From Labor to Intermediates: Firm Growth, Input Substitution, and Monopsony

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We document and dissect a stylized fact about firm growth: the shift from labor to intermediate inputs. This shift occurs in input quantities, cost and output shares, and output elasticities. We establish this regularity in firm data for Germany and in firm (and industry) data for 11 (20) additional countries, and also in response to exogenous product demand shocks. We explain this regularity through a parsimonious model with two features: (i) an elasticity of substitution between intermediates and labor above one, and (ii) an increasing shadow price of labor (monopsony or adjustment costs). Our firm growth regressions identify a labor-intermediates substitution elasticity between 1.8 and 4.2. Labor-intermediates substitution also accounts for much of the labor share decline that we document accompanies firm and industry growth.

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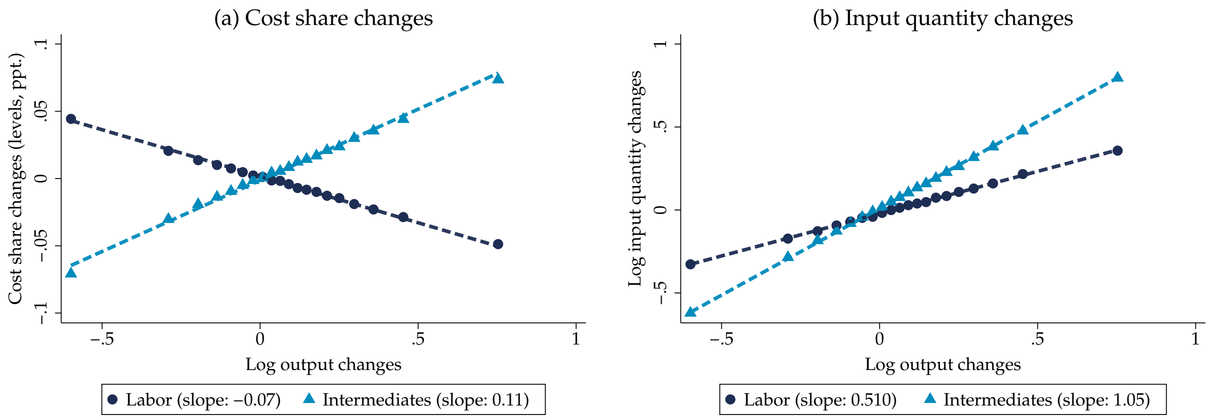
Figure 1: The shift from labor to intermediate inputs: within-firm 4-year changes in cost shares and log input quantities against log output growth.



Notes: The figure reports binned scatter plots of the within-firm changes in labor and intermediate cost shares and log input quantities (labor in headcounts, intermediates in deflated expenditures) against log output changes (deflated sales). All panels depict 4-year differences and control for industry-year fixed effects. The data cover German manufacturing firms from 1995 to 2017; further details on the datasets, samples, and empirical specifications are discussed in the main part of the paper.

1 Introduction

How do firms grow? Standard Cobb-Douglas production and competitive input markets predict that firms simply scale up inputs proportionately, with constant cost shares. Our paper documents, dissects, and rationalizes a systematic departure from this canonical benchmark: as firms grow, they shift from labor to intermediate inputs—as illustrated in Figure 1. This shift holds across the board: in terms of cost shares and input quantities (as depicted) as well as output shares (i.e., the labor share) and output elasticities. It holds across firm types, industries, and countries. Our rationalization is simple but new: as firms grow, labor becomes expensive, and firms substitute toward intermediates.[1] Formally, our *parsimonious* model of firm growth leverages two features: (i) a labor-intermediates substitution elasticity above one, and (ii) an increasing shadow price of labor relative to intermediates, most likely due to monopsony. This paper is the first dedicated study of how these forces jointly shape firm growth and of how firm growth, in turn, offers new insights into these two core parameters.

We primarily study firm growth in German manufacturing micro data.[2] The data include firm-product-specific sales and real quantities, allowing us to construct firm-

[1]Here, economically, firm growth should be thought of as driven by input-neutral shifters, such as in TFP growth or product demand. Our model formalizes this through cost minimization, taking scale as given, and our empirical analyses include an instrument for growth that relies on product demand shifts.

[2]We note that for manufacturing, intermediates primarily consist of product components (e.g., car seats manufactured by other companies) and materials. Temporary agency labor or services play a much smaller role given their small cost shares. In an extension, we study other sectors.

specific output prices and to address the price biases in output elasticities derived from production function estimation (De Loecker et al., 2016, Bond et al., 2021, De Ridder et al., 2026). We draw on OLS regressions as well as an IV strategy using export demand shocks as a firm growth shifter unrelated to factor-augmenting price or technological changes. We also establish the shift from labor to intermediates in nonparametric scatter plots of raw firm-level data and, as a complementary analysis, we also infer output elasticities from cost shares.

Additionally, we confirm our main results in administrative firm-level data from 11 additional countries as well as in industry-level data for 20 European countries using harmonized data from the Competitiveness Research Network (CompNet) for manufacturing *and* non-manufacturing industries—and for a subset of outcomes also for U.S. manufacturing industries. We estimate our regressions at various horizons, from 1- to 10-year changes, and across different size and age classes of firms. As our evidence does not point toward a major role of production function non-homotheticities and/or fixed labor inputs in explaining our results, we rationalize our findings with a parsimonious production perspective on input substitutability as described below.

The shift from labor to intermediates in quantities and the reduction in output elasticities of labor accounts for *about one half to one third* of the negative effect of firm growth on the firm-level labor share. (In log changes, the labor share is equal to the output elasticity minus markups and markdowns.) Hence, our framework provides a novel, technological explanation for the negative association between the labor share and firm growth, complementing existing approaches that focus on large firms' product or labor market power. Importantly, unlike the existing literature on cross-sectional firm *size* gradients and concentration, we focus on *firm growth* in panel data.[3]

To rationalize the *full* set of facts, we propose a deliberately *parsimonious* model of firm growth. It leverages the combination of two features typically studied in isolation: substitutability between intermediates and labor, and an increasing relative shadow price of labor (e.g., from monopsony). In a nutshell, as firms grow, they lower their demand for increasingly expensive labor, shifting to intermediates. If labor and intermediates are substitutes, this shift lowers the output elasticity of labor relative to intermediates. This, in turn, lowers the labor share.

[3]See, e.g., Autor et al., 2020, for a cross-sectional study of markups and labor shares. We find much smaller (but qualitatively similar) cross-sectional differences in output elasticities by firm *size* than by firm *growth*, perhaps due to firm-specific permanent factors shaping input intensities and output elasticities. See our related discussion of Chan et al. (2024) below. Therefore, our additional results on labor shares in growing firms also resonate with the empirical study of Kehrig and Vincent (2021), who study firm growth dynamics in labor shares and highlight the role of demand-side factors (markups).

Substitution elasticities above one imply that as the relative intermediate-labor input quantity ratio increases, their output elasticity ratio falls—consistent with our firm growth facts. In fact, we formally show that this comovement—expressed as the ratio of our reduced-form (OLS or IV) regression coefficients of output elasticities and input quantities on firm growth—*identifies* the substitution elasticity. Our identification strategy—estimating substitution elasticities from within-firm comovement of output elasticities and input quantities—is new. It follows directly from the production function, does not require assumptions about firm optimization, and does not use input prices. Our approach therefore complements existing work that estimates substitution elasticities by relating relative input quantities to input prices under cost minimization.[4] We address the standard concern about unobserved factor-augmenting productivity shocks using an IV strategy based on export demand (but do so for firm growth, whereas the existing literature instruments for input prices). We also support the full identification argument (both production function estimation and our growth regression) in a Monte Carlo simulation.

Quantitatively, our firm growth regressions identify substitution elasticities well above one, ranging from 1.8 to 2.7 (OLS) and from 3.8 to 4.2 (IV). Our paper situates these values in a systematic meta-analysis of existing estimates. We also apply our (OLS) identification of substitution elasticities to firm micro data for additional 11 countries (and for 20 countries in industry data), consistently pointing to substitution elasticities above one.

To account for *why* growing firms choose to shift their input mix from labor to intermediates, our model features an increasing relative shadow price of labor. A natural source is a finitely elastic firm-specific labor supply curve, i.e., firms holding monopsony power over labor. (Alternatively, adjustment costs may play a role, at least in the short run.) In fact, for a given substitution elasticity and under the assumption of firm cost minimization, our input mix estimates identify the elasticity of the labor (shadow) price

[4]We do invoke cost minimization when, additionally, backing out shadow price ratios or in our secondary strategy that proxies for output elasticities with cost shares. Existing studies often combine within-firm/-industry and cross-sectional variation when using arguably exogenous price changes as instruments. Existing firm-level studies are difficult to compare to our approach, as they typically estimate substitution elasticities between intermediates and a capital-labor bundle rather than labor (and combine cross-sectional and within-firm variation). Three studies that directly estimate labor-intermediate substitution elasticities are Doraszelski and Jaumandreu (2018), Huneeus et al. (2022), and Chan (2023), who report estimates ranging from 0.12 to 1.8, 1.05 to 1.62, and 1.6 to 9.6, respectively. Moreover, existing studies also typically assume competitive input markets and do not consider shadow price changes, which are the actual relevant price changes for factor demand adjustments. In this context, Huneeus et al. (2022) also show that accounting for input heterogeneity when constructing intermediate and labor input prices increases estimated substitution elasticities (above unity). We note that our new strategy to identify substitution elasticities, using comovements of input mix and relative output elasticities, identifies the same substitution elasticity that guides the response of input mix to *shadow* input price ratio shifts (due to firms' first order conditions and under cost minimization). We provide further discussion in Section 5.

to the firm (i.e., the inverse labor supply elasticity). We offer a range of implied elasticities across our specifications and wage measurement approaches, and compare them to existing estimates using data from the review of Sokolova and Sorensen (2021).

Our paper focuses on *firm*-level growth. Studying aggregate implications would necessitate input-output network or open-economy perspectives. However, qualitatively, industry-level dynamics, which we study in CompNet data for 20 countries and the United States, echo our firm-level findings: inputs, cost shares, and output elasticities shift from labor to intermediates, implying substitution elasticities above one. Strikingly, the drop in labor output elasticities fully explains why the labor share falls in growing industries, with no role for markups or wage markdowns. This suggests that our production-function-based mechanism may also play a role in aggregate labor share dynamics through changes in output elasticities (see also Mertens, 2022).

Additional related literature. The literature on intermediate-labor substitution is still small compared to that on capital-labor substitution (see e.g., Acemoglu and Restrepo, 2020; Hubmer and Restrepo, 2021; Karabarbounis and Neiman, 2014; Lashkari et al., 2024). Huneeus et al. (2022) estimate that labor and intermediates are substitutes and show that firms with access to cheaper suppliers have lower labor shares. Castro-Vincenzi and Kleinman (2026) use aggregate data to study how rising material prices may lower labor shares if materials and primary inputs (the combination of labor and capital) are complements. We focus on firm-level mechanisms and firm growth, and find empirical evidence showing that intermediates and labor are substitutes and that, as firms and industries grow, shadow prices of labor relative to intermediates increase. In the trade literature, Dhyne et al. (2022) show that labor adjustments to demand shocks are weaker compared to intermediate inputs and emphasize the role of fixed cost in labor. Relatedly, other work in international trade shows that larger/growing firms offshore a greater share of production (Fort, 2017). Finally, in parallel work, Chan et al. (2024) show that larger firms exhibit higher returns to scale and output elasticities of intermediate inputs cross-sectionally and study the resulting efficiency losses under financial frictions. We instead focus on within-firm shifts from labor to intermediates as firms grow, analyze implications for labor share dynamics, and provide a parsimonious theory rationalizing the patterns based on substitutability and monopsony.

Outline. Section 2 provides a formal model and derives predictions. Section 3 describes the German firm-level data, sample, and production function estimation. Section 4 empirically establishes the shift from labor to intermediates. Section 5 interprets the results quantitatively, identifies parameters of interest, and discusses alternative explanations for our findings. Section 6 draws implications for firm-level labor shares. Section 7 uses additional firm- and industry-level data to transfer our analysis to other European countries, sectors outside of manufacturing, and U.S. manufacturing industries. Section 8 concludes.

2 Theory

This section sets up a parsimonious framework for how firm growth affects input intensities, output elasticities, and cost and output shares.

2.1 Firm Optimization

Production function. Firm i in period t transforms labor (L_{it}), intermediates (M_{it}), and capital (K_{it}) into output (Q_{it}) using a constant returns to scale (CRS) production function:

$$Q_{it} = \Omega_{it} \Lambda^K K_{it}^{1-\kappa} \left(\Lambda^{LM} \alpha^L L_{it}^{\frac{\sigma-1}{\sigma}} + \Lambda^{LM} \alpha^M M_{it}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\kappa}. \tag{1}$$

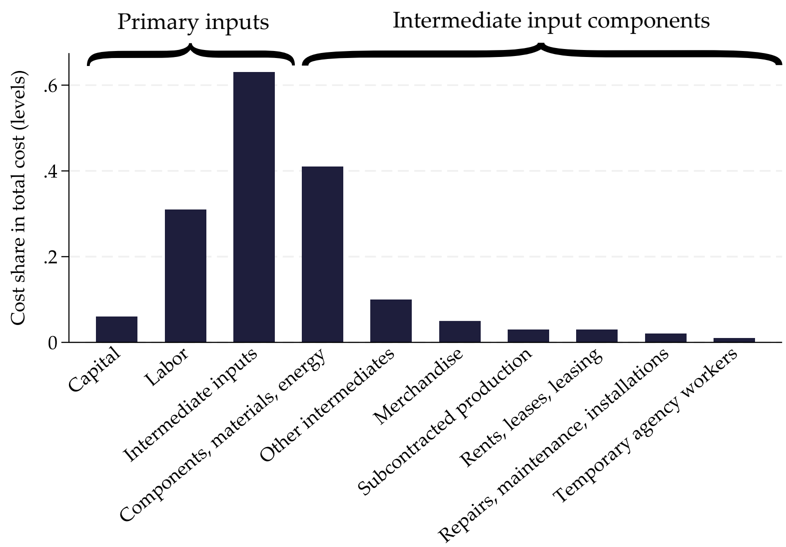
Ω_{it} is total factor productivity (TFP). α^L, α^M, Λ^{LM}, and Λ^K are distribution parameters. Capital enters multiplicatively with a Cobb-Douglas exponent $(1-\kappa)$. Labor and intermediates enter as a constant elasticity of substitution (CES) nest with substitution elasticity σ, forming a labor-intermediate bundle with Cobb-Douglas exponent κ.

We choose this homothetic CES specification for its analytical simplicity and parsimonious account of our findings (Section 5.3 discusses non-homotheticities). It foreshadows our main result, that the substitution from labor to intermediates is the primary pattern accompanying firm growth, with capital intensity shifts being comparatively unimportant. While the capital output elasticity in Eq. (1) is constant, our empirical analysis will permit varying output elasticities (and cost shares) for all inputs (including capital). As the capital output elasticity is small (see below), its level changes remain small despite sizable percent effects, so that a constant $(1-\kappa)$ will be a fair approximation.

Empirical context: relative input intensities in the data. Figure 2 details the typical input structure of German manufacturing firms (data described in Section 3). To compare the intensities of labor, capital, and intermediate inputs, we plot input cost shares (which add up to one). The average capital cost share is just 6% (which equals $1-\kappa$ under perfect input markets).[5] The low average capital share, along with our empirical findings (showing relatively small capital cost share and output elasticity changes in levels), further motivates our focus on labor-intermediate substitution. The figure also decomposes the intermediate

[5]Similarly, under a translog production function, we find a capital output elasticity of 0.12. In our firm-level data, labor's average share of value added is 80%.

Figure 2: Cost shares of production inputs.



Notes: The figure reports average firm-level cost shares for capital, labor, intermediate inputs, and the components of intermediate inputs. Capital costs are approximated as 8% of the capital stock following Dhyne et al. (2024) and the approach used in the CompNet data (CompNet, 2023), which we use in Section 7. Separate information for temporary agency worker cost shares is available from 1999. All other variables are available from 1995. The data cover German manufacturing firms and is described in Section 3.

input cost share. Two thirds of intermediate inputs consist of external product components (e.g., car seats), materials, and energy. Of the remainder, half are classified as "other intermediate inputs," which include services like transport, postage, insurance, and legal services. The other half comprises merchandise, rents, and subcontracted production steps (i.e., a firm contracts another firm to perform specific production tasks). Temporary agency labor represents only about 1% of total costs.

The substitution elasticity: identified off comovement of output elasticities vs. input mix. At the core of our paper is the following insight: Given the production function in Eq. (1), the substitution elasticity (σ) is related to and can be identified by the co-movement of relative output elasticity and input ratio changes (derivation in Appendix D.1):

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\alpha^L}{\alpha^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{-1}{\sigma}} \Leftrightarrow \frac{\theta^L_{it}}{\theta^M_{it}} = \frac{\alpha^L}{\alpha^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}} \tag{2}$$

$$\Rightarrow \frac{\sigma-1}{\sigma} = \frac{\Delta\ln(\theta^L_{it}) - \Delta\ln(\theta^M_{it})}{\Delta\ln(L_{it}) - \Delta\ln(M_{it})}, \tag{3}$$

where, in the last step, we have taken changes within firms (and used the assumption of constant α levels, departures from which we discuss below). $\theta^X = \frac{\partial Q}{\partial X}\frac{X}{Q}$ is the output elasticity of input $X = \{L, K, M\}$. Importantly, Eq. (3) can also be derived by assuming cost minimization (without specifying a production function) as an elasticity of relative input quantity changes to relative shadow prices changes, holding capital and its price fixed (see Appendix D.3).

If labor and intermediates are substitutes ($\sigma > 1$, consistent with our evidence below), Eq. (3) implies that a decrease in the labor-intermediates ratio will decrease the ratio of the labor output elasticity to the intermediate output elasticity. With Cobb-Douglas ($\sigma = 1$), no such change would occur, and complements ($\sigma < 1$) would imply the opposite.

Cost minimization. In what follows, we rely on cost minimization at a given output level, which introduces firm size and growth as quasi-parameters and permits us to cast our reduced-form empirical regressions as structural equations. It also enables us to recover the implied input shadow costs that rationalize observed input mix adjustments. This section focuses on key equations; Appendix D.1 details derivations.

We allow for imperfect product market competition and input market frictions, such as monopsony power and adjustment costs, creating wedges between the marginal costs of production inputs and their unit costs. We discuss this further in Section 2.2. Cost minimization implies a FOC for each input, labor, capital, and intermediates, $X = \{L, K, M\}$,

$$P_{it}^X \underbrace{\left(1 + \frac{\partial P_{it}^X}{\partial X_{it}} \frac{X_{it}}{P_{it}^X} + \frac{\partial \chi^X}{\partial X_{it}}\right)}_{\gamma_{it}^X} = \lambda_{it} \frac{\partial Q_{it}}{\partial X_{it}}. \tag{4}$$

P^X is the price for input X (e.g., wage for labor L), χ^X is an adjustment cost function, λ_{it} is marginal cost, and γ_{it}^X is the input price wedge, such that $P_{it}^X \gamma_{it}^X$ is the input *shadow price*. We express the adjustment cost function in flexible "quasi-static" terms as in Bond et al., 2021 (hence, sidestepping dynamic continuation-related terms; see also Appendix D.1) to highlight the key take-away from Eq. (4): monopsony power and adjustment costs raise marginal input costs (the overall shadow price) beyond an input's price, P_{it}^X.[6]

Inserting Eq. (4) into Eq. (3) recovers the implied shadow price ratio change that rationalizes a given shift in input mix at the firm level for a given level of σ:

$$\Delta \ln\left(P_{it}^L \gamma_{it}^L\right) - \Delta \ln\left(P_{it}^M \gamma_{it}^M\right) = \frac{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}{-\sigma}. \tag{5}$$

Hence, for any $\sigma > 0$, an increase in the shadow price of labor relative to intermediates leads to a decrease in the labor-to-intermediate ratio, with σ scaling this relationship (absent factor-augmenting shifts). Analogously, σ guides the size of the implied input

[6] Under profit maximization, γ_{it}^X equals the wedge between marginal revenue products and input costs.

price ratio shift that must have rationalized a given shift in the input mix.

