longer horizons. (Moreover, when we later estimate substitution elasticities, the returns to scale normalization cancels out—see Equation 3.)

Effects across horizons. Comparing horizons (1, 4, 10 years) across the panels reveals interesting dynamics: the coefficient for labor output elasticities drops from -0.30 to -0.20 and -0.14 for 4- and 10-year horizons, respectively. As we explore in Section 5, the evidence for horizon-dependency could be consistent with short-run labor adjustment costs that shape firms' input mix and production modes.

Table 2: Firm-level adjustments in response to firm growth. OLS regressions.

	$\Delta\ln(L_{it})$	$\Delta\ln(K_{it})$	$\Delta\ln(M_{it})$	$\Delta\ln(CS^L_{it})$	$\Delta\ln(CS^K_{it})$	$\Delta\ln(CS^M_{it})$	$\Delta\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta\ln(\frac{\theta^K_{it}}{RTS_{it}})$	$\Delta\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\Delta\ln(RTS_{it})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.298***	0.0405***	1.025***	-0.376***	-0.728***	0.238***	-0.304***	-0.315***	0.151***	0.0062***
	(0.0040)	(0.0022)	(0.0033)	(0.00393)	(0.0033)	(0.0022)	(0.004)	(0.0063)	(0.0015)	(0.0003)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.215	0.043	0.741	0.343	0.556	0.340	0.230	0.137	0.325	0.053
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.510***	0.206***	1.045***	-0.286***	-0.649***	0.171***	-0.202***	-0.181***	0.0951***	0.0167***
	(0.0064)	(0.0067)	(0.0045)	(0.0062)	(0.0071)	(0.0034)	(0.0060)	(0.0074)	(0.0025)	(0.0006)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.471	0.144	0.860	0.348	0.476	0.355	0.220	0.170	0.268	0.163
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.606***	0.395***	1.041***	-0.219***	-0.490***	0.131***	-0.140***	-0.0720***	0.0582***	0.0278***
	(0.0066)	(0.0082)	(0.0043)	(0.0061)	(0.0083)	(0.0033)	(0.0053)	(0.0069)	(0.0024)	(0.0007)
Observations	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915
N of firms	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595
R^2	0.594	0.256	0.901	0.296	0.329	0.303	0.188	0.149	0.200	0.268

Notes: The table reports OLS regressions from estimating the specification in Equation (10). The dependent variables in Columns (1)-(10) are log changes in labor, capital, intermediates, labor cost shares, capital cost shares, intermediate cost shares, labor output elasticities over returns to scale, capital output elasticities over returns to scale, intermediate input output elasticities over returns to scale, and returns to scale, respectively. Panels A-C report on regressions in those dependent variables on changes in log output for 1-, 4- , and 10-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

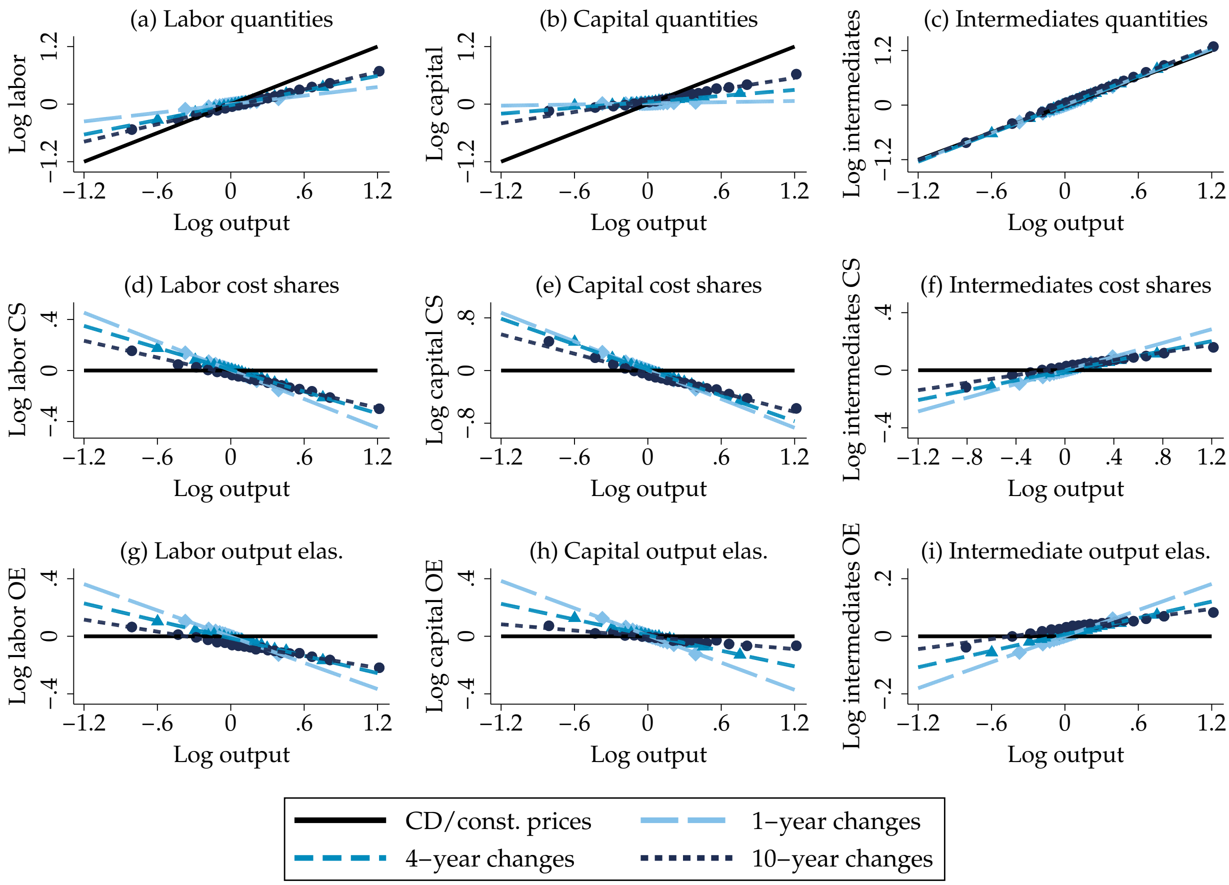
Heterogeneity: types of intermediates. The strong response of intermediates and substitution from labor may partially reflect that intermediate inputs also include intermediate services, unfinished product components, and similar items (see Figure 2). Appendix Table A.5 reports additional regressions for logged cost shares of all available subcategories of intermediates as dependent variables. The effects are as expected. We find strong effects for product components and materials. Effects are negative for intermediate components that are arguably complementary to capital inputs, such as repair, maintenance, and installation services. We find the strongest effects for temporary agency workers, indicating that firms rely on those more flexible labor inputs when growing (see De Leon et al., 2024), particularly in the short run. However, *quantitatively*, this response has a minimal impact on the overall intermediate effect, as, on average, temporary agency workers account for only 1% of total costs (Figure 2). Consistent with this analysis, in an unreported robustness check, we have assigned temporary agency workers and subcontracted production to labor rather than intermediate inputs (and divided those costs by the firm's average wage to approximate imputed labor-equivalent quantities), and found similar results for output elasticity shifts, cost share shifts, and hence implied substitution elasticities.

Heterogeneity: size heterogeneity. Appendix Table A.7 reproduces our regressions splitting firms into five size quintiles. Across all size groups, results align closely with our main results (Table 2). We return to the issue of firm size when discussing non-homotheticities in Section 5.3 below.

Heterogeneity: industries. In Appendix Figure A.2, we report key results by industry. Qualitatively, results are similar across industries—input quantities, cost shares, and output elasticities shift from labor to intermediates. Quantitatively, the extent of these shifts differs somewhat across industries, particularly for output elasticities. We have not systematically examined this heterogeneity for lack of clear variation in the underlying parameters. However, we note that all things equal, one would predict a stronger shift from labor toward intermediate quantities in industries with stronger labor monopsony power or higher labor adjustment costs (relative to intermediates), and similarly for output elasticities and cost shares (provided substitution elasticities above one). Relatedly, we note that firms generally have substantial wage flexibility within the German collective bargaining system, because coverage status is not universal, because it is chosen by firms (by joining employer associations), and because covered firms often pay premia (Jäger et al., 2022). Moreover, this system does not provide clear variation that could systematically mediate monopsony degrees for use in heterogeneity analyses.

Additional analysis: hours worked, capacity utilization, and input quality. From 1999 to 2017, we also observe employment in full-time equivalents (FTE), permitting us to also account for intensive-margin hours adjustments. Appendix Table A.2 shows that headcount and FTE responses are nearly identical, suggesting that intensive margin hours effects are unlikely to confound our estimates. (Capital adjustments are not our focus, and we do not observe capital utilization.) Regarding input *quality* adjustments, we do not have detailed micro-data on labor quality, but we can gauge such effects by studying compositional effects on the basis of average wages. Average wages slightly increase in response to output growth, with coefficients ranging from 0 to 0.1 (Appendix Table A.8). If interpreted as an increase in the labor quality, the efficiency units of labor (quantity plus quality effect)

Figure 3: Firm-level adjustments in response to firm growth (OLS, binned scatter plots).



Notes: The figure reports binned scatter plots from estimating the specification in Equation (10) with OLS for various differences against log output changes for the following dependent variables in log changes: labor, capital, intermediate quantities, output elasticities over returns to scale, and cost shares. It also includes the prediction from a Cobb-Douglas production framework with firms optimizing against constant input price (ratios). All panels report results that are residualized by industry-year fixed effects. German firm-level data.

would only attenuate the implied substitution elasticity in a minor way.[28] The alternative—and our preferred—interpretation of the average wage effect points to upward-sloping labor supply curves (and is quantitatively realistic, see our meta-analysis in Section 5.2). We discuss the limitations of average wages below and instead infer changes in shadow prices as a broader marginal cost measure that is more relevant to our analysis.

Additional analysis: cross-sectional results and firm lifecycle. Our paper focuses on within-firm changes. For completeness, we provide results for the cross-section in Appendix Figure A.1.[29] Finally,

[28]To see this, note that the effect on the wage bill can be imputed as the effect on employment times the average wage, i.e., $\Delta \ln(\bar{w}L) = \Delta \ln(\bar{W}) + \Delta \ln(L)$, such that the labor effect would slightly rise from 0.510 to 0.572 for, e.g., at the 4-year horizon. Foreshadowing our calculation of the substitution elasticity in Section 5.1, and assuming away effects on measured output elasticities, this effect would only slightly change our conclusion about substitution elasticities between intermediates and labor (from 2.25 to 2.29).

[29]The cross-sectional analysis reflects qualitatively different forces such as permanent heterogeneity. Our empirical analysis confirms that large firms have lower labor cost shares and labor output elasticities but paints a less clear picture likely due to input price differences and/or other (permanent) heterogeneities (e.g., the α and Λ terms in Equation (1)). The effect for the labor output elasticity falls to -0.03, again precisely estimated. Scatter plots in Appendix Figure A.1 reveal a slightly concave pattern, consistent with large shorter-run elasticities not extending to cross-sectional variation in firm size,

while we do not have information on firm age in our main data, our extension to CompNet data in other countries in which age is included shows qualitatively similar effects across firm-age groups (Section 7.2 and Appendix Table A.14).

4.2 Causal Effects: IV Strategy using Export Demand Shocks

We now use an instrumental variable strategy that draws on *foreign* product demand shocks as variation in firm growth that is plausibly unrelated to input-biased shifts in production function parameters, input prices, age effects, or input quality (conditional on industry-year fixed effects). This analysis aims to trace firm growth dynamics corresponding to our structural equation in Section 2, where we focused on shifts in firm output while holding such confounding factors constant.

Across outcomes, IV coefficients are similar to the OLS counterparts. This result is consistent with our OLS regressions largely reflecting input-unbiased sources of growth such as shifters in product demand, TFP, or input prices across the board.

Strategy. We follow an established literature using trade shocks as exogenous shifters (see Autor et al., 2016 for a review). In particular, we follow Hummels et al. (2014) and instrument changes in firms' output with changes in world export demand (excluding Germany). The method proceeds in two steps. First, we compute a product-level export-demand shock; second, we translate these shocks into a single share-weighted export-demand shock for each firm. In the first step, we compute the total exports for each product, g, from a country group, n, to the world:

$$EX_{gt} = \sum_{c} ex_{gct}^{n \rightarrow world}, \tag{11}$$

where n contains Australia, Norway, Sweden, Singapore, New Zealand, Great Britain, Canada, Japan, and the US. c denotes the country. Our country selection follows the strategy in Dauth et al. (2014), with the exception that we additionally include the US (results are robust to excluding the US). There are two reasons for this selection. First, we choose other industrialized countries whose economies and export specialization are more plausibly similar to Germany. Second, this selection of farther-away countries (excluding Germany's direct neighbors and members of its EUR currency union) helps mitigate potential endogeneity concerns arising from unobserved shocks that might be correlated between Germany and other nations.

In the second step, we compute firm-specific export demand shocks by aggregating the product-level shocks from Equation (11) to the firm level. We calculate weighted averages of product-level trade flows, using the sales shares of products within firms' product portfolios as weights:

$$INS_{it} = \sum_{g} s_{git=0} \ln EX_{gt}. \tag{12}$$

s_{git} denotes the firm-specific sales weight of product g. To limit anticipatory effects (e.g., product mix adjustments), we fix the weights for each firm to its first year in the sample. This design thus relies on variation from a fixed (internationally tradable) set of products (and including industry-year FEs

which is right-skewed.

implies we use between-firm, within-industry variation).[30] Having constructed these firm-specific instruments, we instrument $\Delta \ln Q_{it}$ in Equation (10) with ΔINS_{it}, i.e., we use export demand shocks to instrument output changes.

Importantly, 80% of firms in our German manufacturing firm sample export in a given year. As we do not observe firm-product-specific export shares, our product sales share weights include domestic sales as well. Furthermore, we restrict our IV analysis to 1- and 4-year differences as the first stage is not sufficiently strong (a low F-statistic) for 10-year changes.

First stage. Table 3 is analogous to Table 2 and reports the corresponding IV results. Column (1) shows the first stage coefficient from regressing output growth on our instrument. We find a statistically significant positive association. The F-statistic is 102.6 for one-year changes and 48.57 for four-year changes. We visualize the first stage regressions in Figure 4 Panel (a).

Results: IV estimates. Table 3 Columns (2)-(11) report the second stage IV results, where we instrument output growth with export demand shocks. Across outcomes, IV coefficients are similar to the OLS counterparts. For instance, the labor output elasticity coefficient is -0.327 (-0.242) at the one-year (four-year) horizon, nearly identical to the OLS results.

Results: Reduced form estimates. Figure 4 Panels (b)-(j) (associated regression tables available on request) report reduced form estimates (i.e., we directly regress the dependent variables from Table 3 on the instrument). Results are fully consistent with our IV regressions and similar for one and four-year changes. For instance, the coefficients for labor output elasticities (divided by returns to scale) are approximately -0.015 for both time horizons. The corresponding intermediate input output elasticity coefficients are both 0.005, and labor cost share coefficients are -0.02 and -0.017.

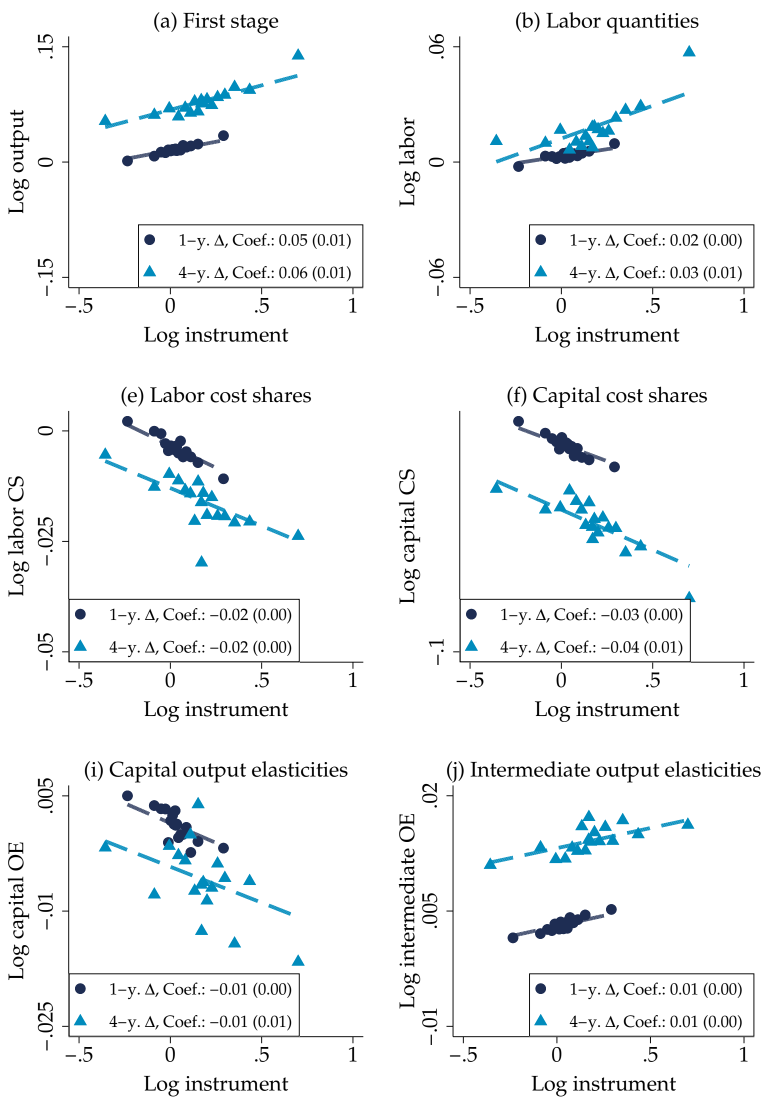
[30]The aggregation is unconditional on export status at the firm-product level, as we only have export information at the overall firm level; in a suggestive unreported check, we have confirmed that the instrument power is derived from exporters.

Table 3: Firm-level adjustments in response to firm growth. IV regressions.

	1st stage	$\Delta \ln(L_{it})$	$\Delta \ln(K_{it})$	$\Delta \ln(M_{it})$	$\Delta \ln(\frac{P_{it}^L L_{it}}{C_{it}})$	$\Delta \ln(\frac{P_{it}^K K_{it}}{C_{it}})$	$\Delta \ln(\frac{P_{it}^M M_{it}}{C_{it}})$	$\Delta \ln(\frac{\theta_{it}^L}{RTS_{it}})$	$\Delta \ln(\frac{\theta_{it}^K}{RTS_{it}})$	$\Delta \ln(\frac{\theta_{it}^M}{RTS_{it}})$	$\Delta \ln(RTS_{it})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Export demand shock	0.0451***										
	(0.0045)										
Log output change		0.324***	0.0612	0.929***	-0.444***	-0.684***	0.171***	-0.327***	-0.229**	0.119***	0.0082
		(0.0543)	(0.0461)	(0.0508)	(0.0521)	(0.0565)	(0.0292)	(0.0623)	(0.0980)	(0.0198)	(0.0053)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
First-stage F-Statistic		102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6
R^2	0.205	0.214	0.042	0.736	0.336	0.554	0.321	0.229	0.132	0.314	0.052
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Export demand shock	0.0635***										
	(0.0091)										
Log output change		0.536***	0.262***	0.963***	-0.269***	-0.570***	0.124***	-0.242***	-0.147	0.0823***	0.0222***
		(0.0737)	(0.0973)	(0.0542)	(0.0664)	(0.0969)	(0.0369)	(0.0706)	(0.0938)	(0.0268)	(0.0075)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-Statistic		48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.183	0.470	0.139	0.855	0.347	0.471	0.339	0.217	0.169	0.266	0.156

Notes: The table reports IV regressions from estimating the specification in Equation (10) using foreign demand shocks as instruments (Equation (12)). Column (1) reports the first-stage regression results. The dependent variables in Columns (2)-(11) are log changes in labor, capital, intermediates, labor cost shares, capital cost shares, intermediate cost shares, labor output elasticities over returns to scale, capital output elasticities over returns to scale, intermediate input output elasticities over returns to scale, and returns to scale, respectively. All columns report regressions of those dependent variables on changes in log output for 1- and 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure 4: First stage and reduced form results, 1- and 4-year changes.



Notes: The figure reports binned scatter plots corresponding to the firm-level first stage (Panel (a)) and reduced form regressions (Panels (b)-(j)). The first stage regresses changes in log output on the instrument, whereas the reduced form regressions do so for the changes in logs of labor, capital, and intermediate quantities, labor, capital, and intermediate cost shares, and labor, capital, and intermediate output elasticities divided by returns to scale (Panels (b)-(j), respectively). Regressions are in one-year and four-year changes. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Table 4: Implied substitution elasticities, effects of firm growth on input ratios and input shadow price ratios, and firm-specific labor supply elasticities.

	OLS				IV			
	σ	$\Delta\ln\left(\frac{L_{it}}{M_{it}}\right)$	$\Delta\ln\left(\frac{P^L_{it}\gamma^L_{it}}{P^M_{it}\gamma^M_{it}}\right)$	$\epsilon^L=\frac{1}{\varepsilon^L}$	σ	$\Delta\ln\left(\frac{L_{it}}{M_{it}}\right)$	$\Delta\ln\left(\frac{P^L_{it}\gamma^L_{it}}{P^M_{it}\gamma^M_{it}}\right)$	$\epsilon^L=\frac{1}{\varepsilon^L}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1-year diff.	2.673	-0.727	0.272	1.10	3.805	-0.605	0.159	2.04
4-year diff.	2.249	-0.535	0.236	2.16	4.158	-0.427	0.103	5.20
10-year diff.	1.837	-0.435	0.237	2.56				

Notes: The table reports substitution elasticities (Columns (1) and (5)) following Equation (14), changes in input factor ratios (Columns (2) and (6)), implied changes in shadow input price ratios (Columns (3) and (7)) following Equation (16), and implied labor supply elasticities following Equation (18), assuming perfectly elastic intermediate input supply (Columns (4) and (8)), based on our OLS (Columns (1)-(4)) and IV (Columns (5)-(8)) regressions from Tables 2 and 3 that regress log output elasticities over returns to scale and log input quantities on log output in within-firm differences. Consequently, Columns (2) and (6) report coefficient ratios for labor and intermediates from these regressions with respect to firm growth, while all other columns report values implied by our regressions as described in the text.

5 Quantitative and Structural Interpretation

We now interpret our reduced form results structurally and quantitatively through the lens of the production model presented in Section 2. Subsequently, we discuss alternative accounts.[31] Importantly, our estimates in this section allow for firm- and year-specific markups and input wedges.

Table 4 summarizes the identified parameters and the mapping from reduced-form empirical moments using the identification arguments from Section 2 (that we further detail below) for the substitution elasticity between labor and intermediates and the firm-specific labor supply elasticity. Figure 5 summarizes existing estimates of the two key parameters, along with the values our study implies. Throughout, we focus on our *firm growth* estimates.

5.1 Identification of Substitution Elasticity

Identification argument. Our identification of the substitution elasticity rests on Equation (3), which identifies the labor-intermediates substitution elasticity, σ, from the co-movement of output elasticities and input quantities:

$$\frac{\sigma-1}{\sigma}=\frac{\Delta\ln(\theta^L_{it})-\Delta\ln(\theta^M_{it})}{\Delta\ln(L_{it})-\Delta\ln(M_{it})}\quad\Rightarrow\quad\sigma=\frac{1}{1-\frac{\Delta\ln(\theta^L_{it})-\Delta\ln(\theta^M_{it})}{\Delta\ln(L_{it})-\Delta\ln(M_{it})}}. \tag{13}$$

To calculate σ, we use within-firm changes (Δ) *conditional on firm growth*, i.e., we insert the estimated OLS and IV coefficients from our firm growth regressions into Equation (13).[32] That is, for each right-hand side component of Equation (13), we plug in the corresponding regression coefficient for

[31] As before, we focus on within-firm changes, which net out fixed, unobserved firm-specific factors, captured by α^L_i and α^M_i in our model (see Equation (3)). Large firms seem to have higher intermediate-labor ratios, with smaller differences in labor and intermediate input output elasticities (Appendix Figure A.1). As our focus is on firm growth, we omit a detailed discussion of how to rationalize those facts with permanent heterogeneity.

[32] Compared to the OLS estimates, the IV estimates can be viewed as additionally removing potential confounders, such as input-biased shifters in prices or production function parameters, that may underlie some of the OLS variation in firm growth.

that outcome estimated in Section 4, separately by time horizon and for OLS or IV:

$$\widehat{\sigma} = \frac{1}{1 - \frac{\rho_{\Delta \ln(\theta^L), \Delta \ln(Q)} - \rho_{\Delta \ln(\theta^M), \Delta \ln(Q)}}{\rho_{\Delta \ln(L), \Delta \ln(Q)} - \rho_{\Delta \ln(M), \Delta \ln(Q)}}}, \tag{14}$$

where the ρ variables correspond to the regression coefficients from Equation (10), for various horizons and outcome variables. Hence, relating our reduced-form difference-based estimates of changes in output elasticities and input factors in response to firm growth to a structural equation, identifies the substitution elasticity, σ, *without any assumptions on input prices* (expect those related to the estimation of output elasticities). Given the precisely estimated underlying coefficients, β, we focus on point estimates.

Implied parameter values: baseline translog estimates. Table 4 Columns (1) and (4) report the implied values of σ based on our OLS and IV estimates. We find that labor and intermediates are substitutes, with σ exceeding unity in all specifications. The OLS estimates range from 2.67 in the short run to 1.84 in the longer run. IV estimates of σ are around 4 for 1- and 4-year changes (IV estimates for 10-year changes are unavailable). These substitution elasticity estimates above one provide a coherent explanation for the decline in relative output elasticities following the reduction in the labor-intermediate quantity ratio.[33]

The relatively high substitution elasticities may also reflect that intermediate inputs include services, product components (e.g., the display of a phone), and similar items. It is plausible that substitution elasticities between raw materials (such as steel) or energy and labor would be lower.[34]

Implied parameter values: cost shares. As alternative specification, we report substitution elasticities using cost shares as our measure of output elasticities in Table A.9. As expected, estimates of substitution elasticities increase, remaining consistently above unity. Our cost share estimates allow for a constant degree of input market imperfections, but, by design, exclude changes in input wedges (γ_{it}^X). Consequently, variation in γ_{it}^X correlated with output growth loads on the substitution elasticity (see Table 6, showing that $\frac{\gamma_{it}^L}{\gamma_{it}^M}$ increases with output growth). This is intuitive: smaller increases in relative shadow prices imply larger substitution elasticities to explain the observed shift from labor toward intermediates. The strong increase in substitution elasticities highlights the importance of accounting for changing input wedges (we show how input wedges change with output growth below).