Our empirical strategy will use Eqs. (3) and (5) to estimate substitution elasticities and the implied shadow price ratios that accompany the production dynamics of firm growth.

Cost and output shares. Changing output elasticities have important implications. Appendix D.1 shows that using Eq. (4) for all inputs determines an input X's cost share:

$$CS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it}^X L_{it} + P_{it}^M M_{it} + P_{it}^K K_{it}} = \frac{\frac{\theta_{it}^X}{\gamma_{it}^X}}{\frac{\theta_{it}^L}{\gamma_{it}^L} + \frac{\theta_{it}^M}{\gamma_{it}^M} + \frac{\theta_{it}^K}{\gamma_{it}^K}}. \tag{6}$$

Similarly, reformulating Eq. (4) defines an input's output share as a function of markups, input wedges, and output elasticities (see Appendix D.1):

$$OS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it} Q_{it}} = \frac{\theta_{it}^X}{\mu_{it}\gamma_{it}^X}, \tag{7}$$

where $\mu_{it} = \frac{P_{it}}{\lambda_{it}}$ is the output price over (total) marginal cost markup. γ_{it}^X is the wage markdown (the wedge between the input's marginal cost and price). Thus, a decrease in an input's output elasticity leads to a reduction in its cost and output shares.[7] Eqs. (2) and (3) highlight that such a decline in the labor output elasticity can result from changes in the labor-intermediate input mix (holding fixed returns to scale and if $\sigma > 1$).

2.2 Predictions for Firm Growth

To make predictions about firm growth, we require additional structure and model *firm-specific* labor and intermediate supply as iso-elastic, $P_{it}^X = a_{it}^X X_{it}^{\varepsilon_i^X}$ for $X = \{L, M\}$, where a_{it}^X is a baseline input price normalization and ε_i^X is the inverse firm-specific input supply elasticity (such that a finite supply elasticity reflects monopsony in input markets).[8] For simplicity, we also assume constant markups, that a firm's capital shadow price does not depend on its own capital demand, and that γ_{it}^L and γ_{it}^M are determined by the supply functions. Importantly, in our empirical analysis, we relax these assumptions and allow for firm- and time-specific markups and wage markdowns (resulting from monopsony/finitely elastic firm-specific labor supply curves, or adjustment costs).

[7]We can express θ_{it}^X in terms of returns to scale ($RTS_{it} = \theta_{it}^L + \theta_{it}^M + \theta_{it}^K$) and the input output elasticity relative to other output elasticities, $\theta_{it}^X = \frac{\theta_{it}^X}{RTS_{it}} RTS_{it}$, which separates returns to scale from the relative technological importance of inputs vis-à-vis other production factors.

[8]Strictly speaking, our analysis does not require firms to exploit upward-sloping firm-specific labor supply; what matters is that shadow input prices increase as output grows. In our empirical analysis we study within-firm changes and include industry-year fixed effects.

Inserting the input supply functions into Eq. (4) determines labor and intermediate demand as functions of marginal products and parameters (see Appendix D.2):

$$X_{it} = \left(\frac{\lambda_{it}\alpha^X}{(1+\varepsilon^X)a_{it}^X}\right)^{\frac{1}{\varepsilon^X}} \left(\frac{\partial Q_{it}}{\partial X_{it}}\right)^{\frac{1}{\varepsilon^X}} \quad \text{for} \quad X = \{L, M\}. \tag{8}$$

Inserting the production function and expressing the resulting equation in terms of the labor-intermediates ratio yields:

$$\frac{L_{it}}{M_{it}} = \varrho_{it}\lambda_{it}^{\frac{\sigma+\kappa-1}{\kappa}\left(\frac{1}{\sigma\varepsilon^L+1}-\frac{1}{\sigma\varepsilon^M+1}\right)} Q_{it}^{\left(\frac{1}{\sigma\varepsilon^L+1}-\frac{1}{\sigma\varepsilon^M+1}\right)}, \tag{9}$$

where ϱ_{it} (expression in Appendix D.2) captures effects unrelated to firm growth (i.e., parameters, baseline prices, and TFP). $\lambda_{it}^{\frac{\sigma+\kappa-1}{\kappa}\left(\frac{1}{\sigma\varepsilon^L+1}-\frac{1}{\sigma\varepsilon^M+1}\right)}$ captures the effect of marginal costs, which increase in output due to increasing supply curves. The key insight from Eq. (9) is that the response of *input ratios* (and thus output elasticities and, in turn, cost and output shares) to an increase in output (i.e., firm growth) depends on $\frac{1}{\sigma\varepsilon^L+1} - \frac{1}{\sigma\varepsilon^M+1}$.

Growth predictions. Table 1 summarizes our predictions for various assumptions about labor-intermediate substitution elasticities and relative input supply elasticities.[9] This section's discussion remains qualitative. Our quantitative interpretation is presented in Section 5, where we back out the implied values for σ and labor supply elasticities, $\epsilon^L = (\varepsilon^L)^{-1}$, identified by our empirical estimates.

Potential cases. We use a CRS Cobb-Douglas production function with perfect markets as a benchmark to fix ideas (CES with $\sigma = 1$), and consider departures under input complementarity ($\sigma < 1$) and substitutability ($\sigma > 1$). We consider three cases for firm-specific input supply elasticities: labor is more or less elastic than intermediates ($(\varepsilon^L)^{-1} > (\varepsilon^M)^{-1}$ and $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$), and the inputs are equally elastic ($(\varepsilon^L)^{-1} = (\varepsilon^M)^{-1}$).

The shaded, top-right area in Table 1 highlights the combination of parameters implied by our empirical evidence: substitutability and labor being less elastically supplied to the firm than intermediates.

Rejected by evidence: equally elastic supply. The benchmark of equal supply elasticities nests the competitive input prices case (middle row of Table 1). In this case, input ratios, output elasticities, cost shares, and output shares are constant as firms grow for any value of the substitution elasticity.

[9] We focus on the empirically relevant case of $\sigma + \kappa > 1$, where κ is approximately the sum of labor and intermediate output elasticities or, under constant returns to scale and perfect input markets, the sum of labor and intermediate cost shares. κ is therefore close to unity (see Figure 2 and Appendix Table A.1). Our estimates of σ are well above unity (see Table 3).

Table 1: Growth predictions: different substitution and supply elasticities (ceteris paribus).

	$\sigma < 1$	$\sigma = 1$	$\sigma > 1$	
L **less** elastic than M $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$	↓	↓	↓	
	↑	=	↓	
	↑	=	↓	
	↑	=	↓	
L **as** elastic as M $(\varepsilon^L)^{-1} = (\varepsilon^M)^{-1}$	=	=	=	$\Delta \ln L_{it}/M_{it}$
	=	=	=	$\Delta \ln \theta^L_{it}/\theta^M_{it}$
	=	=	=	$\Delta \ln CS^L_{it}/CS^M_{it}$
	=	=	=	$\Delta \ln OS^L_{it}/OS^M_{it}$
L **more** elastic than M $(\varepsilon^L)^{-1} > (\varepsilon^M)^{-1}$	↑	↑	↑	
	↓	=	↑	
	↓	=	↑	
	↓	=	↑	

Notes: Ceteris paribus refers to constant returns to scale and non-changing market imperfections with output growth ($\Delta \ln Q_{it}$) (but our empirical treatment will relax those expositional assumptions). The shaded area represents the region of the parameter space consistent with our empirical findings.

Rejected by evidence: Cobb-Douglas and less elastic labor supply. The case of an increasing relative shadow price of labor (top row) paired with Cobb-Douglas ($\sigma = 1$) implies that firms substitute from labor to intermediates, but output elasticities and thus cost and output shares remain unchanged. That is, even though labor becomes relatively more expensive as firms grow, the quantity substitution from labor exactly offsets the price increase, leaving labor cost and output shares constant.

Rejected by evidence: complements and less elastic labor supply. Now, consider that labor and intermediates are complements ($\sigma < 1$). In this case, a relative reduction in labor quantities increases the output elasticity of labor relative to intermediates. Intuitively, firms reduce their labor-intermediate ratio less than one-to-one with the input price ratio increase, causing cost and output shares of labor to *increase*.

Case supported by evidence: substitutes and monopsony. Input substitutability ($\sigma > 1$) implies the opposite: the reduction in the labor-intermediate input quantity ratio translates into declines in output elasticity ratios and cost and output shares of labor. As labor becomes expensive, firms substitute away from it by more than one-to-one. This scenario aligns with our empirical evidence discussed below (and Figure 1).

Rejected by evidence and implausible: more elastic labor supply. For completeness, we include the case where intermediate inputs are less elastically supplied than labor. In this scenario, the opposite sign for quantities emerges across the board, irrespective of σ.

Discussion: non-constant input wedges/supply elasticities and markups. As noted, the above predictions hold ceteris paribus with respect to input wedges and markups. In our empirical analysis, we will measure these factors and discuss their role (Section 6).

Markups tend to rise with firm growth, which dampens input responsiveness to growth relatively more for the more elastically supplied input (this can be seen in Eq. (9) as $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$). The empirical markup-growth relationship is small, so this attenuation is small and does not overturn the predicted reduction in the labor-intermediate input ratio.

Regarding changing input wedges, i.e., markdowns, in the data, we will find that the wage markdown (relative to the markdown on intermediates) rises with growth. As a result, the relative shadow price of labor increases by even more than plain wages. This *amplifies* the shift from labor to intermediates. One implication is that, in this case, even under Cobb-Douglas (i.e., under constant output elasticities), we may observe declines in labor shares. We formally discuss this case in Section 6 and differentiate between changes in wedges and output elasticities by directly measuring output elasticities using the production function estimation methods outlined below.

Discussion: factor-augmenting productivity. In Eq. (1), α^L and α^M are assumed constant (within the firm); otherwise, the estimation of the substitution elasticity would need to account for such changes (i.e., they would enter Eqs. (3) and (5)).[10] Similar to other work, in our empirical section, we address potential biases from unobserved factor-augmenting productivity shocks. We control for industry-year FE to absorb these shocks at the industry level. To account for firm-specific factor-augmenting productivity shocks, we use an IV approach based on export-related *product demand* shocks—where causality goes from plausibly factor-neutral output changes to output elasticity changes (we also discuss non-homotheticities)—see Section 5.3 and Appendix D.4 for details. We also illustrate this identification argument using a Monte Carlo simulation in Section 5.1.

3 Firm-level Data

We now describe the German firm-level data and the production function estimation.

Production data. The German manufacturing firm-product-level panel data cover the period of 1995-2017. The data are collected and supplied by the German Statistical Offices. The unit of observation is firms (not establishments or plants, although 90% of firms are single-plant firms).[11] The data include employment, investment (includes tangible and intangible assets), intermediate input costs, wage bills, depreciation, and total gross

[10]For instance, with time-varying factor-specific technologies, α^L_{it} and α^M_{it}, Eq. (3) becomes $\frac{\Delta \ln(\theta^L_{it}) - \Delta \ln(\theta^M_{it})}{\Delta \ln(\alpha^L_{it}) - \ln(\alpha^M_{it}) + \Delta \ln(L_{it}) - \Delta \ln(M_{it})}$ (where for clean exposition α^L_{it} and α^M_{it} are normalized to enter CES Eq. (1) with exponent $\frac{\sigma-1}{\sigma}$, i.e., the exponent on labor and intermediates).

[11]In this dataset, firms are defined as legal units, referring to the smallest legally independent unit that keeps accounts for commercial or tax purposes.

output. A rare feature is that the data include *real product quantities and sales* at a ten-digit product classification, allowing us to compute firm-product-level prices.[12] The data cover a stratified 40% sample of firms with at least 20 employees and consist of a rotating, stratified (industry and firm size classes) panel that is redrawn every 4-5 years (due to stratification, firms above 500 employees are always included).[13] Labor is defined as the number of employees on September 30th. All other variables pertain to the full calendar year.[14] We provide details on data access and preparation, capital stock construction, variable definitions, and summary statistics in Appendix E.

Summary statistics. Appendix Table A.1 provides summary statistics for our sample.

Supplementary trade data. To provide *causal* evidence on the relationship of our variables of interest with output growth (i.e., growth in response to an input-neutral shifter), we will draw on the IV approach by Hummels et al. (2014) to instrument output changes with foreign export demand. To do so, we merge bilateral trade flows from the United Nations Comtrade Database at the firm-product-year level (using the PRODCOM2002 product codes and the official concordance tables following Bräuer et al., 2023).

Production function estimation and output elasticities: CES and cost shares. Our identification of the substitution elasticity σ requires time-varying and firm-specific output elasticities (see Eq. (3)). We use two complementary approaches. First, cost shares equal output elasticities, $\theta_{it}^{X} = \frac{P^X X}{\sum_{X'} P^{X'} X'}$ (see Eq. (6)) under the usual identification assumptions of CRS and perfect input markets in cross-sectional analyses. However, for our within-firm growth analyses (see Eq. (11)), we can substantially relax this assumption, requiring only *non-changing* returns to scale and input wedges (e.g., from iso-elastic labor supply as in Section 2). We will find that input wedges increase with firm growth, causing an upward bias in estimated substitution elasticities in the cost share approach—and we provide a more flexible and robust approach next.

Production function estimation and output elasticities: translog. While we produce all results with the cost share specification (i.e., the standard method for CES-type functions, including Eq. (1)), our preferred and primary approach estimates a translog production function. This approach permits input wedges and returns to scale to vary, and it can approximate more flexible production functions, while nesting a CES function, as it can be viewed as an approximation to any twice-differentiable production function (which,

[12]Product examples: "Tin sheets and tapes, thicker than 0.2mm", "Workwear: long trousers, men, cotton."

[13]Using other country data that features smaller firms (see Section 7), we confirm robustness to including smaller firms in an unreported robustness check.

[14]We will estimate results for 1-, 4-, and 10-year changes—assuaging concerns about timing differences between labor and other variables.

among other properties, we will confirm in a Monte Carlo simulation in Section 5.1 and Figure 5 below, and as noted in Caves et al., 1982).[15] Importantly, as discussed above, the identification argument in Eq. (3) remains valid as it does not rely on a specific production function and follows directly from cost minimization (see Appendix D.3).[16] The translog specification is given by:

$$\begin{aligned}\tilde{q}_{it} = \beta_j^l l_{it} + \beta_j^k k_{it} + \beta_j^m m_{it} + \beta_j^{ll} l_{it}^2 + \beta_j^{kk} k_{it}^2 + \beta_j^{mm} m_{it}^2 + \\ \beta_j^{lk} l_{it} k_{it} + \beta_j^{lm} l_{it} m_{it} + \beta_j^{km} k_{it} m_{it} + \beta_j^{lkm} l_{it} k_{it} m_{it} + \omega_{it} + \nu_{it},\end{aligned} \tag{10}$$

where lower case letters denote logs. ν_{it} is an i.i.d. error term. $\tilde{q}_{it}$ denotes log firm sales deflated by a *firm-specific* price index that we derive from firm-product-level price information. ω_{it} is log total factor productivity. Labor enters Eq. (10) in quantities, intermediates and capital enter as expenditures deflated by 2-digit-industry-year-specific deflators. The production function is estimated for each industry, j, separately (NACE Rev. 1.1 two-digit). Using the estimated industry-specific coefficients from Eq. (10) and firm-specific input levels, we compute firm-year output elasticities as $\theta_{it}^X = \frac{\partial \tilde{q}_{it}}{\partial x_{it}}$ for input $X = \{L, M, K\}$.

We detail our production function estimation approach in Appendix F, where we also discuss how we address recently highlighted biases to production function estimation. Most importantly, we apply a *correction for price biases (De Loecker et al., 2016) by constructing a firm-specific output price index from our firm-product-level price data* as in Eslava et al. (2004) and by additionally including an input price control function, which proxies for unobserved input price variation using information on firms' output prices and market shares. The latter follows the firm-level adaptation of the control function method in De Loecker et al. (2016) by Mertens (2022).[17] As described in detail in Appendix F, including the input price control function captures, among others, quality differences in inputs that create (unobserved) input price variation across firms. Importantly, we explicitly define labor in terms of quantities rather than expenditures in Eq. (10) as labor markets are imperfect

[15]Using simulated data, Figure 5 shows that a translog approach with our identification of substitution elasticities described below recovers the true substitution (and thus output) elasticities if there is an underlying true structural CES production function with Hicks-neutral productivity.

[16]In principle, the translog allows for varying substitution elasticities across firms and time. We focus on an average elasticity and show that substitution elasticities are similar across firm output quintiles.

[17]This approach relies on the positive correlation between output and input prices (e.g., due to high quality outputs requiring high quality inputs). To account for the dependence of input decisions on productivity, we utilize a control function approach similar to Wooldridge (2009) and proxy productivity with information on expenditures on materials, product components, and energy. The control function for productivity also includes (lagged) wages (in addition to the input price control function), which helps address some of the identification concerns raised by Doraszelski and Jaumandreu (2021), particularly relevant in our context, where labor markets are imperfect.

(but we control for lagged wages, see also Footnote 17).

While estimating the translog production function allows for imperfect product and input markets and non-constant returns to scale, the identification relies on Hicks-neutrality to formulate a control function for productivity based on flexible production inputs (materials, product components, and energy expenditures; see Appendix F).[18] This is exactly where the main advantage of our alternative cost share approach comes in: it can account for non-Hicks-neutral productivity (at the cost of imposing a constant degree of input market imperfections). It is reassuring that the cost share and translog approach both yield qualitatively similar results, although they identify σ under very different assumptions. We also demonstrate these properties and identification results in a Monte Carlo simulation in Section 5.1.

4 Firm-level Evidence: Reduced Form Analysis

We now report empirical patterns of firm growth and associated dynamics of input use, output elasticities, and cost and output shares in the micro data. Section 5 will interpret these reduced form moments structurally and argue how they identify the implied substitution elasticities and input supply elasticities.

4.1 OLS Regressions

Strategy. For each firm i in year t and outcome $\mathbb{O}_{it}$ (input quantities, cost shares, output shares, output elasticities, and, later, markups and markdowns), we estimate OLS regressions in log differences across h years (i.e., $\Delta^h x_{it} = x_{it+h} - x_{it}$) of the form:

$$\Delta^h \ln(\mathbb{O}_{it}) = \rho^h_{\Delta \ln(\mathbb{O}), \Delta \ln(Q)} \cdot \Delta^h \ln Q_{it} + \upsilon_{jt} + \nu_{it}. \tag{11}$$

Q_{it} is deflated sales, and υ_{jt} captures 4-digit NACE rev. 1.1 industry-year fixed effects.[19] The difference specification accounts for unobserved constant firm characteristics. That

[18]Recent studies (e.g., Demirer, 2025) estimate production functions allowing for labor-augmenting productivity, though at the cost of imposing competitive labor markets. Demirer (2025) shows that biases from ignoring non-Hicks-neutral productivity are significantly smaller under a flexible translog specification compared to Cobb-Douglas—in some cases, differences in estimated output elasticities between Hicks-neutral and non-Hicks-neutral production functions become statistically insignificant when considering the translog specification.

[19]For this regression, we use the 2-digit industry output price deflator provided by the Statistical Office (but also include industry-year fixed effects). Using our own firm-specific output price index yields, however, similar results. The former may be preferable if there is measurement error in the firm-level output price index or under correlation between input and output prices.

is, we focus on within-firm and within-industry idiosyncratic firm growth, rather than industry growth or industry responses to shocks. We will study industry-aggregated analogs of this specification in Section 7.[20] ν_{it} is an error term. We cluster standard errors at the firm level. The regression is unweighted. We conduct robustness checks by size (sales) quintiles below, finding similar results for all key outcomes. The coefficient of interest is $\rho^h_{\Delta \ln(\mathbb{O}),\Delta \ln(Q)}$. It captures the percent effect (co-movement) of a one percent change in firms' output on firm-level outcome $\mathbb{O}_{it}$ (compared to the industry-year mean growth). We examine time differences h ranging from one year to ten years.

Results. The first row of each panel in Table 2 presents OLS regression results (1- to 10-year changes across panels). As a companion exhibit, Figure 3 visualizes firm-level relationships from Table 2 using binned scatter plots and residualizing variables by industry-year fixed effects as in Eq. (11)—revealing linear relationships that support the specification. As OLS effects are precisely estimated, we focus on point estimates.

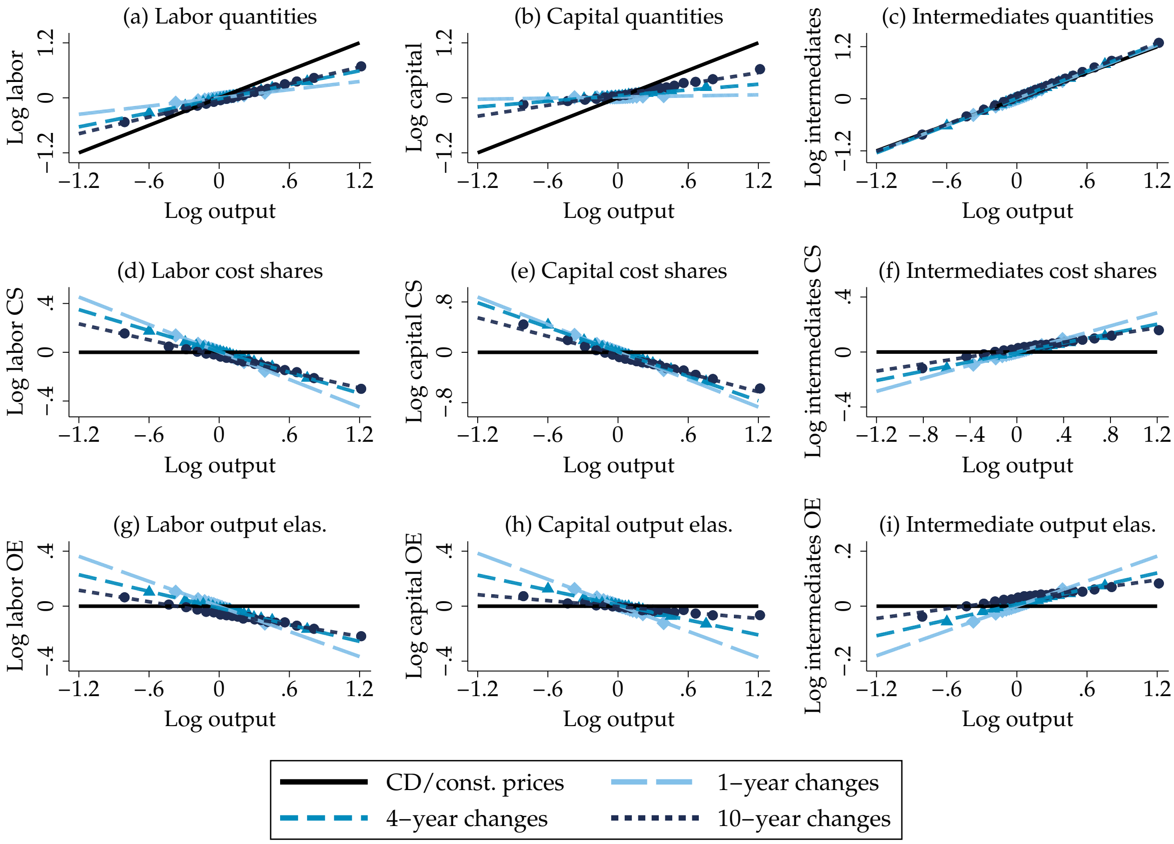
Input quantities. Table 2 Columns (2)-(4) and Figure 3 Panels (a)-(c) report effects on input use. We measure labor in head counts. Capital and intermediates are measured as industry-deflated monetary values (Appendix E describes the capital stock calculation).[21] We discuss hours worked, capacity utilization, and input quality below. (We omit TFP as another factor that accounts for firm growth with inputs scaling (weakly) less than one to one with output, and below, test and find no evidence for increasing returns to scale.)

Intermediate inputs exhibit approximately a unit elasticity with output growth; labor and capital increase by much less. These relative slopes result in a declining ratio of labor (and capital) to intermediate inputs. The small standard errors allow us to reject the

[20] In unreported robustness checks, we also ran specifications with firm effects to take out firm-specific trends. This specification leads to slightly higher coefficient estimates, particularly at longer horizons, likely due to its remaining variation capturing more transitory fluctuations around the firm-specific trends. This consistency of results suggests that our findings are not merely responses to transitory shocks.

[21] Not accounting for firm-specific intermediate input prices may slightly attenuate our results on real intermediate input quantities. This bias is likely small, and we can ballpark it as follows. Percent changes can be expressed as $\frac{dM_{it}}{M_{it}} + \frac{d\widetilde{P}^M_{it}}{\widetilde{P}^M_{it}} = \frac{d\widetilde{M}_{it}}{\widetilde{M}_{it}}$, where M_{it} is real quantities (our object of interest), $\widetilde{P}_{it}$ is the *firm-specific* intermediate price deviation from the industry intermediate input price index, and $\widetilde{M}_{it}$ is the measured real intermediate input quantity adjusted for the industry index. Assuming that firm-specific price movements are linked to firm-specific real quantities through firm-specific elasticity ϵ', such that $\frac{d\widetilde{P}^M_{it}}{\widetilde{P}^M_{it}} = \epsilon' \cdot \frac{dM_{it}}{M_{it}}$, we can write: $\frac{dM_{it}}{M_{it}} = \frac{d\widetilde{M}_{it}/\widetilde{M}_{it}}{1+\epsilon'}$. Thus, the estimated regression coefficient on the basis of our industry-deflated intermediate input measure is related to the object of interest by $\rho_{M,Q} = \frac{\rho_{\widetilde{M},Q}}{1+\epsilon'}$. The attenuation is small; for instance, even if we assumed that intermediate inputs were as inelastically supplied as labor, with $\epsilon' \approx 0.1$, then latent real intermediate inputs would only move slightly less elastically with output than suggested by our coefficient estimate—i.e., our overall results are unlikely to be significantly affected. Intermediate inputs are likely more elastically supplied than this benchmark suggests, implying negligible bias in our results. We see value in obtaining better evidence on firm-specific supply elasticities of intermediates.

Figure 3: Firm-level adjustments in response to firm growth (OLS, binned scatter plots).



Notes: The figure reports binned scatter plots from estimating the specification in Eq. (11) with OLS for various differences against log output changes for the following dependent variables in log changes: labor, capital, intermediate quantities, output elasticities over returns to scale, and cost shares. It also includes the prediction from a Cobb-Douglas production framework with firms optimizing against constant input price (ratios). All panels report results that are residualized by industry-year fixed effects. German firm-level data.

hypothesis of proportionate input growth, which would be expected in a Cobb-Douglas model with constant or uniformly shifting (shadow) input prices.

Cost shares. One possible explanation for the shift from labor to intermediates is divergence of input prices, while production remains consistent with a Cobb-Douglas model. In this scenario, cost shares would remain stable, as firms adjust input quantities inversely proportionately to rising input prices. (Changing input wedges are not captured by monetary costs and hence cost shares—we explore their role in Section 6.)

Table 2 Columns (5)-(7) and Figure 3 Panels (d)-(f) report effects on cost shares, i.e., input expenditures divided by total costs.[22] The data reveal a striking shift toward intermediate

[22]We approximate capital costs as 8 percent of the capital stock, following Dhyne et al. (2024) and CompNet (2023). For logged specifications, this homogeneous multiplicative factor is absorbed by industry-year fixed effects. Similarly, an industry-specific capital deflator would drop out. For that reason, we also do not attempt to provide more granular heterogeneous cost of capital measures.

costs, away from labor (and capital), indicating increased outsourcing.

Complementing the cost share analysis in logs, we also run the specification in level (ppt.) changes (as in Figure 1), showing that intermediate cost shares absorb the decrease in the labor cost shares and, with a quantitatively much smaller role, the shift in capital cost shares (see Appendix Table A.3). (We also study the labor income share in Section 6.)

Output elasticities and returns to scale. The prediction of constant input cost shares in a Cobb-Douglas model relies on the assumption of fixed input wedges and stable output elasticities. Indeed, Eq. (6) shows that with constant input wedges and returns to scale, shifts in cost shares correspond directly to changes in output elasticities. To account for the possibility of varying input wedges, Table 2 Columns (8)-(10) and Figure 3 Panels (g)-(i) analyze output elasticities based on our translog production function estimates.

We find a strong negative relationship between output growth and labor output elasticities. Table 2 Column (8) reveals that an additional 10 percentage point output growth reduces the (growth of) labor output elasticity by 3 percentage points, with a coefficient of -0.30 (SE 0.004) at a one-year horizon. Figure 3 Panel (g) visualizes the underlying relationship. This shift away from labor output elasticities is accompanied by a significant increase in intermediate output elasticities, as shown in Column (10) and Panel (i). Notably, capital output elasticities decline as well (Column (9) and Panel (h)).

Again, Appendix Table A.3 reproduces our findings in levels rather than logs for the dependent variables. Level changes in labor and capital output elasticities offset those in intermediate output elasticities, with labor and intermediates driving most of the variation. This result is consistent with the much smaller capital cost shares and output elasticities (Figure 2 and Appendix Table A.1).

We normalize the output elasticities in our regressions by returns to scale. The results remain consistent without this adjustment (Appendix Table A.3). Column (11) shows that returns to scale are relatively stable in the short term and only change slightly over longer horizons. Moreover, when we later estimate substitution elasticities, the returns to scale normalization cancels out—see Eq. (3).

Table 2: Firm-level adjustments in response to firm growth: OLS and IV estimates.

	1st stage	$\Delta\ln(L_{it})$	$\Delta\ln(K_{it})$	$\Delta\ln(M_{it})$	$\Delta\ln\left(\frac{P^L_{it}L_{it}}{C_{it}}\right)$	$\Delta\ln\left(\frac{P^K_{it}K_{it}}{C_{it}}\right)$	$\Delta\ln\left(\frac{P^M_{it}M_{it}}{C_{it}}\right)$	$\Delta\ln\left(\frac{\theta^L_{it}}{RTS_{it}}\right)$	$\Delta\ln\left(\frac{\theta^K_{it}}{RTS_{it}}\right)$	$\Delta\ln\left(\frac{\theta^M_{it}}{RTS_{it}}\right)$	$\Delta\ln(RTS_{it})$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Panel A: 1-year differences (29,950 firms; 183,813 firm-year observations)											
OLS estimates											
Log output change		0.298***	0.0405***	1.025***	-0.376***	-0.728***	0.238***	-0.304***	-0.315***	0.151***	0.0062***
		(0.0040)	(0.0022)	(0.0033)	(0.0039)	(0.0033)	(0.0022)	(0.0040)	(0.0063)	(0.0015)	(0.0003)
R^2		0.215	0.043	0.741	0.343	0.556	0.340	0.230	0.137	0.325	0.053
IV estimates											
Export demand shock											
(first stage)	0.0451***										
	(0.0045)										
Log output change		0.324***	0.0612	0.929***	-0.444***	-0.684***	0.171***	-0.327***	-0.229**	0.119***	0.0082
		(0.0543)	(0.0461)	(0.0508)	(0.0521)	(0.0565)	(0.0292)	(0.0623)	(0.0980)	(0.0198)	(0.0053)
R^2	0.205	0.214	0.042	0.736	0.336	0.554	0.321	0.229	0.132	0.314	0.052
First-stage F-stat		102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6
Panel B: 4-year differences (11,492 firms; 70,936 firm-year observations)											
OLS estimates											
Log output change		0.510***	0.206***	1.045***	-0.286***	-0.649***	0.171***	-0.202***	-0.181***	0.0951***	0.0167***
		(0.0064)	(0.0067)	(0.0045)	(0.0062)	(0.0071)	(0.0034)	(0.0060)	(0.0074)	(0.0025)	(0.0006)
R^2		0.471	0.144	0.860	0.348	0.476	0.355	0.220	0.170	0.268	0.163
IV estimates											
Export demand shock											
(first stage)	0.0635***										
	(0.0091)										
Log output change		0.536***	0.262***	0.963***	-0.269***	-0.570***	0.124***	-0.242***	-0.147	0.0823***	0.0222***
		(0.0737)	(0.0973)	(0.0542)	(0.0664)	(0.0969)	(0.0369)	(0.0706)	(0.0938)	(0.0268)	(0.0075)
R^2	0.183	0.470	0.139	0.855	0.347	0.471	0.339	0.217	0.169	0.266	0.156
First-stage F-stat		48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57
Panel C: 10-year differences (10,595 firms; 49,915 firm-year observations)											
OLS estimates											
Log output change		0.606***	0.395***	1.041***	-0.219***	-0.490***	0.131***	-0.140***	-0.0720***	0.0582***	0.0278***
		(0.0066)	(0.0082)	(0.0043)	(0.0061)	(0.0083)	(0.0033)	(0.0053)	(0.0069)	(0.0024)	(0.0007)
R^2		0.594	0.256	0.901	0.296	0.329	0.303	0.188	0.149	0.200	0.268

Notes: The table reports OLS and IV regressions from estimating the specification in Eq. (11). IV uses foreign demand shocks as instruments (Eq. (12)). Column (1) reports the first-stage regression results. The dependent variables in Columns (2)–(11) are log changes in labor, capital, intermediates, labor, capital, and intermediate cost shares, labor, capital, and intermediate output elasticities over returns to scale, and returns to scale, respectively. All columns report regressions of those dependent variables on changes in log output for 1-, 4-year, and 10-year differences (10-year differences are not available for IV). All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Effects across horizons. Comparing horizons across the panels reveals interesting dynamics: the coefficient for labor output elasticities drops from -0.30 to -0.20 and -0.14 when considering the 1, 4, 10 years (Column (8)). As we explore in Section 5, such horizon-dependency could be consistent with short-run labor adjustment costs.

Heterogeneity: types of intermediates. The strong shift from labor to intermediates may reflect either outsourcing of employment through services or of entire production steps (see Figure 2). Appendix Table A.4 reports additional regressions for logged cost shares of all available subcategories of intermediates. We find strong effects for the main category product components, materials, and energy, possibly capturing offshoring/outsourcing of labor-intensive production steps/components. Effects are negative for intermediates arguably complementary to capital, such as repair, maintenance, and installation services. In percent, we find the strongest effects for temporary agency workers, indicating that firms rely on those more flexible labor inputs when growing (see Bilal and Lhuillier, 2022; De Leon et al., 2024), particularly in the short run. However, *quantitatively*, in percentage points, this response has a minimal overall impact, as the average cost share of temporary agency workers is only 1% (Figure 2).[23]

Heterogeneity: size heterogeneity. Appendix Table A.6 reruns the analysis by quintiles of firm size. The subsample effects align closely with our main results (Table 2). We return to the issue of firm size when discussing non-homotheticities in Section 5.3.

Heterogeneity: industries. Appendix Figure A.3 shows that results are qualitatively robust across industry. Quantitatively, these shifts differ somewhat across industries, particularly for output elasticities. We have not systematically examined this heterogeneity but note that all things equal, stronger effects should emerge in industries with stronger labor monopsony power or adjustment costs. Relatedly, we note that firms generally have substantial wage flexibility within the German collective bargaining system, because coverage status is not universal but is chosen by firms (by joining employer associations), and covered firms often pay premia (Jäger et al., 2022).

Additional analysis: hours worked, capacity utilization, and input quality. From 1999 to 2017, we also observe employment in full-time equivalents (FTE) that capture intensive-margin hours adjustments. Appendix Table A.2 shows that headcount and FTE responses are nearly identical. (Capital adjustments are not our focus, and we do not observe capital utilization.) While we do not have detailed micro-data on labor quality, we can gauge compositional effects on the basis of average wages. Average wages slightly increase in

[23]Indeed, in an unreported robustness check, we have assigned temporary agency workers and subcontracted production to labor rather than intermediate inputs (and divided those costs by the firm's average wage to approximate labor-equivalent quantities) and found similar results across the board.

output growth, with coefficients ranging from 0 to 0.1 (Appendix Table A.9). If interpreted as an increase in labor quality, the efficiency units of labor (quantity plus quality effect) would only slightly attenuate the implied substitution elasticity.[24] The alternative—and our preferred—interpretation of the average wage effect points to upward-sloping labor supply curves (and is quantitatively realistic, see our meta-analysis in Section 5.2). We discuss the limitations of average wages below and instead infer changes in shadow prices as a broader marginal cost measure that is more relevant to our analysis.

Additional analysis: cross-sectional results and firm lifecycles. Our paper focuses on within-firm changes. For completeness, we provide results for the cross-section in Appendix Figure A.2.[25] Finally, while we do not have information on firm age in our main data, our extension to other countries in which age is included shows qualitatively similar effects across firm-age groups (Section 7.2 and Appendix Table A.13).

4.2 Causal Effects: IV Strategy using Export Demand Shocks

We now instrument for firm growth with foreign product demand shocks that are plausibly unrelated to factor-augmenting productivity, input price or quality, and age effects (conditional on industry-year effects)—tightening the link to the model analysis in Section 2 that held constant such confounders. Across outcomes, IV and OLS yield similar coefficients. This is consistent with the OLS regressions largely reflecting input-neutral growth sources, such as shifters in product demand, TFP, or prices of all inputs.

Strategy. We follow an established literature using trade shocks as exogenous shifters (see Autor et al., 2016 for a review). In particular, we follow Hummels et al. (2014) and instrument changes in firms' output with changes in world export demand. The method proceeds in two steps. First, we compute the total exports for each product, g, from each country c in a proxy country group $\mathcal{N}$ (to any destination country): $ex_{gt} = \sum_{c \in \mathcal{N}} ex_{gct}$. In studying such export shocks in the German context, we follow Dauth et al. (2014) and define $\mathcal{N}$ to include Australia, Norway, Sweden, Singapore, New Zealand, Great Britain,

[24]The effect on the wage bill can be imputed as the effect on employment times the average wage, i.e., $\Delta \ln(\bar{w}L) = \Delta \ln(\bar{W}) + \Delta \ln(L)$, such that, e.g., at the 4-year horizon, the labor effect would slightly rise from 0.510 to 0.572. Foreshadowing our calculation of the substitution elasticity in Section 5.1, and assuming away effects on measured output elasticities, this effect would only slightly change our conclusion about substitution elasticities between intermediates and labor (from 2.25 to 2.29).

[25]The cross-sectional analysis reflects qualitatively different forces, such as permanent heterogeneity. Our empirical analysis confirms that large firms have lower labor cost shares and labor output elasticities but paints a less clear picture likely due to input price differences and/or other (permanent) heterogeneities (e.g., the α and Λ terms in Eq. (1)). The effect for the labor output elasticity falls to -0.03, again precisely estimated. Scatter plots in Appendix Figure A.2 reveal a slightly concave pattern, consistent with large shorter-run elasticities not extending to cross-sectional variation in firm size, which is right-skewed.

Canada, Japan, and, additionally, the U.S. (but results are robust to dropping the U.S.). These industrialized economies have export structures plausibly similar to Germany's. Excluding Germany's direct neighbors and members of its EUR currency union helps mitigate potential endogeneity concerns (Dauth et al., 2014).