Meta-analysis of existing substitution elasticity estimates. Figure 5 Panel (a) situates our baseline translog estimates in a systematic meta-analysis of existing estimates (we indicate our own estimates by "MS"). Existing estimates of the intermediate-labor substitution elasticity rely on disparate identification strategies and production model assumptions. We therefore differentiate between approaches

[33] We do not focus on comparing short- vs. long-run substitution elasticities or attempt to interpret these dynamics in context of the Le Châtelier principle (Samuelson, 1947, Milgrom and Roberts, 1996), although we note that both labor and intermediates are presumably quite flexible compared to capital. However, OLS (IV) estimates indicate somewhat smaller (slightly larger) long- than short-run elasticities. These patterns must be interpreted in the context of intermediate-input mix adjustments: in the short run, firms increase inputs *more substitutable with labor*, such as temporary agency workers or subcontracted work, relatively more strongly, while in the longer run, firms increase inputs like rents and leases, repairs and maintenance, or other intermediates relatively more strongly (see Appendix Tables A.5 and A.6).

[34] As an aggregate perspective, consistent with labor and intermediates being substitutes, manufacturing intermediate to labor expenditure ratios increased while intermediate input to labor price ratios declined. See Appendix Figure A.3 for evidence on Germany and the US.

that estimate substitution elasticities between intermediates and a capital-labor bundle (diamonds) and between intermediates and labor (triangles). The former approach restricts the substitution elasticity between intermediates and capital to be identical to that between intermediates and labor. It typically identifies the substitution elasticity using the ratio between intermediates and the capital-labor bundle (value added). The latter approach directly relates intermediates to labor and aligns with our methodology. These direct estimates of labor-intermediates substitution elasticities tend to be higher and exceed unity in several studies. Appendix B provides details on the estimates included in Figure 5 Panel (a); we report mean values and estimates often vary significantly within a given study (for instance, values in Chan (2023) range from 1.6 to 9.6). We also note that our estimates are derived from within-firm changes.

5.2 Identification of Firm-Specific (Relative) Labor Supply Elasticities

Why do firms change their input mix (and thus output elasticities) as they grow? Firms' cost minimization provides a natural answer, as it ties firms' optimal input mix to input prices. We now trace out the implied input price ratio, and translate it into an implied labor supply elasticity to the firm.

Step 1: identifying implied input price ratio changes w.r.t. firm growth. Using our estimates of σ and our firm growth regression coefficients, we can infer the implied (average) change in the shadow price ratio (specifically, the change that is caused by firm growth in our empirical specification). We insert our regression coefficients into Equation (4) to back out the implied effect of firm growth on within-firm changes in the shadow price ratio:

$$\Delta \ln\left(\frac{P^L_{it}\gamma^L_{it}}{P^M_{it}\gamma^M_{it}}\right) = \frac{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}{-\sigma} \tag{15}$$

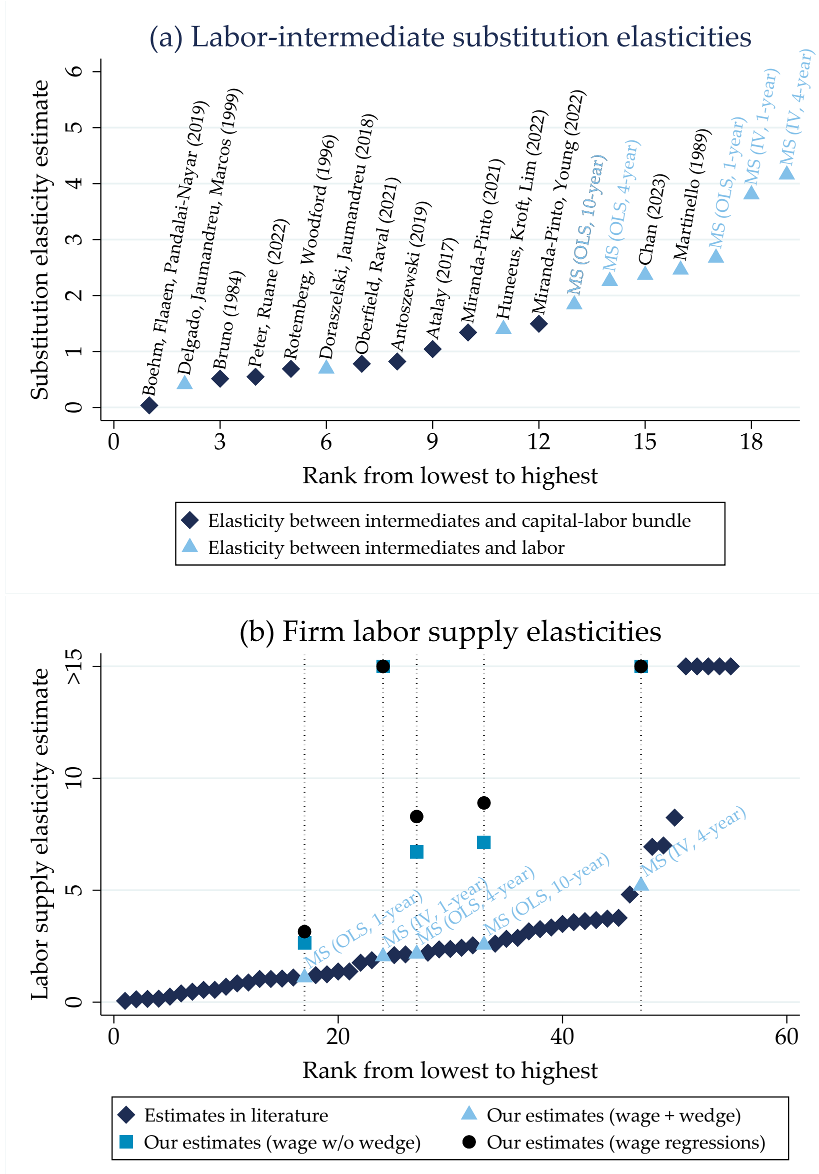
$$\Rightarrow \widehat{\rho}_{\Delta \ln(\frac{P^L\gamma^L}{P^M\gamma^M}),\Delta \ln(Q)} = \frac{\rho_{\Delta \ln(L),\Delta \ln(Q)} - \rho_{\Delta \ln(M),\Delta \ln(Q)}}{-\widehat{\sigma}}. \tag{16}$$

To our knowledge, our paper is the first to combine estimates of output elasticities with a standard production model to measure unobserved input price ratios.

We report results for $\widehat{\rho}_{\Delta \ln(\frac{P^L\gamma^L}{P^M\gamma^M}),\Delta \ln(Q)}$ in Table 4 Columns (3) and (6). OLS estimates indicate changes in the log price ratio of 0.27 in the short run and 0.23 in the long run. IV estimates are 0.16 and 0.10 for short- and longer-run changes, respectively. Consequently, as firms grow the shadow price of labor increases relative to that of intermediate inputs. This increase is stronger in the short run than in the long run. As a result, firms substitute labor with intermediates and change their modes of production as reflected in their output elasticities.

Potential sources of the relative increase in labor prices. What forces cause relative labor costs to rise as firms grow? One natural explanation is that the firm-specific labor supply elasticity is low (compared to that of intermediate inputs), suggesting the presence of monopsony power in labor markets, which raises wages as firms hire more workers. The persistence of rising relative labor costs even over a long-term (10-year) horizon reinforces the monopsony-based explanation. However, the fact that relative labor costs are notably higher in the short run also points to short-run adjustment costs, which are plausibly more significant for labor than for intermediate inputs. Adjustment costs

Figure 5: Meta-analysis: substitution and labor supply elasticities vs. literature.



Notes: The figure reports estimates of labor-intermediate substitution elasticities (Panel (a)) and labor supply elasticities (Panel (b)) from the literature and from our analysis (our own estimates are indicated by "MS" and in light blue). For substitution elasticity estimates, we focus on samples that encompass the most firms or report mean values (excluding negative estimates when computing the means). Appendix B provides details on the displayed substitution elasticities. Data on labor supply elasticities come from the meta-study by Sokolova and Sorensen (2021). We exclude negative estimates. If available, we report median IV estimates. If no IV approach is used, we report medians of all other estimates. (We prefer medians due to outliers.) We do not implement any alternative selection methods (e.g., best practice corrections). Sokolova and Sorensen (2021) note that best practice correction estimates vary by methodology: separations-based approaches yield elasticity estimates between 6.4 (not identified) and 9.9 (identified), while stock-based inverse approaches produce estimates above 20.For a list of studies entering the firm-specific labor supply elasticity estimates, we refer to Sokolova and Sorensen (2021).

can thus be an important driver of (short-run) changes in input price ratios, and, through that, input quantity and output elasticity ratios.

Step 2: Identifying the firm-specific factor supply elasticities. Although not crucial for our key results (we only require increasing *relative* shadow prices for labor compared to intermediates), we find it useful to discuss implied estimates of the labor supply elasticity. We acknowledge that, compared to the identification of σ, the identification of the labor supply elasticity requires stronger assumptions. As our estimates identify the relative elasticity of labor to that of intermediates, we can infer the *absolute* firm-specific labor supply elasticity under two natural assumptions (which are not required for identifying substitution elasticities). First, intermediate inputs are perfectly elastically supplied.[35] Second, the markdown, γ^L_{it}, is constant in firm growth (we still permit a baseline wedge)—an assumption we later relax by measuring γ^L_{it} and studying its firm growth gradient directly. Those two assumptions imply that the input shadow price gradient is solely due to wage increases, allowing us to infer the implied firm-specific labor supply elasticities from our estimated input price changes and the employment effects:

$$\widetilde{\epsilon}^L = \frac{\Delta \ln(L_{it})}{\Delta \ln\left(\frac{P^L_{it}\gamma^L_{it}}{P^M_{it}\gamma^M_{it}}\right)} = \frac{\Delta \ln(L_{it})}{\Delta \ln(P^L_{it})}, \tag{17}$$

which we identify on the basis of the following ratio:

$$\widehat{\widetilde{\epsilon}}^L = \frac{\rho_{\Delta \ln(L), \Delta \ln(Q)}}{\widehat{\rho}_{\Delta \ln(\frac{P^L\gamma^L}{P^M\gamma^M}), \Delta \ln(Q)}}. \tag{18}$$

That is, we identify the labor supply elasticity by dividing the changes in labor quantities from Tables 2 and 3 by the changes in input price ratios from Table 4 as constructed above. We emphasize that this parameter, $\widetilde{\epsilon}^L$, is the *inverse* of the supply elasticity, ε^L, in Section 2. Moreover, the $\widehat{\rho}$ term in the denominator is *not* a regression coefficient but the inferred input shadow price ratio gradient backed out in Equation (15) above.

Results: implied firm-specific labor supply elasticities. We report implied labor supply elasticities, ϵ^L, in Table 4. The values range from 1.10 to 2.56 for our OLS and from 2.04 to 5.20 for our IV results. The higher long-run than short-run elasticities either reflect horizon-dependence of labor supply elasticities, or may reflect firm-side adjustment costs (non-constant γ^L_{it}). Nevertheless, even in the long run, supply elasticities remain relatively low. These estimates showcase the quantitative basis for the monopsony-driven incentive for firms to shift from labor to intermediate inputs.

Robustness: permitting markdowns and direct wage estimates. By using the shadow price of labor (the product of the wage and wage markdown), our initial estimates of the labor supply elasticity may be confounded by the markdown variation. We now calculate firm-specific labor supply elasticities with respect to wages, under two alternative assumptions. We report these alternative values vertically above our initial estimates in Figure 5 Panel (b).

First, we report results that subtract the measured markdown effect on $\gamma^L_{it}/\gamma^M_{it}$ that we estimate in

[35]We are not aware of comparable estimates for the firm-specific supply elasticities for intermediates. Bilal and Lhuillier (2022) make an analogous assumption for a labor-only model involving labor service purchases (such as temp work agencies), although our intermediates are largely made of goods rather than services. Similarly, Dobbelaere and Mairesse (2013) and subsequent studies assume perfect elastic intermediate supply to back out markdowns and, on the basis of markdown levels, the implied labor supply elasticities underlying those markdowns under the assumption of them exclusively and precisely reflecting wage setting power. For temporary agency workers as one facet of intermediates, Drenik et al. (2023) show that the prices of these intermediates covary with firm-specific regular workers' wages at a rate of much less than one-to-one (with much of the positive covariance likely reflecting labor-specific mechanisms unlikely to extend to other intermediates, such as equity constraints or the presence of equal-pay regulations). Goldschmidt and Schmieder (2017) document that outsourcing of low-skill service tasks is motivated by and associated with cost savings in Germany. Huneeus et al. (2022) focus on intermediate input price variation from production networks as a driver of cross-sectional outsourcing differentials.

Section 6 when studying the labor share implications (squares). As we find that markdowns *increase* as firms grow, this calculation pushes up the labor supply elasticities in our study. Intuitively, by eliminating the effect of increasing wage markdowns, a given shift in labor now corresponds to a smaller change in labor prices, implying higher supply elasticities.

Second, we also measure wages directly, using the average firm wage (wage bill per head) from the firm-level data and run regressions of wage changes on output growth as before (dots). These additional wage regressions are reported in Appendix Table A.8. Particularly for the OLS strategy, average wages increase in firm growth (consistent with monopsony), which, together with the labor quantity changes, yields a direct estimate of the wage change along the labor supply curve. Since those wages move less than the inferred shadow price ratios above, the implied firm-specific labor supply elasticities are again higher than our baseline measure. For the IV effects, the wage effect point estimates are close to zero; hence, the associated elasticities would be large, but the wider confidence intervals for the wages also accommodate elasticities more consistent with the literature. Overall, we note that average wages are subject to composition bias such as from worker quality (and may be confounded by hours responses) and that we cannot merge matched employer-employee data or data on worker skills to our firm data to use cleaner wage measures.[36]

Meta-analysis of existing parameter estimates. Figure 5 Panel (b) summarizes existing estimates of labor supply elasticities based on the meta-analysis of Sokolova and Sorensen (2021), along with the values our study implies, including the additional estimates from the robustness checks discussed above. While estimates are ranked by size, for our own estimates, we provide their respective robustness checks vertically stacked. Overall, our estimates fall well into the range of existing estimates and highlight a lower short- than long-run labor supply elasticity. The figure also illustrates that supply elasticities are higher for our IV estimates, and when we net out markdown shifts or use direct (average) wage estimates. We note again that our estimates are based on within-firm growth regressions.

5.3 Alternative Mechanisms

Automation. Recent work has highlighted capital-based technological change as a factor reducing labor's importance to firms (Acemoglu and Restrepo, 2020, Hubmer and Restrepo, 2021). Such technological change could explain a decline in labor's output elasticity with firm growth. However, rather than an increase, we document a decline in capital output elasticities with firm growth. Instead, intermediate inputs *unrelated to capital services* gain in importance (as discussed above, maintenance and repair services decline, Appendix Table A.5). Therefore, while automation is an important process shaping changes in firms' production and aggregate labor market trends, it appears not to rationalize our findings in the context of idiosyncratic firm growth. Additionally, we reiterate that our causality runs from plausibly input-neutral output shifters to changes in output elasticities.

Biased technological change. A potential concern is that our output elasticities may be mismeasured due to the assumption of Hicks-neutrality. However, we do not believe that measurement error,

[36]For instance, over a comparable period, Lochner et al. (2020) report a German large-firm wage premium of 10–11 log points, of which 4–6 points are due to person fixed effects, which would suggest higher labor supply elasticities. However, we also note that these regressions capture long-run elasticities and note an ongoing debate about the level of the elasticity.

particularly factor-biased technological shocks, can explain our findings.

First, our results for cost shares and output elasticities are quantitatively similar (see Tables 2 and 3). This similarity is reassuring because these metrics reflect firms' output elasticities under different assumptions. Our direct output elasticity estimates rely on Hicks-neutrality and a specific model of firm behavior, while allowing for varying input market imperfections and returns to scale. In contrast, changes in cost shares capture changes in output elasticities under non-changing input market imperfections and non-changing returns to scale, but accommodate non-Hicks-neutral productivity processes without estimating the underlying production function. In Appendix Table A.9, we compute substitution elasticities from cost share estimates, which suggest even stronger substitutability between labor and intermediates.

Second, we employ an instrumental variable approach, using foreign export demand shocks as instruments for firm growth. These shocks are plausibly largely independent of firm-specific factor-biased technological shocks that could influence output elasticities and overall output. Again, here, our causality runs from firm growth to changes in output elasticities rather than from changes in technology to changes in output.

Fixed costs and size dependence. Our data do not allow us to differentiate between fixed and flexible labor inputs. If firms' production involves fixed labor costs (or shared labor inputs) that scale up less than proportionately with flexible labor, labor output elasticities decline as firms grow. Effectively, fixed costs break the homotheticity of a production function (Dhyne et al., 2022, Savagar and Kariel, 2024).

We expect fixed costs to be particularly relevant for small firms (e.g., fixed overhead costs).[37] However, splitting our firm sample into size (sales) quintiles (by year and industry), yields similar results across size quintiles for our regressions (Appendix Table A.7). As a result, Table 5 shows that also substitution and supply elasticities appear similar across firms of different sizes (we rely on OLS as IV first stages become weak in this sample split).[38] Therefore, we do not find clear evidence for fixed labor inputs in explaining the decline in relative labor quantities and output elasticities with firm growth. (In Appendix Table A.7, we find that declines in capital output elasticities and increases in intermediate inputs are somewhat moderated in larger firms. This result is consistent with large initial capital fixed costs.)

To further guide our assessment of the role of fixed costs, we additionally formalize a basic fixed-cost model in Appendix C.6. This basic model permits us to quantify the fixed cost share needed to rationalize our regression results absent the substitution mechanism we explore in this paper. For 1-year changes, our OLS (IV) estimates imply a required fixed labor cost share in total labor costs of 0.51 (0.50) for the average firm, while for 4-year changes, the required fixed labor cost share is 0.28 (0.31).[39]

[37]We emphasize that we refer to true fixed costs, i.e., the part of labor that does not scale with size.

[38]Despite theoretical predictions that large firms exert greater influence over wages, direct empirical evidence on how labor supply elasticities (i.e., firm-specific wage levels) vary with firm size remains limited. In a tentative empirical check, our data indicate a wage-size (output) gradient that is constant in logs, which is inconsistent with higher monopsony power among larger firms (Appendix Figure A.4).

[39]These numbers are huge because they refer to the share of fixed labor in total labor over a given horizon. That is, 30% (50%) of labor cost would need to be fixed over a horizon of 4 years (1 year). To put this into perspective: using U.S. census data for manufacturing plants during 1974-2011, Ederhof et al. (2021) report a total fixed cost share in total cost of 20%, where they consider depreciation, rental payments, non-production worker salaries, and fringe benefits as fixed cost

Table 5: Implied substitution elasticities, effects of firm growth on input ratios and input shadow price ratios, and firm-specific labor supply elasticities, by size quintiles (OLS, 4-year changes).

	σ	$\Delta \ln \left(\frac{L_{it}}{M_{it}}\right)$	$\Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M}\right)$	
	(1)	(2)	(3)	(4)
Overall	2.259	-0.535	0.236	2.16
1st quintile	2.007	-0.546	0.272	1.86
2nd quintile	2.236	-0.550	0.246	1.97
3rd quintile	2.186	-0.540	0.247	1.99
4th quintile	2.327	-0.519	0.223	2.37
5th quintile	2.277	-0.526	0.231	2.29

Notes: The table replicates Table 4 by size quintiles for OLS. Size quintiles are computed by year and industry. Size is measured by sales.

Additionally, the model reveals that labor fixed costs (or broader fixed costs) cannot rationalize the positive effect of firm growth on intermediate output elasticities. While our paper does not aim to adjudicate between fixed costs and our substitution mechanism, we cautiously argue that the data leave substantial room for the mechanism we explore, in addition to a fixed-costs account.[40]

Non-homothetic CES. Fixed costs are not the only factor that can break the homotheticity of a production function. Our paper proposes a homothetic CES production function that rationalizes changes in relative output elasticities through changes in relative input quantities (and prices). Alternatively, intermediate inputs might become more efficient at larger scale, such that firms' relative labor output elasticities decline as a *direct* result of firm growth. Recently, Lashkari et al. (2024) discussed such a mechanism in the context of IT inputs becoming more efficient at larger scale. An adaptation of their framework to our CES model above and intermediates instead of IT can be described by the following non-homothetic production function:

$$Q_{it} = \Omega_{it} \Lambda_i^K K_{it}^{1-\kappa} \left(\Lambda_i^{LM} \alpha_i^L L_{it}^{\frac{\sigma-1}{\sigma}} + \Lambda_i^{LM} \alpha_i^M \left(\frac{M_{it}}{Q_{it}^{\eta}} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\kappa}, \quad (19)$$

where $\eta > -1$ is a firm-size scaling parameter that captures the non-homotheticity. If $\eta = 0$, Equation (19) collapses to Equation (1). Under this non-homothetic CES, the ratio of output elasticities is (derived in Appendix C.4):

$$\frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha_i^L}{\alpha_i^M} \left(\frac{L_{it}}{M_{it}} \right)^{\frac{\sigma-1}{\sigma}} Q_{it}^{\frac{-\eta}{\sigma}}. \quad (20)$$

Compared to the baseline homothetic production function in Equation (3), the ratio of output elasticities now features an additional *direct* effect of firm size on the output elasticity ratio: growing firms' labor output elasticity relative to their intermediate input output elasticity increases (decreases) if $\eta < 0$ ($\eta > 0$).[41]

(without discussing the horizon over which these costs are fixed). We note that a definitive benchmarking remains an open question due to lack of a consensus estimate of fixed cost shares.

[40] Additionally, we do not find striking differences across industries, which arguably significantly differ in fixed cost, e.g., due to exporting activity, product introduction costs, etc. (Appendix Figure A.2).

[41] Note that the translog production function generally permits non-homotheticities. Here, we focus on a specific form

Equation (20) suggests an empirical horse race testing between the relevance of the non-homotheticity in the form of direct output effect versus the input factor ratio in regulating the observed changes in output elasticities. Specifically, we regress log changes in labor/intermediate output elasticity and cost share ratios on log changes in input factor ratios *and* on log changes in output. We consider both output elasticity ratios and cost share ratios as dependent variables.[42] We report results from this exercise in Appendix Table A.10. We find that once we control for the input factor ratio, there is no statistically or economically significant negative effect of log output changes on relative output elasticity and cost share ratio changes (we do find a statistically significant but economically small negative effect on cost share ratios with OLS). This informal test suggests that a non-homotheticity in output is unlikely to be a primary factor behind shifts in labor relative to intermediate output elasticities with firm growth. Instead, the changes in the input ratio and relative input prices appear to explain why growing firms move toward less labor- and more intermediate-intensive production modes, and also offer a natural dynamic explanation for the stronger short-run effects. In addition, we reiterate that across firm size groups, we find similar results (discussed above).

6 Implications for Firm-level Labor Shares

We now examine a key implication of firm growth on labor's output elasticity, shaped by factor substitution and monopsony: the decline in the labor share, driven by production function properties rather than direct market power. The distinction is essential: while monopsony plays a role in this mechanism by raising labor prices in growing firms, leading to substitution from labor to intermediates, the novel effect on the labor share that our framework uncovers works through output elasticities and hence production function properties rather than through increasing markdowns.

6.1 Labor Share Decomposition and Identification

We adapt our framework from Section 2 to conceptualize this link and clarify the identification of the additional underlying drivers of the labor share.

The labor share. Taking logs of Equation (6) yields a log decomposition of firms' labor shares in output into three terms: output elasticities, markups, and markdowns:

$$\ln(LS_{it}) = \ln(\theta^L_{it}) - \ln(\mu_{it}) - \ln(\gamma^L_{it}). \tag{21}$$

Identifying price markups and wage markdowns. To separate the three determinants of the labor share, we construct measures of markups and markdowns from our production function estimation. Due to the translog structure of our production function, these variables are *firm- and time-specific*.

of non-homotheticity in output in context of our CES model. Nonetheless, the empirical analysis below can also be more broadly interpreted as testing the role of a direct output dependence (non-homotheticity) vs. changing input ratios in shaping changes in output elasticities and cost shares (and thus output shares).

[42]Translog output elasticities depend directly on input levels, which could limit their variation once we include input levels as regressors. We do not include input levels (to avoid limiting their variation in quantity of output), but we still include the labor-intermediate input ratio. That said, our alternative analysis of cost shares as a proxy for output elasticities would not be subject to any such concerns.

We derive output price markups using the production approach of Hall (1986) and De Loecker and Warzynski (2012). Assuming that intermediate inputs are a flexible input and that firms take intermediate input prices as given, such that $\gamma_{it}^M = 1$, markups, μ_{it}, can be derived from the first-order condition for intermediates within our framework (see Appendix C.5):

$$\mu_{it} = \frac{P_{it}}{\lambda_{it}} = \theta_{it}^M \frac{P_{it}Q_{it}}{P_{it}^M M_{it}}. \tag{22}$$

Following recent work (e.g., Dobbelaere and Mairesse, 2013, Yeh et al., 2022), we define an expression for firms' wage markdowns, γ_{it}^L, by combining the first-order conditions for intermediates and labor and again assuming that $\gamma_{it}^M = 1$ (see Appendix C.5):

$$\gamma_{it}^L = \frac{\theta_{it}^L}{\theta_{it}^M} \frac{P_{it}^M M_{it}}{P_{it}^L L_{it}}. \tag{23}$$

Importantly, we acknowledge that Equation (23) also captures any biased technological differences between firms that are not captured by output elasticities (but they will be captured by cost shares as discussed above).

Relaxing $\gamma_{it}^M = 1$: absolute vs. relative wedges. While we follow the conventional argument that $\gamma_{it}^M = 1$ is required for these two identification strategies, we note that γ_{it}^L is effectively a measure of *relative* input cost markdowns for labor vs. intermediate inputs. Assuming exogenous intermediate input prices and flexible intermediate inputs imposes $\gamma_{it}^M = 1$ and hence $\gamma_{it}^L = \frac{\gamma_{it}^L}{\gamma_{it}^M}$. In fact, the relative wedge exactly aligns with our theory as we focus on the reallocation between intermediates and labor.

Relaxing $\gamma_{it}^M = 1$: levels vs. changes. Moreover, for our specification in *changes*, we can relax the assumption that $\gamma_{it}^M = 1$; instead, it suffices to assume that within a firm, the intermediate wedge is constant ($\gamma_{it}^M = \gamma_i^M$). This is a considerably weaker assumption.

Total labor wedge. Finally, as a complement (and again assuming $\gamma_{it}^M = 1$), we also study the total labor wedge, which is the combined distortion from markup and labor market imperfections (this is sometimes used as a markup measure, assuming perfect labor markets):

$$\mu_{it}\gamma_{it}^L = \theta_{it}^L \frac{P_{it}Q_{it}}{P_{it}^L L_{it}}. \tag{24}$$

6.2 Results

We now show and dissect the empirical results applying the decomposition and identification from above.

Strategy. To measure and decompose the effects of firm growth on firm-level labor shares, we estimate the regression model in Equation (10) for changes in log labor shares, log markups, log markdowns, and log labor wedges as outcome variables.

Throughout, we focus on the sales labor share because this is the relevant labor share definition that corresponds to the output elasticity of labor and our firm-level production function framework. However, we also include the labor share in value added ($\frac{P_{it}^L L_{it}}{VA_{it}}$) as a robustness check (value added,

Table 6: Labor share, markup, markdown, and output elasticity changes in response to firm growth.

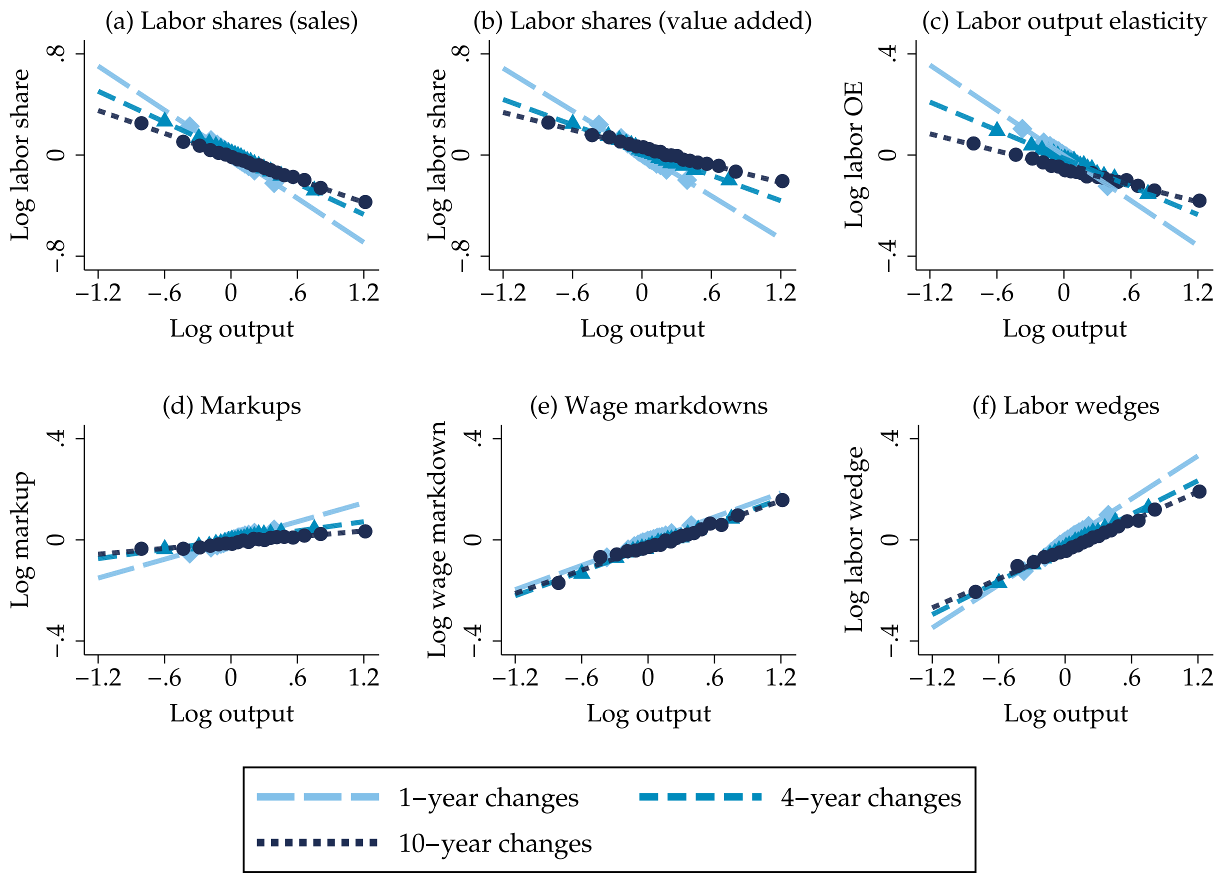
	$\Delta\ln(LS_{it})$	$\Delta\ln(\mu_{it})$	$\Delta\ln(\gamma^L_{it})$	$\Delta\ln(\mu_{it}\gamma^L_{it})$	$\Delta\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta\ln(\theta^L_{it})$	$\Delta\ln(\frac{P^L_{it}L_{it}}{VA_{it}})$
Panel A: OLS, 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	-0.581***	0.124***	0.159***	0.283***	-0.304***	-0.298***	-0.561***
	(0.0040)	(0.0025)	(0.0048)	(0.0044)	(0.005)	(0.0046)	(0.0076)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.534	0.096	0.115	0.191	0.230	0.215	0.190
Panel B: OLS, 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	-0.406***	0.0608***	0.160***	0.220***	-0.202***	-0.185***	-0.334***
	(0.0061)	(0.0032)	(0.0065)	(0.0055)	(0.0060)	(0.0063)	(0.0081)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.461	0.126	0.193	0.239	0.220	0.201	0.188
Panel C: OLS, 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	-0.303***	0.0383***	0.152***	0.191***	-0.140***	-0.112***	-0.231***
	(0.0062)	(0.0030)	(0.0060)	(0.0048)	(0.0053)	(0.0057)	(0.0068)
Observations	49,915	49,915	49,915	49,915	49,915	49,915	49,915
N of firms	10,595	10,595	10,595	10,595	10,595	10,595	10,595
R^2	0.378	0.125	0.213	0.253	0.188	0.161	0.178
Panel D: IV, 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	-0.674***	0.186***	0.169**	0.355***	-0.327***	-0.318***	-0.842***
	(0.0521)	(0.0418)	(0.0759)	(0.0697)	(0.0623)	(0.0645)	(0.120)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950
First-stage F-Statistic	102.6	102.6	102.6	102.6	102.6	102.6	102.6
R^2	0.524	0.084	0.115	0.185	0.229	0.215	0.160
Panel E: IV, 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	-0.443***	0.154***	0.0688	0.223***	-0.242***	-0.220***	-0.603***
	(0.0726)	(0.0444)	(0.0800)	(0.0713)	(0.0706)	(0.0742)	(0.119)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-Statistic	48.57	48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.459	0.072	0.180	0.239	0.217	0.199	0.135

Notes: The table reports OLS and IV regressions estimating the specification in Equation (10). The dependent variables in Columns (1)-(7) are log changes of the labor share in sales, markups, wage markdowns, labor wedge, labor output elasticity divided by returns to scale, labor output elasticity, and labor share in value added, respectively. Panels A-C rely on OLS and regress those dependent variables on changes in log output for 1-, 4- , and 10-year differences. Panels D-E rely on IV and regress those dependent variables on changes in log output for 1-, and 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

VA_{it}, is revenue minus intermediate input expenditures). Again, we present OLS and IV results and estimate regressions for 1-, 4-, and 10-year differences.

Results. Table 6 is organized as its counterparts, Tables 2 and 3, showing the OLS (Panels A-C) and IV (Panels D-E) effects of firm output growth on changes in labor shares, markups, markdowns, labor wedges, and value added labor shares by time horizon. We additionally add our previous estimates

Figure 6: Labor share, markup, markdown, and output elasticity changes in response to firm growth (OLS, binned scatter plots).



Notes: The figure reports binned scatter plots estimating the specification in Equation (10) with OLS for various differences. The graphs report binned scatter plots for changes in the logs of labor shares in sales and value added, labor output elasticities (not divided by returns to scale), markups, wage markdowns, and the labor wedge against log output changes. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

on labor output elasticities to facilitate the analysis. As indicated in Equation (21), the coefficients on markups, wage markdowns, and output elasticities *sum to* that on the labor share. As before, we also visualize the variation underlying the regression estimates in Figure 6.

Declining labor shares. We find a strong negative effect of firm output on labor shares. OLS results for sales labor shares range from -0.581 to -0.303 for 1- to 10-year differences. Results for value added labor shares, as well as IV estimates, are similar. Again, as for output elasticities, effects somewhat shrink as we widen the time horizons.

Half to one third of the effect: declining output elasticity of labor. In the short and medium run, up to the 4-year differences, *half* of this negative effect of output growth on labor shares is driven by the declining output elasticity of labor. This result holds for OLS and IV regressions. For 10-year differences, the declining output elasticity still accounts for one third of the effect (as changing returns to scale become increasingly important over longer time horizons).

The remainder: conventional market power effects. The remaining half (short- to medium-run) or two thirds (long-run) of the within-firm effect of firm growth on labor share changes is explained by increasing markups and wage markdowns. As we control for industry fixed effects, output growth reflects an increase in firms' market shares, and a firm's market power is expected to increase with its market share (as in Atkeson and Burstein, 2008 for product markets and Berger et al., 2022 for labor markets). Notably, over longer horizons, wage markdowns become much more important than product markups for explaining the decline in labor shares as firms grow.[43]

7 Evidence from other Countries and Aggregate Industry Implications

We now extend the analysis to other countries and sectors using firm and industry data. First, we replicate our baseline OLS regressions on administrative data (i.e., the equivalents to the German micro data) in 11 additional European countries. Subsequently, we study aggregate industry dynamics using a micro-aggregated dataset for 20 European countries and for the United States to show that our firm-level results carry over to industry aggregates (despite aggregation biases). Section 7.1 presents the data. Section 7.2 reproduces key results from our firm-level analysis for 11 other countries. Section 7.3 extends our firm-level regressions to the industry level and approximates industry substitution elasticities based on our firm-level equations. Section 7.4 discusses implications for industry labor shares.

7.1 International Industry Panel Data: CompNet Data

CompNet data. We use the 9th (most recent) vintage of the CompNet data (CompNet, 2023), which is a micro-aggregated database for 22 European countries.[44] The data are collected and provided by the Competitiveness Research Network (henceforth, CompNet). CompNet sources its data from representative administrative firm-level records located within European national statistical institutes and central banks (akin to the US Census data). The CompNet team distributes harmonized data collection protocols (i.e., Stata codes) across the data providers and invests significant efforts in harmonizing the input data to maximize comparability across countries. These protocols compute micro-aggregated results. From these results, the CompNet team constructs the CompNet database. The data are aggregated at various levels. We use the country-industry-level data (NACE Rev. 2, two-digit industries), which is the most detailed aggregation level available.

The data contain, among other features, country-industry-level information on firms' sales, inputs, expenditures, markups, wage markdowns, and output elasticities. They cover 1999-2021, and we focus on manufacturing (NACE Rev. 2 two-digit industries 10-33) which has the best coverage.[45] Yet, we will also present key results for non-manufacturing industries.[46] Yearly coverage varies across countries

[43] Also in the cross section, market power effects (particularly markdowns) are more important, consistent with Autor et al. (2020) and De Loecker et al. (2020) (see Figure A.5).

[44] We drop Malta, due to insufficient observations for output elasticities, and the UK, which did not provide industry-level data to CompNet.

[45] Manufacturing drives much of the decline in labor shares in most developed countries (Dao et al., 2019).

[46] This includes the NACE rev. 2 industries 41-43 (construction), 45-47 (wholesale/retail trade and repair of motor vehicles and motorcycles), 49-53 (transportation/storage), 55-56 (accommodation/food services), 58-63 (information and communication technology), 68 (real estate), 69-75 (professional/scientific/technical activities), and 77-82 (administrative/support service activities).

as shown in Appendix Table A.11. We focus on the data containing firms with at least 20 employees as this is available for more countries and is consistent with our German micro data (and we note our results hold for the subset of countries for which we have data based on firms of all size classes.) To ensure representativeness, the data are weighted by firm population weights. For further details on the data, we refer to CompNet's User Guide (CompNet, 2023).

While no comparable US data are available, we provide a limited set of results for the US based on the NBER-CES Manufacturing Industry Database (which lacks output elasticities) for 1958-2016.[47]

Production function estimation. The CompNet data provide information on industry-level markups, wage markdowns, and output elasticities using various types of estimation approaches. We rely on measures derived from industry-specific translog specifications, estimated using the two-step control function approach of Ackerberg et al. (2015) (in our German firm-level data we instead use a one-step approach similar to Wooldridge, 2009).[48] The log production function in the CompNet data is almost identical to Equation (9) with exception of excluding the triple interaction term. It is specified as follows and estimated in the micro firm-level panel data separately by country-industry cells (i.e., coefficients differ by industry-country):

$$q_{it} = \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \beta_{l2} l_{it}^2 + \beta_{k2} k_{it}^2 + \beta_{m2} m_{it}^2 + \beta_{lk} l_{it} k_{it} + \beta_{lm} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \omega_{it} + \nu_{it}. \quad (25)$$

Lower case letters denote logs. As there is no firm-specific price information observed in the firm-level data underlying the CompNet data, output, q_{it}, is measured as deflated sales using industry-specific deflators. To account for firm-specific price variation within industries, the CompNet estimation routine additionally controls for firms' market shares when estimating the production function.[49] Labor is defined as number of employees. Intermediates and capital are defined as deflated intermediate input expenditures and deflated capital stocks (book values) using country-industry-year-specific deflators. Markups and wage markdowns are computed as described in Equations (22) and (23). Output elasticities are computed as in the German micro data: $\theta_{it}^X = \frac{\partial \ln(Q_{it})}{\partial \ln(X_{it})}$ for each input $X = \{L, K, M\}$.

Industry aggregation. Industry-level values are constructed as weighted means of the firm-level micro variables, using sales weights for output elasticities, labor cost weights for markdowns, and intermediate input cost weights for markups (i.e., the denominators of the markup and markdown expressions).[50] This weighted aggregation differs from our previous unweighted firm-level analysis and allows us to focus on representative industry aggregates. Consistently, all other variables are computed as country-industry totals and their ratios.

Industry summary statistics and sample. To construct a harmonized sample, we only keep country-

[47]While the NBER CES database currently ends in 2018, capital, which is required to construct cost shares, is missing after 2016.

[48]Using the alternative approach could be viewed as another form of robustness test. See Ackerberg et al. (2015) for details on this estimation strategy. In summary, the estimator controls for unobserved productivity (ω_{it}) using a control function containing intermediate inputs, labor, and capital. The approach assumes a simple firm decision model where intermediate inputs and labor are flexible and intermediate input demand depends on capital, labor, and productivity. Under certain restrictions, one can invert the intermediate input demand function to approximate productivity as a function of intermediates, labor, and capital.

[49]This approach follows De Loecker et al. (2020). Under a Cournot model, market shares perfectly capture markup variation. Therefore, market shares can help absorb some of the unobserved price variation.

[50]Results are robust to using sales weights for all variables, which is also available in the CompNet data (unreported).

industry pairs for which we observe labor shares, output elasticities, cost shares, markups, and wage markdowns.[51] We provide summary statistics of key variables by country from the country-industry-level data in Appendix Table A.11. The table also reports the yearly coverage for each country in CompNet.

Commissioning our own firm-level regressions in the CompNet micro data. To supplement the country-industry-level CompNet data, we collaborated with the CompNet team to incorporate additional firm-level regressions into their *10th* vintage data collection. These regressions replicate our firm-level analysis in 11 countries and extend our initial analysis beyond manufacturing, which we could not do with our German manufacturing micro data. The 10th vintage data include a slightly different sample of countries and covers more recent years than the currently available 9th vintage country-industry data (we document yearly coverage below). Due to the time-intensive nature of running our codes, we have, so far, only received results for a subset of data providers: France, Hungary, Poland, Portugal, Slovakia, Slovenia, Estonia, Latvia, The Netherlands, Romania, and Switzerland.[52]

7.2 Firm-level Results for 11 other European Countries

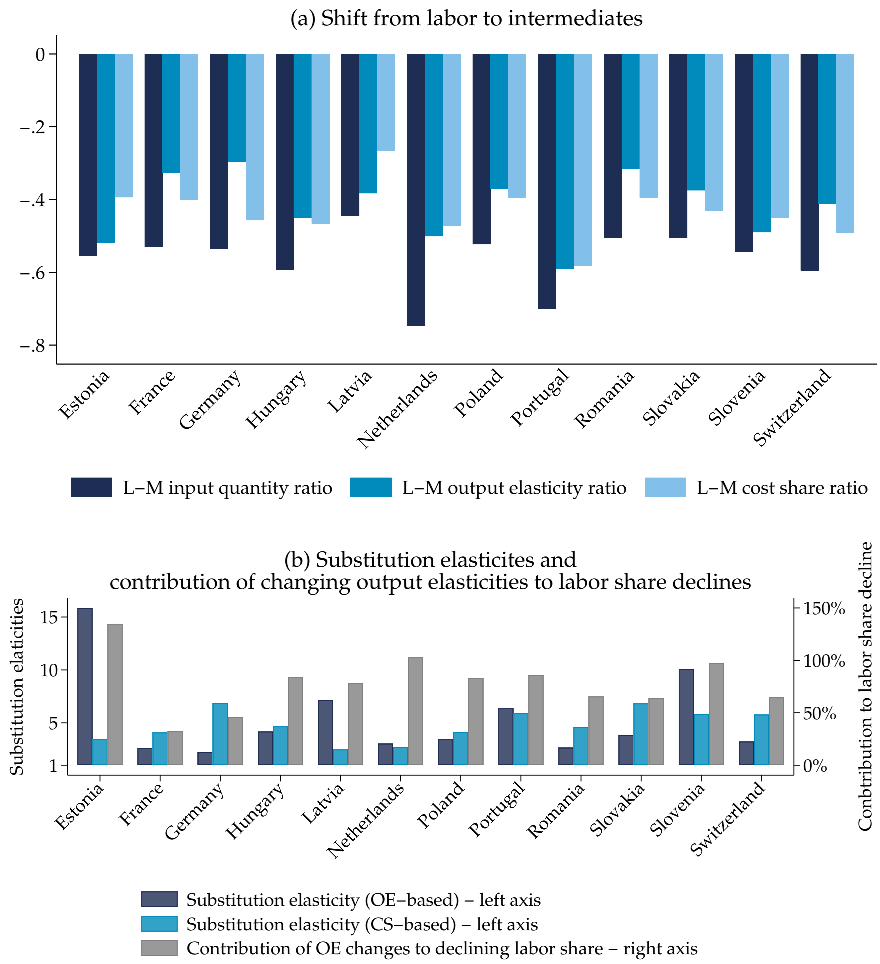
Figure 7 summarizes firm-level manufacturing sector results (OLS) for the 11 countries using the CompNet infrastructure. We replicate our previous firm-level analysis across these countries. Panel (a) reports the *ratio* of regression coefficients for labor and intermediate input quantities, cost shares, and output elasticities. Panel (b) reports implied substitution elasticities and the contribution of changes in labor output elasticities to declines in firm-level labor shares with firm growth. In Panel (b), we display substitution elasticities based on output elasticity (OE) and cost share (CS) coefficients. Beyond the advantages already discussed in our German analysis, the cost share approach provides an important robustness check in the CompNet data, where firm-specific price data are unavailable for production function estimation. To facilitate comparison, we include our previous results for German manufacturing firms. Appendix Tables A.12 and A.13 detail all underlying regression results and additionally report results including non-manufacturing sectors (all results are highly statistically significant).

Results. Our previous firm-level findings for Germany are confirmed across all 11 other countries: input quantities, cost shares, and output elasticities shift from labor to intermediates, with substitution elasticities consistently exceeding unity. Results align closely with our findings for Germany, with the quantitative intensity of the shift from labor to intermediates differing somewhat across countries (we are not aware of consistently estimated cross-country estimates of the labor supply elasticity to the firm). Also, labor output elasticities decline more sharply with firm growth in most countries. As a result, with the exception of France, the contribution of changes in labor output elasticities to declines in firm-level labor shares with firm growth is higher than in the German data.

[51]Some country-industry-year cells report missing values in the CompNet data because the number of firms in the underlying micro data was too small either for passing country-specific disclosure rules or for estimating the production functions.

[52]With further cooperation from data providers, we can extend our analysis to additional countries and provide these results upon request.

Figure 7: Country-specific results from firm-level regressions (manufacturing).



Notes: The figures reports OLS estimates of the specification in Equation (10) for different countries (manufacturing sector firm micro data). Panel (a) report ratios of coefficients on labor and intermediate input quantities, cost shares, and output elasticities. A negative value indicates a shift from labor to intermediates. Panel (b) reports implied substitution elasticities (left axis) based on the shifts in input quantity ratios and (i) output elasticity ratios (OE-based), and (ii) cost share ratios (CS-based). Additionally, Panel (b) reports the contribution of changes in firms' output elasticities to labor share shares with firm growth (right axis). This is computed by dividing regression coefficients on output elasticities by regression coefficients on labor shares (as in Section 6.2). Appendix Table A.12 reports the underlying regression results in more detail and outcome variable by outcome variable. Results for Germany come from our previous firm-level analysis on the German firm-level data. Other results are based on firm-level data from CompNet data providers.

Manufacturing vs. all sectors. Results for all sectors (Table A.13) are similar to manufacturing (Table A.12), with some countries showing slightly stronger or weaker changes. In both samples, substitution elasticities always exceed unity.

Young and mature firms. Our German firm data lack information on firms' registration years. However, in six of the eleven countries included in the firm-level CompNet data analysis, this information is available. In Appendix Table A.14, we replicate our analysis for young (no older than five years) and mature (older than five years) manufacturing sector firms. The results are similar between the two groups. If anything, the shift from labor toward intermediates appears somewhat stronger for mature firms. Similar findings by age hold if considering all sectors (not reported).

7.3 Industry-level Dynamics in 20 European Countries and the United States

We now study the industry-level analog of our firm-level analysis. We expect divergence in the effect sizes given aggregation biases, reallocation and compositional changes, intra-industry intermediate input patterns now being absorbed, and because market- vs. firm-specific labor supply elasticities may differ (see, e.g., Berger et al., 2022).

Industry growth regressions. We transfer our firm-level growth analyses to the industry level. We regress changes in log input quantities, log cost shares, and log output elasticities on industry output growth in the aggregated *country-industry* data provided by CompNet. The horizons are 1-, 4-, and 8-year changes (10-year changes are not feasible with CompNet data), controlling for country-year and industry-year fixed effects as we pool country-industry pairs (denoted by c and j, respectively). Compared to the firm-level analysis, this industry analysis could also be read as studying the impact of more aggregate shocks. (Results are similar with country-industry fixed effects, but we omit them to approximate the firm-level specification.)

Results. Table 7 and Figure 8 present industry-level results akin to the firm-level results in Table 2 and Figure 3. Our analysis shows that the relationships observed in firm-level data also hold at the European industry level: as industries expand, labor-intermediate ratios, labor cost shares, and labor output elasticities decrease, while intermediate cost shares and output elasticities rise. The point estimates are precisely estimated and indicate that 10% higher industry output growth leads to a 0.5% reduction in the labor output elasticity.[53] The binned scatter plots illustrate clear monotonic, nearly linear relationships between output growth and the outcome variables.

Country analysis and substitution elasticities. Table 8 Columns (1)-(3) present results for changes in industry-level labor-intermediate quantity, output elasticity, and cost share ratios in response to industry growth, based on country-specific *regression coefficients* from reproducing Table 7 by countries (4-year changes). As before, we also estimate substitution elasticities using Equation (14). Column (4) reports these substitution elasticity estimates based on the coefficients on output elasticities and input quantities (σ^{OE}) from our industry growth regressions. Column (5) uses cost share coefficients instead of output elasticity coefficients (σ^{CS}).

Results. In almost all countries, industry growth is associated with a shift from industry-level labor to intermediates in terms of quantities, cost shares, and output elasticities (with some quantitative differences). Industry-level labor-intermediate substitution elasticities exceed unity in almost all countries. European-level values, derived from Table 7, and US values derived from running the same regressions with the NBER-CES Manufacturing Industry Database (1958-2016) are reported at the bottom of Table 8.[54] The US data lack output elasticity estimates, but changes in labor and intermediate quantities and cost shares are similar in Europe and the US. Also, substitution elasticities based on cost shares are similar (approximately 4 in both regions). Substitution elasticity estimates based on direct output elasticity estimates are 1.34 in Europe (manufacturing).

[53] Without taking logs, coefficients on industry-level labor, capital, and intermediate output elasticities divided by returns to scale are -0.0112, -0.00003, and 0.0148, respectively (4-year changes). Hence, the increase in intermediate input output elasticities is again largely rationalized by the decline in labor output elasticities.