In the second step, we aggregate the product-level shifts ex_{gt} into firm-specific export demand shocks using firm-specific product sales weights, s_{git}, measured in each firm's first sample year to reduce anticipatory effects such as product-mix adjustments:

$$Z_{it} = \sum_{g} s_{gi,t=0} \ln ex_{gt}. \tag{12}$$

We thus rely on variation from a fixed (internationally tradable) set of products (and including industry-year FEs implies we use between-firm, within-industry variation). Importantly, 80% of German manufacturers in our sample export in a given year.[26] We then instrument output changes, $\Delta \ln Q_{it}$, in Eq. (11) with export demand shocks, ΔZ_{it}. Furthermore, we restrict our IV analysis to 1- and 4-year differences as the first stage for the 10-year changes falls below conventional F-statistic thresholds. (However, in unreported checks we confirmed that overall point estimates and resulting substitution elasticities remain similar, albeit less precisely estimated.)

Results. Table 2 Column (1) shows the first stage effect from regressing output growth on our instrument. We find a statistically significant positive effect, and an F-statistic of 102.6 (48.57) for one-year (four-year) changes. Columns (2)-(11) report the second stage IV results, instrumenting output growth with export demand shocks. Across outcomes, IV and OLS coefficients are similar, with IV results implying a somewhat smaller shift from labor to intermediate input quantities compared to the associated output elasticity shift. Below, in Section 5.1 (see also Table 3), we will show that these slightly different estimates will imply a larger substitution elasticity with IV than with OLS.

5 Quantitative and Structural Interpretation

We now interpret our reduced form results structurally and quantitatively through the theoretical lens of Section 2, backing out the labor-intermediates substitution elasticity and the firm-specific labor supply elasticity. Table 3 reports the key moments and identified

[26]As export information is reported at the firm level only, the aggregation is unconditional on export status at the firm-product level; in unreported analyses, we confirmed that exporters drive the instrument power. Moreover, our product sales share weights therefore include domestic sales.

Table 3: Identification of the labor-intermediate substitution elasticity and the labor supply elasticities based on firm growth regression coefficients.

	$L-M$ **differences in growth regression coefficients (Table 2)**		**Estimates based on growth regression coefficient differences**		
	Input quantities: $\rho_{\Delta\ln(L),\Delta\ln(Q)}$ $-\rho_{\Delta\ln(M),\Delta\ln(Q)}$	Output elasticities: $\rho_{\Delta\ln(\theta^L),\Delta\ln(Q)}$ $-\rho_{\Delta\ln(\theta^M),\Delta\ln(Q)}$	$L-M$ price reg. effect (implied): $\widehat{\rho}_{\Delta\ln(\frac{P^L\gamma^L}{P^M\gamma^M}),\Delta\ln(Q)}$	$L-M$ subst. el.: $\widehat{\sigma}$	L.S. elasticity: $\widehat{\epsilon}^L=\frac{1}{\widehat{\varepsilon}^L}$
	(1)	(2)	(3)	(4)	(5)
Panel A: OLS					
1-year diff.	-0.73	-0.46	0.27	2.67	1.10
4-year diff.	-0.54	-0.30	0.24	2.25	2.16
10-year diff.	-0.44	-0.20	0.24	1.84	2.56
Panel B: IV					
1-year diff.	-0.61	-0.45	0.16	3.81	2.04
4-year diff.	-0.43	-0.32	0.10	4.16	5.20

Notes: Columns (1) and (2) report differences of the coefficients on labor and intermediates (Column (1)) and on labor and intermediate output elasticities (Column (2)) from our growth regressions following the specification in Eq. (11) and the results reported in Table 2. Column (3) uses Eq. (17) to recover changes in shadow input price ratios from Columns (1) and (2) in combination with the estimate of σ in Column (4). Column (4) recovers the labor-intermediate substitution elasticity (σ) from Columns (1) and (2) using Eq. (15). Column (5) recovers the elasticity of labor supply with respect to the relative shadow price of labor using Eq. (19) (i.e., assuming constant wedges and intermediate input shadow prices, so that the latter reflects only wage changes). Panel A (B) reports results based on OLS (IV) growth regressions reported in Table 2.

parameters.[27] We then discuss alternative accounts.

5.1 Identification of Substitution Elasticity

Identification argument. Using Eq. (3), we identify the labor-intermediates substitution elasticity, σ, from the co-movement of output elasticities and input quantities:

$$\frac{\sigma-1}{\sigma}=\frac{\Delta\ln(\theta^L_{it})-\Delta\ln(\theta^M_{it})}{\Delta\ln(L_{it})-\Delta\ln(M_{it})} \tag{13}$$

$$\Rightarrow \sigma=\frac{1}{1-\frac{\Delta\ln(\theta^L_{it})-\Delta\ln(\theta^M_{it})}{\Delta\ln(L_{it})-\Delta\ln(M_{it})}}. \tag{14}$$

Specifically, we use within-firm changes (Δ) *conditional on firm growth*, i.e., we insert into this structural equation the reduced form regression coefficients estimated using firm

[27] We focus on within-firm changes, which net out fixed, unobserved firm-specific factors—including input wedges that are constant within the firm over time. Large firms seem to have higher intermediate-labor ratios, with smaller differences in labor and intermediate input output elasticities (Appendix Figure A.2). As our focus is on firm growth, we omit a detailed discussion of how to rationalize those facts with permanent heterogeneity.

growth variation in Section 4:

$$\widehat{\sigma} = \frac{1}{1 - \frac{\rho_{\Delta \ln(\theta^L), \Delta \ln(Q)} - \rho_{\Delta \ln(\theta^M), \Delta \ln(Q)}}{\rho_{\Delta \ln(L), \Delta \ln(Q)} - \rho_{\Delta \ln(M), \Delta \ln(Q)}}}, \tag{15}$$

where the ρ correspond to the regression coefficients from Eq. (11), for OLS and IV, various horizons, and the respective outcome variables.[28] Given the precisely estimated underlying coefficients, ρ, we focus on point estimates. (Bootstrapping is unfortunately not feasible at the Statistical Office due to computational resource limits.)

Next, we present the estimates of $\widehat{\sigma}$ implied by our regression estimates (the ρ coefficients). Then, at the end of this section, we will support the *full* identification argument—both production function estimation and our growth-based regression identifying the elasticity in Eq. (15)—in a series of Monte Carlo simulation exercises (see Figure 5 below).

Implied parameter values: translog estimates. Table 3 Column (4) reports the implied values of σ using output elasticities based on our translog estimates. We find that labor and intermediates are substitutes, with σ exceeding one in all specifications. The OLS estimates range from 2.67 in the short run to 1.84 in the longer run. IV estimates of σ are around 4 for 1- and 4-year changes (to reiterate, IV estimates for 10-year changes are unavailable due to a weak first stage at this longer horizon).

A simple decomposition (inspecting the empirical counterparts of the variables in Eq. (15) reported in Table 3 Columns (1) and (2)) reveals that the somewhat weaker quantity shift accounts for the IV estimates exceeding OLS estimates while the output elasticity shift is comparable.[29] Substantively, higher IV than OLS estimates of σ may reflect measurement error or may reflect labor-augmenting productivity shocks that bias OLS substitution elasticities *downward* under $\sigma > 1$—hence, a bias working *against* our proposed substitution mechanism. This bias is derived analytically in Appendix D.4 and studied in our Monte Carlo simulations in Figure 5 Panel (c) below.

Implied parameter values: cost shares. As alternative specification, we report substitu-

[28]Compared to the OLS estimates, the IV estimates can be viewed as additionally removing potential confounders, such as factor-augmenting shifters in prices or production function parameters, that may underlie some of the OLS variation in firm growth.

[29]We do not focus on comparing short- vs. long-run substitution elasticities or attempt to interpret these dynamics in context of the Le Châtelier principle, although we note that both labor and intermediates are presumably quite flexible compared to capital. However, OLS (IV) estimates indicate somewhat smaller (slightly larger) long- than short-run elasticities. These patterns must be interpreted in the context of intermediate-input mix adjustments: in the short run, firms increase inputs *more substitutable with labor*, such as temporary agency workers or subcontracted work, relatively more strongly, while in the longer run, firms increase inputs like rents and leases, repairs and maintenance, or other intermediates relatively more strongly (see Appendix Tables A.4 and A.5).

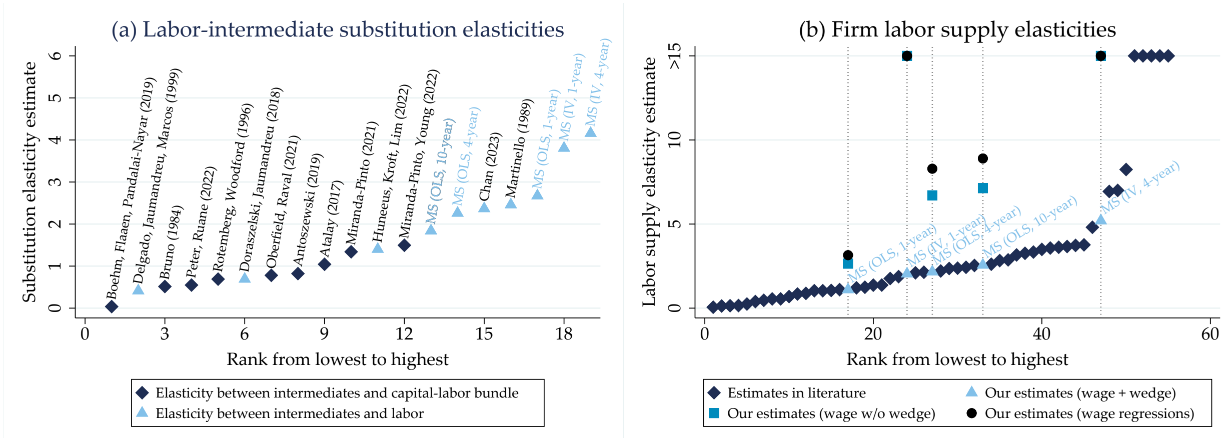
tion elasticities using cost shares to measure output elasticities in Appendix Table A.8 (also consistent with the CES production function in Section 2). These cost share estimates are robust to labor augmenting productivity shocks, but biased under size-dependent input wedges. As expected, estimates of substitution elasticities *increase*. Cost share estimates allow for a constant but not changing degree of labor wedges (relative to intermediates), $\frac{\gamma^L_{it}}{\gamma^M_{it}}$. If this wedge (ratio) increases with output growth (as shown in Section 6, Table 4), it biases substitution elasticities upward in the cost share approach (also illustrated in our Monte Carlo simulation below, and summarized in Figure 5 Panel (b)).

Price and quantity comovement. Due to firms' first-order conditions, our identification of σ can be viewed as equivalent to using comovements in input quantities and (shadow) prices as described in Eq. (5): $\Delta \ln \left(\frac{P^L_{it}\gamma^L_{it}}{P^M_{it}\gamma^M_{it}} \right) = -\frac{1}{\sigma}\Delta \ln \left(\frac{L_{it}}{M_{it}} \right)$. A key challenge in directly using this equation is that *firm-specific* (shadow) prices are unobserved. Nonetheless, to understand if price changes are generally consistent with our estimates, we report average wage responses to firm growth in Appendix Table A.9, which (under the assumption that intermediate-input prices are absorbed by industry–time fixed effects) is equivalent to studying relative input price changes. These measured average wage responses are much smaller in absolute value than the corresponding changes in labor-to-intermediate input ratios reported in Table 3 Column (1). This amplification of responses of input quantity ratios compared to input price ratios is reassuring and consistent with $\sigma > 1$.[30]

Meta-analysis of substitution elasticity estimates. Figure 4 Panel (a) situates our baseline translog estimates (denoted by "MS") in a meta-analysis of existing estimates. We differentiate between studies that estimate substitution elasticities between intermediates and a capital-labor bundle (diamonds) and between intermediates and labor (triangles). The former approach restricts the substitution elasticity between intermediates and capital to be identical to that between intermediates and labor. It typically identifies the substitution elasticity using the ratio between intermediates and the capital-labor bundle (value added). The latter approach directly relates intermediates to labor, aligns with our methodology, and tends to yield higher substitution elasticities that exceed unity in several studies. Appendix C provides details on the estimates included in Figure 4 Panel (a). We report mean values and estimates often vary significantly within a given study (e.g., values in Chan (2023) range from 1.6 to 9.6). We also note that our estimates are derived from within-firm changes only (many studies use cross-sectional variation, too—e.g., Do-

[30]In Section 6, we estimate input wedges, $\frac{\gamma^L_{it}}{\gamma^M_{it}}$, and study their changes in response to output growth. Adding changes in input wedges to changes in wages in response to growth will by design yield the reported shadow price changes in Table 3, Column (3) (this is because of firms' first-order conditions) that are smaller than the input-quantity ratio changes.

Figure 4: Meta-analysis: substitution and labor supply elasticities vs. literature.



Notes: The figure reports estimates of labor-intermediate substitution elasticities (Panel (a)) and labor supply elasticities (Panel (b)) from the literature and from our analysis (our own estimates are indicated by "MS" and in light blue). For substitution elasticity estimates, we focus on samples that encompass the most firms or report mean values (excluding negative estimates when computing the means). Appendix C provides details on the displayed substitution elasticities. Data on labor supply elasticities come from the meta-study by Sokolova and Sorensen (2021). We exclude negative estimates. If available, we report median IV estimates. If no IV approach is used, we report medians of all other estimates. (We prefer medians due to outliers.) We do not implement alternative selection methods (e.g., best practice corrections). Sokolova and Sorensen (2021) note that best practice correction estimates vary by methodology: separations-based approaches yield elasticity estimates between 6.4 (not identified) and 9.9 (identified), while stock-based inverse approaches produce estimates above 20. For a list of studies entering the firm-specific labor supply elasticity estimates, we refer to Sokolova and Sorensen (2021).

raszelski and Jaumandreu, 2018). To our knowledge, the strategy to relate relative output elasticities to input quantity ratios using within-firm changes is new to the literature.

Supporting the identification argument: Monte Carlo simulations. We support our growth-based identification of σ in a Monte Carlo simulation, generating a panel data set of firms with a CES production function (in labor and intermediates) with known substitution elasticity σ, which varies across simulations. We first recover output elasticities using three distinct approaches (see below). We then recover σ using our growth regression framework (Eq. (11)) in combination with the estimator in Eq. (15), using both OLS (all growth variation) and IV strategies (isolating Hicks-neutral sources).[31] We consider the following three economic environments for firm growth: (i) Hicks-neutral productivity shocks and constant labor wedges (markdowns), (ii) Hicks-neutral productivity shocks and labor wedges (relative to intermediates, $\frac{\gamma_{it}^L}{\gamma_{it}^M}$) growing in output,[32] and (iii) labor-augmenting productivity (LAP) shocks and constant labor wedges.

[31]We first generate output from a reduced form output policy rule in which output is determined by a random demand shifter and a random (potentially non-Hicks-neutral) productivity shifter (AR (1) with persistence of 0.7). We recover input decisions consistent with this output policy rule based on a CES production function in labor and intermediates (i.e., output is given as under cost minimization). Labor supply is upward sloping and iso-elastic (though we let markdowns vary as described below), while intermediate input prices are constant. Firms minimize cost. We simulate 8 periods with 1,500 firms per period in each simulation.

[32]Empirically, input wedges are correlated with firm size (e.g., Mertens, 2023). See also in Section 6.

To obtain output elasticities, we consider three alternative measures for each firm-year observation: (i) the true structural output elasticities implied directly by the CES production function for the given parameters and inputs, (ii) output elasticities derived from an estimated Hicks-neutral translog specification (analogous to Eq. (10)),[33] and (iii) output elasticities inferred from observed input cost shares. The latter two are model analogs of our actual empirical strategy; the first is a theoretical benchmark to establish the fundamental identification result.

Figure 5 plots the resulting $\widehat{\sigma}$ estimates against each structural σ. The 45-degree line corresponds to perfect identification; with under- (over-)estimates to the right (left).

Panel (a) considers the baseline case with Hicks-neutral productivity and a constant labor wedge (e.g., monopsony with iso-elastic labor supply). In this setting, our growth-regression approach recovers the true (CES-based) σ reliably, even for cost shares (as growth regressions in differences eliminate the constant labor wedge). This also shows that the translog function can recover the underlying CES output and substitution elasticities.

Panel (b) allows labor wedges to increase with firm size.[34] The growth regressions themselves remain unbiased, as shown by the σ estimates based on the true structural CES output elasticities. But cost share-based elasticities are upward biased because this approach understates shadow price changes (by assuming a constant labor wedge) for a given quantity shift. Importantly, the translog output elasticities continue to recover σ.

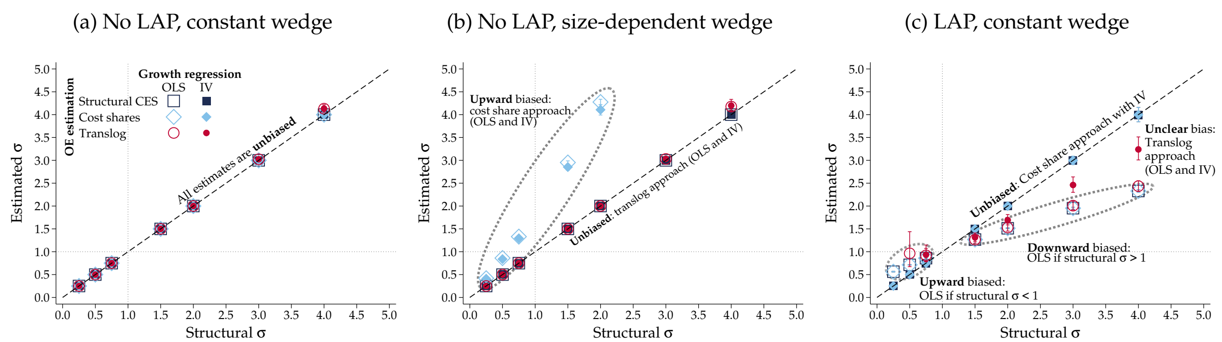
Panel (c) introduces LAP shocks (a standard concern in the literature). Here, translog-based output elasticities are biased (with ambiguous direction and magnitude), but cost share-based ones remain unbiased. Additionally, LAP shocks bias OLS growth regression *toward unity*, as also shown in Appendix D.4—a bias that disappears under IV. Hence, estimates based on output elasticities derived from cost shares combined with our IV growth regression still reliably identify σ. We discuss LAP again in Section 5.3 below.

In summary, in our baseline setting with Hicks-neutral productivity and isoelastic labor supply (as in Section 2), we can identify σ with either translog- or cost share-based output elasticities and our growth regressions. Additionally, while the translog approach always recovers σ under Hicks-neutral productivity, but fails under LAP, the cost share-based approach recovers σ under constant labor wedges and labor augmenting (or Hicks-neutral) productivity. We find it reassuring that both approaches produce qualitatively consistent results that indicate $\sigma > 1$. We prefer the translog as it yields more conservative

[33]In the simulation, productivity is observed and can therefore be controlled for directly rather than using a productivity control function.

[34]We model this by introducing an ad hoc size-dependent labor cost shifter, $\mathcal{T}(Q_{it}) = Q_{it}^{\tau}$, that scales with output: $Cost_{it} = \mathcal{T}(Q_{it}) P_{it}^{L}(L_{it}) L_{it} + P^{M} M_{it}$.

Figure 5: Monte Carlo simulation to support the identification strategy: substitution elasticities σ estimated in growth regressions (OLS and IV), using estimated output elasticities (translog, cost share, and benchmark structural CES), for three environments (labor-augmenting productivity (LAP) shocks, constant labor wedges, and size-dependent labor wedges).



Notes: The figure reports Monte Carlo simulations results (500 repeats) that illustrate identification of the labor–intermediates substitution elasticity (σ). In each firm-period, output is generated by exogenous demand and productivity shocks. Firms minimize the cost of producing given output using a given CES technology in labor and intermediates. Labor supply is upward sloping. Three scenarios are considered: (a) no labor-augmenting productivity (LAP) and a constant labor wedge, (b) no LAP and an output-size-dependent labor wedge, and (c) LAP with a constant labor wedge. The simulation generates input choices, output elasticities (translog production function and cost shares), and growth rates, and then recovers σ from growth regressions (as in Eq. (11)) using three approaches to estimate the required output elasticities: as a benchmark, the structural CES output elasticities (structural CES), cost share-based output elasticities, and translog-estimated output elasticities, each under OLS and IV. The IV uses the simulated Bartik-style demand shifter as an exogenous source of output variation. The figure plots these estimated σ values against their underlying structural values. The error bands indicate the 5^{th} and 95^{th} percentile values from the repeated Monte-Carlo simulations. For visualization purposes, we omit estimates outside the 0-5 interval (i.e., large cost share estimates in Panel (b), negative translog estimates in Panel (c)). Further simulation details are described in the text and available upon request.

(smaller) estimates of σ.[35] *Taken together, the plausible scenarios provide little support for the concern that our strategies could identify $\widehat{\sigma} > 1$ while masking a true structural elasticity $\sigma < 1$ in our firm-growth setting.*

5.2 Identification of Firm-Specific (Relative) Labor Supply Elasticities

Why do firms change their input mix as they grow? The natural candidate is the price signal. We now trace out the implied input price ratio, and translate it into an implied firm-specific labor supply elasticity—building on monopsony models.

Step 1: identifying implied input price ratio changes w.r.t. firm growth. Using our estimates of σ and our firm growth regression coefficients, we can infer the implied (average) change in the shadow price ratio (i.e., the change that is caused by firm growth). We insert our regression coefficients into Eq. (5) to back out the implied effect of firm

[35]To our knowledge, no study (and approach) can jointly address factor-augmenting productivity and size-dependent monopsony wedges. Our emphasis is on the latter, and we note that $\sigma \leq 1$ appears hard to square with our evidence (as shown below, also across different countries), the Monte Carlo simulations, and the bias suggested by OLS. Recall that our regressions always absorb industry-specific LAP shocks.

growth on within-firm changes in the shadow price ratio:

$$\Delta \ln\left(\frac{P^L_{it}\gamma^L_{it}}{P^M_{it}\gamma^M_{it}}\right) = \frac{\Delta\ln(L_{it}) - \Delta\ln(M_{it})}{-\sigma} \tag{16}$$

$$\Rightarrow \widehat{\rho}_{\Delta\ln(\frac{P^L\gamma^L}{P^M\gamma^M}),\Delta\ln(Q)} = \frac{\rho_{\Delta\ln(L),\Delta\ln(Q)} - \rho_{\Delta\ln(M),\Delta\ln(Q)}}{-\widehat{\sigma}}. \tag{17}$$

To our knowledge, our paper is the first to attempt to combine estimates of output elasticities with a standard production model to back out unobserved input price ratios.[36]

We report results for $\widehat{\rho}_{\Delta\ln(\frac{P^L\gamma^L}{P^M\gamma^M}),\Delta\ln(Q)}$ in Table 3 Column (3). OLS and IV estimates indicate implied changes in the log price ratio of 0.27 (0.23) and 0.16 (0.10) in the short (long) run, respectively. Hence, as firms grow, the shadow price of labor increases relative to that of intermediates, prompting substitution.

Potential sources of the relative increase in labor prices. One natural explanation for the wage effect of firm growth is monopsony, i.e., a low, finitely elastic firm-specific labor supply (here: compared to that of intermediates). The persistence of the (relative) wage effect over 10 years (Table 3, Column 3) reinforces this view, although the higher short run effects may reflect larger short-run adjustment costs for labor than intermediates.

Step 2: Identifying firm-specific factor supply elasticities. While our mechanism requires only increasing *relative* shadow prices for labor compared to intermediates, we find it useful to discuss implied estimates of the firm-specific labor supply elasticity. This identification requires stronger assumptions than the identification of σ above. First, to identify absolute rather than relative supply elasticities, we assume that intermediate inputs are perfectly elastically supplied.[37] Second, the markdown, γ^L_{it}, is constant in firm growth (we still permit a baseline wedge)—an assumption we later relax by measuring γ^L_{it} and studying its firm growth gradient directly. Those two assumptions imply that the input shadow price gradient is solely due to wage increases, allowing us to infer the implied firm-specific labor supply elasticities from our estimated input price changes and

[36]Using Eq. (15) in Eq. (17), the predicted shadow price ratio changes can be expressed as: $\widehat{\rho}_{\Delta\ln\left(\frac{p^L_{it}l^L_{it}}{p^M_{it}m^M_{it}}\right),\Delta\ln(Q)} = \left(\rho_{\Delta\ln(\theta^L_t),\Delta\ln(Q)} - \rho_{\Delta\ln(\theta^M_t),\Delta\ln(Q)}\right) - \left(\rho_{\Delta\ln(L_t),\Delta\ln(Q)} - \rho_{\Delta\ln(M_t),\Delta\ln(Q)}\right)$.

[37]We are not aware of comparable estimates of firm-specific supply elasticities for intermediate inputs. Bilal and Lhuillier (2022) make a similar assumption in a labor-only model with labor service purchases (although our intermediates are largely made of goods), while Dobbelaere and Mairesse (2013) and subsequent studies assume perfectly elastic intermediate supply when backing out markdowns and implied labor supply elasticities. One exception is Treuren (2025), who studies input wedges for labor and intermediates in a rent-sharing setting.

the employment effects:

$$\widetilde{\epsilon}^L = \frac{\Delta \ln(L_{it})}{\Delta \ln\left(\frac{P^L_{it}\gamma^L_{it}}{P^M_{it}\gamma^M_{it}}\right)} = \frac{\Delta \ln(L_{it})}{\Delta \ln(P^L_{it})}, \tag{18}$$

which we, again, identify feeding in the coefficient analogs from the growth regressions:

$$\Rightarrow \widehat{\widetilde{\epsilon}}^L = \frac{\rho_{\Delta \ln(L), \Delta \ln(Q)}}{\widehat{\rho}_{\Delta \ln(\frac{P^L \gamma^L}{P^M \gamma^M}), \Delta \ln(Q)}}. \tag{19}$$

That is, we identify the labor supply elasticity, $\widetilde{\epsilon}^L$, by dividing changes in labor quantity ratios (Table 3 Column (1)) by changes in input price ratios (Column (3)). Hence, $\widetilde{\epsilon}^L$ is the *inverse* of the wage-employment elasticity, ε^L, in Section 2. Moreover, $\widehat{\rho}$ is not a regression coefficient but the inferred input shadow price ratio gradient from Eq. (17) above.

Results: implied firm-specific labor supply elasticities. We report implied labor supply elasticities, ϵ^L, in Table 3. The values range from 1.10 to 2.56 for our OLS and from 2.04 to 5.20 for our IV results. The higher long-run than short-run elasticities either reflect horizon-dependence of labor supply elasticities, or may reflect firm-side adjustment costs (non-constant γ^L_{it}). Nevertheless, even in the long run, supply elasticities remain relatively low. These estimates showcase the quantitative basis for the monopsony-driven incentive for firms to shift from labor to intermediate inputs.

Robustness: permitting markdowns and direct wage estimates. By using the shadow price of labor (the product of the wage and wage markdown), our initial labor supply elasticity estimates may be confounded by markdown variation. We now calculate firm-specific labor supply elasticities under two alternative assumptions. We report these alternative values in our meta-analysis in Figure 4 Panel (b).

First, we report results that subtract the measured (relative) markdown effect on $\gamma^L_{it}/\gamma^M_{it}$ that we estimate in Section 6 below. As we find that (relative) markdowns *increase* as firms grow, netting them out raises labor supply elasticities.

Second, we also measure wages directly, using the average firm wage (wage bill per head), and run regressions of wage changes on output growth as before—although we caveat that average wages are subject to composition bias.[38] These additional wage regressions are reported in Appendix Table A.9. Using OLS, average wages increase in firm

[38]We cannot merge matched employer-employee data or data on worker skills to our firm data to use cleaner wage measures. For instance, over a comparable period, Lochner et al. (2020) report a German large-firm wage premium of 10–11 log points, of which 4–6 points are due to person fixed effects, which would suggest higher labor supply elasticities. However, we also note that these regressions capture long-run elasticities. There is an ongoing debate about the level of the elasticity.

(employment) growth (consistent with monopsony). Since those wages move less than the inferred shadow price ratios above, the implied firm-specific labor supply elasticities are again higher. The IV point estimates for wages are close to zero; hence, the associated elasticities would be large, but the wider confidence intervals for the wages also accommodate elasticities more consistent with the literature.

Meta-analysis of existing parameter estimates and our full set of estimates. Figure 4 Panel (b) summarizes existing estimates of labor supply elasticities based on the meta-analysis of Sokolova and Sorensen (2021), along with our identified values (including the robustness checks discussed above). While estimates are ranked by size, for our own estimates, we provide robustness check results vertically stacked. Overall, our estimates fall well into the range of existing estimates and highlight a lower short- than long-run labor supply elasticity. Supply elasticities are higher for our IV estimates, and when we net out markdown shifts (squares) or use direct (average) wage estimates (dots).

5.3 Discussion: Alternative Mechanisms

Automation. Recent work has highlighted *capital*-based technological change, such as automation, as a factor reducing labor's importance to firms (Acemoglu and Restrepo, 2020, Hubmer and Restrepo, 2021). Such technological change could not explain the firm growth patterns we study, specifically, capital output elasticities fall with firm growth and instead intermediate input output elasticities rise.

Unobserved factor-augmenting productivity shocks. An important concern in the production function literature is the presence of unobserved factor-augmenting productivity shocks (typically labor-augmenting productivity, LAP). In our setting, LAP may matter through two channels: it may bias translog-based output elasticities, and it may bias the growth regression-based identification of σ. We argue that neither channel is likely to overturn our conclusion that labor and intermediates are substitutes.

We address the first concern (biased output elasticities) by additionally drawing on cost share-based output elasticities (which are robust to LAP) and by assessing the bias in a Monte Carlo simulation (Figure 5 Panel (c)). If anything, cost share-based estimates yield even *higher* σ (Appendix Table A.8).

We address the second concern (bias in growth regressions) in three ways. Intuitively, LAP positively correlated with firm growth, rather than our mechanisms based on substitution under Hicks-neutral productivity in response to increasingly costly labor, could only explain the shift from labor to intermediates if $\sigma < 1$.[39]

[39]Formally, to see the role of the condition $\sigma < 1$, note that, under LAP and constant input prices, it holds

First, our specifications use within-firm changes and industry-year fixed effects, absorbing persistent firm-specific technology differences and industry-level LAP shocks. Second, the bias from firm-specific LAP can be viewed as an omitted variable bias (OVB) in the growth regressions and the estimating equation for σ (see Appendix D.4). Our IV strategy addresses this OVB by isolating output variation from export demand shocks, which are also plausibly orthogonal to LAP (and other confounders).[40] Third, in Appendix D.4, we draw on our structural model equations and regression arguments to further examine the role of LAP. While we do back out a (moderate) degree of LAP in firm growth, we conclude that it is unlikely to explain our findings and, if anything, introduces a *downward* OVB in our estimate of $\sigma^{OLS} > 1$ (with the true σ hence lying above one).

Fixed costs. If production involves fixed labor costs (or shared labor inputs) that scale up less than proportionately with flexible labor, output elasticities with respect to total labor should decline as firms grow. Effectively, fixed costs break the homotheticity of a production function (Dhyne et al., 2022, Savagar and Kariel, 2026). Rather than adjudicating between fixed costs and our substitution mechanism, we cautiously conclude that the data leave substantial scope for substitution to account for our facts—particularly as fixed costs likely play a smaller role after 10 years. We study the role of fixed costs in two tests.

First, we estimate our effects by size-groups (Appendix Tables A.6 and A.7) and find similar patterns for labor and intermediates. As fixed costs are more relevant for small firms (e.g., fixed overhead costs), the stability of results across firm size groups may leave little room for this explanation.[41] While we do not find clear evidence for differential labor adjustments, Appendix Table A.6 shows that declines in capital output elasticities are moderated in larger firms, consistent with initial capital fixed costs.

Second, to quantify the fixed cost share needed to rationalize our regression results absent our substitution mechanism, we formalize a basic fixed-cost model in Appendix D.7. For 1-year changes, our OLS (IV) estimates imply a required fixed labor cost share in total labor costs of 0.51 (0.50) for the average firm, while for 4-year changes, the required fixed labor cost share is 0.28 (0.31).[42]

that $\Delta \ln (L_{it}/M_{it}) = (\sigma - 1)\Delta \ln \left(\alpha^L_{it}/\alpha^M_{it}\right)$, where $\alpha^L_{it}/\alpha^M_{it}$ denotes LAP (similarly for output elasticities, see Eq. (C.18) in Appendix D.4).

[40] The logic echoes that of more standard input price instruments (e.g., Raval, 2019, Huneeus et al., 2022).

[41] Relatedly, in a tentative empirical check, our data indicate a stable wage-size (output) gradient in logs, which does not suggest obviously higher monopsony power among larger firms (Appendix Figure A.1).

[42] These numbers are huge because they refer to the share of fixed labor in total labor over a given horizon. To put this into perspective: using U.S. Census data for manufacturing plants during 1974-2011, Ederhof et al. (2021) define depreciation, rental payments, fringe benefits, *and even non-production worker salaries* as fixed cost and report a fixed cost share in total cost of 20%. They do not specify the time horizon over which these costs are fixed; we believe that over our multi-year horizons many of these cost components—such as non-production worker salaries—are adjustable. We note that a definitive benchmarking remains an open

Importantly, the model also clarifies that, at least in its basic version, labor fixed costs (or broader fixed costs) cannot rationalize the positive *absolute* (rather than relative to labor) effect of firm growth on intermediate output elasticities. They can solely account for an absolute decline in labor output elasticities.

Non-homothetic CES. Our paper rationalizes changes in relative output elasticities through changes in relative input quantities (and prices). Alternatively, intermediate inputs might become more efficient at larger scale, such that firms' relative labor output elasticities decline as a *direct* result of firm growth. To explore this possibility, we adapt a version of the production function in Lashkari et al. (2024) (who study how non-homotheticities in the production function can explain why larger firms are more IT-intensive) to study the role of non-homotheticities in our context:

$$Q_{it} = \Omega_{it}\Lambda^K K_{it}^{1-\kappa}\left(\Lambda^{LM}\alpha^L L_{it}^{\frac{\sigma-1}{\sigma}} + \Lambda^{LM}\alpha^M\left(\frac{M_{it}}{Q_{it}^{\eta}}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\kappa}, \tag{20}$$

where $\eta > -1$ is a firm-size scaling parameter that captures the non-homotheticity. If $\eta = 0$, Eq. (20) collapses to Eq. (1). Under this non-homothetic CES, the ratio of output elasticities now features an additional *direct* size effect shaped by η (see Appendix D.5):[43]

$$\frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha^L}{\alpha^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}} Q_{it}^{\frac{-\eta}{\sigma}}. \tag{21}$$

Eq. (21) suggests a mediation analysis of the direct output effect (non-homotheticity) versus the input factor ratio (our substitution channel) in regulating the observed changes in output elasticities: a "horse race" regression of log changes in the labor-intermediate output elasticity ratio on log changes in input quantity ratios *and* on log changes in output. We also run a regression using cost shares as dependent variables instead of output elasticities.[44] Appendix Table A.10 reports results from this exercise. Once we control for the input factor ratio, there is no statistically or economically significant negative effect of output growth on relative output elasticity and cost share ratio changes. This informal test points away from a significant non-homotheticity in output and toward the substitution channel as the key force. We also reiterate the stable estimates across size groups.

question due to lack of a consensus estimate of fixed cost shares.

[43]The translog production function permits general non-homotheticities. Here, we focus on a specific form of non-homotheticity. This can be more broadly interpreted as testing the role of a direct output dependence (non-homotheticity) vs. changing input ratios as mediator.

[44]Translog output elasticities depend directly on input levels, which could limit their variation once we include input quantity ratios as regressors. Our alternative analysis of cost shares as a measure for output elasticities is not subject to this concern.

6 Implications for Firm-level Labor Shares

A key implication of the substitution from labor to intermediates is the decline in growing firms' labor shares through production function properties—rather than higher markdowns. (Monopsony remains relevant, insofar as it initiates this substitution by increasing the cost of labor.)

6.1 Labor Share Decomposition and Identification

The labor share. Building on the framework in Section 2, we use Eq. (7) to decompose firms' labor shares in output into output elasticities, markups, and markdowns:

$$\ln(LS_{it}) = \ln(\theta_{it}^{L}) - \ln(\mu_{it}) - \ln(\gamma_{it}^{L}). \tag{22}$$

Identifying price markups and wage markdowns. To separate the three determinants of the labor share, we construct firm-year-specific measures of markups and markdowns from our translog production function estimation. We derive output price markups using the production approach (Hall, 1986, De Loecker and Warzynski, 2012). Assuming that $\gamma_{it}^{M} = 1$, i.e., intermediate inputs are flexible and that firms take their price as given, the first-order condition for intermediates yields (see Appendix D.6):

$$\mu_{it} = \frac{P_{it}}{\lambda_{it}} = \theta_{it}^{M} \frac{P_{it} Q_{it}}{P_{it}^{M} M_{it}}, \tag{23}$$

where μ_{it} is the markup. Following existing work (e.g., Dobbelaere and Mairesse, 2013, Yeh et al., 2022), we derive firms' wage markdowns, γ_{it}^{L}, by combining the first-order conditions for intermediates and labor, again assuming that $\gamma_{it}^{M} = 1$ (see Appendix D.6):

$$\gamma_{it}^{L} = \frac{\theta_{it}^{L}}{\theta_{it}^{M}} \frac{P_{it}^{M} M_{it}}{P_{it}^{L} L_{it}}. \tag{24}$$

Importantly, if $\gamma_{it}^{M} \neq 1$, Eq. (24) recovers the relative markdown or wedge $\widehat{\gamma}_{it}^{L} = \frac{\gamma_{it}^{L}}{\gamma_{it}^{M}}$, which determines the relative shadow price of labor versus intermediates. When focusing on within-firm changes, this becomes $\Delta \ln \widehat{\gamma}_{it}^{L} = \Delta \ln \gamma_{it}^{L} - \Delta \ln \gamma_{it}^{M}$. As a result, any *constant* markdown on intermediates drops out in our growth-based setting. A similar logic holds for the markup Eq. (23) and for unobserved factor-augmenting productivity levels.

Total labor wedge. Finally, as a complement, we also study the total labor wedge, which is the combined distortion from markup and labor market imperfections (this is sometimes

Table 4: Firm growth and labor share, markup, markdown, and output elasticity changes.