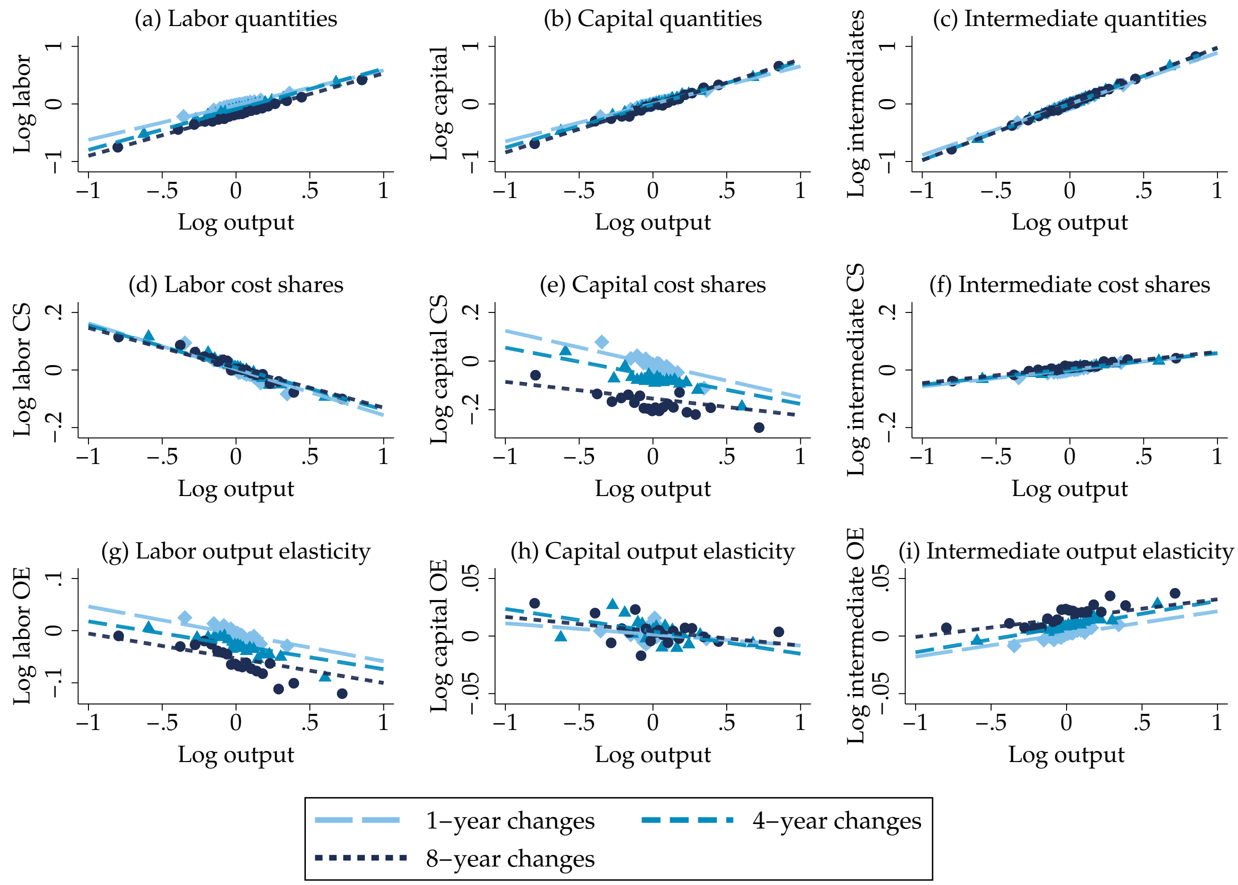
[54] For US cost shares, capital costs are approximated as 8% of the real capital stock, as nominal values are not reported; labor and material expenditures are reported in nominal terms.

Table 7: Industry-level adjustments in response to industry growth (Europe). OLS regressions.

	$\Delta\ln(L_{cjt})$	$\Delta\ln(K_{cjt})$	$\Delta\ln(M_{cjt})$	$\Delta\ln(CS^L_{cjt})$	$\Delta\ln(CS^K_{cjt})$	$\Delta\ln(CS^M_{cjt})$	$\Delta\ln(\frac{\theta^L_{cjt}}{RTS_{cjt}})$	$\Delta\ln(\frac{\theta^K_{cjt}}{RTS_{cjt}})$	$\Delta\ln(\frac{\theta^M_{cjt}}{RTS_{cjt}})$	$\Delta\ln(RTS_{cjt})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.604***	0.652***	0.888***	-0.159***	-0.137***	0.058***	-0.053***	-0.001	0.020**	0.0016
	(0.038)	(0.048)	(0.036)	(0.020)	(0.028)	(0.012)	(0.013)	(0.016)	(0.008)	(0.0014)
Industry-year FE	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Country-year FE	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Observations	5,233	5,233	5,233	5,233	5,233	5,233	5,233	5,233	5,233	5,233
R^2	0.780	0.584	0.813	0.313	0.396	0.221	0.217	0.178	0.212	0.153
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.705***	0.749***	0.974***	-0.147***	-0.116***	0.055***	-0.046***	-0.020	0.022**	0.0038
	(0.034)	(0.034)	(0.023)	(0.024)	(0.036)	(0.015)	(0.013)	(0.015)	(0.009)	(0.0023)
Industry-year FE	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Country-year FE	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Observations	4,207	4,207	4,207	4,207	4,207	4,207	4,207	4,207	4,207	4,207
R^2	0.880	0.746	0.907	0.399	0.545	0.279	0.252	0.229	0.243	0.164
Panel C: 8-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.716***	0.809***	0.979***	-0.139***	-0.069*	0.054***	-0.047***	-0.012	0.016*	0.007**
	(0.026)	(0.043)	(0.016)	(0.022)	(0.038)	(0.001)	(0.016)	(0.019)	(0.008)	(0.004)
Industry-year FE	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Country-year FE	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Observations	2,904	2,904	2,904	2,904	2,904	2,904	2,904	2,904	2,904	2,904
R^2	0.911	0.815	0.946	0.451	0.581	0.322	0.298	0.278	0.241	0.183

Notes: The table reports OLS regressions from estimating industry versions of the specification in Equation (10). The dependent variables in Columns (1)-(10) are log changes in country-industry-level total labor, capital, and intermediates, industry-level labor, capital, and intermediate cost shares, country-industry-level labor, capital, and intermediate input output elasticities over returns to scale, and country-industry-level returns to scale, respectively. Panels A-C report on regressions of those industry-level dependent variables on changes in industry-level log output for 1-, 4- , and 8-year differences. All regressions control for country-year and industry-year fixed effects and we pool country-industry pairs. Standard errors are clustered at the country-industry level. Significance: 0.01***, 0.05**, 0.1*. CompNet manufacturing data.

Figure 8: Industry-level adjustments in response to industry growth (OLS, binned scatter plots).



Notes: The figure reports binned scatter plots from estimating industry versions of the specification in Equation (10) with OLS for various differences. Panels (a)-(i) report on regressions of changes in the logs of country-industry-level labor, capital, and intermediate quantities, country-industry-level labor, capital, and intermediate cost shares, and country-industry-level labor, capital, and intermediate output elasticities over returns to scale against country-industry-level log output changes, respectively. All panels control for country-year and industry-year fixed effects and we pool country-industry pairs. CompNet manufacturing data.

Manufacturing vs. non-manufacturing sectors. The last row extends the CompNet data analysis to non-manufacturing country-industry pairs as a robustness test. Substitution elasticities exceed unity in manufacturing and non-manufacturing industries, indicating that our findings are consistent across different sectors of the economy. This generalization echoes the robustness check using non-manufacturing firms in firm-level micro data across several CompNet countries reported in Table A.13.

7.4 Industry Labor Share Implications

As with the firm-level analysis, we now study the industry labor share consequences of the decline of industry labor output elasticities with industry growth. We reiterate that our main focus is on firm-level growth, and drawing aggregate implications would necessitate an input-output network analysis or an open economy perspective.

Labor shares and their drivers. Table 9 and Figure 9 are European country-industry counterparts

Table 8: Implied industry-level substitution elasticities and changes in inputs, output elasticities, and cost share ratios based on 4-year differences.

	$\Delta \ln \left(\frac{L_{cjt}}{M_{cjt}}\right)$	$\Delta \ln \left(\frac{\theta^L_{cjt}}{\theta^M_{cjt}}\right)$	$\Delta \ln \left(\frac{CS^L_{cjt}}{CS^M_{cjt}}\right)$	σ^{OE}	σ^{CS}
	(1)	(2)	(3)	(4)	(5)
Belgium (2000-2020)	-0.23	-0.10	-0.16	1.80	3.36
Croatia (2002-2021)	-0.70	-0.16	-0.64	1.31	12.71
Czech Republic (2005-2020)	-0.40	-0.01	-0.31	1.03	4.67
Denmark (2001-2020)	-0.13	-0.01	-0.09	1.12	3.13
Finland (1999-2020)	-0.09	-0.08	-0.13	5.72	-2.45
France (2004-2020)	-0.40	-0.05	-0.22	1.13	2.25
Germany (2001-2018)	-0.06	0.01	-0.05	0.85	6.59
Hungary (2003-2020)	-0.51	-0.09	-0.38	1.21	4.03
Italy (2006-2020)	-0.18	-0.09	-0.12	2.11	2.89
Latvia (2007-2019)	-0.43	-0.11	-0.25	1.33	2.46
Lithuania (2000-2020)	-0.15	-0.08	-0.11	2.04	3.59
Netherlands (2007-2019)	-0.11	0.03	-0.06	0.81	2.51
Poland (2002-2020)	-0.13	-0.01	-0.09	1.09	2.98
Portugal (2010-2020)	-0.68	0.02	-0.48	0.98	3.42
Romania (2005-2020)	-0.22	-0.03	-0.13	1.17	2.58
Slovakia (2000-2020)	-0.18	-0.09	-0.16	1.98	12.73
Slovenia (2002-2021)	-0.09	0.01	-0.13	0.92	-2.49
Spain (2008-2020)	-0.44	-0.23	-0.16	2.11	1.60
Sweden (2003-2020)	-0.03	-0.04	0.02	-5.77	0.64
Switzerland (2009-2020)	-0.31	-0.19	-0.24	2.63	4.58
Europe (1999-2021, manufac.)	-0.27	-0.07	-0.20	1.34	4.01
USA (1958-2016, manufac.)	-0.31		-0.23		3.88
Europe (1999-2021, non-manufac.)	-0.23	-0.08	-0.15	1.48	2.72

Notes: The table reports implied changes in labor-intermediate input ratios (Column (1)), output elasticity ratios (Column (2)), cost shares (Column (3)), and substitution elasticities based on output elasticity coefficients (Column (4)) and cost share coefficients (Column (5)) as estimated from country-specific versions of the country-industry-level regressions reported in Table 7 (4-year changes). All results are based on manufacturing industries, except for the last row, which uses non-manufacturing industries. Industries are 2-digit NACE rev. 2 industries for Europe and 6-digit NAICS industries for the US. CompNet data and NBER-CES Manufacturing Industry Database.

to the firm-level analyses shown in Table 2 and Figure 3. As before, we regress country-industry log changes in outcome variables on output growth, controlling for country-year and industry-year fixed effects. Due to aggregation bias, we do not expect that coefficients on output elasticities, markups, and wage markdowns sum to the coefficient on labor shares as they did in our firm-level analysis.[55]

Results. The results qualitatively align with our firm-level findings: as industries grow, industry labor shares decline, primarily due to a reduction in labor output elasticities. However, markups also *decline*, while changes in wage markdowns are statistically insignificant, which is inconsistent with a decline in labor shares. Therefore, increases in markups or wage markdowns cannot explain the negative relationship between labor shares and growth at the industry level. Instead, the industry analysis points to a substantial role for labor output elasticities. Quantitatively, the slope of the aggregate output elasticity is about a quarter of the slope of the labor share in sales (4-year specification).[56]

[55]Aggregation biases may arise from compositional effects, weighting choices, Jensen's inequality, intra-industry trade, and shifts in labor demand and supply.

[56]In addition to classical aggregation biases, note that under competitive markets, the industry labor share equals the

Table 9: Industry labor share, market imperfection, and output elasticity changes in response to industry growth (Europe). OLS regressions.

	$\Delta\ln(LS_{cjt})$	$\Delta\ln(\mu_{cjt})$	$\Delta\ln(\gamma_{cjt})$	$\Delta\ln(\frac{\theta^L_{cjt}}{RTS_{cjt}})$	$\Delta\ln(\theta^L_{cjt})$	$\Delta\ln(\frac{P^L_{cjt}L_{cjt}}{VA_{cjt}})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output change	-0.319***	-0.031***	0.004	-0.053***	-0.051***	-0.162***
	(0.041)	(0.009)	(0.013)	(0.013)	(0.014)	(0.029)
Observations	5,233	5,233	5,233	5,233	5,233	5,233
R^2	0.544	0.257	0.332	0.217	0.214	0.330
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output change	-0.216***	-0.032***	0.006	-0.046***	-0.042***	-0.116***
	(0.029)	(0.007)	(0.015)	(0.013)	(0.014)	(0.023)
Observations	4,207	4,207	4,207	4,207	4,207	4,207
R^2	0.547	0.341	0.438	0.252	0.243	0.383
Panel C: 8-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output change	-0.194***	-0.031***	0.026	-0.047***	-0.040**	-0.128***
	(0.024)	(0.009)	(0.021)	(0.0048)	(0.017)	(0.021)
Observations	2,904	2,904	2,904	2,904	2,904	2,904
R^2	0.516	0.400	0.412	0.298	0.282	0.406

Notes: The table reports OLS regressions estimating industry versions of the specification in Equation (10). The dependent variables in Columns (1)-(6) are log changes of country-industry-level labor shares in sales, markups, wage markdowns, labor output elasticities divided by returns to scale, labor output elasticities, and labor shares in value added, respectively. Panels A-C report on regressions of those dependent variables on changes in country-industry-level log output for 1-, 4-, and 8-year differences. All regressions control for country-year and industry-year fixed effects. Standard errors are clustered at the country-industry level. Significance: 0.01***, 0.05**, 0.1*. CompNet manufacturing data.

Notably, as with the firm-level analysis, the value added labor share also declines, highlighting that the rise in intermediates mainly reflects a shift away from labor.[57]

Simple check: industry splits. We close our paper with one more check at the industry level. We split our manufacturing data into growing and shrinking industries and study how labor shares, output elasticities, markups, and wage markdowns have changed between countries' first and last years in the CompNet data and for the US (NBER-CES data, where we focus on the period 1998–2016 to approximate the CompNet years and study only labor shares due to lack of output elasticities).[58] Figure 10 reports the result.

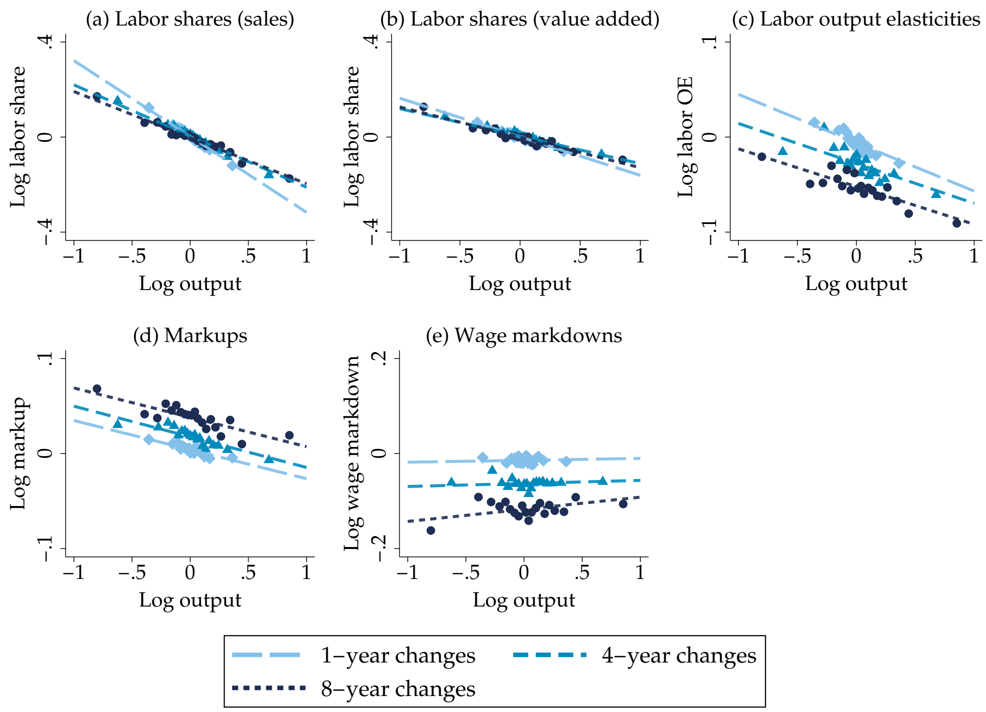
Examining the European results, the first striking result is that on average, labor shares decreased

industry output elasticity of labor: $\sum_n s_{it}LS_{it} = \sum_n s_{it}\theta^L_{it}$, where s_{it} is the sales weight. Under imperfect markets, the pass-through from changes in industry output elasticities of labor to changes in industry labor shares is additionally shaped by the distribution of firms' markups and wage markdowns. Nevertheless, qualitatively, it remains true that the partial effect of a change in the aggregate labor-output elasticity on the labor share, holding fixed market imperfections, is positive. Equally, the partial effect (ceteris paribus) of changes in industry markups and wage markdowns on industry labor shares is negative. Importantly, "holding fixed" refers here not only to the industry level, but also to the firm distribution.

[57]This observation aligns with previous research that identifies the offshoring of labor-intensive tasks as a major factor contributing to the declining labor share (e.g., Elsby et al., 2013). Ruzic (2024) also documents that, on average, intermediate inputs displace labor more strongly than capital.

[58]As yearly coverage varies between countries in CompNet, we first calculate log changes in each of these variables for each country-industry pair between the first and last year in the data, i.e., we pool differences for country-industry pairs of different lengths (see Table 8 for the yearly country coverage). Subsequently, we take weighted averages (as described in the figure note) of these changes across all country-industry-pairs.

Figure 9: Labor share, market imperfection, and output elasticity changes in response to industry growth (OLS, binned scatter plots).

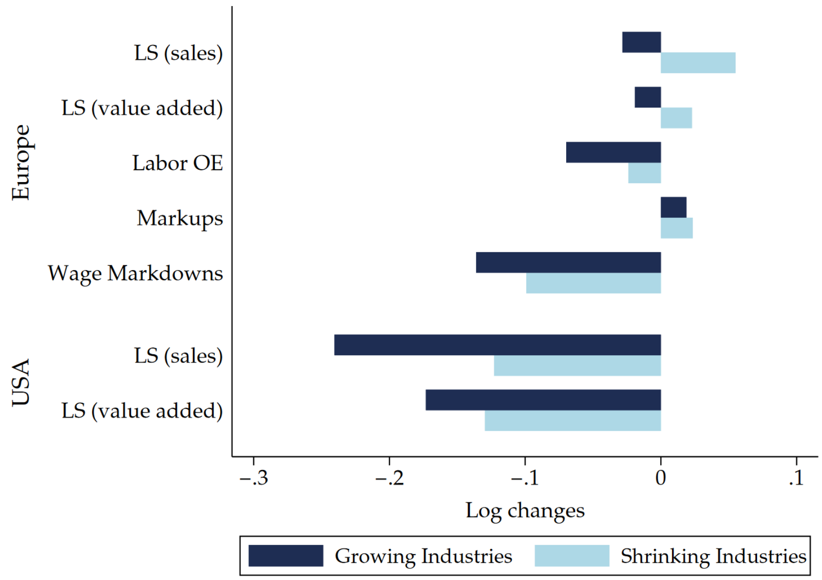


Notes: The figure reports binned scatter plots estimating industry versions of the specification in Equation (10) with OLS for various differences. The graphs relate changes in country-industry-level logs of labor shares in sales and value added, labor output elasticities (not divided by returns to scale), markups, and wage markdowns to country-industry-level log output changes. All regressions control for country-year and industry-year fixed effects. CompNet data.

within expanding industries, but remained stable in contracting industries. Expanding industries also experienced a much stronger decline in labor output elasticities. Markups and wage markdowns exhibit trends that should, in theory, *increase* labor shares in growing industries relative to shrinking ones—specifically, markups increase less and wage markdowns decrease more in expanding industries. Through the lens of our framework, changes in labor output elasticities are therefore the only factor that can account for the observation that, on average, labor shares declined in growing but not in shrinking industries (to recap, in the aggregate, changes in markups, markdowns, and output elasticities need not, and do not, sum to changes in labor shares).

In the US, we see a marked decline in labor shares, particularly within expanding industries—consistent with the output elasticity channel (but we lack direct output elasticity measures). Additionally, the decline in labor shares within shrinking US industries may point to market power playing a more substantial role in the US (see De Loecker et al., 2020 and Autor et al., 2020 for that channel).

Figure 10: Changes in labor shares, output elasticities, markups, and wage markdowns in growing and shrinking industries (manufacturing)



Notes: The figure reports weighted average changes in the logs of labor shares, labor output elasticities, markups, and wage markdowns across country-industry pairs for growing and shrinking industries. Changes are first computed for each pair and then averaged across all pairs. Time spans differ across country-industry pairs and not all industries are available for all years within a country. Industry-weights are based on the first year for each country-industry pair, and we apply the same weights as CompNet uses for aggregation, i.e., labor cost weights for markdowns, intermediate input weights, sales weights for labor shares in sales, and value added weights for labor shares in value added. European results are reported in the top five bars and based on CompNet data (1999-2021) and NACE Rev. 2 two-digit industries. US results are reported in the bottom two bars and based on the NBER-CES Manufacturing Industry Database (1998-2016) and NAICS 6-digit industries. There are 154 shrinking and 196 growing country-industry pairs in the CompNet data. For the US data, we observe 209 shrinking and 155 growing industries.

8 Conclusion

We have documented and dissected a stylized fact about firm growth: the declining importance of labor and the increasing importance of intermediate inputs in firms' production. As labor and intermediates function as substitutes, this shift in the input mix reduces (increases) the output elasticity of labor (intermediates). As a result, the labor (intermediates) share falls (increases) in growing firms. This shift from labor to intermediates explains between one-third (10-year horizon) and one-half (1- and 4-year horizon) of the decline in labor shares in growing firms and accounts for most of the decline in labor shares within growing industries. We establish these patterns using OLS and IV regression in rich German firm-level micro data, administrative firm-level data from 11 additional countries, as well as in micro-aggregated industry data for 20 European countries. Our findings hold for small and large and young and mature growing firms. We rationalize the facts with a parsimonious production function framework characterized by (i) an elasticity of substitution between intermediates and labor that exceeds one, and (ii) an increasing shadow price of labor (e.g., due to monopsony or adjustment costs).

The findings also have broader implications. For instance, many current estimates of factor misallocation and productivity rely on Cobb-Douglas production functions, which assume constant output elasticities. Moreover, our results imply that, generally, any shocks that affect output growth will alter output elasticities, input mixes, and cost shares, with large magnitudes for horizons of up to 10 years. Ignoring this regularity may confound analyses of firm-level effects from productivity shocks, trade, competition, or subsidies on a wide range of related outcomes in the short and medium term.

Our findings also show how monopsony not only distorts the steady state firm sizes but also firm growth. We also trace how firms respond to these growth constraints by intensifying their use of intermediate inputs—i.e., by outsourcing production—for which we estimate a high elasticity of substitution with labor. In our case, these intermediate inputs are supplied by other firms and are largely product components and materials (rather than, e.g., temporary agency work). It thus appears that monopsony lengthens the supply chain as firms, or, more broadly, industries and perhaps the aggregate economy, grow.

Finally, the mechanism we highlight in this paper offers avenues for future research, such as firms' life cycles, reallocation, potential implications for aggregate labor shares, global production networks, directed innovation, disaggregated analyses to specific subcategories of intermediate inputs or labor types, and a deeper treatment of the role of capital.

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Appendix of:
From Labor to Intermediates: Firm Growth, Input Substitution, and Monopsony

Matthias Mertens and Benjamin Schoefer

Contents

A Appendix Tables and Figures

Table A.1: Summary statistics of the German manufacturing sample.

	Mean (1)	**p25** (2)	**Median** (3)	**p75** (4)	**St.Dev.** (5)	**Obs.** (6)
Number of employees	366.63	53	109	272	2559.55	183,813
Real wage (1995 values)	34,162	25,972	33,823	41,485	11,492	183,813
Labor share (sales)	0.30	0.21	0.29	0.38	0.12	183,813
Labor share (value added)	0.80	0.63	0.76	0.88	2.89	183,813
Labor cost share	0.31	0.22	0.30	0.39	0.13	183,813
Capital cost share	0.06	0.03	0.05	0.07	0.04	183,813
Intermediates cost share	0.63	0.54	0.64	0.73	0.14	183,813
Materials, energy, ext. components cost share	0.41	0.29	0.41	0.53	0.17	183,813
Merchandise cost share	0.05	0.00	0.00	0.05	0.10	183,813
Subcontracted production by other companies cost share	0.03	0.00	0.00	0.03	0.06	183,813
Repairs, maintenance, installations cost share	0.02	0.01	0.03	0.04	0.02	183,813
Rents, leases, leasing cost share	0.03	0.01	0.02	0.04	0.03	183,813
Temporary agency worker cost share	0.01	0.00	0.00	0.01	0.03	164,410
Other intermediates cost share	0.10	0.05	0.08	0.12	0.06	183,813
Markup	1.09	0.97	1.05	1.17	0.20	183,813
Wage markdown	1.08	0.71	0.96	1.32	0.55	183,813
Output elasticity of labor	0.31	0.23	0.31	0.38	0.11	183,813
Output elasticity of capital	0.12	0.08	0.11	0.15	0.06	183,813
Output elasticity of intermediates	0.64	0.57	0.64	0.71	0.10	183,813
Returns to scale	1.06	0.98	1.05	1.13	0.12	183,813

Notes: This table presents summary statistics for selected variables from the German manufacturing sector firm-level data. Columns (1)-(5) show the mean, 25th percentile, median, 75th, and standard deviation, respectively. Column (6) reports the number of non-missing observations. German micro-data.

Table A.2: Firm-level adjustments in employment, measured in heads and in full time equivalents (FTE), to firm growth. OLS regressions.

	OLS	OLS	IV	IV
	Log FTE changes	Log head changes	Log FTE changes	Log head changes
Panel A: 1-year changes	(1)	(2)	(3)	(4)
Log output change	0.285***	0.284***	0.259***	0.264***
	(0.00421)	(0.00419)	(0.0531)	(0.0518)
Observations	160,764	160,764	160,764	160,764
N of firms	28,969	28,969	28,969	28,969
First-stage F-Statistic			98.66	98.66
R^2	0.189	0.205	0.188	0.205
Panel B: 4-year changes	(1)	(2)	(3)	(4)
Log output change	0.490***	0.487***	0.500***	0.504***
	(0.00747)	(0.00745)	(0.0811)	(0.0788)
Observations	53,106	53,106	53,106	53,106
N of firms	7,879	7,879	7,879	7,879
First-stage F-Statistic			38.17	38.17
R^2	0.458	0.464	0.458	0.464

Notes: The table reports OLS and IV regressions from estimating the specification in Equation (10) for 1- and 4-year differences. The dependent variable is logged employment in changes, once measured in head counts, once measured as full time equivalents. All columns report regressions of those dependent variables on output growth for 1- and 4-year changes. Panels A-B report results for 1- and 4-year differences, respectively. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data. A future version of the paper will be updated to also include the results for 10-year changes (OLS), which remained under disclosure review at the point of circulation.

Table A.3: Firm-level adjustments in cost shares and output elasticities to firm growth. Different specifications: not logged, not dividing by RTS. OLS regressions.

	$\Delta \frac{w_{it}L_{it}}{C_{it}}$	$\Delta \frac{r_{it}K_{it}}{C_{it}}$	$\Delta \frac{z_{it}M_{it}}{C_{it}}$	$\Delta \frac{\theta^L_{it}}{RTS_{it}}$	$\Delta \frac{\theta^K_{it}}{RTS_{it}}$	$\Delta \frac{\theta^M_{it}}{RTS_{it}}$	$\Delta \ln(\theta^L_{it})$	$\Delta \ln(\theta^K_{it})$	$\Delta \ln(\theta^M_{it})$	$\Delta\theta^L_{it}$	$\Delta\theta^K_{it}$	$\Delta\theta^M_{it}$	ΔRTS_{it}
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Log output change	-0.103***	-0.0380***	0.141***	-0.0710***	-0.0201***	0.0911***	-0.298***	-0.309***	0.157***	-0.0723***	-0.0203***	0.0991***	0.00647***
	(0.0011)	(0.0003)	(0.0011)	(0.0007)	(0.0003)	(0.0009)	(0.0046)	(0.0064)	(0.00129)	(0.0008)	(0.0004)	(0.0007)	(0.0004)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.303	0.419	0.413	0.266	0.191	0.277	0.215	0.132	0.397	0.278	0.183	0.425	0.157
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Log output change	-0.0692***	-0.0366***	0.106***	-0.0438***	-0.0136***	0.0574***	-0.185***	-0.164***	0.112***	-0.0420***	-0.0124***	0.0729***	0.0185***
	(0.00157)	(0.0006)	(0.00179)	(0.00108)	(0.000525)	(0.0014)	(0.0063)	(0.0077)	(0.0021)	(0.0014)	(0.0007)	(0.0013)	(0.0008)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.303	0.419	0.413	0.266	0.191	0.277	0.201	0.160	0.369	0.220	0.154	0.388	0.157
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Log output change	-0.0535***	-0.0275***	0.0810***	-0.0305***	-0.0051***	0.0356***	-0.112***	-0.0441***	0.0860***	-0.0236***	-0.0018***	0.0559***	0.0304***
	(0.0016)	(0.0006)	(0.0018)	(0.0011)	(0.0005)	(0.0014)	(0.00569)	(0.0073)	(0.0021)	(0.00137)	(0.0007)	(0.0013)	(0.0008)
Observations	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915
N of firms	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595
R^2	0.259	0.318	0.347	0.222	0.155	0.201	0.161	0.141	0.322	0.174	0.142	0.334	0.262

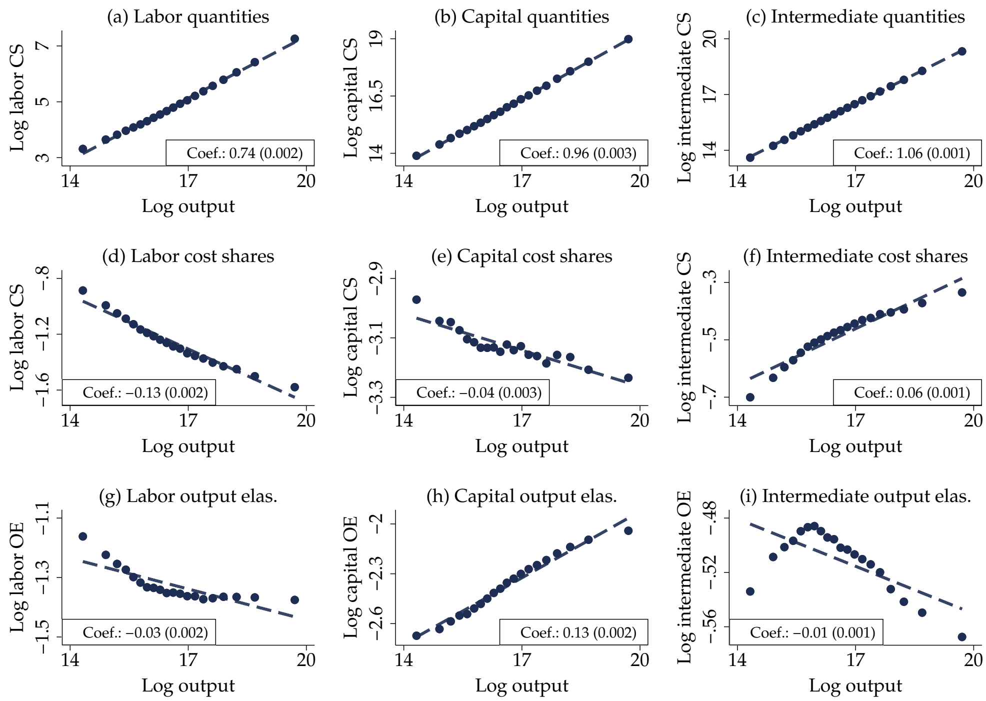
Notes: The table reports OLS regressions from estimating the specification in Equation (10). The dependent variables in Columns (1)-(13) are non-logged changes in labor, capital, and intermediate input cost shares, output elasticities divided by returns to scale, and logged and non-logged labor, capital, and intermediate input output elasticities not divided by returns to scale, respectively. Panel A-C report on regressions of those dependent variables on changes in log output for 1-, 4- , and 10-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.4: Firm-level adjustments in cost shares and output elasticities to firm growth. Different specifications: not logged, not dividing by RTS. IV regressions.

	1st stage	$\Delta \frac{w_{it}L_{it}}{C_{it}}$	$\Delta \frac{r_{it}K_{it}}{C_{it}}$	$\Delta \frac{z_{it}M_{it}}{C_{it}}$	$\Delta \frac{\theta_{it}^L}{RTS_{it}}$	$\Delta \frac{\theta_{it}^K}{RTS_{it}}$	$\Delta \frac{\theta_{it}^M}{RTS_{it}}$	$\Delta \ln(\theta_{it}^L)$	$\Delta \ln(\theta_{it}^K)$	$\Delta \ln(\theta_{it}^M)$	$\Delta\theta_{it}^L$	$\Delta\theta_{it}^K$	$\Delta\theta_{it}^M$	ΔRTS_{it}
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Export demand shock	0.0451***													
	(0.0045)													
Log output change		-0.0933***	-0.0283***	0.122***	-0.0626***	-0.0133***	0.0759***	-0.318***	-0.220**	0.127***	-0.0672***	-0.0135***	0.0914***	0.0107*
		(0.0144)	(0.0042)	(0.0153)	(0.0098)	(0.0039)	(0.0116)	(0.0645)	(0.0995)	(0.0170)	(0.0113)	(0.00455)	(0.0102)	(0.00575)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
First-stage F-Statistic		102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6
R^2	0.205	0.294	0.427	0.400	0.309	0.208	0.329	0.215	0.127	0.387	0.277	0.171	0.424	0.049
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Export demand shock	0.0635***													
	(0.0091)													
Log output change		-0.0555***	-0.0280***	0.0836***	-0.0465***	-0.0070	0.0536***	-0.220***	-0.125	0.105***	-0.0445***	-0.0005	0.0739***	0.0289***
		(0.0177)	(0.0062)	(0.0201)	(0.0127)	(0.0058)	(0.0157)	(0.0742)	(0.0973)	(0.0227)	(0.0153)	(0.0076)	(0.0143)	(0.00876)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-Statistic		48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.297	0.403	0.402	0.266	0.175	0.276	0.183	0.199	0.158	0.368	0.220	0.121	0.388	0.139

Notes: The table reports IV regressions from estimating the specification in Equation (10) using foreign demand shocks as instruments (Equation (12)). Column (1) reports the first-stage regression results. The dependent variables in Columns (2)-(14) are non-logged changes in labor, capital, and intermediate input cost shares, output elasticities divided by returns to scale, and logged and non-logged labor, capital, and intermediate input output elasticities not divided by returns to scale, respectively. Panel A-C report on regressions of those dependent variables on changes in log output for 1-, 4- , and 10-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure A.1: The cross-sectional relationships: output elasticities and cost shares.



Notes: The figure reports binned scatter plots estimating the specification in Equation (10) in levels with OLS. Panels (a)-(i) report on regressions of the logs of labor, capital, and intermediate quantities, labor, capital, and intermediate cost shares, and labor, capital, and intermediate output elasticities over returns to scale on log output, respectively. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Table A.5: Firm-level adjustments in intermediate cost shares by intermediate type (OLS).

	$\Delta\ln(E_{it}^{CS})$	$\Delta\ln(Merch_{it}^{CS})$	$\Delta\ln(Sub_{it}^{CS})$	$\Delta\ln(Rep_{it}^{CS})$	$\Delta\ln(Rent_{it}^{CS})$	$\Delta\ln(Temp_{it}^{CS})$	$\Delta\ln(Other_{it}^{CS})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	0.297***	0.408***	0.388***	-0.104***	-0.417***	1.165***	-0.107***
	(0.001)	(0.029)	(0.022)	(0.011)	(0.011)	(0.0329)	(0.008)
Observations	183,807	83,975	96,062	176,845	175,391	86,927	183,813
N of firms	29,950	15,277	19,842	29,598	29,340	18,048	29,950
R^2	0.108	0.069	0.061	0.029	0.055	0.133	0.030
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	0.187***	0.429***	0.218***	-0.113***	-0.281***	0.578***	-0.054***
	(0.001)	(0.044)	(0.035)	(0.014)	(0.018)	(0.0431)	(0.011)
Observations	70,933	36,495	35,962	68,859	67,827	32,512	70,936
N of firms	11,492	6,096	6,737	11,366	11,149	5,640	11,492
R^2	0.128	0.117	0.115	0.072	0.079	0.150	0.065
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	0.137***	0.217***	0.148***	-0.072***	-0.218***	0.434***	0.011
	(0.008)	(0.051)	(0.038)	(0.015)	(0.021)	(0.0416)	(0.012)
Observations	48,950	21,102	22,376	46,996	46,143	19,064	48,953
N of firms	10,381	4,542	5,743	10,189	9,996	5,647	10,381
R^2	0.130	0.126	0.120	0.074	0.080	0.124	0.068

Notes: The table reports on OLS regressions from estimating the specification in Equation (10) for 1-year (Panel A), 4-year (Panel B), and 10-year differences. The dependent variables in Columns (1)-(7) are log changes of raw materials, energy, and external components cost shares, merchandise cost shares, subcontracted production performed by other companies cost shares, repairs, maintenance, and installation cost shares, rents, leases, and leasing cost shares, temporary agency worker cost shares, and other intermediate inputs cost shares, respectively. All columns report regressions of those dependent variables on log output changes for 1-, 4-, and 10-year changes. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

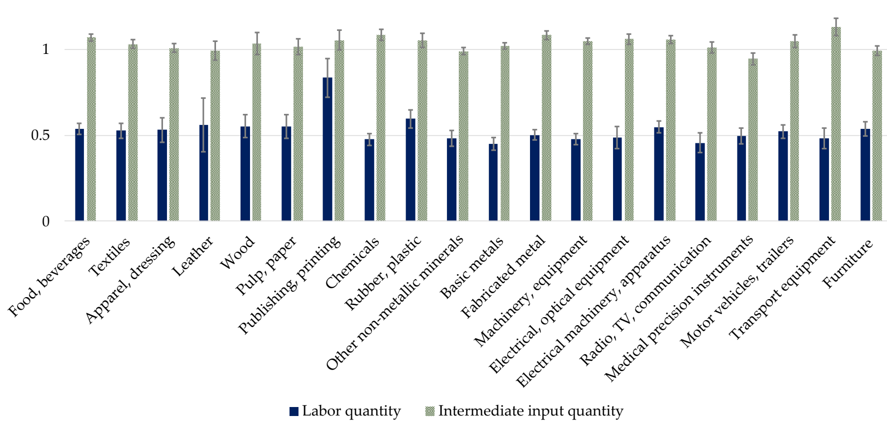
Table A.6: Firm-level adjustments in intermediate input quantities by intermediate type (OLS).

	$\Delta \ln(E_{it})$	$\Delta \ln(Merch_{it})$	$\Delta \ln(Sub_{it})$	$\Delta \ln(Rep_{it})$	$\Delta \ln(Rent_{it})$	$\Delta \ln(Temp_{it})$	$\Delta \ln(Other_{it})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	1.066***	1.179***	1.151***	0.664***	0.352***	1.932***	0.661***
	(0.007)	(0.003)	(0.0217)	(0.0113)	(0.0107)	(0.0323)	(0.008)
Observations	183,807	83,975	96,062	176,845	175,391	86,927	183,813
N of firms	29,950	15,277	19,842	29,598	29,340	18,048	29,950
R^2	0.467	0.116	0.117	0.077	0.055	0.205	0.109
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	1.044***	1.281***	1.062***	0.744***	0.576***	1.427***	0.804***
	(0.011)	(0.044)	(0.0359)	(0.014)	(0.018)	(0.042)	(0.0116)
Observations	70,933	36,495	35,962	68,859	67,827	32,512	70,936
N of firms	11,492	6,096	6,737	11,366	11,149	5,640	11,492
R^2	0.600	0.187	0.182	0.180	0.103	0.232	0.264
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	1.030***	1.107***	1.026***	0.820***	0.675***	1.322***	0.904***
	(0.009)	(0.051)	(0.039)	(0.015)	(0.022)	(0.041)	(0.012)
Observations	48,950	21,102	22,376	46,996	46,143	19,064	48,953
N of firms	10,381	4,542	5,743	10,189	9,996	5,647	10,381
R^2	0.690	0.207	0.211	0.252	0.134	0.224	0.386

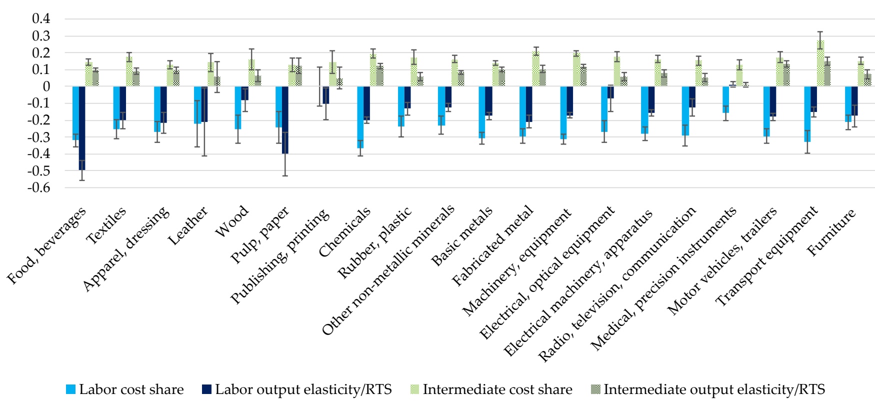
Notes: The table reports on OLS regressions from estimating the specification in Equation (10) for 1-year (Panel A), 4-year (Panel B), and 10-year differences. The dependent variables in Columns (1)-(7) are log changes of raw materials, energy, and external components, merchandise, subcontracted production performed by other companies, repairs, maintenance, and installation, rents, leases, and leasing, temporary agency worker, and other intermediate inputs, respectively. All columns report regressions of those dependent variables on log output changes for 1-, 4-, and 10-year changes. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure A.2: Firm-level adjustments separately by industry, 4-year changes, OLS.

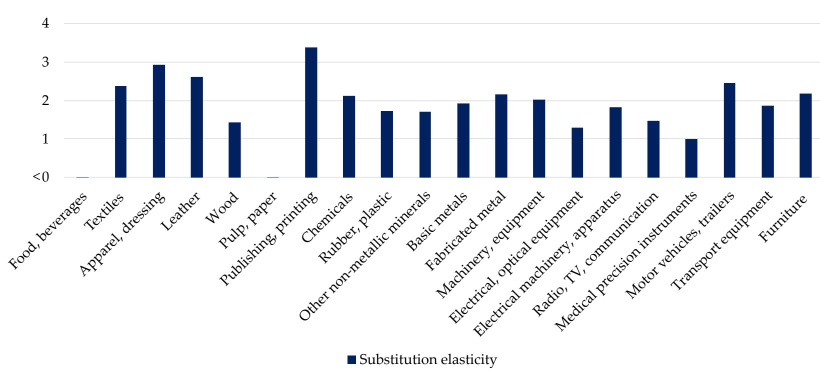
(a) Input quantities



(b) Output elasticities and cost shares

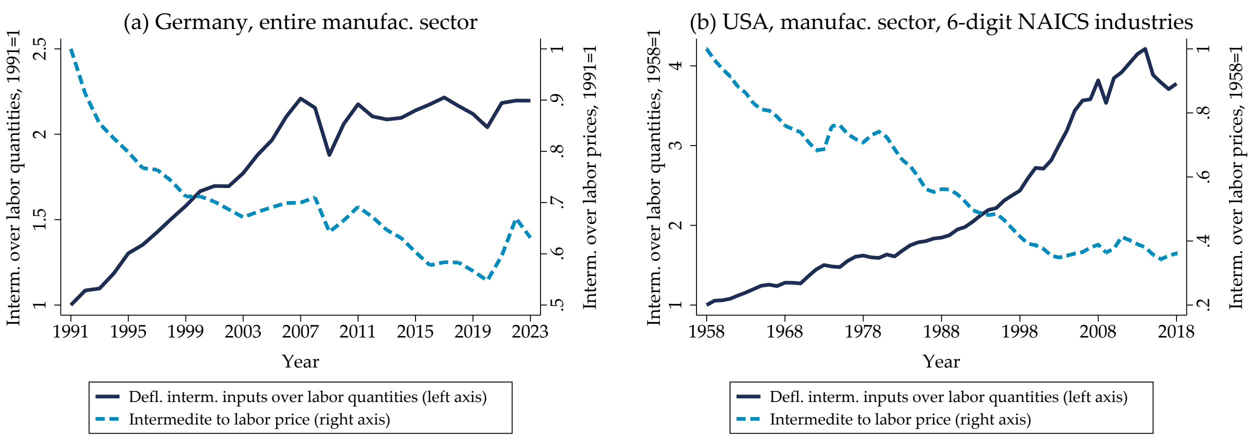


(c) Substitution elasticities



Notes: The figure reports OLS regressions from estimating Equation (10) in 4-year changes by two-digit NACE manufacturing industries. The dependent variables are logged labor and intermediate input quantities (Panel (a)), cost shares and labor and intermediate output elasticities divided by returns to scale ((Panel (b)), and implied substitution elasticities derived as described in Section 5 (Panel (c)). In Panel (c), values for "Food, beverages" and "Pulp, paper" are truncated at zero as these two estimates are negative (-8.25 and -7.61, respectively) and should not be interpreted, as they result from output elasticity responses exceeding input quantity responses (likely due to measurement errors or small sample sizes). All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. The error bars indicate 95% confidence intervals. Results for 1- and 10-year changes are similar. German firm-level data.

Figure A.3: Changes in aggregate intermediate to labor prices and quantities.



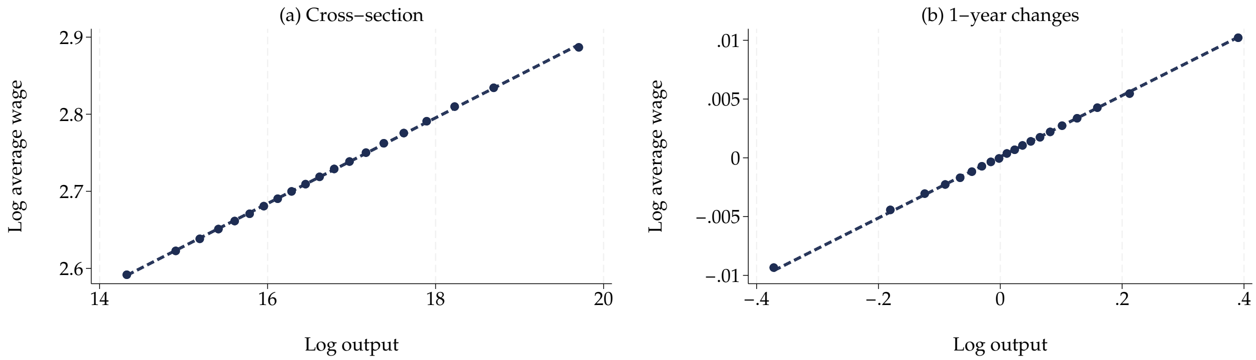
Notes: The figures report changes in prices for labor (wages) and intermediate inputs (price index) and the ratio of deflated intermediate inputs to labor quantities for Germany (Panel (a)) and the US (Panel (b)). Values are normalized to unity in the first year. German data refer to the aggregate manufacturing sector. US data refer to averages across 6-digit NAICS industries. Data from the Federal Statistical Office of Germany and the NBER-CES Manufacturing Industry Database.

Table A.7: Firm-level adjustments in output elasticities in response to firm growth: effects by size quintiles (4-year changes).

	$\Delta\ln(L_{it})$	$\Delta\ln(K_{it})$	$\Delta\ln(M_{it})$	$\Delta\ln(CS^L_{it})$	$\Delta\ln(CS^K_{it})$	$\Delta\ln(CS^M_{it})$	$\Delta\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta\ln(\frac{\theta^K_{it}}{RTS_{it}})$	$\Delta\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\Delta\ln(RTS_{it})$
Panel A: 1st size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.505***	0.259***	1.051***	-0.248***	-0.594***	0.183***	-0.179***	-0.183***	0.095***	0.024***
	(0.011)	(0.016)	(0.009)	(0.0111)	(0.0172)	(0.0072)	(0.010)	(0.017)	(0.005)	(0.001)
Observations	14,984	14,984	14,984	14,984	14,984	14,984	14,984	14,984	14,984	14,984
N of firms	4,408	4,408	4,408	4,408	4,408	4,408	4,408	4,408	4,408	4,408
R^2	0.558	0.290	0.862	0.426	0.492	0.421	0.337	0.255	0.351	0.357
Panel B: 2nd size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.484***	0.217***	1.034***	-0.280***	-0.625***	0.174***	-0.206***	-0.175***	0.098***	0.018***
	(0.012)	(0.015)	(0.009)	(0.0106)	(0.0153)	(0.0062)	(0.011)	(0.014)	(0.004)	(0.001)
Observations	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276
N of firms	4,508	4,508	4,508	4,508	4,508	4,508	4,508	4,508	4,508	4,508
R^2	0.540	0.268	0.872	0.456	0.505	0.457	0.383	0.295	0.384	0.325
Panel C: 3rd size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.492***	0.208***	1.032***	-0.299***	-0.630***	0.169***	-0.200***	-0.164***	0.093***	0.017***
	(0.012)	(0.013)	(0.010)	(0.0131)	(0.0134)	(0.0071)	(0.011)	(0.013)	(0.005)	(0.001)
Observations	13,211	13,211	13,211	13,211	13,211	13,211	13,211	13,211	13,211	13,211
N of firms	4,265	4,265	4,265	4,265	4,265	4,265	4,265	4,265	4,265	4,265
R^2	0.546	0.295	0.877	0.461	0.559	0.463	0.353	0.355	0.394	0.312
Panel D: 4th size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.528***	0.178***	1.047***	-0.295***	-0.687***	0.160***	-0.202***	-0.169***	0.094***	0.014***
	(0.015)	(0.012)	(0.009)	(0.0138)	(0.0129)	(0.0063)	(0.013)	(0.010)	(0.005)	(0.001)
Observations	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276
N of firms	3,532	3,532	3,532	3,532	3,532	3,532	3,532	3,532	3,532	3,532
R^2	0.576	0.287	0.905	0.484	0.620	0.503	0.384	0.417	0.427	0.288
Panel E: 5th size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.529***	0.150***	1.055***	-0.307***	-0.722***	0.162***	-0.196***	-0.206***	0.099***	0.008***
	(0.016)	(0.012)	(0.012)	(0.0165)	(0.0138)	(0.0085)	(0.015)	(0.021)	(0.006)	(0.002)
Observations	11,648	11,648	11,648	11,648	11,648	11,648	11,648	11,648	11,648	11,648
N of firms	2,115	2,115	2,115	2,115	2,115	2,115	2,115	2,115	2,115	2,115
R^2	0.596	0.292	0.910	0.501	0.686	0.537	0.363	0.359	0.456	0.310

Notes: The table reports on OLS regressions from estimating the specification in Equation (10) for 4-year differences. The dependent variables in Columns (1)-(10) are log changes in labor, capital, and intermediates quantities, labor, capital, and intermediate cost shares, and labor, capital, and intermediate input output elasticities divided by returns to scale, respectively. Panels A-E report results for the 1st, 2nd, 3rd , 4th, and 5th quintile of the firm-level output distribution (computed within year and industry). All columns report regressions of those dependent variables on log output changes for 4-year changes. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Size is measured by sales. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure A.4: Wage-size gradient (OLS).



Notes: The figures report binned scatter plots from regressing log wages on log firm size (output). Panel (a) reports cross-sectional results in levels; Panel (b) reports first-difference results (1-year changes). All regressions control for industry-year fixed effects. German firm-level data.

Table A.8: Effect of firm growth on average wages. German micro data. (OLS).

	OLS	IV
	Log real wage change	Log real wage change
Panel A: 1-year changes	(1)	(2)
Log output change	0.095***	-0.024
	(0.003)	(0.0562)
Observations	183,813	183,813
N of firms	29,950	29,950
First-stage F-Statistic		102.6
R^2	0.062	0.035
Panel B: 4-year changes	(1)	(2)
Log output change	0.062***	0.031
	(0.003)	(0.046)
Observations	70,936	70,936
N of firms	11,492	11,492
First-stage F-Statistic		48.57
R^2	0.126	0.122
Panel C: 10-year changes	(1)	
Log output change	0.068***	
	(0.003)	
Observations	48,953	
N of firms	10,381	
R^2	0.156	

Notes: The table reports OLS and IV regressions from estimating the specification in Equation (10) for 1-, 4-, and 10-year differences. The dependent variable is the logged wage in changes computed as the wage bill divided by the number of employees. Panels A-C report results for 1-, 4-, and 10-year differences, respectively. All columns report regressions of those dependent variables on output growth for 1-, 4-, and 10-year changes. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.9: Implied substitution elasticities, effects of firm growth on input ratios and input shadow price ratios, and firm-specific labor supply elasticities, based on cost shares as output elasticity measures.