	$\Delta\ln(LS_{it})$	$\Delta\ln(\mu_{it})$	$\Delta\ln(\gamma_{it}^L)$	$\Delta\ln(\mu_{it}\gamma_{it}^L)$	$\Delta\ln\left(\frac{\theta_{it}^L}{RTS_{it}}\right)$	$\Delta\ln(\theta_{it}^L)$	$\Delta\ln\left(\frac{P_{it}^L L_{it}}{VA_{it}}\right)$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: 1-year diff. (29,950 firms; 183,813 firm-year observations							
OLS estimates							
Log output change	-0.581***	0.124***	0.159***	0.283***	-0.304***	-0.298***	-0.561***
	(0.0040)	(0.0025)	(0.0048)	(0.0044)	(0.0050)	(0.0046)	(0.0076)
R^2	0.534	0.096	0.115	0.191	0.230	0.215	0.190
IV estimates							
Log output change	-0.674***	0.186***	0.169**	0.355***	-0.327***	-0.318***	-0.842***
	(0.0521)	(0.0418)	(0.0759)	(0.0697)	(0.0623)	(0.0645)	(0.120)
R^2	0.524	0.084	0.115	0.185	0.229	0.215	0.160
First-stage F-statistic	102.6	102.6	102.6	102.6	102.6	102.6	102.6
Panel B: 4-year diff. (11,492 firms; 70,936 firm-year observations							
OLS estimates							
Log output change	-0.406***	0.0608***	0.160***	0.220***	-0.202***	-0.185***	-0.334***
	(0.0061)	(0.0032)	(0.0065)	(0.0055)	(0.0060)	(0.0063)	(0.0081)
R^2	0.461	0.126	0.193	0.239	0.220	0.201	0.188
IV estimates							
Log output change	-0.443***	0.154***	0.0688	0.223***	-0.242***	-0.220***	-0.603***
	(0.0726)	(0.0444)	(0.0800)	(0.0713)	(0.0706)	(0.0742)	(0.119)
R^2	0.459	0.072	0.180	0.239	0.217	0.199	0.135
First-stage F-statistic	48.57	48.57	48.57	48.57	48.57	48.57	48.57
Panel C: 10-year diff. (10,595 firms; 49,915 firm-year observations							
OLS estimates							
Log output change	-0.303***	0.0383***	0.152***	0.191***	-0.140***	-0.112***	-0.231***
	(0.0062)	(0.0030)	(0.0060)	(0.0048)	(0.0053)	(0.0057)	(0.0068)
R^2	0.378	0.125	0.213	0.253	0.188	0.161	0.178

Notes: The table reports OLS and IV regressions estimating the specification in Eq. (11). The dependent variables in Columns (1)–(7) are log changes of the labor share in sales, markups, wage markdowns, labor wedge, labor output elasticity divided by returns to scale, labor output elasticity, and labor share in value added, respectively. Panel A reports 1-year differences and Panel B reports 4-year differences, each showing OLS and IV estimates. Panel C reports 10-year differences (OLS only). All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

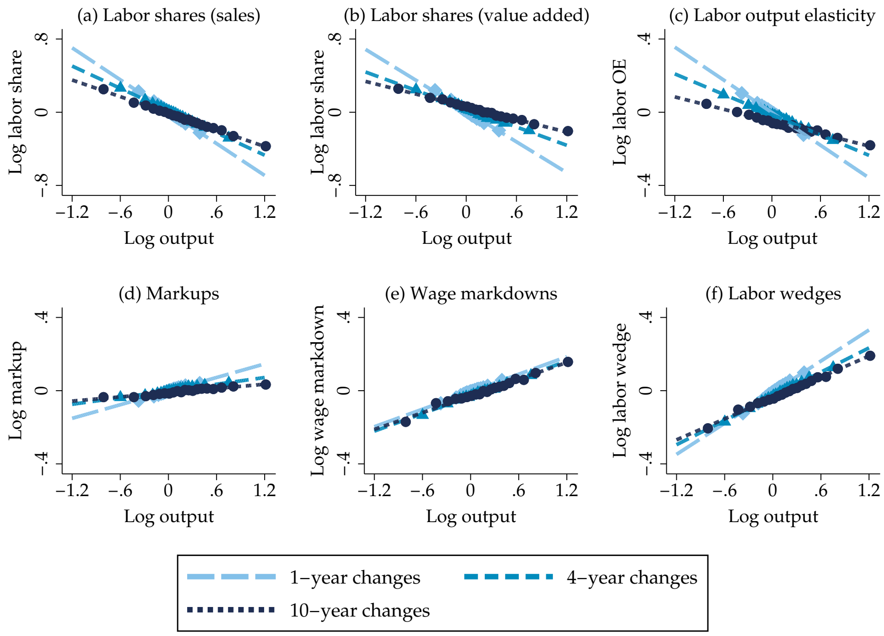
used as a markup measure, assuming perfect labor markets):

$$\mu_{it}\gamma_{it}^L = \theta_{it}^L \frac{P_{it}Q_{it}}{P_{it}^L L_{it}}. \tag{25}$$

6.2 Implementation and Results

Strategy. To measure and decompose the effects of firm growth on firm-level labor shares, we estimate the regression model in Eq. (11) for log changes in labor shares, markups, markdowns, and labor wedges as outcomes. While we focus on the sales labor share (as it corresponds to the output elasticity of labor and our firm-level production function framework), we also include the value added labor share ($\frac{P_{it}^L L_{it}}{VA_{it}}$) as a robustness check (value added, VA_{it}, is revenue minus intermediate input expenditures).

Figure 6: Firm growth and labor share, markup, markdown, and output elasticity changes (OLS, binned scatter plots).



Notes: The figure reports binned scatter plots estimating the specification in Eq. (11) with OLS for various differences. The graphs report binned scatter plots for changes in the logs of labor shares in sales and value added, labor output elasticities (not divided by returns to scale), markups, wage markdowns, and the labor wedge against log output changes. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Results. Table 4 reports the OLS and IV results by time horizon. We include our previous estimates on labor output elasticities to facilitate the analysis. As indicated in Eq. (22), the coefficients on markups, wage markdowns, and output elasticities *sum to* that on the labor share. As before, Figure 6 visualizes the variation underlying the regression estimates.

Declining labor shares. We find a strong negative effect of firm growth on labor shares. OLS results for sales labor shares range from -0.581 to -0.303 for 1- to 10-year differences. Results for value added labor shares, as well as IV estimates, are similar. Again, as for output elasticities, effects somewhat shrink as we widen the time horizons.

Half to one third of the effect: declining output elasticity of labor. In the short and medium runs (1 and 4 years), the output elasticity coefficient is about half the size of the labor share coefficient. Hence, the declining output elasticity of labor drives *half* of the negative effect of output growth on labor shares. For 10 year differences, it still accounts for one third of the effect (as returns to scale shifts become more important).

The remainder: conventional market power effects. Higher markups and wage markdowns account for the remaining half (short to medium run) or two thirds (long run) of the effect of firm growth on labor shares. This pattern would be consistent with some oligopsony models, in which a firm's product demand or labor supply elasticities fall in its product or labor market shares (e.g., Atkeson and Burstein, 2008; Berger et al., 2022).[45]

7 Other Sectors and Countries, and Industry Aggregation

We close our paper by extending the analysis to 11 additional European countries with similar administrative micro data and to aggregate industry data using a micro-aggregated dataset for 20 European countries and for the U.S.

7.1 International Industry Panel Data: CompNet Data

CompNet industry-level data. We use the 9th vintage of the Competitiveness Research Network (CompNet) data (CompNet, 2023), which is a population-weighted micro-aggregated database for 22 European countries.[46] We use the country-industry-level data (NACE Rev. 2, two-digit industries), which is the most detailed aggregation level available. The data contain, among other features, country-industry-level information on firms' sales, inputs, expenditures, markups, wage markdowns, and output elasticities (estimated with a translog production function that does not incorporate firm-level price data, see CompNet, 2023). Values are available as size-weighted and unweighted averages at the industry level. We rely on size-weighted averages. The data cover 1999-2021, and we focus on manufacturing (NACE Rev. 2 two-digit industries 10-33) which has the best coverage. Yet, we also present key results for non-manufacturing industries.[47] Yearly coverage varies across countries (see Appendix Table A.11—where we provide CompNet summary statistics). We focus on the data with firms with at least 20 employees, which is available for more countries and consistent with our German micro data (we confirmed robustness to including smaller firms wherever covered). For further details, see CompNet (2023).

[45]In the cross section (see Appendix Figure A.2), market power effects appear more important, consistent with Autor et al. (2020) and De Loecker et al. (2020), although we further decompose the standard total labor wedge into markup and markdown components.

[46]CompNet sources its data from representative administrative firm-level records located within European national statistical institutes and central banks (akin to the U.S. Census data). The CompNet team distributes harmonized data collection protocols (i.e., Stata codes) across the data providers and invests significant efforts in harmonizing the input data to maximize comparability across countries. These protocols compute micro-aggregated results. From these results, the CompNet team constructs the CompNet database. The data are aggregated at various levels. We drop Malta, due to insufficient observations for output elasticities, and the UK, which did not provide industry-level data to CompNet.

[47]This includes the NACE rev. 2 industries 41-43 (construction), 45-47 (wholesale/retail trade and repair of motor vehicles and motorcycles), 49-53 (transportation/storage), 55-56 (accommodation/food services), 58-63 (information and communication technology), 68 (real estate), 69-75 (professional/scientific/technical activities), and 77-82 (administrative/support service activities).

U.S. NBER CES data. For the U.S., we draw on the NBER-CES Manufacturing Industry Database for a limited analysis (which lacks output elasticities) for 1958-2016 (after which capital, required to construct cost shares, is missing).

Commissioning our own firm-level regressions in the CompNet micro data. To replicate our firm-level growth regressions, we also collaborated with the CompNet team to replicate our firm-level main OLS regressions in 11 countries (listed in Figure 7) and extend our initial analysis beyond manufacturing and for different firm age groups.[48]

7.2 Firm-level Results for 11 other European Countries

Figure 7 summarizes the results of our replication of the firm-level manufacturing sector analysis (OLS) for the 11 countries using the CompNet infrastructure. Panel (a) reports the *ratio* of regression coefficients for labor and intermediate input quantities, cost shares, and output elasticities. Panel (b) reports implied substitution elasticities and the contribution of changes in labor output elasticities to declines in firm-level labor shares with firm growth. Substitution elasticities are reported based on output elasticity (OE) and cost share (CS) coefficients. To facilitate comparison, we include our previous results for German manufacturing firms. Appendix Table A.12 details all underlying regression results and additionally reports results including non-manufacturing sectors.

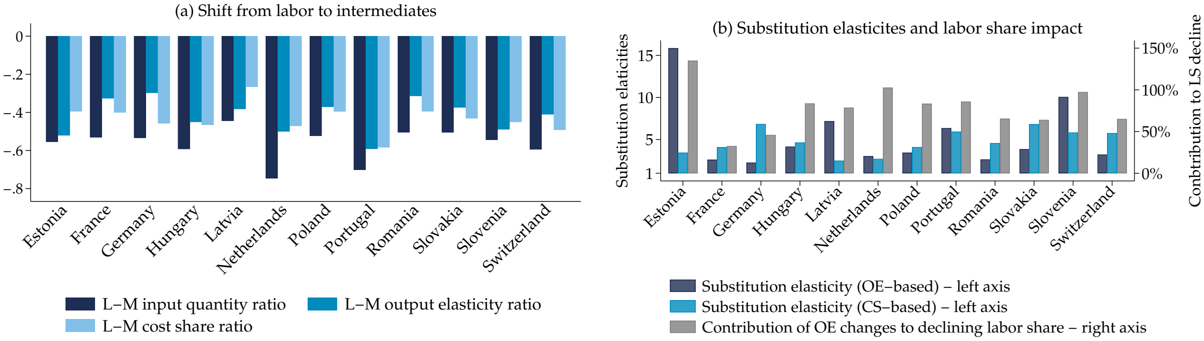
Manufacturing. Our firm-level findings from Germany are confirmed across all 11 other countries: input quantities, cost shares, and output elasticities shift from labor to intermediates, with substitution elasticities consistently exceeding unity. The magnitudes of the shift vary somewhat across countries (we are not aware of firm-specific labor supply elasticity estimates by country). Also, labor output elasticities decline more sharply with firm growth in most countries and, therefore, contribute more strongly to declining labor shares in growing firms, compared to our German results.

All sectors. The patterns extend to all sectors rather than just manufacturing, which our Germany-based analysis was restricted to. See Appendix Table A.12.

Young and mature firms. In six of the eleven countries, data on firm age is available (but not in Germany). In Appendix Table A.13, we replicate our analysis for young (five years or younger) and mature manufacturing firms. The results are similar. If anything, the shift from labor toward intermediates appears somewhat stronger for mature firms. Similar findings by age hold if considering all sectors (not reported).

[48]Specifically, CompNet incorporated these regressions into their *10th* vintage data collection. The 10th vintage data include a different sample of countries and covers more recent years than the 9th vintage country-industry data (we document yearly coverage in regression tables). Due to the long running time of codes, we have only received results for the subset of countries reported on below.

Figure 7: Country-specific results from firm-level regressions (manufacturing).



Notes: The figure reports OLS estimates of the specification in Eq. (11) for different countries (manufacturing sector firm micro data). Panel (a) reports ratios of coefficients on labor and intermediate input quantities, cost shares, and output elasticities. A negative value indicates a shift from labor to intermediates. Panel (b) reports implied substitution elasticities (left axis) based on the shifts in input quantity ratios and (i) output elasticity ratios (OE-based), and (ii) cost share ratios (CS-based). Additionally, Panel (b) reports the contribution of changes in firms' output elasticities to labor shares with firm growth (right axis). This is computed by dividing regression coefficients on output elasticities by regression coefficients on labor shares (as in Section 6.2). Appendix Table A.12 reports the underlying regression results in more detail and by outcome variable. Results for Germany come from our previous firm-level analysis on the German firm-level data. Other results are based on firm-level data from CompNet data providers.

7.3 Industry-level Dynamics in 20 European Countries and the U.S.

We now study the industry-level analog of our firm-level analysis—providing an aggregate perspective that will remain suggestive. Many caveats apply: aggregation biases, reallocation, compositional changes, intra-industry intermediate input patterns now being absorbed, and differences between market- and firm-specific labor supply elasticities.

European industry growth regressions. We focus on manufacturing industries. We regress changes in log input quantities, log cost shares, and log output elasticities on industry output growth in the aggregated *country-industry* data provided by CompNet. The horizons are 1-, 4-, and 8-year changes (10-year changes are not feasible with CompNet data), controlling for country-year and industry-year fixed effects as we pool country-industry pairs (results are similar with country-industry fixed effects.)

European pooled results. Figure 8 presents manufacturing industry-level results (OLS) akin to our previous firm-level results. Our analysis shows that the relationships observed in firm-level data also hold at the European industry level: as industries expand, labor-intermediate ratios, labor cost shares, and labor output elasticities decrease, while intermediate cost shares and output elasticities rise. The point estimates are precisely estimated and indicate that 10% higher industry output growth leads to a 0.5% reduction in the labor output elasticity (adjusted for RTS changes).[49]

[49]Without taking logs, coefficients on industry-level labor, capital, and intermediate output elasticities divided by returns to scale are -0.0112, -0.00003, and 0.0148, respectively (4-year changes). Hence, the increasing intermediate input output elasticities are largely rationalized by declining labor output elasticities.

Europe and the U.S.: Country-specific results. Appendix Table A.14 presents results for changes in industry-level labor-intermediate quantity, output elasticity, and cost share ratios in response to industry growth, based on country-specific *regression coefficients* from reproducing Figure 8 by countries (4-year changes). As before, we estimate substitution elasticities using Eq. (15), first, based on coefficients on output elasticities from our industry growth regressions, and second, using cost share coefficients instead. U.S. values are derived from running the same regressions that we run in the European CompNet data with the NBER-CES Manufacturing Industry Database (1958-2016) using cost shares (the data do not include output elasticities).[50]

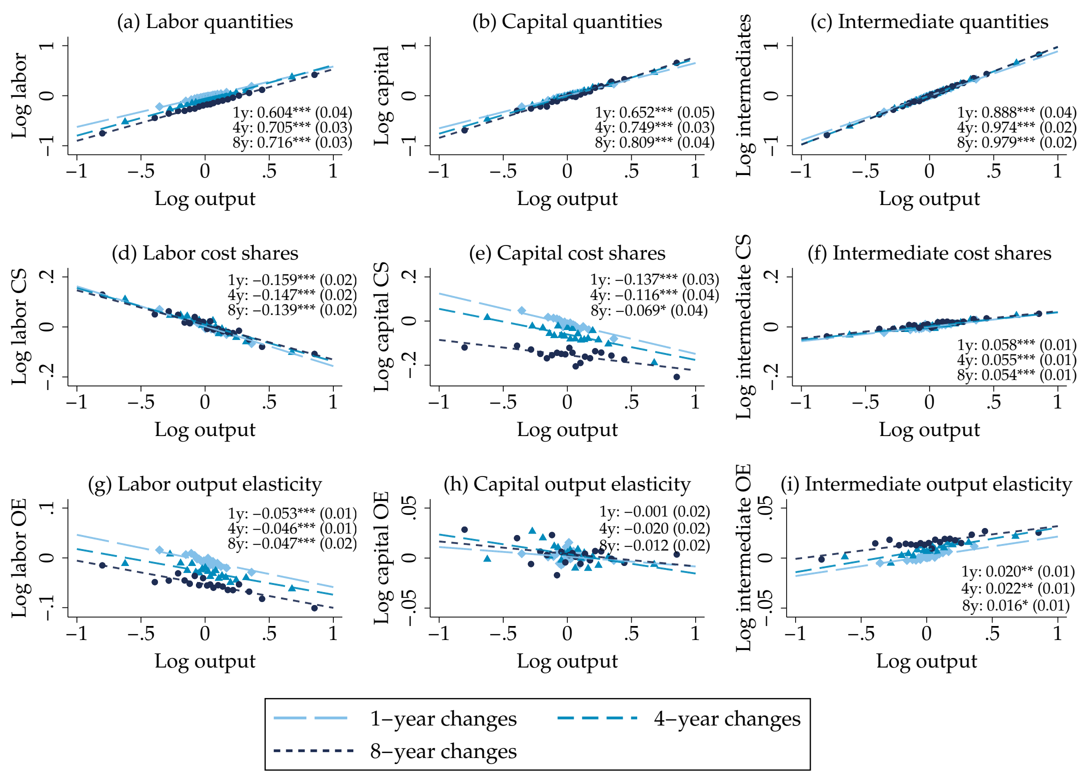
In almost all countries, industry growth is associated with a shift from industry-level labor to intermediates in terms of quantities, cost shares, and output elasticities (with some quantitative differences). Industry-level labor-intermediate substitution elasticities exceed unity in almost all countries. The U.S. data lack output elasticity estimates, but changes in labor and intermediate quantities and cost shares are similar in Europe and the U.S (Appendix Table A.14). Also, substitution elasticities based on cost shares are similar—approximately 4 in both regions. The substitution elasticity estimate based on directly computed output elasticity movements in Europe is 1.34 for manufacturing industries and 1.48 for non-manufacturing industries.

Europe: Industry labor share implications. In Figure 9, we now attempt a suggestive industry-country analog of the labor share analysis. We caveat that fully drawing aggregate implications would necessitate input-output network or open-economy perspectives. Qualitatively, the patterns align with the firm-level ones: as industries grow, industry labor shares decline. The largest contribution is the reduction in labor output elasticities. Importantly, markups somewhat *decline*, while markdowns are approximately unchanged. These changes in markups and markdowns are inconsistent with the declining labor share in industry growth. Hence, despite the caveats in the industry-level aggregation, in Figure 9, the decline in the labor output elasticity appears as the only consistent pattern that aligns with the observed changes industry labor shares. Quantitatively, the slope of the aggregate output elasticity is meaningful and about a fifth of the slope of the labor share in sales (4-year specification). Importantly, due to aggregation bias, we do not expect that coefficients on output elasticities, markups, and wage markdowns sum to labor share coefficients (as they did in our firm analysis), and our analysis remains suggestive.[51]

[50]For U.S. cost shares, capital costs (which are ultimately not relevant for studying relative shifts from labor to intermediates) are approximated as 8% of the real capital stock, as nominal values are not reported; labor and material expenditures are reported in nominal terms.

[51]Aggregation biases may arise from compositional effects, weighting choices, Jensen's inequality, intra-industry trade, and shifts in labor demand and supply. In addition to classical aggregation biases, note

Figure 8: Europe: Industry-level adjustments in response to industry growth (OLS).