	OLS			IV		
	σ	$\Delta \ln \left(\frac{w_{it}\gamma_{it}^L}{z_{it}\gamma_{it}^M} \right)$	ϵ_{it}^L	σ	$\Delta \ln \left(\frac{w_{it}\gamma_{it}^L}{z_{it}\gamma_{it}^M} \right)$	ϵ_{it}^L
	(1)	(2)	(3)	(4)	(5)	(6)
1-year diff.	6.434	0.113	2.64	2.28 - inf.	0.00 - 0.35	0.62 - inf.
4-year diff.	6.859	0.078	6.54	12.559	0.034	15.765
10-year diff.	5.118	0.085	7.13			

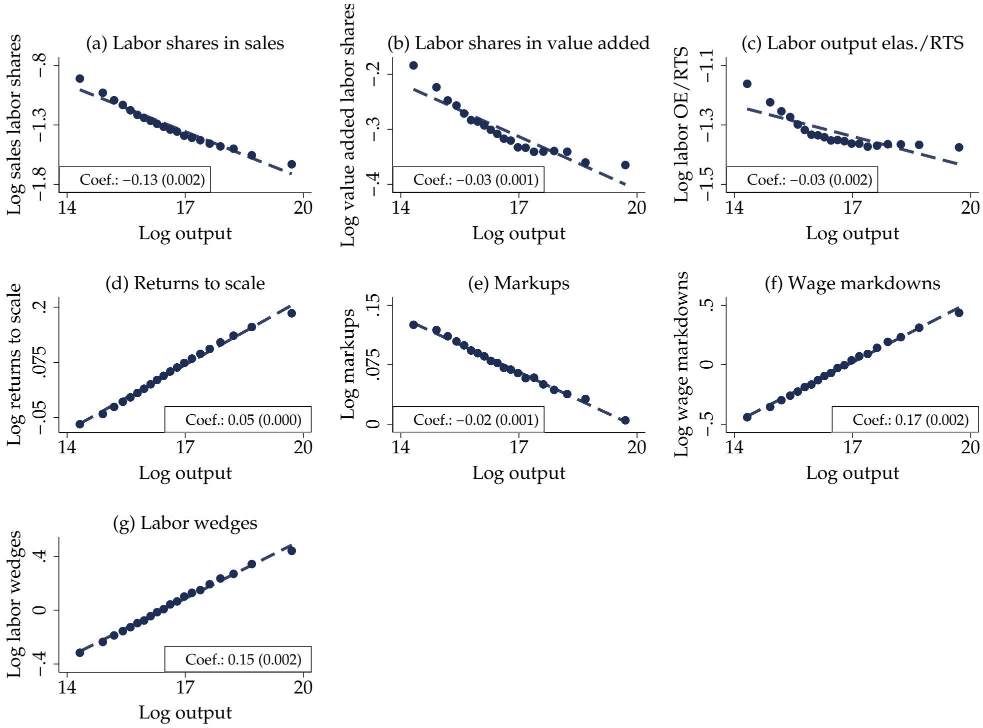
Notes: The table reports substitution elasticities (Columns (1) and (5)) following Equation (14), changes in input factor ratios (Columns (2) and (6)), implied changes in shadow input price ratios (Columns (3) and (7)) following Equation (16), and implied labor supply elasticities following Equation (18), assuming perfectly elastic intermediate input supply (Columns (4) and (8)), based on our OLS (Columns (1)-(4)) and IV (Columns (5)-(8)) regressions from Tables 2 and 3 that regress log cost shares and log input quantities on log output in within-firm differences. Consequently, Columns (2) and (6) report coefficient ratios for labor and intermediates from these regressions with respect to firm growth, while all other columns report values implied by our regressions as described in the text. As IV point estimates suggest a substitution elasticity of infinity for 1-year changes, we report intervals using 95% confidence intervals from all point estimates entering the computation for this specification. This yields much larger intervals than directly computing confidence intervals for substitution elasticities and other values.

Table A.10: Testing for the role of non-homotheticities (direct output dependence), OLS results, 4-year changes.

	$\Delta\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta\ln(CS^L_{it})$	$\ln(CS^L_{it})$	$\Delta\ln(CS^L_{it})$	$\Delta\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$	$\Delta\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$	$\Delta\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$	$\Delta\ln(\frac{CS^L_{it}}{CS^M_{it}})$	$\Delta\ln(\frac{CS^L_{it}}{CS^M_{it}})$	$\Delta\ln(\frac{CS^L_{it}}{CS^M_{it}})$
Panel A: 4-year diff. OLS	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Log output change	-0.202***		0.027***	-0.286***		0.003	-0.297***		0.0512***	-0.457***		-0.020***
	(0.006)		(0.005)	(0.006)		(0.003)	(0.00785)		(0.00555)	(0.009)		(0.004)
Log labor-interm. input ratio		0.412***	0.427***		0.538***	0.540***		0.622***	0.651***		0.826***	0.815***
		(0.006)	(0.007)		(0.003)	(0.004)		(0.00656)	(0.00726)		(0.004)	(0.005)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.220	0.462	0.463	0.348	0.765	0.765	0.245	0.595	0.597	0.373	0.810	0.810
Panel B: 4-year diff. IV	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)			
Log output change	-0.242***		-0.087	-0.269***		-0.052	-0.324***		-0.0772	-0.393***		-0.053
	(0.071)		(0.075)	(0.066)		(0.048)	(0.0911)		(0.0857)	(0.097)		(0.061)
Log labor-interm. input ratio		0.412***	0.364***		0.538***	0.509***		0.622***	0.579***		0.826***	0.797***
		(0.006)	(0.042)		(0.003)	(0.027)		(0.00656)	(0.0482)		(0.004)	(0.035)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-value	48.57		39	48.57		39	48.57		39	48.57		39
R^2	0.217	0.462	0.445	0.347	0.765	0.761	0.244	0.595	0.583	0.369	0.810	0.809

Notes: The table reports OLS (Panel A) and IV (Panel B) regressions from estimating the specification in Equation (10) for 4-year differences while additionally controlling for log changes of the labor-intermediate input ratio. The IV specification uses foreign demand shocks as instruments (Equation (12)). The dependent variable in Columns (1)-(3) is the log change in the labor output elasticity divided by returns to scale (we have checked that output elasticities unadjusted for RTS yield very similar results). The dependent variable in Columns (4)-(6) is the log changes in the labor cost share. The dependent variable in Columns (7)-(9) is the log change in the labor output elasticity divided by the intermediate input output elasticity. The dependent variable in Columns (10)-(12) is the log change in the labor output cost share divided by the intermediate input cost share. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure A.5: The cross-sectional relationships: labor shares, output elasticities, markups, markdowns, labor wedges, and returns to scale.



Notes: The figure reports binned scatter plots estimating the specification in Equation (10) in levels with OLS. Panels (a)-(g) report on regressions of the logs of labor shares in sales and value added, labor output elasticities divided by returns to scale, returns to scale, markups, wage markdowns, and labor wedges, respectively. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Table A.11: CompNet summary statistics, averages of industry averages (manufacturing).

Country	Lab. share (sales)	Lab. share (value added)	Lab. cost share	Lab. output elast.	Interm. cost share	Interm. output elast.	Markup	Wage Markdown
Belgium (2000-2020)	0.17	0.42	0.21	0.31	0.74	0.67	1.23	1.26
Croatia (2002-2021)	0.19	0.40	0.25	0.28	0.67	0.64	1.32	1.05
Czech Republic (2005-2020)	0.16	0.59	0.17	0.18	0.78	0.79	1.19	0.75
Denmark (2001-2020)	0.25	0.72	0.26	0.26	0.69	0.74	1.18	0.80
Finland (1999-2020)	0.21	0.69	0.22	0.21	0.75	0.77	1.15	0.84
France (2004-2020)	0.21	0.39	0.31	0.38	0.62	0.57	1.51	1.37
Germany (2001-2018)	0.22	0.74	0.21	0.36	0.69	0.77	1.13	1.53
Hungary (2003-2020)	0.17	0.61	0.17	0.19	0.77	0.77	1.12	0.86
Italy (2006-2020)	0.17	0.63	0.17	0.17	0.78	0.82	1.19	0.77
Latvia (2007-2019)	0.18	0.53	0.21	0.23	0.73	0.75	1.25	0.99
Lithuania (2000-2020)	0.19	0.66	0.20	0.20	0.74	0.76	1.15	1.14
Netherlands (2007-2019)	0.21	0.68	0.22	0.22	0.74	0.75	1.11	0.85
Poland (2002-2020)	0.16	0.54	0.17	0.17	0.78	0.78	1.18	0.79
Portugal (2010-2020)	0.19	0.62	0.20	0.25	0.74	0.74	1.12	1.09
Romania (2005-2020)	0.18	0.61	0.17	0.20	0.77	0.77	1.14	0.90
Slovakia (2000-2020)	0.16	0.50	0.18	0.19	0.76	0.76	1.14	1.09
Slovenia (2002-2021)	0.20	0.64	0.21	0.20	0.74	0.74	1.12	0.83
Spain (2008-2020)	0.17	0.64	0.19	0.21	0.79	0.77	1.17	0.96
Sweden (2003-2020)	0.22	0.38	0.32	0.41	0.62	0.56	1.31	1.52
Switzerland (2009-2020)	0.26	0.70	0.28	0.25	0.68	0.72	1.18	0.79

Notes: The table reports country-level averages of industry-level labor shares (in sales and value added), labor cost shares, labor output elasticities, intermediate input cost shares, intermediate input output elasticities, markups, and wage markdowns, i.e., for each country, we average across country-industry-level values. Country-industry-level aggregates of labor shares are computed from firms' total labor costs, total sales, and total value added. Country-industry cost shares are computed as total input expenditures divided by total costs. Country-industry-level averages of labor output elasticities are sales-weighted aggregates of firm-level values. Country-industry-level average markups are computed as intermediate input cost-weighted averages of firm-level values. Country-industry-level average wage markdowns are derived as labor cost-weighted averages of firm-level values. CompNet manufacturing data. Firms with at least 20 employees.

Table A.12: Firm-level results for other countries: coefficients on log output changes, substitution elasticities, and contribution of labor output elasticity changes to labor share changes (manufacturing, 4-year changes, OLS).

	$\Delta\ln(L_{it})$	$\Delta\ln(M_{it})$	$\Delta\ln(CS^L_{it})$	$\Delta\ln(CS^M_{it})$	$\Delta\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\Delta\ln(\theta^L_{it})$	$\Delta\ln(LS_{it})$	$\Delta\ln(\frac{P^L_{it}L_{it}}{VA_{it}})$	σ^{OE}	σ^{CS}	LS contrib. of $\Delta\theta^L_{it}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
France (1995-21)	0.496***	1.027***	-0.0882***	0.313****	-0.131***	0.196***	-0.122***	-0.373***	-0.350***	2.60	4.01	33%
Obs.: 282,245	(0.004)	(0.004)	(0.003)	(0.004)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)			
Hungary (2003-22)	0.464***	1.057***	-0.307***	0.159***	-0.330***	0.121***	-0.343***	-0.409***	-0.340***	4.18	4.67	84%
Obs.: 35,902	(0.007)	(0.007)	(0.008)	(0.006)	(0.007)	(0.003)	(0.007)	(0.008)	(0.010)			
Poland (2002-22)	0.535***	1.058***	-0.262***	0.134***	-0.281***	0.0901***	-0.281***	-0.337***	-0.259***	3.44	4.12	83%
Obs.: 98,385	(0.004)	(0.003)	(0.004)	(0.003)	(0.004)	(0.001)	(0.004)	(0.004)	(0.004)			
Portugal (2004-22)	0.414***	1.115***	-0.347***	0.236***	-0.393***	0.198***	-0.403***	-0.469***	-0.352***	6.37	5.94	86%
Obs.: 33,478	(0.007)	(0.007)	(0.007)	(0.006)	(0.007)	(0.005)	(0.008)	(0.007)	(0.009)			
Slovakia (2000-23)	0.513***	1.019***	-0.266***	0.166***	-0.273***	0.102***	-0.264***	-0.412***	-0.381***	3.86	6.84	64%
Obs.: 9,605	(0.017)	(0.020)	(0.015)	(0.010)	(0.013)	(0.007)	(0.014)	(0.016)	(0.025)			
Slovenia (2002-23)	0.547***	1.091***	-0.285***	0.166***	-0.355***	0.135***	-0.351***	-0.360***	-0.236***	10.07	5.85	98%
Obs.: 7,452	(0.016)	(0.013)	(0.015)	(0.011)	(0.017)	(0.007)	(0.017)	(0.016)	(0.015)			
Estonia (2004-23)	0.547***	1.102***	-0.243***	0.151***	-0.393***	0.127***	-0.394***	-0.292***	-0.192***	15.86	3.45	135%
Obs.: 1,627	(0.031)	(0.024)	(0.029)	(0.014)	(0.033)	(0.01)	(0.034)	(0.031)	(0.034)			
Latvia (2005-21)	0.495***	0.940***	-0.239***	0.0272***	-0.258***	0.125***	-0.257***	-0.327***	-0.423***	7.18	2.49	79%
Obs.: 2,760	(0.023)	(0.022)	(0.028)	(0.0055)	(0.022)	(0.010)	(0.022)	(0.022)	(0.039)			
Netherlands (2007-22)	0.369***	1.116***	-0.303***	0.169***	-0.358***	0.143***	-0.363***	-0.353***	-0.185***	3.04	2.72	103%
Obs.: 30,210	(0.009)	(0.008)	(0.011)	(0.007)	(0.008)	(0.004)	(0.008)	(0.011)	(0.008)			
Romania (2005-23)	0.526***	1.031***	-0.269***	0.126***	-0.232***	0.0827***	-0.238***	-0.363***	-0.337***	2.65	4.59	66%
Obs.: 21,642	(0.0076)	(0.005)	(0.008)	(0.004)	(0.006)	(0.003)	(0.007)	(0.008)	(0.013)			
Switzerland (2009-22)	0.460***	1.055***	-0.303***	0.189***	-0.287***	0.124***	-0.283***	-0.434***	-0.365***	3.23	5.78	65%
Obs.: 11,875	(0.016)	(0.013)	(0.015)	(0.010)	(0.013)	(0.0057)	(0.014)	(0.016)	(0.017)			
Germany (1995-17)	0.510***	1.045***	-0.286***	0.171***	-0.202***	0.095***	-0.185***	-0.406***	-0.334***	2.26	6.86	46%
Obs.: 70,936	(0.006)	(0.005)	(0.006)	(0.003)	(0.006)	(0.003)	(0.006)	(0.006)	(0.008)			

Notes: The table reports OLS regressions from estimating the specification in Equation (10) for different countries by OLS (Columns (1)-(9)) using 4-year differences. The table reports coefficients on log output change in Columns (1)-(9). The dependent variables in Columns (1)-(9) are log changes in labor and intermediate quantities, cost shares, and output elasticities divided by returns to scale, labor output elasticities, labor shares in sales, and labor shares in value added, respectively. All columns report regressions of those dependent variables on changes in log output for 4-year differences. All regressions control for two-digit industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data. Columns (10) and (11) report estimates of the substitution elasticity between labor and intermediates based on Equation (14) using output elasticity coefficients (Column (10)) and cost share coefficients (Column (11)). Column (12) reports the contribution of changes in labor output elasticities (Column (7)) to changes in labor shares (Column (8)). Firm-level manufacturing data from CompNet data providers. Firm-level data from CompNet data providers and our German manufacturing firm-level data.

Table A.13: Firm-level results for other countries: Coefficients on log output changes, substitution elasticities, and contribution of labor output elasticity changes to labor share changes (all sectors, 4-year changes, OLS).

	$\Delta\ln(L_{it})$	$\Delta\ln(M_{it})$	$\Delta\ln(CS_{it}^{L})$	$\Delta\ln(CS_{it}^{M})$	$\Delta\ln(\frac{\theta_{it}^{L}}{RTS_{it}})$	$\Delta\ln(\frac{\theta_{it}^{M}}{RTS_{it}})$	$\Delta\ln(\theta_{it}^{L})$	$\Delta\ln(LS_{it})$	$\Delta\ln(\frac{P_{it}^{L}L_{it}}{VA_{it}})$	σ^{OE}	σ^{CS}	LS contrib. of $\Delta\theta_{it}^{L}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
France (1995-21)	0.511***	0.975***	-0.049***	0.268***	-0.095***	0.183***	-0.087***	-0.340****	-0.333***	2.50	3.15	26%
Obs.: 589,669	(0.003)	(0.004)	(0.002)	(0.004)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)			
Hungary (2003-22)	0.497***	1.039***	-0.270***	0.125***	-0.276***	0.094***	-0.292***	-0.356***	-0.297***	3.15	3.69	82%
Obs.: 82,062	(0.005)	(0.004)	(0.005)	(0.004)	(0.005)	(0.002)	(0.005)	(0.005)	(0.007)			
Poland (2002-22)	0.535***	1.064***	-0.273***	0.128***	-0.276***	0.090***	-0.278***	-0.336***	-0.231***	3.25	4.13	83%
Obs.: 229,205	(0.003)	(0.002)	(0.003)	(0.002)	(0.003)	(0.001)	(0.003)	(0.003)	(0.003)			
Portugal (2004-22)	0.501***	1.044***	-0.240***	0.165***	-0.293***	0.149***	-0.301***	-0.361***	-0.305***	5.38	3.93	83%
Obs.: 74,869	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.003)	(0.005)	(0.005)	(0.006)			
Slovakia (2000-23)	0.493***	0.916***	-0.200***	0.130***	-0.202***	0.076***	-0.211***	-0.412***	-0.420***	2.93	4.55	51%
Obs.: 25,395	(0.010)	(0.012)	(0.009)	(0.006)	(0.009)	(0.005)	(0.009)	(0.009)	(0.014)			
Slovenia (2002-23)	0.591***	1.080***	-0.259***	0.142***	-0.294***	0.116***	-0.298***	-0.321***	-0.191***	6.19	5.57	93%
Obs.: 17,696	(0.014)	(0.008)	(0.011)	(0.007)	(0.012)	(0.006)	(0.012)	(0.011)	(0.012)			
Estonia (2004-23)	0.493***	1.079***	-0.261***	0.142***	-0.348***	0.151***	-0.344***	-0.323***	-0.219***	6.74	3.20	107%
Obs.: 4,020	(0.023)	(0.016)	(0.021)	(0.011)	(0.022)	(0.011)	(0.022)	(0.024)	(0.026)			
Latvia (2005-21)	0.529***	0.859***	-0.172***	0.0081**	-0.145***	0.074***	-0.132***	-0.320***	-0.361***	2.97	2.20	41%
Obs.: 13,405	(0.012)	(0.013)	(0.015)	(0.003)	(0.010)	(0.007)	(0.010)	(0.012)	(0.015)			
Netherlands (2007-22)	0.497***	1.089***	-0.230***	0.130***	-0.257***	0.107***	-0.249***	-0.270***	-0.144***	2.60	2.55	92%
Obs.: 142,659	(0.005)	(0.004)	(0.005)	(0.003)	(0.004)	(0.002)	(0.004)	(0.005)	(0.004)			
Romania (2005-23)	0.523***	1.012***	-0.266***	0.096***	-0.153***	0.044***	-0.163***	-0.350***	-0.328***	1.67	3.86	50%
Obs.: 58,441	(0.005)	(0.003)	(0.005)	(0.002)	(0.004)	(0.002)	(0.005)	(0.005)	(0.008)			
Switzerland (2009-22)	0.548***	1.058***	-0.232***	0.172***	-0.215***	0.119***	-0.220***	-0.347***	-0.277***	2.90	4.81	63%
Obs.: 28,572	(0.011)	(0.012)	(0.01)	(0.01)	(0.009)	(0.005)	(0.009)	(0.010)	(0.011)			

Notes: The table reports OLS regressions from estimating the specification in Equation (10) for different countries by OLS (Columns (1)-(9)) using 4-year differences. The table reports coefficients on log output change in Columns (1)-(9). The dependent variables in Columns (1)-(9) are log changes in labor and intermediate quantities, cost shares, and output elasticities divided by returns to scale, labor output elasticities, labor shares in sales, and labor shares in value added, respectively. All columns report regressions of those dependent variables on changes in log output for 4-year differences. All regressions control for two-digit industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data. Columns (10) and (11) report estimates of the substitution elasticity between labor and intermediates based on Equation (14) using output elasticity coefficients (Column (10)) and cost share coefficients (Column (11)). Column (12) reports the contribution of changes in labor output elasticities (Column (7)) to changes in labor shares (Column (8)). Firm-level data on all sectors from CompNet data providers. Firm-level data from CompNet data providers.

Table A.14: Firm-level results for other countries: Coefficients on output growth, substitution elasticities, and contribution of labor output elasticity changes to labor share changes (4-year changes, OLS). Young and mature manufacturing firms.

	$\Delta\ln(L_{it})$	$\Delta\ln(M_{it})$	$\Delta\ln(CS^L_{it})$	$\Delta\ln(CS^M_{it})$	$\Delta\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\Delta\ln(\theta^L_{it})$	$\Delta\ln(LS_{it})$	$\Delta\ln(\frac{P^L_{it}L_{it}}{VA_{it}})$	σ^{OE}	σ^{CS}	LS contrib. of $\Delta\theta^L_{it}$
Panel A: young firms	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
France (1995-21)	0.477***	1.018***	-0.109***	0.227***	-0.131***	0.195***	-0.122***	-0.318***	-0.305***	2.52	2.64	38%
Obs.: 21,988	(0.008)	(0.010)	(0.007)	(0.008)	(0.004)	(0.006)	(0.005)	(0.007)	(0.008)			
Hungary (2003-22)	0.488***	1.052***	-0.249***	0.134***	-0.287***	0.116***	-0.299***	-0.330***	-0.274***	3.50	3.12	91%
Obs.: 3,966	(0.016)	(0.013)	(0.015)	(0.011)	(0.015)	(0.006)	(0.015)	(0.015)	(0.019)			
Slovakia (2000-23)	0.570***	1.014***	-0.255***	0.134***	-0.248***	0.0830***	-0.242***	-0.375***	-0.370***	3.93	8.07	65%
Obs.: 1,157	(0.031)	(0.031)	(0.031)	(0.021)	(0.028)	(0.013)	(0.029)	(0.031)	(0.051)			
Slovenia (2002-23)	0.590***	1.097***	-0.220***	0.140***	-0.279***	0.140***	-0.273***	-0.266***	-0.163***	5.76	3.45	103%
Obs.: 719	(0.036)	(0.026)	(0.032)	(0.023)	(0.038)	(0.017)	(0.038)	(0.033)	(0.029)			
Latvia (2005-21)	0.563***	0.973***	-0.224**	0.022	-0.205***	0.125***	-0.208***	-0.272***	-0.409***	5.13	2.49	76%
Obs.: 236	(0.053)	(0.053)	(0.090)	(0.0140)	(0.054)	(0.031)	(0.058)	(0.062)	(0.126)			
Netherlands (2007-22)	0.439***	1.107***	-0.251***	0.152***	-0.308***	0.126***	-0.313***	-0.293***	-0.141***	2.85	2.52	107%
Obs.: 1,774	(0.030)	(0.028)	(0.032)	(0.024)	(0.024)	(0.012)	(0.024)	(0.033)	(0.035)			
Panel B: mature firms	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
France (1995-21)	0.498***	1.029***	-0.0824***	0.330***	-0.131***	0.199***	-0.122***	-0.382***	-0.357***	2.64	4.48	32%
Obs.: 253,635	(0.004)	(0.004)	(0.003)	(0.004)	(0.002)	(0.002)	(0.002)	(0.003)	(0.004)			
Hungary (2003-22)	0.454***	1.060***	-0.326***	0.168***	-0.343***	0.124***	-0.356***	-0.434***	-0.354***	4.36	5.41	82%
Obs.: 31,916	(0.008)	(0.007)	(0.009)	(0.006)	(0.007)	(0.003)	(0.008)	(0.009)	(0.011)			
Slovakia (2000-23)	0.491***	1.018***	-0.272***	0.176***	-0.283***	0.108***	-0.274***	-0.428***	-0.391***	3.88	6.67	64%
Obs.: 8,399	(0.018)	(0.022)	(0.015)	(0.012)	(0.015)	(0.008)	(0.016)	(0.018)	(0.028)			
Slovenia (2002-23)	0.533***	1.097***	-0.306***	0.179***	-0.376***	0.137***	-0.371***	-0.387***	-0.244***	11.06	7.14	96%
Obs.: 6,656	(0.017)	(0.014)	(0.016)	(0.012)	(0.019)	(0.008)	(0.019)	(0.016)	(0.016)			
Latvia (2005-21)	0.479***	0.938***	-0.245***	0.0305***	-0.269***	0.127***	-0.267***	-0.338***	-0.424***	7.29	2.50	80%
Obs.: 2,482	(0.026)	(0.024)	(0.029)	(0.006)	(0.024)	(0.010)	(0.025)	(0.024)	(0.042)			
Netherlands (2007-22)	0.363***	1.117***	-0.308***	0.171***	-0.364***	0.145***	-0.369***	-0.359***	-0.188***	3.14	2.74	103%
Obs.: 28,411	(0.010)	(0.008)	(0.012)	(0.007)	(0.008)	(0.004)	(0.009)	(0.012)	(0.008)			

Notes: The table reports OLS regressions from estimating the specification in Equation (10) for different countries (Columns (1)-(9)) using 4-year differences. The table reports coefficients on log output changes in Columns (1)-(9). The dependent variables in Columns (1)-(9) are log changes in labor and intermediate quantities, cost shares, and output elasticities divided by returns to scale, labor output elasticities, labor shares in sales, and labor shares in value added, respectively. All columns report regressions of those dependent variables on changes in log output for 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data. Columns (10) and (11) report estimates of the substitution elasticity between labor and intermediates based on Eq. (14) using output elasticity coefficients (Column (10)) and cost share coefficients (Column (11)). Column (12) reports the contribution of changes in labor output elasticities (Column (7)) to changes in labor shares (Column (8)). Panel A reports results for firms not older than five years. Panel B reports results for firms older than five years. Firm-level micro data from CompNet data providers for a subset of CompNet countries with information on registration years.

B Details on Substitution Elasticities in Meta-Analysis (Figure 5 Panel (a))

We provide details on the studies in our meta-analysis on substitution elasticities (Figure 5 Panel (a)), distinguishing between those estimating elasticities between intermediates and a capital-labor bundle and those directly estimating elasticities between intermediates and labor. Estimating substitution elasticities for a capital-labor bundle imposes the restriction that intermediates substitute for capital and labor identically. Moreover, these elasticities are typically derived from the ratio between intermediates and the capital-labor bundle rather than the intermediates-to-labor ratio. We believe that this restriction can help explain why studies estimating substitution elasticities between intermediates and a capital-labor bundle tend to yield lower values (often below unity), while the few studies that directly estimate intermediate-labor substitution elasticities tend to find larger values. In our study, we use the latter approach and directly estimate the intermediate-labor substitution elasticity.

We do not review studies that focus on substitution between energy and labor (such as Bretschger and Jo, 2024), as energy constitutes a relatively small portion of intermediate inputs. We emphasize that the definition of intermediates can matter for substitution elasticities and expect the substitution elasticity between raw materials, such as steel, or energy and labor, to be smaller than the substitution elasticity between labor and unfinished product components or intermediate services. We estimate an overall substitution elasticity and leave a more detailed analysis of subcategories of intermediates for future work.

Substitution elasticities between intermediates and a capital-labor bundle:

- Peter and Ruane (2022). Plant-level data, India, manufacturing. Identification from input price variation. We compute the mean of the OLS (0.479) and IV (0.618) estimates in Table 6: 0.5485.

- Boehm et al. (2019). Firm-level data, US, manufacturing and non-manufacturing. Identification from short-run output and input variation under constant input prices. We take the value for all firms in Table 2: 0.037.

- Bruno (1984). Country-level data, US, UK, Germany, Japan, manufacturing and non-manufacturing. Identification from input price variation. We compute the mean of all values in Table 3 except the negative ones (0.196, 0.57, 0.337, 0.46, 0.35, 0.132, 0.472, 0.649, 0.812, 0.766, 0.91): 0.514.

- Oberfield and Raval (2021). Firm-level data, US, manufacturing. Identification from input price variation. We compute the mean of all values in Table 3 (1.03, 0.83, 0,69, 0,78, 0,57): 0.78.

- Antoszewski (2019). Industry-level data (WIOD), many OECD countries, manufacturing and non-manufacturing. Identification from input price variation. We compute the mean of all values in Table 8 (too many to report, ranging from 0.4324 to 1.1288): 0.8211.

- Atalay (2017). Industry-level data, US, manufacturing and non-manufacturing. Identification from input price variation. We compute the mean of all values in Table 8 (1.18, 1.27, 0.84, 0.88): 1.043.

- Rotemberg and Woodford (1996). Industry-level data, US, manufacturing. Identification from input price variation. We take the value from Table 2: 0.69.

- Miranda-Pinto (2021). Industry-level data, several countries, manufacturing and non-manufacturing. Identification from input price variation. We take the value for all firms in Table 4: 1.34.

- Miranda-Pinto and Youngs (2022). Industry-level data, US, manufacturing and non-manufacturing. Identification from input price variation. We compute the mean of the means of the fixed effects (0.85) and IV (2.14) specifications in Table 2 (we do not use the value by industries as several industries were pooled and share the same estimates): 1.495.

Substitution elasticities between intermediates and labor:

- Delgado et al. (1999). Firm-level data, Spain, manufacturing. Identification from input price variation. We compute the mean of all estimates of $1 - \beta$ in Table 2 (too many to report, ranging from 0.03 to 1.00): 0.414.

- Doraszelski and Jaumandreu (2018). Firm-level data, Spain, manufacturing. Identification from input price variation. We compute the mean of all values in Tables 3, 4, and 5 (too many values to report, ranging from 0.12 to 1.846): 0.691.

- Huneeus et al. (2022). Firm-level data, Chile, manufacturing and non-manufacturing. Identification from input price variation. We compute the mean of all estimates in Table 5 (1.55, 1.62, 1.05): 1.401. The authors emphasize in Footnote 38 that using their more sophisticated price measures that account for input heterogeneity yield higher estimates of the substitution elasticity (1.55, 1.62) compared to existing price measures (1.05) as typically used in the literature, such as by Doraszelski and Jaumandreu (2018) or Oberfield and Raval (2021).

- Chan (2023). Firm-task-level data, Denmark, manufacturing and non-manufacturing. Identification from input price variation. We compute the mean of all estimates in Table 6 (too many to report, ranging from 1.65 to 9.62): 2.368.

- Martinello (1989). Industry-level data, Canada, US, wood products industry. Identification from input price variation and a structural labor market model. We compute the mean of the estimates in Table 4 (2.662, 2.257): 2.46.

C Theoretical Appendix

C.1 Derivations for Section 2.1

The notation follows the main text. We can define the production function in general terms, $Q_{it}(K_{it}, L_{it}, M_{it}, \Omega_{it}) = Q_{it}(.)$ and write firms' cost minimization as a Lagrangian function:

$$\begin{aligned}\mathcal{L}_{it} = P_{it}^L(L_{it})L_{it} + \chi^L(L_{it})P_{it}^L + P_{it}^M(M_{it})M_{it} + \chi^M(M_{it})P_{it}^M \\ +P_{it}^K(K_{it})K_{it} + \chi^K(K_{it})P_{it}^K - \lambda_{it}(Q_{it} - Q_{it}(.)),\end{aligned} \tag{C.1}$$

where, for simplicity, we model input adjustment costs through convex functions $\chi^X(X_{it})$ with $X = \{L, M, K\}$ and write the optimization in a quasi-static way as in Bond et al. (2021) (such that we avoid writing out the dynamic problem). One interpretation of this setting is that each input is associated with a baseline quantity, $\bar{X}_{it}$, and that firms incur adjustment costs when choosing an input quantity that differs from $\bar{X}_{it}$. The first-order conditions for each production input lead to Equation (2) from the main text:

$$\frac{\partial \mathcal{L}_{it}}{\partial X_{it}} = 0 \quad \Rightarrow \quad P_{it}^X \underbrace{\left(1 + \frac{\partial P_{it}^X}{\partial X_{it}} \frac{X_{it}}{P_{it}^X} + \frac{\partial \chi^X}{\partial L_{it}}\right)}_{\gamma_{it}^X} = \lambda_{it} \frac{\partial Q_{it}}{\partial X_{it}}. \tag{C.2}$$

Multiplying Equation (C.2) by $\frac{X_{it}}{Q_{it}}$ using the definition of the output elasticity, $\theta^X = \frac{\partial Q}{\partial X}\frac{X}{Q}$, and noting that $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$ (price over markup) leads to:

$$X_{it}P_{it}^X = P_{it}Q_{it}\frac{\theta_{it}^X}{\mu_{it}\gamma_{it}^X}. \tag{C.3}$$

Rearranging this equation yields the share of input expenditures in sales as a function of output elasticities, markups, and input price markdowns, γ_{it}^X, which is also Equation (6) of the main text:

$$OS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it}Q_{it}} = \frac{\theta_{it}^X}{\mu_{it}\gamma_{it}^X}. \tag{C.4}$$

Similarly, we can define Equation (C.3) for each input and recover expressions for input cost shares (which equal Equation (5) of the main text):

$$CS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it}^X L_{it} + P_{it}^M M_{it} + P_{it}^K K_{it}} = \frac{\frac{\theta_{it}^X}{\gamma_{it}^X}}{\frac{\theta_{it}^L}{\gamma_{it}^L} + \frac{\theta_{it}^M}{\gamma_{it}^M} + \frac{\theta_{it}^K}{\gamma_{it}^K}}. \tag{C.5}$$

For the next derivations, we rely on the production function (Equation (1)). The partial derivatives with respect to labor and intermediate yield the first part of Equation (3):

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} (\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}} \Lambda_i^{LM} \alpha_i^L \kappa L_{it}^{\frac{-1}{\sigma}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}}{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} (\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}} \Lambda_i^{LM} \alpha_i^M \kappa M_{it}^{\frac{-1}{\sigma}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{-1}{\sigma_{it}}}. \tag{C.6}$$

Multiplying this expression by $\frac{L_{it}}{Q_{it}}/\frac{M_{it}}{Q_{it}}$ yields the second part of Equation (3):

$$\frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}}. \tag{C.7}$$

Finally, taking logs and differences (the latter eliminates the firm fixed effect, $\frac{\alpha_i^L}{\alpha_i^M}$) yields the last part of Equation (3):

$$\frac{\sigma-1}{\sigma} = \frac{\Delta\ln(\theta_{it}^L) - \Delta\ln(\theta_{it}^M)}{\Delta\ln(L_{it}) - \Delta\ln(M_{it})}. \tag{C.8}$$

C.2 Derivations for Section 2.2

The notation follows the main text. To recover Equation (7) from the main text, we impose more structure on our model by defining inverse input supply functions as $P_{it}^X = a_{it}^X X_{it}^{\varepsilon^X}$ for $X = \{L, M\}$ and abstract from adjustment costs for labor and intermediates. For simplicity, we also assume that markups and capital shadow costs do not depend on firm scale. Inserting the input supply functions into Equation (C.1) and dropping adjustment cost terms (except for capital) yields the following Lagrangian:

$$\mathcal{L}_{it} = a_{it}^L L_{it}^{1+\varepsilon^L} + a_{it}^M M_{it}^{1+\varepsilon^M} + P_{it}^K(K_{it})K_{it} + \chi^K(K_{it})P_{it}^K - \lambda_{it}(Q_{it} - Q_{it}(.)). \tag{C.9}$$

The first-order conditions for labor and intermediates yield Equation (7) from the main text:

$$\frac{\partial\mathcal{L}_{it}}{\partial L_{it}} = 0 \quad\Rightarrow\quad \left(1+\varepsilon^L\right)a_{it}^L L_{it}^{\varepsilon^L} = \lambda_{it}\frac{\partial Q_{it}}{\partial L_{it}} \quad\Rightarrow\quad L_{it} = \left(\frac{\lambda_{it}}{(1+\varepsilon^L)\,a_{it}^L}\frac{\partial Q_{it}}{\partial L_{it}}\right)^{\frac{1}{\varepsilon^L}} \tag{C.10}$$

$$\frac{\partial\mathcal{L}_{it}}{\partial M_{it}} = 0 \quad\Rightarrow\quad M_{it} = \left(\frac{\lambda_{it}}{(1+\varepsilon^M)\,a_{it}^M}\frac{\partial Q_{it}}{\partial M_{it}}\right)^{\frac{1}{\varepsilon^M}}. \tag{C.11}$$

The first-order condition for capital is still Equation (C.2):

$$\frac{\partial\mathcal{L}_{it}}{\partial K_{it}} = 0 \quad\Rightarrow\quad P_{it}^K\gamma_{it}^K = \lambda_{it}\frac{\partial Q_{it}}{\partial K_{it}}. \tag{C.12}$$

Inserting the derivative of the production function with respect to labor (see Equation (C.6)) into Equation (C.10) yields:

$$L_{it} = \left(\frac{\lambda_{it}(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha_i^L\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^L)\,a_{it}^L}K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}L_{it}^{\frac{-1}{\sigma}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{1}{\varepsilon^L}} \tag{C.13}$$

$$= \left(\frac{\lambda_{it}(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha_i^L\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^L)\,a_{it}^L}K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}. \tag{C.14}$$

Deriving the same expression for intermediates and combining it with Equation (C.14) yields:

$$\frac{L_{it}}{M_{it}} = \frac{\left(\frac{\lambda_{it}(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha_i^L\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^L)a_{it}^L}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(\frac{\lambda_{it}(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha_i^M\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^M)a_{it}^M}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}} \frac{\left(K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}}. \tag{C.15}$$

The derivative of the production function with respect to capital is:

$$\frac{\partial Q_{it}}{\partial K_{it}} = (1-\kappa)\frac{Q_{it}}{K_{it}}. \tag{C.16}$$

Inserting this expression into Equation (C.12) allows us to write capital demand as:

$$K_{it} = \left(\frac{\lambda_{it}(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)Q_{it}. \tag{C.17}$$

Inserting this capital demand equation into Equation (C.15) yields Equation (8) from the main text:

$$\frac{L_{it}}{M_{it}} = \frac{\left(\frac{\lambda_{it}(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha_i^L\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^L)a_{it}^L}\left(\frac{\lambda_{it}(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)^{\frac{(1-\kappa)(\sigma-1)}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(\frac{\lambda_{it}(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha_i^M\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^M)a_{it}^M}\left(\frac{\lambda_{it}(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)^{\frac{(1-\kappa)(\sigma-1)}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}} \frac{\left(Q_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(Q_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}} \tag{C.18}$$

$$= \varrho_{it}\lambda_{it}^{\frac{\sigma+\kappa-1}{\kappa}\left(\frac{1}{\sigma\varepsilon^L+1}-\frac{1}{\sigma\varepsilon^M+1}\right)}Q_{it}^{\frac{(1-\kappa)(\sigma-1)+\sigma\kappa-\sigma+1}{\sigma\kappa}\left(\frac{\sigma}{(\sigma\varepsilon^L+1)}-\frac{\sigma}{(\sigma\varepsilon^M+1)}\right)} \tag{C.19}$$

$$= \varrho_{it}\lambda_{it}^{\left(\frac{\sigma+\kappa-1}{\kappa(\sigma\varepsilon^L+1)}-\frac{\sigma+\kappa-1}{\kappa(\sigma\varepsilon^M+1)}\right)}Q_{it}^{\left(\frac{1}{\sigma\varepsilon^L+1}-\frac{1}{\sigma\varepsilon^M+1}\right)}, \tag{C.20}$$

where $\varrho_{it} = \frac{\left(\frac{(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha_i^L\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^L)a_{it}^L}\left(\frac{(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)^{\frac{(1-\kappa)(\sigma-1)}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(\frac{(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha_i^M\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^M)a_{it}^M}\left(\frac{(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)^{\frac{(1-\kappa)(\sigma-1)}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}}$ is a function of parameters and markdowns only.

C.3 Deriving the Substitution Elasticity of the Main Text without Imposing Functional Form Assumptions on the Production Function

We focus on how firm growth induces a shift from labor to intermediates. In our setting, growing firms move along the supply curves of labor and intermediate inputs, which affects relative input (shadow) prices. The shift from labor to intermediates is governed by the elasticity of relative input quantities with respect to relative shadow price changes. We can write this elasticity formally as:

$$\sigma_{it}^{REL} = \frac{\Delta\ln(\frac{M_{it}}{L_{it}})}{\Delta\ln(\frac{P_{it}^L\gamma_{it}^L}{P_{it}^M\gamma_{it}^M})}. \tag{C.21}$$

Assuming that firms' minimize cost, it holds that $P_{it}^X = \frac{P_{it}Q_{it}}{X_{it}} \frac{\theta_{it}^X}{\mu_{it}\gamma_{it}^X}$, where γ_{it}^X, μ_{it}, and $P_{it}Q_{it}$ are the input wedge, markup, and sales. Using this, we can rewrite Equation (C.21) as:

$$\sigma_{it}^{REL} = \frac{\Delta \ln(\frac{M_{it}}{L_{it}})}{\Delta \ln\left(\frac{\theta_{it}^L}{L_{it}} / \frac{\theta_{it}^M}{M_{it}}\right)} \quad \Leftrightarrow \quad \frac{\sigma_{it}^{REL}-1}{\sigma_{it}^{REL}} \frac{\Delta \ln(\theta_{it}^L) - \Delta \ln(\theta_{it}^M)}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}. \tag{C.22}$$

Equation (C.22) is identical to the substitution elasticity that we use in the main part of the paper (Equation (3)), and we derived it here without any assumptions on the functional form of the production function. This equation measures the elasticity of substitution as the responsiveness of input ratios to changes in input shadow price ratios. It provides an intuitive measure of substitution in our context, as firms move along the input supply curves for labor and intermediates (possibly shifting two input prices at the same time), and shadow prices reflect the relevant price ratios for firms' optimization decisions.

C.4 Non-homothetic CES Production Function: Derivations

Notation follows the main text. To derive Equation (20), we follow the same derivation steps as for Equations (C.6) and (C.7) in Appendix C.1. Specifically, the ratio of marginal products from the non-homothetic production function in Equation (19) from the main text is written:

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} (\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}} \Lambda_i^{LM} \alpha_i^L \kappa L_{it}^{\frac{-1}{\sigma}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}}{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} (\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}} \Lambda_i^{LM} \alpha_i^M \kappa \left(\frac{M_{it}}{Q_{it}^{\eta}}\right)^{\frac{-1}{\sigma}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}} = \frac{\alpha_i^L}{\alpha_i^M} \left(\frac{L_{it}}{M_{it}}\right)^{\frac{-1}{\sigma_{it}}} Q_{it}^{\frac{-\eta}{\sigma}}. \tag{C.23}$$

Multiplying this expression by $\frac{L_{it}}{Q_{it}} / \frac{M_{it}}{Q_{it}}$ yields Equation (20) of the main text:

$$\frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha_i^L}{\alpha_i^M} \left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}} Q_{it}^{\frac{-\eta}{\sigma}}. \tag{C.24}$$

C.5 Markups and Wage Markdowns

The notation follows the main text. We now show how we derive the equations for firms' markups and wage markdowns in terms of observables and output elasticities. This follows Hall (1986), De Loecker and Warzynski (2012), and Dobbelaere and Mairesse (2013). As discussed in the main text, we assume that intermediates are supplied perfectly elastically and that there are no adjustment costs (as standard in the literature for this derivation). *Strictly speaking, for our firm-level analysis in changes, a weaker assumption is sufficient: we can permit intermediate input market imperfections γ_{it}^M but those must be constant over time within a firm, i.e., $\gamma_{it}^M = \gamma_i^M$* (indeed, our results provide suggestive evidence for that property). This implies the following cost-minimization problem:

$$\mathcal{L}_{it} = P_{it}^L(L_{it})L_{it} + P^M M_{it} + P^K K_{it} - \lambda_{it}(Q_{it} - Q_{it}(.)). \tag{C.25}$$

where, for simplicity, we also abstract from capital market imperfections, and where $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$. The first-order conditions for labor and intermediates are:

$$P_{it}^M = \frac{P_{it}}{\mu_{it}} \frac{\partial Q_{it}}{\partial M_{it}} \tag{C.26}$$

$$P_{it}^L \gamma_{it}^L = \frac{P_{it}}{\mu_{it}} \frac{\partial Q_{it}}{\partial M_{it}}. \tag{C.27}$$

Rearranging Equation (C.26) yields an expression for the markup that we can identify in the data on the basis of the intermediate output share and output elasticity:

$$\mu_{it} = \frac{P_{it}}{MC_{it}} = \theta_{it}^{M} \frac{P_{it}Q_{it}}{P_{it}^{M} M_{it}}. \tag{C.28}$$

Combining Equations (C.27) and (C.26) recovers an expression for the wage markdown:

$$\gamma_{it}^{L} = \frac{\theta_{it}^{L}}{\theta_{it}^{M}} \frac{P_{it}^{M} M_{it}}{P_{it}^{L} L_{it}}. \tag{C.29}$$

Rearranging Equation (C.27) also yields the equation for the measure of combined distortions from markup and labor market imperfections:

$$\mu_{it}\gamma_{it}^{L} = \theta_{it}^{L} \frac{P_{it}Q_{it}}{P_{it}^{L} L_{it}}. \tag{C.30}$$

C.6 Fixed-Cost Model

We now provide a simple assessment of how fixed costs could account for our findings. We do so in a simple model in which a portion of the labor is fixed. To assess the capacity of fixed costs to explain our findings, we compute the latent fixed labor cost share that would be required to explain our results without labor-intermediate substitution and imperfect input markets.

To assume away the role of intermediate-labor input substitutability, we rely on a Cobb-Douglas production function (i.e., we impose substitution elasticity of one). Hence, this model variant shuts down our mechanism entirely.

Firms produce output using labor, intermediates, and capital. Now, total labor L_{it} consists of fixed overhead labor L_{it}^{F} (where the time index in principle permits non-constant amounts, but the key point is that it is not a choice variable) and variable production labor $L_{it} - L_{it}^{F}$. The production function is:

$$Q_{it} = (L_{it} - L_{it}^{F})^{\Phi^{L}} M_{it}^{\Phi^{M}} K_{it}^{\Phi^{K}}, \tag{C.31}$$

Differentiating Q_{it} with respect to L_{it} permits us to construct the reduced-form output elasticity to *total* labor:

$$\underbrace{\frac{dQ_{it}}{Q_{it}} \frac{L_{it}}{dL_{it}}}_{=\theta_{it}^{L}} = \Phi^{L} \frac{L_{it}}{L_{it} - L_{it}^{F}}. \tag{C.32}$$

Hence, the output elasticity of total labor—the object we study in the main text—can vary due to the presence of fixed labor costs, even though the structural output elasticity of variable production labor is constant at Φ^{L}.

In the fixed-cost model, the fixed portion of labor governs the relationship between firm growth and the output elasticity of total labor:

$$\frac{d\theta_{it}^L}{\theta_{it}^L} = - \underbrace{\frac{L_{it}^F}{L_{it} - L_{it}^F}}_{\substack{\text{Fixed labor to} \\ \text{variable labor ratio}}} \frac{dL_{it}}{L_{it}} \tag{C.33}$$

$$\Leftrightarrow \quad \underbrace{\frac{L_{it}^F}{L_{it}}}_{\substack{\text{Share of fixed} \\ \text{labor in total labor}}} = \frac{-\frac{d\theta_{it}^L}{\theta_{it}^L} \Big/ \frac{dL_{it}}{L_{it}}}{1 - \frac{d\theta_{it}^L}{\theta_{it}^L} \Big/ \frac{dL_{it}}{L_{it}}}. \tag{C.34}$$

We now insert the coefficients from Tables 2 and 3 for $\frac{d\theta_{it}^L}{\theta_{it}^L}$ and $\frac{dL_{it}}{L_{it}}$, as they capture the average change in labor output elasticities (θ_{it}^L) and labor quantities in response to output growth, into Equations (C.33) and (C.34). This allows us to infer the fixed labor share required to fully explain our findings without relying on factor substitution and imperfect input markets. For 1-year changes, the OLS (IV) estimates suggest a required fixed labor to variable labor ratio of 1.02 (1.01), and for a 4-year horizon, the ratio is 0.40 (0.45). These values imply a portion of fixed labor in total labor of 0.51 (0.50) for the one-year horizon and 0.28 (0.31) at the four-year horizon.

In addition, the nature of fixed input requirements—strictly speaking, not adjusting in the short run and remaining constant across firm sizes—suggest that the shrinking output elasticity of labor with firm growth should be strongly sized-based.[59] However, we do not find evidence supporting this notion; instead, we document stable effects of firm growth on inputs and output elasticities by firm size classes (Table A.7).

Finally, fixed costs cannot explain the shift from the output elasticity of labor to the output elasticity of intermediates. In the above model, the intermediate input output elasticity is fixed at Φ^M:

$$\theta_{it}^M = \Phi^M \tag{C.35}$$

$$\frac{d\theta_{it}^M}{\theta_{it}^M} = 0. \tag{C.36}$$

Our findings that intermediate input output elasticities increase and absorb the labor output elasticity declines contradict this prediction of a fixed-cost model, whereas our proposed substitution mechanism can fully account for our findings.[60] (And if anything, fixed costs in intermediates, too, would predict a decline rather than, as we find, an increase in output elasticities of intermediates.)

We caveat that our simple fixed-cost setup may not capture richer versions of fixed costs, including those that may scale with firm size, multiple fixed costs across products or business activities/markets/export status (although we do not find striking heterogeneity across industries as shown in Figure A.2) or fixed costs in other factors, but we believe that our findings above suggest that fixed costs are unlikely to be a parsimonious explanation of the full set of our findings. However, we do not aim to definitively adjudicate between our mechanism and the fixed-cost view.

[59] To see this, fix L^F and let L grow, such that the output elasticity becomes constant and converges to Φ^L.

[60] Of course, *cost shares* would shift towards intermediates, but we directly study output elasticities as well.

D Further Details on the German Firm-Product Level data

Table D.1: Variable definition in the German micro data.

Variable	Definition
L_{it}	Labor in headcounts.
P_{it}^{L}	Firm wage (firm average), defined as gross salary before taxes (including mandatory social costs) + "other social expenses" (including expenditures for company outings, advanced training, and similar costs) divided by the number of employees.
K_{it}	Capital (including tangible and intangible assets) derived by a perpetual inventory method as described below. Intangibles include, software, patents, licenses, brand/trademark value, and similar items.
M_{it}	Deflated total intermediate input expenditures, defined as expenditures for raw materials, energy, intermediate services, goods for resale, renting, repairs, and contracted work conducted by other firms.
E_{it}	Deflated expenditures for raw, auxiliary, and operating materials and energy inputs (includes external product components). E_{it} is part of M_{it}.
$Merch_{it}$	Deflated expenditures for merchandise. $Merch_{it}$ is part of M_{it}.
Sub_{it}	Deflated expenditures for subcontracted production performed by other companies. Sub_{it} is part of M_{it}.
Rep_{it}	Deflated expenditures for repairs, maintenance, installation, and assembly. Rep_{it} is part of M_{it}.
$Temp_{it}$	Deflated expenditures for temporary agency workers. $Temp_{it}$ is part of M_{it}.
$Rent_{it}$	Deflated expenditures for rent, leases, leasing. Rep_{it} is part of M_{it}.
$Other_{it}$	Deflated expenditures for Other intermediate costs (insurance, postage, transport, etc.). $Other_{it}$ is part of M_{it}.
$P_{it}^{M} M_{it}$	Nominal values of total intermediate input expenditures.
$P_{it}Q_{it}$	Nominal total revenue, defined as total gross output, including, among others, sales from own products, sales from intermediate goods, revenue from offered services, and revenue from commissions/brokerage.
Q_{it}	Quasi-quantity measure of physical output, i.e., $P_{it}Q_{it}$ deflated by a firm-specific price index (denoted by PI_{it}).
PI_{it}	Firm-specific Törnqvist price index, derived as in Eslava et al., 2004. See Appendix E.1 for its construction.
P_{igt}	Price of a product g.
$share_{igt}$	Revenue share of a product g in total firm revenue.
ms_{it}	Weighted average of firms' product market shares in terms of revenues. The weights are the sales of each product in firms' total product market sales.
G_{it}	Headquarter location of the firm. 90% of firms in our sample are single-plant firms.
D_{it}	A four-digit industry indicator variable. The industry of each firm is defined as the industry in which the firm generates most of its sales.
Exp_{it}	Dummy-variable being one, if firms generate export market sales.
$NumP_{it}$	The number of products a firm produces.

Notes: The table lists all variables and variable definitions used in the paper. Nominal values are deflated by a 2-digit industry-level deflator for intermediate inputs which is supplied by the Federal Statistical Office of Germany.