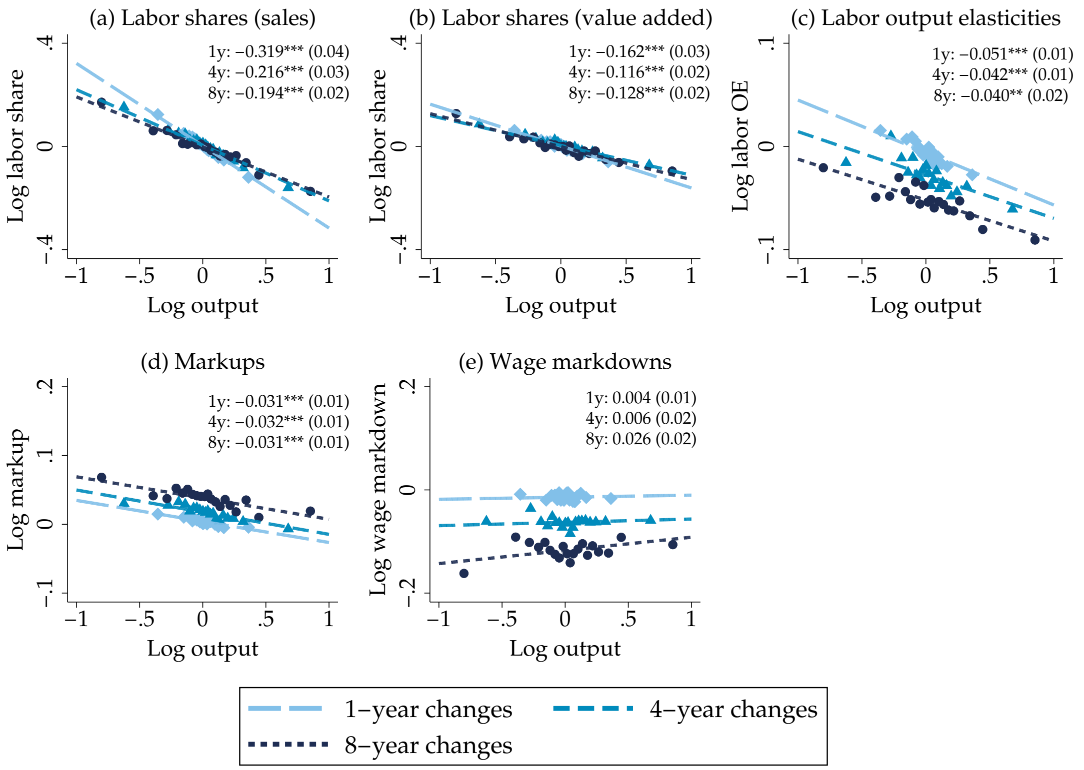
Notes: The figure reports binned scatter plots from estimating industry versions of the specification in Eq. (11) with OLS for various differences. Panels (a)-(i) report on regressions of changes in the logs of country-industry-level labor, capital, and intermediate quantities, country-industry-level labor, capital, and intermediate cost shares, and country-industry-level labor, capital, and intermediate output elasticities over returns to scale against country-industry-level log output changes, respectively. All panels control for country-year and industry-year fixed effects and we pool country-industry pairs. Country-industry observations: 5,233 (1-year), 4,207 (4-year) 2,904 (8-year). CompNet manufacturing data.

8 Conclusion

We have documented and dissected a regularity about firm growth: the shift from labor to intermediate inputs in firms' production. As labor and intermediates function as substitutes, this shift in the input mix reduces (increases) the output elasticity of labor (intermediates). As a result, as firms grow, their labor shares fall, and their intermediate input shares increase, with our novel production-function-based account being the pri-

that under competitive markets, the industry labor share equals the industry output elasticity of labor: $\sum_n s_{it} LS_{it} = \sum_n s_{it}\theta^L_{it}$, where s_{it} is the sales weight (i.e., the weight used in our industry aggregates in the CompNet data). Under imperfect markets, the pass-through from changes in industry output elasticities of labor to changes in industry labor shares is additionally shaped by the distribution of firms' markups and wage markdowns. Nevertheless, qualitatively, it remains true that the partial effect of a change in the aggregate labor-output elasticity on the labor share, holding fixed market imperfections, is positive. Equally, the partial effect (ceteris paribus) of changes in industry markups and wage markdowns on industry labor shares is negative. Importantly, "holding fixed" refers here not only to aggregates, but also to firm distributions.

Figure 9: Europe: Labor share, markup, markdown, and output elasticity changes in response to industry growth (OLS, binned scatter plots).



Notes: The figure reports binned scatter plots estimating industry versions of Eq. (11) for various differences (OLS). The graphs relate changes in logs of country-industry-level labor shares in sales and value added, labor output elasticities (not divided by returns to scale), markups, and wage markdowns to country-industry-level log output changes. All regressions control for country-year and industry-year fixed effects. Country-industry observations: 5,233 (1-year), 4,207 (4-year) 2,904 (8-year). CompNet data.

mary driver rather than markups or wage markdowns. We establish these patterns using OLS and IV regression in rich German firm-level micro data, administrative firm-level data from 11 other countries, and micro-aggregated industry data for 20 European countries. We rationalize the facts with a parsimonious production function framework characterized by (i) an elasticity of substitution between intermediates and labor that exceeds one, and (ii) an increasing shadow price of labor (e.g., due to monopsony or adjustment costs). The latter is the incipient mechanism that motivates firms to substitute away from labor.

The findings also have broader implications. For instance, many current estimates of factor misallocation and productivity rely on Cobb-Douglas production functions, which assume constant output elasticities. Moreover, our results imply that, generally, any shocks that affect output growth will alter output elasticities, input mixes, and cost shares, with large magnitudes for horizons of up to 10 years. Ignoring this regularity may confound analyses of firm-level effects from productivity shocks, trade, competition, or subsidies on various related outcomes.

Our findings also show how monopsony not only distorts steady state firm sizes but also firm growth. We trace how firms respond to these growth constraints by intensifying their use of intermediate inputs—i.e., by outsourcing production—for which we estimate a high elasticity of substitution with labor. In our case, these intermediate inputs are supplied by other firms and are largely product components and materials (rather than, e.g., temporary agency work). It thus appears that monopsony lengthens the supply chain as firms, or, more broadly, industries and perhaps the aggregate economy, grow.

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Online Appendix

A Additional Tables

Table A.1: Summary statistics of the German manufacturing sample.

	Mean (1)	p25 (2)	Median (3)	p75 (4)	St.Dev. (5)	Obs. (6)
Number of employees	366.63	53	109	272	2559.55	183,813
Real wage (1995 values)	34,162	25,972	33,823	41,485	11,492	183,813
Labor share (sales)	0.30	0.21	0.29	0.38	0.12	183,813
Labor share (value added)	0.80	0.63	0.76	0.88	2.89	183,813
Labor cost share	0.31	0.22	0.30	0.39	0.13	183,813
Capital cost share	0.06	0.03	0.05	0.07	0.04	183,813
Intermediates cost share	0.63	0.54	0.64	0.73	0.14	183,813
Materials, energy, ext. components cost share	0.41	0.29	0.41	0.53	0.17	183,813
Merchandise cost share	0.05	0.00	0.00	0.05	0.10	183,813
Subcontracted production by other companies cost share	0.03	0.00	0.00	0.03	0.06	183,813
Repairs, maintenance, installations cost share	0.02	0.01	0.03	0.04	0.02	183,813
Rents, leases, leasing cost share	0.03	0.01	0.02	0.04	0.03	183,813
Temporary agency worker cost share	0.01	0.00	0.00	0.01	0.03	164,410
Other intermediates cost share	0.10	0.05	0.08	0.12	0.06	183,813
Markup	1.09	0.97	1.05	1.17	0.20	183,813
Wage markdown	1.08	0.71	0.96	1.32	0.55	183,813
Output elasticity of labor	0.31	0.23	0.31	0.38	0.11	183,813
Output elasticity of capital	0.12	0.08	0.11	0.15	0.06	183,813
Output elasticity of intermediates	0.64	0.57	0.64	0.71	0.10	183,813
Returns to scale	1.06	0.98	1.05	1.13	0.12	183,813

Notes: This table presents summary statistics for selected variables from the German manufacturing sector firm-level data. Columns (1)-(5) show the mean, 25^{th} percentile, median, 75^{th}, and standard deviation, respectively. Column (6) reports the number of non-missing observations. German micro-data.

Table A.2: Firm-level adjustments in heads and full time equivalents to firm growth (OLS).

	OLS	OLS	IV	IV
Panel A: 1-year changes	Log FTE changes (1)	Log head changes (2)	Log FTE changes (3)	Log head changes (4)
Log output change	0.285***	0.284***	0.259***	0.264***
	(0.00421)	(0.00419)	(0.0531)	(0.0518)
Observations	160,764	160,764	160,764	160,764
N of firms	28,969	28,969	28,969	28,969
First-stage F-Statistic			98.66	98.66
R^2	0.189	0.205	0.188	0.205
Panel B: 4-year changes	(1)	(2)	(3)	(4)
Log output change	0.490***	0.487***	0.500***	0.504***
	(0.00747)	(0.00745)	(0.0811)	(0.0788)
Observations	53,106	53,106	53,106	53,106
N of firms	7,879	7,879	7,879	7,879
First-stage F-Statistic			38.17	38.17
R^2	0.458	0.464	0.458	0.464

Notes: The table reports OLS and IV regressions from estimating the specification in Eq. (11) for 1- and 4-year differences. The dependent variable is logged employment in changes, once measured in head counts, once measured as full time equivalents. All columns report regressions of those dependent variables on output growth for 1- and 4-year changes. Panels A-B report results for 1- and 4-year differences, respectively. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data. A future version of the paper will be updated to also include the results for 10-year changes (OLS), which remained under disclosure review at the point of circulation.

Table A.3: Firm-level adjustments in cost shares and output elasticities to firm growth. Different specifications: not logged, not dividing by RTS. OLS and IV regressions.

	1st stage	$\Delta \frac{w_{it}L_{it}}{C_{it}}$	$\Delta \frac{r_{it}K_{it}}{C_{it}}$	$\Delta \frac{z_{it}M_{it}}{C_{it}}$	$\Delta \frac{\theta^L_{it}}{RTS_{it}}$	$\Delta \frac{\theta^K_{it}}{RTS_{it}}$	$\Delta \frac{\theta^M_{it}}{RTS_{it}}$	$\Delta \ln(\theta^L_{it})$	$\Delta \ln(\theta^K_{it})$	$\Delta \ln(\theta^M_{it})$	$\Delta\theta^L_{it}$	$\Delta\theta^K_{it}$	$\Delta\theta^M_{it}$	ΔRTS_{it}
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Panel A: 1-year differences (29,950 firms; 183,813 firm-year observations)														
OLS estimates														
Log output change		-0.103***	-0.0380***	0.141***	-0.0710***	-0.0201***	0.0911***	-0.298***	-0.309***	0.157***	-0.0723***	-0.0203***	0.0991***	0.00647***
		(0.0011)	(0.0003)	(0.0011)	(0.0007)	(0.0003)	(0.0009)	(0.0046)	(0.0064)	(0.00129)	(0.0008)	(0.0004)	(0.0007)	(0.0004)
R^2		0.303	0.419	0.413	0.266	0.191	0.277	0.215	0.132	0.397	0.278	0.183	0.425	0.157
IV estimates														
Export demand shock														
(first stage)	0.0451***													
	(0.0045)													
Log output change		-0.0933***	-0.0283***	0.122***	-0.0626***	-0.0133***	0.0759***	-0.318***	-0.220**	0.127***	-0.0672***	-0.0135***	0.0914***	0.0107*
		(0.0144)	(0.0042)	(0.0153)	(0.0098)	(0.0039)	(0.0116)	(0.0645)	(0.0995)	(0.0170)	(0.0113)	(0.00455)	(0.0102)	(0.00575)
R^2	0.205	0.294	0.427	0.400	0.309	0.208	0.329	0.215	0.127	0.387	0.277	0.171	0.424	0.049
First-stage F-stat		102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6
Panel B: 4-year differences (11,492 firms; 70,936 firm-year observations)														
OLS estimates														
Log output change		-0.0692***	-0.0366***	0.106***	-0.0438***	-0.0136***	0.0574***	-0.185***	-0.164***	0.112***	-0.0420***	-0.0124***	0.0729***	0.0185***
		(0.00157)	(0.0006)	(0.00179)	(0.00108)	(0.000525)	(0.0014)	(0.0063)	(0.0077)	(0.0021)	(0.0014)	(0.0007)	(0.0013)	(0.0008)
R^2		0.303	0.419	0.413	0.266	0.191	0.277	0.201	0.160	0.369	0.220	0.154	0.388	0.157
IV estimates														
Export demand shock														
(first stage)	0.0635***													
	(0.0091)													
Log output change		-0.0555***	-0.0280***	0.0836***	-0.0465***	-0.0070	0.0536***	-0.220***	-0.125	0.105***	-0.0445***	-0.0005	0.0739***	0.0289***
		(0.0177)	(0.0062)	(0.0201)	(0.0127)	(0.0058)	(0.0157)	(0.0742)	(0.0973)	(0.0227)	(0.0153)	(0.0076)	(0.0143)	(0.00876)
R^2	0.297	0.403	0.402	0.266	0.175	0.276	0.183	0.199	0.158	0.368	0.220	0.121	0.388	0.139
First-stage F-stat		48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57
Panel C: 10-year differences (10,595 firms; 49,915 firm-year observations)														
OLS estimates														
Log output change		-0.0535***	-0.0275***	0.0810***	-0.0305***	-0.0051***	0.0356***	-0.112***	-0.0441***	0.0860***	-0.0236***	-0.0018***	0.0559***	0.0304***
		(0.0016)	(0.0006)	(0.0018)	(0.0011)	(0.0005)	(0.0014)	(0.00569)	(0.0073)	(0.0021)	(0.00137)	(0.0007)	(0.0013)	(0.0008)
R^2		0.259	0.318	0.347	0.222	0.155	0.201	0.161	0.141	0.322	0.174	0.142	0.334	0.262

Notes: This table reports OLS and IV regressions of the specification in Eq. (11). The dependent variables in Columns (2)–(14) are non-logged changes in labor, capital, and intermediate-input cost shares; output elasticities divided by returns to scale; and logged and non-logged elasticities not divided by RTS. Panels A–C correspond to 1-, 4-, and 10-year differences. IV uses foreign demand shocks as instruments (Eq. (12)); Column (1) reports the first stage. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*.

Table A.4: Firm-level adjustments in intermediate cost shares by intermediate type (OLS).

	$\Delta\ln(CS_{it}^{Ene})$	$\Delta\ln(CS_{it}^{Merch})$	$\Delta\ln(CS_{it}^{Sub})$	$\Delta\ln(CS_{it}^{Rep})$	$\Delta\ln(CS_{it}^{Rent})$	$\Delta\ln(CS_{it}^{Temp})$	$\Delta\ln(CS_{it}^{Other})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	0.297***	0.408***	0.388***	-0.104***	-0.417***	1.165***	-0.107***
	(0.001)	(0.029)	(0.022)	(0.011)	(0.011)	(0.0329)	(0.008)
Observations	183,807	83,975	96,062	176,845	175,391	86,927	183,813
N of firms	29,950	15,277	19,842	29,598	29,340	18,048	29,950
R^2	0.108	0.069	0.061	0.029	0.055	0.133	0.030
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	0.187***	0.429***	0.218***	-0.113***	-0.281***	0.578***	-0.054***
	(0.001)	(0.044)	(0.035)	(0.014)	(0.018)	(0.0431)	(0.011)
Observations	70,933	36,495	35,962	68,859	67,827	32,512	70,936
N of firms	11,492	6,096	6,737	11,366	11,149	5,640	11,492
R^2	0.128	0.117	0.115	0.072	0.079	0.150	0.065
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	0.137***	0.217***	0.148***	-0.072***	-0.218***	0.434***	0.011
	(0.008)	(0.051)	(0.038)	(0.015)	(0.021)	(0.0416)	(0.012)
Observations	48,950	21,102	22,376	46,996	46,143	19,064	48,953
N of firms	10,381	4,542	5,743	10,189	9,996	5,647	10,381
R^2	0.130	0.126	0.120	0.074	0.080	0.124	0.068

Notes: The table reports on OLS regressions from estimating the specification in Eq. (11) for 1-year (Panel A), 4-year (Panel B), and 10-year differences. The dependent variables in Columns (1)-(7) are log changes of raw materials, energy, and external components cost shares, merchandise cost shares, subcontracted production performed by other companies cost shares, repairs, maintenance, and installation cost shares, rents, leases, and leasing cost shares, temporary agency worker cost shares, and other intermediate inputs cost shares, respectively. All columns report regressions of those dependent variables on log output changes for 1-, 4-, and 10-year changes. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.5: Firm-level adjustments in intermediate type quantities (OLS).

	$\Delta\ln(Ene_{it})$	$\Delta\ln(Merch_{it})$	$\Delta\ln(Sub_{it})$	$\Delta\ln(Rep_{it})$	$\Delta\ln(Rent_{it})$	$\Delta\ln(Temp_{it})$	$\Delta\ln(Other_{it})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	1.066***	1.179***	1.151***	0.664***	0.352***	1.932***	0.661***
	(0.007)	(0.003)	(0.0217)	(0.0113)	(0.0107)	(0.0323)	(0.008)
Observations	183,807	83,975	96,062	176,845	175,391	86,927	183,813
N of firms	29,950	15,277	19,842	29,598	29,340	18,048	29,950
R^2	0.467	0.116	0.117	0.077	0.055	0.205	0.109
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	1.044***	1.281***	1.062***	0.744***	0.576***	1.427***	0.804***
	(0.011)	(0.044)	(0.0359)	(0.014)	(0.018)	(0.042)	(0.0116)
Observations	70,933	36,495	35,962	68,859	67,827	32,512	70,936
N of firms	11,492	6,096	6,737	11,366	11,149	5,640	11,492
R^2	0.600	0.187	0.182	0.180	0.103	0.232	0.264
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output change	1.030***	1.107***	1.026***	0.820***	0.675***	1.322***	0.904***
	(0.009)	(0.051)	(0.039)	(0.015)	(0.022)	(0.041)	(0.012)
Observations	48,950	21,102	22,376	46,996	46,143	19,064	48,953
N of firms	10,381	4,542	5,743	10,189	9,996	5,647	10,381
R^2	0.690	0.207	0.211	0.252	0.134	0.224	0.386

Notes: The table reports on OLS regressions from estimating the specification in Eq. (11) for 1-year (Panel A), 4-year (Panel B), and 10-year differences. The dependent variables in Columns (1)-(7) are log changes of raw materials, energy, and external components, merchandise, subcontracted production performed by other companies, repairs, maintenance, and installation, rents, leases, and leasing, temporary agency worker, and other intermediate inputs, respectively. All columns report regressions of those dependent variables on log output changes for 1-, 4-, and 10-year changes. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.6: Firm-level adjustments in output elasticities in response to firm growth: effects by size quintiles (4-year changes).