Data access. The data can be accessed at the Research Data Centres of the Federal Statistical Office of Germany and the Statistical Offices of the German Länder (states). Data request can be made at: https://www.forschungsdatenzentrum.de/en/request. The statistics we used are: "AFiD-Modul Produkte," "AFiD-Panel Industriebetriebe," "AFiD-Panel Industrieunternehmen," "Investitionserhebung im Bereich Verarbeitendes Gewerbe, Bergbau und Gewinnung von Steinen und Erden," "Panel der Kostenstrukturerhebung im Bereich Verarbeitendes Gewerbe, Bergbau und Gewinnung von Steinen und Erden." The data are combined by the statistical offices and provided as a merged dataset.

Variable definitions. Table D.1 presents an overview of the variable definitions of all variables used in this article. This includes variables used in other sections of the appendix.

Outlier cleaning. We exclude the top and bottom two percent outliers with respect to value added over revenue and revenue over labor, capital, intermediate input expenditures, and labor costs. We replace quantity and price information with missing values for products displaying a price deviation from the average price in the top and bottom one percent. We also drop the sectors 16 (tobacco), 23 (mineral oil and coke), and 37 (recycling) as the observation count is insufficient to derive estimates of firms' production functions in these industries.

Capital stock estimation. As capital stocks are not directly observed, we calculate a time series of capital stocks for every firm using the perpetual inventory method of Bräuer et al. (2023):

$$K_{it} = K_{it-1}(1 - depr_{jt-1}) + I_{it-1}, \tag{D.1}$$

where K_{it}, $depr_{jt-1}$ and I_{it-1} denote firm i's capital stock, the depreciation rate of capital, and investment. Investment captures firms' total investment in buildings, equipment, machines, and other investment goods (including intangible assets). Nominal values are deflated by a two-digit industry-level deflator supplied by the German Statistical Office.

We derive the industry- and year-specific depreciation rate from official information on the expected lifetime of capital goods (supplied by the statistical offices). To do so, we define the lifetime of a capital good LT as a function of its depreciation rate:

$$LT = depr \int_0^\infty (1 - depr)^t t dt. \tag{D.2}$$

Using partial integration gives:

$$LT = depr \left[\frac{(1 - depr)^t}{\ln(1 - depr)} t \right]_0^\infty - depr \int_0^\infty \frac{(1 - depr)^t}{\ln(1 - depr)} dt, \tag{D.3}$$

where the first term on the right-hand side equals zero because $0 < depr < 1$. Integrating the remaining expression yields:

$$LT = \frac{depr}{\ln(1 - depr) \times \ln(1 - depr)}, \tag{D.4}$$

which we can numerically solve for $depr$. As the lifetime of capital goods is separately given for years and capital good types (buildings and equipment), we derive a depreciation rate for each year and capital good type separately. To derive a single industry-specific depreciation rate, we weight the depreciation rates for buildings and equipment respectively with the industry-level share of building capital in total capital and equipment capital in total capital (this information is supplied by the statistical offices). For the practical implementation, we assume that the depreciation rate of a firm's whole capital stock equals the depreciation rate of newly purchased capital.

The initial capital stock for the perpetual inventory method is derived from reported tax depreciation. We do not use the reported tax depreciation when calculating capital stock series as tax depreciation may vary due to state-induced tax incentives and might therefore not reliably reflect the true amount of depreciated capital. Given that firms likely report too high depreciation levels due to such tax incentives, our first capital values within a capital series are likely overestimated. However, over time, observed investment decisions gradually receive a larger weight in estimated capital stocks, mitigating the impact of the first capital stock. Given that we estimate very reasonable output elasticities (see Table A.1), we are confident that our capital variables reliably reflect firms' true capital stocks.[61]

Deriving a time-consistent industry classification. During our long time-series, the NACE classification of industries (and thus firms into industries) changed twice. Once in 2002 and once in 2008. Because our estimation of the production function requires a time-consistent industry classification at the firm level (as we allow for industry-specific production functions), it is crucial to recover a time-consistent NACE industry classification. Recovering such a time-consistent industry classification from official concordance tables is, however, problematic as they contain many ambiguous sector reclassifications. To address this issue, we follow the procedure described in Mertens (2022) and use information on firms' product mix to classify firms into NACE Rev 1.1 industries based on their main production activities. This procedure exploits the fact that the first four digits of the ten-digit GP product classification reported in the German data are identical to the NACE classification (i.e., they indicate the industry of the product). Applying this method demands a consistent reclassification of all products into the GP2002 scheme (which corresponds to the NACE Rev 1.1 scheme). Reclassifying products is, due to the granularity of the ten-digit classification, less ambiguous than reclassifying industries. In the few ambiguous cases, we can follow the firms' product mix over the reclassification periods and unambiguously reclassify most products (i.e., we observe what firms produce before and after reclassification years). Having constructed a time-consistent product-industry classification according to the GP2002 scheme, we attribute every firm to the NACE Rev 1.1 industry in which it generates most of its revenue. When comparing the classification with the one of the statistical offices for the years 2002-2008 (years in which industries are already reported in NACE Rev 1.1), Mertens (2022) finds that this two-digit and four-digit classification of firms into industries matches the classification of the statistical offices in 95% and 86% of all cases, respectively.

[61] As firms likely tend to overstate their capital depreciation, our capital stocks are likely a closer approximation of the true capital stock used in firms' production processes than capital measures based on book values.

E Production Function Estimation in the German Data

We follow Mertens (2022) in estimating the production function. This approach yields firm- and time-specific output elasticities by assuming the following translog production function (throughout, lower case letter denote logs):

$$\tilde{q}_{it} = \boldsymbol{\phi}_{it}' \boldsymbol{\beta} + \omega_{it} + \epsilon_{it} \, . \tag{E.1}$$

$\tilde{q}_{it}$ denotes the log of produced quantities measured as sales deflated by a firm-specific input price index as described below and ϕ_{it}' captures the production inputs capital (k_{it}), labor (l_{it}), and intermediates (m_{it}) and its interactions. The industry-specific production function that we will estimate for each two-digit Nace rev. 1.1 industry is specified in logs as:

$$\begin{aligned} \tilde{q}_{it} =& \beta_l l_{it} + \beta_m m_{it} + \beta_k k_{it} + \beta_{ll} l_{it}^2 + \beta_{mm} m_{it}^2 + \beta_{kk} k_{it}^2 \\ &+ \beta_{lk} l_{it} k_{it} + \beta_{lm} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \beta_{lkm} l_{it} k_{it} m_{it} + \omega_{it} + \epsilon_{it}. \end{aligned} \tag{E.2}$$

The output elasticity of labor is

$$\frac{\partial \tilde{q}_{it}}{\partial l_{it}} = \beta_l + 2\beta_{ll} l_{it} + \beta_{lm} m_{it} + \beta_{lk} k_{it} + \beta_{lkm} k_{it} m_{it} \, . \tag{E.3}$$

ϵ_{it} is an i.i.d. error term and ω_{it} denotes Hicks-neutral productivity and follows a Markov process. ω_{it} is unobserved to the econometrician, yet firms know ω_{it} before making input decisions for flexible inputs (intermediates in our case). We assume that only firms' input decision for intermediates depends on productivity shocks. Labor and capital do not respond to contemporary productivity shocks. However, our results are similar when allowing labor to respond to productivity innovations. In fact, the CompNet routine for the production function estimation models labor as flexible and we find consistent results in both datasets. In Equation (E.2), we explicitly include labor quantities rather than labor expenditure, which is preferable in case of imperfect labor markets. Intermediates and capital enter as deflated monetary values, and we account for input price variation through the input price control function as described below.

There are three issues preventing us from estimating the production function in Equation (E.1) using OLS:

(1) We need to estimate a physical production model to recover the relevant output elasticities. Although we observe product quantities, quantities cannot be aggregated across the various products of multi-product firms. Relying on the standard practice to apply sector-specific output deflators does not solve this issue if output prices vary within industries. While we address this bias below, we note that Yeh et al. (2022) argue it does not affect ratios of output elasticities, since the elasticities themselves are biased to the same extent.

(2) We do not observe firm-specific input prices for capital and intermediate inputs. If input prices are correlated with input decisions and output levels, an endogeneity issue arises. Unlike the previous identification issue, this bias affects output elasticities differentially.

(3) The fact that productivity is unobserved and that firms' flexible input decisions depend on productivity shocks, creates another endogeneity problem.

We now discuss how we solve these three identification problems.

E.1 Solving Challenge (1) by Deriving a Firm-specific Output Price Index

As we cannot aggregate output quantities across different products of a firm (a common problem), we follow Eslava et al. (2004) and construct a firm-specific price index from observed output prices. We use this price index to purge observed firm revenue from price variation by deflating firm revenues with this price index.[62] We construct firm-specific Törnqvist price indices for each firm's composite revenue from its various products in the following way:

$$PI_{it} = \prod_{g=1}^{n} \frac{p_{igt}}{p_{igt-1}}^{1/2(share_{igt}+share_{igt-1})} PI_{it-1} \,. \tag{E.4}$$

PI_{it} is the price index, p_{igt} is the firm-specific price of good, g, that we observe in the data, and $share_{igt}$ is the share of this good in total product market sales of firm i in period t. The growth of the index value is the product of the individual products' price growths, weighted with the average sales share of that product over the current and the last year. The first year available in the data is the base year (i.e., $PI_{it=1995} = 100$). If firms enter after 1995, we follow Eslava et al. (2004) and use an industry average of the computed firm price indices as a starting value. Similarly, we impute missing product price growth information in other cases with an average of product price changes within the same industry.[63] After deflating firm revenue with this price index, we have a quasi-quantity measure of output, which we highlight with a tilde: $\tilde{q}_{it}$.[64]

E.2 Solving Challenge (2) by Accounting for Unobserved Input Price Variation

While the recent literature stresses the so-called "output-price bias" when estimating production functions, previous work has also highlighted that unobserved input prices introduce another identification problem. To control for input price variation across firms, we use a firm-level analog of De Loecker et al. (2016) and define a price-control function from firm-product-level output price information that we add to the production function in Equation (E.1):

$$\tilde{q}_{it} = \boldsymbol{\phi}'_{it}\boldsymbol{\beta} + B_{it}((pi_{it}, ms_{it}, G_{it}, D_{it}) \times \boldsymbol{\phi}^c_{it}) + \omega_{it} + \epsilon_{it} \,. \tag{E.5}$$

$B_{it}(.) = B_{it}((pi_{it}, ms_{it}, G_{it}, D_{it}) \times \boldsymbol{\phi}^c_{it})$ is the price control function consisting of our logged firm-specific output price index (pi_{it}), a logged sales-weighted average of firms' product market sales shares (ms_{it}), a headquarter location dummy (G_{it}) and a four-digit industry dummy (D_{it}). $\boldsymbol{\phi}^c_{it} = [1; \boldsymbol{\phi}_{it}]$, where $\boldsymbol{\phi}_{it}$ includes the production function input terms as specified in Equation (E.2). These are either in monetary terms and deflated by an industry-level deflator (capital and intermediates) or already reported in quantities (labor). The constant entering $\boldsymbol{\phi}^c_{it}$ highlights that elements of $B(.)$ enter the price control function linearly and interacted with $\boldsymbol{\phi}_{it}$ (a consequence of the translog production function). The idea behind the price-control function $B(.)$ is that output prices, product market shares, firm location, and firms' industry affiliation are informative about firms' input prices. Particularly, we assume that product prices and market shares contain information about product quality and that producing high-quality products requires expensive high-quality inputs. As De Loecker et al. (2016) discuss, this reasoning motivates the addition of a control function containing output price and market share information to the right-hand side of the production function to control for unobserved input price variation emerging from input quality differences across firms. We also include year, location,

[62] This approach has also been applied in various other studies, such as Smeets and Warzynski (2013).

[63] For roughly 30% of all product observations in the data, firms do not have to report quantities as the statistical office views them as not being meaningful.

[64] Note that, as discussed in Bond et al. (2021), using an output price index does not fully purge firm-specific price variation. There remains a base year difference in prices. Yet, using a firm-specific price index follows the usual practice of using price indices to deflate nominal values, we are thus following the best practice. Moreover, it is the only available approach when pooling multi- and single-product firms. Estimating the production function separately by single-plant firms requires other strong assumptions like perfect input divisibility of all inputs across all products. Finally, our results are also robust to using cost-share approaches to estimate the production function, which requires other assumptions (constant returns to scale, competitive input markets, and the absence of adjustment costs).

and four-digit industry dummies into $B(.)$ to further absorb the remaining differences in local and four-digit industry-specific input prices.

Conditional on elements in $B(.)$, we assume that there are no remaining input price differences across firms. Although restrictive, this assumption is more general than the ones employed in most other studies estimating production functions without having access to firm-specific price data and which implicitly assume that firms face identical input and output prices within industries.

A notable difference between the original approach of De Loecker et al. (2016) and our version is that they estimate product-level production functions, whereas we transfer their framework to the firm level. For that, we use firm-product-specific sales shares in firms' total product market sales to aggregate firm-product-level information to the firm-level. This implicitly assumes that i) such firm aggregates of product quality increase in firm aggregates of product prices and input quality, ii) firm-level input costs for inputs entering as deflated expenditures increase in firm-level input quality, and iii) product price elasticities are equal across the various products of a firm. These or even stricter assumptions are always implicitly invoked when estimating firm-level production functions.

Finally, note that even if some of the above assumptions do not hold, including the price control function is still preferable to omitting it. This is because the price control function can nevertheless absorb some of the unobserved price variations and does not require that input prices vary between firms with respect to all elements of $B_{it}(.)$. The estimation can regularly result in coefficients implying that there is no price variation at all. The attractiveness of a price control function lies in its agnostic view about the existence and degree of input price variation.

E.3 Solving Challenge (3) by Controlling for Unobserved Productivity

To address the dependence of firms' intermediate input decisions on unobserved productivity, we follow Olley and Pakes (1996) and Levinsohn and Petrin (2003) and employ a control function approach. We base our control function on firms' consumption of energy and materials, which we denote by e_{it} and which are components of total intermediate inputs. Inverting the demand function for e_{it} defines an expression for productivity:

$$\omega_{it} \equiv g_{it}(.) = g_{it}(e_{it}, k_{it}, l_{it}, \mathbf{\Gamma}_{it}). \tag{E.6}$$

$\mathbf{\Gamma}_{it}$ captures state variables of the firm, that in addition to k_{it} and l_{it} affect firms' demand for e_{it}. Ideally, $\mathbf{\Gamma}_{it}$ should include a wide set of variables affecting productivity and demand for e_{it}. We include dummy variables for export (EX_{it}) activities, the log of the number of products a firm produces ($NumP_{it}$), and the average wage a firm pays (P^L_{it}) into $\mathbf{\Gamma}_{it}$. The latter absorbs unobserved quality and price differences that shift input demand for e_{it}. Including wages also helps to absorb variation in marginal costs and wage markdowns, which addresses some of the critique raised by Doraszelski and Jaumandreu (2021) because wages are a serially correlated input price that is also correlated with other input prices. Remember that productivity follows a first-order Markov process. Firms can shift this Markov process as described in Doraszelski and Jaumandreu (2013) and De Loecker (2013), giving rise to the following law of motion for productivity: $\omega_{it} = h_{it}(\omega_{it-1}, \mathbf{T}_{it-1}) + \xi_{it} = h_{it}(.) + \xi_{it}$, where ξ_{it} denotes the innovation in productivity and $\mathbf{T}_{it} = (EX_{it}, NumP_{it})$ reflects the fact that we allow for learning effects from export market participation and (dis)economies of scope through adding and dropping products to influence firm productivity.[65] Plugging Equation (E.6) and the law of motion for productivity into Equation (E.5) gives

$$\tilde{q}_{it} = \boldsymbol{\phi}'_{it}\boldsymbol{\beta} + B_{it}(.) + h_{it}(.) + \epsilon_{it} + \xi_{it}\,, \tag{E.7}$$

[65]Doraszelski and Jaumandreu (2013) also highlight the role of R&D investment in shifting firms' productivity process. We would also like to add this information to the productivity model but do not observe R&D expenditures for the early years in our data.

which constitutes the basis of our estimation.

E.4 Identifying Moments

We estimate Equation (E.7) separately by two-digit NACE Rev. 1.1 industries using a one-step estimator as in Wooldridge (2009).[66] Our estimator uses lagged values of flexible inputs (i.e., intermediates) as instruments for their contemporary values to address the dependence of firms' flexible input decisions on realizations of ξ_{it}. Similarly, we use lagged values of terms including firms' market share and output price index as instruments for their contemporary values as we consider these to be flexible variables.[67] We define identifying moments jointly on ϵ_{it} and ξ_{it}:

$$E[(\epsilon_{it} + \xi_{it})\mathbf{Y}_{it}] = 0. \tag{E.8}$$

$\mathbf{Y}_{it}$ includes lagged interactions of intermediate inputs with labor and capital, contemporary interactions of labor and capital, contemporary location and industry dummies, the lagged output price index, lagged market shares, lagged elements of $h_{it}(.)$, and lagged interactions of the output price index with production inputs. Formally this implies:

$$\mathbf{Y}'_{it} = (J_{it}(.), A_{it-1}(.), T_{it-1}(.), \Psi_{it}(.), \boldsymbol{\vartheta}_{it-1}) \ , \tag{E.9}$$

where we defined:

- $J_{it}(.) = (l_{it},\, k_{it},\, l_{it}^2,\, k_{it}^2,\, l_{it}k_{it},\, G_{it},\, D_{it})$,
- $A_{it}(.) = (m_{it},\, m_{it}^2,\, l_{it}m_{it},\, k_{it}m_{it},\, l_{it}k_{it}m_{it},\, ms_{it},\, \pi_{it})$,
- $T_{it}(.) = \big((l_{it},\, k_{it},\, l_{it}^2,\, k_{it}^2,\, l_{it}k_{it},\, m_{it},\, m_{it}^2,\, l_{it}m_{it},\, k_{it}m_{it},\, l_{it}k_{it}m_{it}) \times \pi_{it}\big)$,
- $\Psi_{it}(.) = \sum_{n=0}^{3} \sum_{w=0}^{3-b} \sum_{h=0}^{3-n-b} \, l_{it-1}^n \, k_{it-1}^b \, e_{it-1}^h$, and
- $\boldsymbol{\vartheta}_{it-1} = (Exp_{it-1},\, NumP_{it-1},\, P_{it-1}^L)$,
- with P_{it}^L denoting the average wage a firm pays.

E.5 Results

Table E.2 reports the industry-specific coefficients of the translog production function (due to space constraints, we do not report coefficients on the control functions). In Table, A.1 we additionally report output elasticities derived from these coefficients and firms' input decisions. Average labor, intermediate, and capital output elasticities equal 0.31 (0.11), 0.64 (0.12), and 0.12 (0.10) with standard errors in parentheses.

E.6 Limitations

Our approach addresses biases stemming from input and output prices, which frequently affect existing production function estimates. Nonetheless, we acknowledge the remaining inherent limitations

[66] We approximate $h_{it}(.)$ by a third-order polynomial in all of its elements, except for the variables in $\boldsymbol{\Gamma}_{it}$. Those we add linearly. $B_{it}(.)$ is approximated by a flexible polynomial where we interact the output price index with elements in $\boldsymbol{\phi}_{it}$ and add the vector of market shares, the output price index, and the location and industry dummies linearly. Interacting further elements of $B_{it}(.)$ with $\boldsymbol{\phi}_{it}$ creates too many parameters to be estimated. This implementation is similar to De Loecker et al. (2016).

[67] These timing assumptions also address any simultaneity concerns with respect to the price variables entering the right-hand side of our estimation.

Table E.2: Estimated production coefficients, by industries.

	β_l	β_m	β_k	β_{ll}	β_{mm}	β_{kk}	β_{lk}	β_{lm}	β_{km}	β_{lmk}
Industry	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Food, beverages	0.355	-0.382**	-0.006	0.042***	0.065***	0.016***	0.058	-0.073*	-0.036***	-0.001
	(0.591)	(0.189)	(0.187)	(0.007)	(0.004)	(0.004)	(0.036)	(0.038)	(0.012)	(0.187)
Textiles	1.162	-0.704**	-0.694**	0.081***	0.073***	0.036***	0.030	-0.116	-0.020	-0.001
	(1.202)	(0.295)	(0.331)	(0.017)	(0.008)	(0.007)	(0.068)	(0.082)	(0.021)	(0.331)
Apparel, dressing	2.797**	0.344	0.669	0.046**	0.065***	0.023**	-0.106	-0.208**	-0.085***	0.008*
	(1.202)	(0.383)	(0.408)	(0.020)	(0.008)	(0.010)	(0.071)	(0.085)	(0.025)	(0.408)
Leather	-0.199	-1.368**	-0.617	0.139***	0.085***	0.010	0.146	-0.088	0.018	-0.007
	(2.215)	(0.693)	(0.833)	(0.034)	(0.014)	(0.018)	(0.151)	(0.140)	(0.040)	(0.833)
Wood	-0.773	-0.581*	-0.604*	0.088***	0.058***	0.024***	0.121*	-0.005	-0.008	-0.006
	(1.181)	(0.313)	(0.321)	(0.016)	(0.006)	(0.007)	(0.072)	(0.074)	(0.019)	(0.321)
Pulp, paper	1.586	-0.497	0.264	0.067***	0.096***	0.015**	0.056	-0.206**	-0.067**	0.002
	(1.304)	(0.404)	(0.426)	(0.021)	(0.011)	(0.007)	(0.071)	(0.089)	(0.030)	(0.426)
Publishing, printing	-0.0391	-0.880**	0.126	0.059***	0.083***	0.006	0.108	-0.087	-0.036	-0.003
	(1.356)	(0.428)	(0.400)	(0.018)	(0.009)	(0.006)	(0.084)	(0.085)	(0.025)	(0.400)
Chemicals	-0.983	-0.614***	-0.243	0.020	0.063***	0.016**	0.136***	0.014	-0.025*	-0.005***
	(0.615)	(0.235)	(0.210)	(0.0139)	(0.008)	(0.006)	(0.034)	(0.040)	(0.015)	(0.210)
Rubber, plastic	1.208*	-0.211	0.0724	0.058***	0.057***	0.012***	0.006	-0.126***	-0.031**	0.002
	(0.625)	(0.202)	(0.185)	(0.014)	(0.006)	(0.005)	(0.034)	(0.044)	(0.013)	(0.185)
Other non-metallic minerals	-0.556	-0.521*	-0.563**	0.048***	0.052***	0.022***	0.106*	0.009	-0.007	-0.006
	(0.942)	(0.284)	(0.270)	(0.011)	(0.007)	(0.005)	(0.055)	(0.061)	(0.017)	(0.270)
Basic Metals	-0.432	-0.658*	-0.115	0.050*	0.063***	0.007	0.114*	-0.025	-0.016	-0.004
	(1.199)	(0.380)	(0.405)	(0.028)	(0.010)	(0.008)	(0.063)	(0.084)	(0.030)	(0.405)
Fabricated metal	1.271*	-0.659***	-0.0280	0.102***	0.071***	0.010***	0.024	-0.146***	-0.020	0.000
	(0.703)	(0.214)	(0.210)	(0.011)	(0.005)	(0.004)	(0.042)	(0.047)	(0.013)	(0.210)
Machinery, equipment	0.596	-0.670***	-0.263	0.064***	0.068***	0.019***	0.058*	-0.084**	-0.023**	-0.002
	(0.531)	(0.165)	(0.165)	(0.012)	(0.004)	(0.004)	(0.030)	(0.035)	(0.011)	(0.165)
Electrical, optical equipment	1.691	-0.112	-0.705	0.137***	0.048***	0.041**	-0.033	-0.114	-0.011	-0.001
	(2.303)	(1.073)	(1.080)	(0.049)	(0.018)	(0.019)	(0.15)	(0.156)	(0.069)	(1.080)
Electrical machinery, apparatus	0.453	-0.231	-0.018	0.035**	0.059***	0.015**	0.063	-0.088*	-0.038**	-0.000
	(0.781)	(0.248)	(0.248)	(0.015)	(0.007)	(0.006)	(0.045)	(0.051)	(0.015)	(0.248)
Radio, TV, communication	2.066	-0.533	-1.357**	0.129***	0.053***	0.051***	-0.035	-0.150	0.005	0.000
	(1.639)	(0.456)	(0.552)	(0.036)	(0.013)	(0.015)	(0.088)	(0.107)	(0.031)	(0.552)
Medical precision instruments	-0.0189	-0.0313	-0.286	0.051*	0.024***	0.014	0.032	-0.002	-0.002	-0.002
	(1.014)	(0.320)	(0.347)	(0.026)	(0.007)	(0.009)	(0.061)	(0.067)	(0.021)	(0.347)
Motor vehicles, trailers	1.389*	-0.124	0.360	0.041*	0.071***	0.019**	0.009	-0.130**	-0.066***	0.002
	(0.756)	(0.281)	(0.258)	(0.023)	(0.011)	(0.008)	(0.036)	(0.056)	(0.017)	(0.258)
Transport equipment	1.583	0.137	0.885*	0.038	0.056***	-0.003	-0.021	-0.139	-0.060**	0.004
	(1.731)	(0.492)	(0.512)	(0.028)	(0.013)	(0.012)	(0.098)	(0.114)	(0.030)	(0.512)
Furniture	1.701	-0.755**	-0.325	0.103***	0.079***	0.025***	0.004	-0.168**	-0.025	0.001
	(1.267)	(0.348)	(0.362)	(0.018)	(0.006)	(0.006)	(0.077)	(0.082)	(0.022)	(0.362)

Notes: The table reports the estimated industry-specific production function coefficients from estimating Equation (E.7) using the control function approach described in Appendix E. German firm-level data.

in the general methodology for estimating production functions, which remain subjects of active research. For example, Bond et al. (2021) highlight potential biases in output elasticities if inputs directly influence demand (without observing these inputs), such as intermediates used in marketing. Similarly, aggregating production and non-production labor into a single labor measure can bias output elasticities, though this concern might be less critical in manufacturing contexts. Furthermore, incomplete specification of the productivity control function may violate the monotonicity assumption crucial for identification doraszelski2021reexamining.[68] Although we extend traditional methods by incorporating wages, export status, and the number of products into our productivity control function, important unobserved factors may still exist that cause the monotonicity assumption to fail. Due

[68] Gandhi et al. (2020) also critique the control function approach. An alternative is to estimate a dynamic panel model, as in Blundell and Bond (2000), which assumes an AR(1) productivity process.

to these concerns, we complement our parametric approach with a non-parametric identification of output elasticities based on cost shares (as detailed in the main text).

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