	$\Delta\ln(L_{it})$	$\Delta\ln(K_{it})$	$\Delta\ln(M_{it})$	$\Delta\ln(CS^{L}_{it})$	$\Delta\ln(CS^{K}_{it})$	$\Delta\ln(CS^{M}_{it})$	$\Delta\ln(\frac{\theta^{L}_{it}}{RTS_{it}})$	$\Delta\ln(\frac{\theta^{K}_{it}}{RTS_{it}})$	$\Delta\ln(\frac{\theta^{M}_{it}}{RTS_{it}})$	$\Delta\ln(RTS_{it})$
Panel A: 1st size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.505***	0.259***	1.051***	-0.248***	-0.594***	0.183***	-0.179***	-0.183***	0.095***	0.024***
	(0.011)	(0.016)	(0.009)	(0.0111)	(0.0172)	(0.0072)	(0.010)	(0.017)	(0.005)	(0.001)
Observations	14,984	14,984	14,984	14,984	14,984	14,984	14,984	14,984	14,984	14,984
N of firms	4,408	4,408	4,408	4,408	4,408	4,408	4,408	4,408	4,408	4,408
R^2	0.558	0.290	0.862	0.426	0.492	0.421	0.337	0.255	0.351	0.357
Panel B: 2nd size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.484***	0.217***	1.034***	-0.280***	-0.625***	0.174***	-0.206***	-0.175***	0.098***	0.018***
	(0.012)	(0.015)	(0.009)	(0.0106)	(0.0153)	(0.0062)	(0.011)	(0.014)	(0.004)	(0.001)
Observations	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276
N of firms	4,508	4,508	4,508	4,508	4,508	4,508	4,508	4,508	4,508	4,508
R^2	0.540	0.268	0.872	0.456	0.505	0.457	0.383	0.295	0.384	0.325
Panel C: 3rd size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.492***	0.208***	1.032***	-0.299***	-0.630***	0.169***	-0.200***	-0.164***	0.093***	0.017***
	(0.012)	(0.013)	(0.010)	(0.0131)	(0.0134)	(0.0071)	(0.011)	(0.013)	(0.005)	(0.001)
Observations	13,211	13,211	13,211	13,211	13,211	13,211	13,211	13,211	13,211	13,211
N of firms	4,265	4,265	4,265	4,265	4,265	4,265	4,265	4,265	4,265	4,265
R^2	0.546	0.295	0.877	0.461	0.559	0.463	0.353	0.355	0.394	0.312
Panel D: 4th size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.528***	0.178***	1.047***	-0.295***	-0.687***	0.160***	-0.202***	-0.169***	0.094***	0.014***
	(0.015)	(0.012)	(0.009)	(0.0138)	(0.0129)	(0.0063)	(0.013)	(0.010)	(0.005)	(0.001)
Observations	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276
N of firms	3,532	3,532	3,532	3,532	3,532	3,532	3,532	3,532	3,532	3,532
R^2	0.576	0.287	0.905	0.484	0.620	0.503	0.384	0.417	0.427	0.288
Panel E: 5th size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output change	0.529***	0.150***	1.055***	-0.307***	-0.722***	0.162***	-0.196***	-0.206***	0.099***	0.008***
	(0.016)	(0.012)	(0.012)	(0.0165)	(0.0138)	(0.0085)	(0.015)	(0.021)	(0.006)	(0.002)
Observations	11,648	11,648	11,648	11,648	11,648	11,648	11,648	11,648	11,648	11,648
N of firms	2,115	2,115	2,115	2,115	2,115	2,115	2,115	2,115	2,115	2,115
R^2	0.596	0.292	0.910	0.501	0.686	0.537	0.363	0.359	0.456	0.310

Notes: The table reports on OLS regressions from estimating the specification in Eq. (11) for 4-year differences. The dependent variables in Columns (1)-(10) are log changes in labor, capital, and intermediates quantities, labor, capital, and intermediate cost shares, and labor, capital, and intermediate input output elasticities divided by returns to scale, respectively. Panels A-E report results for the 1st, 2nd, 3rd , 4th, and 5th quintile of the firm-level output distribution (computed within year and industry). All columns report regressions of those dependent variables on log output changes for 4-year changes. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Size is measured by sales. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.7: Substitution elasticities, effects of firm growth on input ratios and input shadow price ratios, and firm-specific labor supply elasticities (OLS, 4-year changes, size quintiles).

	σ	$\Delta \ln \left(\frac{L_{it}}{M_{it}}\right)$	$\Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M}\right)$	
	(1)	(2)	(3)	(4)
Overall	2.259	-0.535	0.236	2.16
1st quintile	2.007	-0.546	0.272	1.86
2nd quintile	2.236	-0.550	0.246	1.97
3rd quintile	2.186	-0.540	0.247	1.99
4th quintile	2.327	-0.519	0.223	2.37
5th quintile	2.277	-0.526	0.231	2.29

Notes: The table replicates Table 3 by size (i.e., sales) quintiles for OLS. Size quintiles are computed by year and industry.

Table A.8: Substitution elasticities, effects of firm growth on input ratios and input shadow price ratios, and firm-specific labor supply elasticities, based on cost shares.

	OLS			IV		
	σ	$\Delta \ln \left(\frac{w_{it}\gamma_{it}^L}{z_{it}\gamma_{it}^M}\right)$	ϵ_{it}^L	σ	$\Delta \ln \left(\frac{w_{it}\gamma_{it}^L}{z_{it}\gamma_{it}^M}\right)$	ϵ_{it}^L
	(1)	(2)	(3)	(4)	(5)	(6)
1-year diff.	6.434	0.113	2.64	2.28 - inf.	0.00 - 0.35	0.62 - inf.
4-year diff.	6.859	0.078	6.54	12.559	0.034	15.765
10-year diff.	5.118	0.085	7.13			

Notes: The table reports substitution elasticities (Columns (1) and (5)) following Eq. (15), changes in input factor ratios (Columns (2) and (6)), implied changes in shadow input price ratios (Columns (3) and (7)) following Eq. (17), and implied labor supply elasticities following Eq. (19), assuming perfectly elastic intermediate input supply (Columns (4) and (8)), based on our OLS (Columns (1)-(4)) and IV (Columns (5)-(8)) regressions from Table 2 that regress log cost shares and log input quantities on log output in within-firm differences. Consequently, Columns (2) and (6) report coefficient ratios for labor and intermediates from these regressions with respect to firm growth, while all other columns report values implied by our regressions as described in the text. As IV point estimates suggest a substitution elasticity of infinity for 1-year changes, we report intervals using 95% confidence intervals from all point estimates entering the computation for this specification. This yields much larger intervals than directly computing confidence intervals for substitution elasticities and other values.

Table A.9: Effect of firm growth on wages. German micro data.

	Dependent variable: Log real wage changes				
	OLS			IV	
	1-year	4-year	10-year	1-year	4-year
Log output change	0.095***	0.062***	0.068***	-0.024	0.031
	(0.003)	(0.003)	(0.003)	(0.056)	(0.046)
Observations	183,813	70,936	48,953	183,813	70,936
N of firms	29,950	11,492	10,381	29,950	11,492
First-stage F-statistic				102.6	48.57
R^2	0.062	0.126	0.156	0.035	0.122

Notes: The table reports OLS and IV regressions of changes in relative shadow prices (Panel A) and average wages (Panel B) on output growth for 1-, 4-, and 10-year differences. Wages are defined as the wage bill divided by the number of employees. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance levels: *** 1%, ** 5%, * 10%. German firm-level data.

Table A.10: Testing for the role of non-homotheticities (direct output dependence), OLS and IV results, 4-year differences.

	$\Delta \ln\left(\frac{\theta^L_{it}}{RTS_{it}}\right)$	$\Delta \ln\left(\frac{\theta^L_{it}}{RTS_{it}}\right)$	$\Delta \ln\left(\frac{\theta^L_{it}}{RTS_{it}}\right)$	$\Delta \ln(CS^L_{it})$	$\ln(CS^L_{it})$	$\Delta \ln(CS^L_{it})$	$\Delta \ln\left(\frac{\theta^L_{it}}{\theta^M_{it}}\right)$	$\Delta \ln\left(\frac{\theta^L_{it}}{\theta^M_{it}}\right)$	$\Delta \ln\left(\frac{\theta^L_{it}}{\theta^M_{it}}\right)$	$\Delta \ln\left(\frac{CS^L_{it}}{CS^M_{it}}\right)$	$\Delta \ln\left(\frac{CS^L_{it}}{CS^M_{it}}\right)$	$\Delta \ln\left(\frac{CS^L_{it}}{CS^M_{it}}\right)$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
4-year differences (11,492 firms; 70,936 firm-year observations)												
OLS estimates												
Log output change	-0.202***		0.027***	-0.286***		0.003	-0.297***		0.0512***	-0.457***		-0.020***
	(0.006)		(0.005)	(0.006)		(0.003)	(0.00785)		(0.00555)	(0.009)		(0.004)
Log labor-intermediate input ratio		0.412***	0.427***		0.538***	0.540***		0.622***	0.651***		0.826***	0.815***
		(0.006)	(0.007)		(0.003)	(0.004)		(0.00656)	(0.00726)		(0.004)	(0.005)
R^2	0.220	0.462	0.463	0.348	0.765	0.765	0.245	0.595	0.597	0.373	0.810	0.810
IV estimates												
Log output change	-0.242***		-0.087	-0.269***		-0.052	-0.324***		-0.0772	-0.393***		-0.053
	(0.071)		(0.075)	(0.066)		(0.048)	(0.0911)		(0.0857)	(0.097)		(0.061)
Log labor-intermediate input ratio		0.412***	0.364***		0.538***	0.509***		0.622***	0.579***		0.826***	0.797***
		(0.006)	(0.042)		(0.003)	(0.027)		(0.00656)	(0.0482)		(0.004)	(0.035)
R^2	0.217	0.462	0.445	0.347	0.765	0.761	0.244	0.595	0.583	0.369	0.810	0.809
First-stage F-stat	48.57		39	48.57		39	48.57		39	48.57		39

Notes: The table reports OLS and IV regressions from estimating the specification in Eq. (11) for 4-year differences while additionally controlling for log changes in the labor-intermediate input ratio. The IV specification uses foreign demand shocks as instruments (Eq. (12)). The dependent variable in Columns (1)–(3) is the log change in the labor output elasticity divided by returns to scale (results are very similar when output elasticities are not adjusted for RTS). The dependent variable in Columns (4)–(6) is the log change in the labor cost share. The dependent variable in Columns (7)–(9) is the log change in the labor output elasticity divided by the intermediate-input output elasticity. The dependent variable in Columns (10)–(12) is the log change in the labor cost share divided by the intermediate-input cost share. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.11: CompNet summary statistics, averages of industry averages (manufacturing).

Country	**Lab. share (sales)**	**Lab. share (value added)**	**Lab. cost share**	**Lab. output elast.**	**Interm. cost share**	**Interm. output elast.**	**Markup**	**Wage Markdown**
Belgium (2000-2020)	0.17	0.42	0.21	0.31	0.74	0.67	1.23	1.26
Croatia (2002-2021)	0.19	0.40	0.25	0.28	0.67	0.64	1.32	1.05
Czech Republic (2005-2020)	0.16	0.59	0.17	0.18	0.78	0.79	1.19	0.75
Denmark (2001-2020)	0.25	0.72	0.26	0.26	0.69	0.74	1.18	0.80
Finland (1999-2020)	0.21	0.69	0.22	0.21	0.75	0.77	1.15	0.84
France (2004-2020)	0.21	0.39	0.31	0.38	0.62	0.57	1.51	1.37
Germany (2001-2018)	0.22	0.74	0.21	0.36	0.69	0.77	1.13	1.53
Hungary (2003-2020)	0.17	0.61	0.17	0.19	0.77	0.77	1.12	0.86
Italy (2006-2020)	0.17	0.63	0.17	0.17	0.78	0.82	1.19	0.77
Latvia (2007-2019)	0.18	0.53	0.21	0.23	0.73	0.75	1.25	0.99
Lithuania (2000-2020)	0.19	0.66	0.20	0.20	0.74	0.76	1.15	1.14
Netherlands (2007-2019)	0.21	0.68	0.22	0.22	0.74	0.75	1.11	0.85
Poland (2002-2020)	0.16	0.54	0.17	0.17	0.78	0.78	1.18	0.79
Portugal (2010-2020)	0.19	0.62	0.20	0.25	0.74	0.74	1.12	1.09
Romania (2005-2020)	0.18	0.61	0.17	0.20	0.77	0.77	1.14	0.90
Slovakia (2000-2020)	0.16	0.50	0.18	0.19	0.76	0.76	1.14	1.09
Slovenia (2002-2021)	0.20	0.64	0.21	0.20	0.74	0.74	1.12	0.83
Spain (2008-2020)	0.17	0.64	0.19	0.21	0.79	0.77	1.17	0.96
Sweden (2003-2020)	0.22	0.38	0.32	0.41	0.62	0.56	1.31	1.52
Switzerland (2009-2020)	0.26	0.70	0.28	0.25	0.68	0.72	1.18	0.79

Notes: The table reports country-level averages of industry-level labor shares (in sales and value added), labor cost shares, labor output elasticities, intermediate input cost shares, intermediate input output elasticities, markups, and wage markdowns, i.e., for each country, we average across country-industry-level values. Country-industry-level aggregates of labor shares are computed from firms' total labor costs, total sales, and total value added. Country-industry cost shares are computed as total input expenditures divided by total costs. Country-industry-level averages of labor output elasticities are sales-weighted aggregates of firm-level values. Country-industry-level average markups are computed as intermediate input cost-weighted averages of firm-level values. Country-industry-level average wage markdowns are derived as labor cost-weighted averages of firm-level values. CompNet manufacturing data. Firms with at least 20 employees.

Table A.12: Firm-level results for other countries: coefficients on log output changes, substitution elasticities, and contribution of labor output elasticity changes to labor share changes (manufacturing separately and all sectors, 4-year changes, OLS).

	$\Delta\ln(L_{it})$	$\Delta\ln(M_{it})$	$\Delta\ln(CS^L_{it})$	$\Delta\ln(CS^M_{it})$	$\Delta\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\Delta\ln(\theta^L_{it})$	$\Delta\ln(LS_{it})$	$\Delta\ln(\frac{P^L_{it}L_{it}}{VA_{it}})$	σ^{OE}	σ^{CS}	LS contrib.	Obs.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Panel A: Manufacturing													
France (1995–21)	0.496***	1.027***	-0.088***	0.313***	-0.131***	0.196***	-0.122***	-0.373***	-0.350***	2.60	4.01	33%	282,245
Hungary (2003–22)	0.464***	1.057***	-0.307***	0.159***	-0.330***	0.121***	-0.343***	-0.409***	-0.340***	4.18	4.67	84%	35,902
Poland (2002–22)	0.535***	1.058***	-0.262***	0.134***	-0.281***	0.090***	-0.281***	-0.337***	-0.259***	3.44	4.12	83%	98,385
Portugal (2004–22)	0.414***	1.115***	-0.347***	0.236***	-0.393***	0.198***	-0.403***	-0.469***	-0.352***	6.37	5.94	86%	33,478
Slovakia (2000–23)	0.513***	1.019***	-0.266***	0.166***	-0.273***	0.102***	-0.264***	-0.412***	-0.381***	3.86	6.84	64%	9,605
Slovenia (2002–23)	0.547***	1.091***	-0.285***	0.166***	-0.355***	0.135***	-0.351***	-0.360***	-0.236***	10.07	5.85	98%	7,452
Estonia (2004–23)	0.547***	1.102***	-0.243***	0.151***	-0.393***	0.127***	-0.394***	-0.292***	-0.192***	15.86	3.45	135%	1,627
Latvia (2005–21)	0.495***	0.940***	-0.239***	0.027***	-0.258***	0.125***	-0.257***	-0.327***	-0.423***	7.18	2.49	79%	2,760
Netherlands (2007–22)	0.369***	1.116***	-0.303***	0.169***	-0.358***	0.143***	-0.363***	-0.353***	-0.185***	3.04	2.72	103%	30,210
Romania (2005–23)	0.526***	1.031***	-0.269***	0.126***	-0.232***	0.083***	-0.238***	-0.363***	-0.337***	2.65	4.59	66%	21,642
Switzerland (2009–22)	0.460***	1.055***	-0.303***	0.189***	-0.287***	0.124***	-0.283***	-0.434***	-0.365***	3.23	5.78	65%	11,875
Germany (1995–17)	0.510***	1.045***	-0.286***	0.171***	-0.202***	0.095***	-0.185***	-0.406***	-0.334***	2.26	6.86	46%	70,936
Panel B: All sectors													
France (1995–21)	0.511***	0.975***	-0.049***	0.268***	-0.095***	0.183***	-0.087***	-0.340***	-0.333***	2.50	3.15	26%	589,669
Hungary (2003–22)	0.497***	1.039***	-0.270***	0.125***	-0.276***	0.094***	-0.292***	-0.356***	-0.297***	3.15	3.69	82%	82,062
Poland (2002–22)	0.535***	1.064***	-0.273***	0.128***	-0.276***	0.090***	-0.278***	-0.336***	-0.231***	3.25	4.13	83%	229,205
Portugal (2004–22)	0.501***	1.044***	-0.240***	0.165***	-0.293***	0.149***	-0.301***	-0.361***	-0.305***	5.38	3.93	83%	74,869
Slovakia (2000–23)	0.493***	0.916***	-0.200***	0.130***	-0.202***	0.076***	-0.211***	-0.412***	-0.420***	2.93	4.55	51%	25,395
Slovenia (2002–23)	0.591***	1.080***	-0.259***	0.142***	-0.294***	0.116***	-0.298***	-0.321***	-0.191***	6.19	5.57	93%	17,696
Estonia (2004–23)	0.493***	1.079***	-0.261***	0.142***	-0.348***	0.151***	-0.344***	-0.323***	-0.219***	6.74	3.20	107%	4,020
Latvia (2005–21)	0.529***	0.859***	-0.172***	0.008**	-0.145***	0.074***	-0.132***	-0.320***	-0.361***	2.97	2.20	41%	13,405
Netherlands (2007–22)	0.497***	1.089***	-0.230***	0.130***	-0.257***	0.107***	-0.249***	-0.270***	-0.144***	2.60	2.55	92%	142,659
Romania (2005–23)	0.523***	1.012***	-0.266***	0.096***	-0.153***	0.044***	-0.163***	-0.350***	-0.328***	1.67	3.86	50%	58,441
Switzerland (2009–22)	0.548***	1.058***	-0.232***	0.172***	-0.215***	0.119***	-0.220***	-0.347***	-0.277***	2.90	4.81	63%	28,572

Notes: OLS estimates of Eq. (11) using 4-year differences. Columns (1)–(9) report coefficients on log output growth. Columns (10) and (11) report substitution elasticities based on output elasticities and cost shares, respectively. Column (12) reports the contribution of changes in labor output elasticities (Column 7) to labor share changes (Column 8). Column (13) reports the number of firm-year observations. Panel A reports manufacturing firms only; Panel B reports all sectors. Firm-level data from CompNet data providers and German firm-level data. All regressions include two-digit industry-year fixed effects. Standard errors are clustered at the firm level. We omit standard errors due to space constraints and only report significance levels by stars: Significance: 0.01***, 0.05**, 0.1*. The table with values for all standard errors is available on request.

Table A.13: Firm-level results for other countries: Coefficients on output growth, substitution elasticities, and contribution of labor output elasticity changes to labor share changes (4-year changes, OLS). Young and mature manufacturing firms.

	$\Delta\ln(L_{it})$	$\Delta\ln(M_{it})$	$\Delta\ln(CS^L_{it})$	$\Delta\ln(CS^M_{it})$	$\Delta\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\Delta\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\Delta\ln(\theta^L_{it})$	$\Delta\ln(LS_{it})$	$\Delta\ln(\frac{P^L_{it}L_{it}}{VA_{it}})$	σ^{OE}	σ^{CS}	LS contrib. of $\Delta\theta^L_{it}$
Panel A: young firms	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
France (1995-21)	0.477***	1.018***	-0.109***	0.227***	-0.131***	0.195***	-0.122***	-0.318***	-0.305***	2.52	2.64	38%
Obs.: 21,988	(0.008)	(0.010)	(0.007)	(0.008)	(0.004)	(0.006)	(0.005)	(0.007)	(0.008)			
Hungary (2003-22)	0.488***	1.052***	-0.249***	0.134***	-0.287***	0.116***	-0.299***	-0.330***	-0.274***	3.50	3.12	91%
Obs.: 3,966	(0.016)	(0.013)	(0.015)	(0.011)	(0.015)	(0.006)	(0.015)	(0.015)	(0.019)			
Slovakia (2000-23)	0.570***	1.014***	-0.255***	0.134***	-0.248***	0.0830***	-0.242***	-0.375***	-0.370***	3.93	8.07	65%
Obs.: 1,157	(0.031)	(0.031)	(0.031)	(0.021)	(0.028)	(0.013)	(0.029)	(0.031)	(0.051)			
Slovenia (2002-23)	0.590***	1.097***	-0.220***	0.140***	-0.279***	0.140***	-0.273***	-0.266***	-0.163***	5.76	3.45	103%
Obs.: 719	(0.036)	(0.026)	(0.032)	(0.023)	(0.038)	(0.017)	(0.038)	(0.033)	(0.029)			
Latvia (2005-21)	0.563***	0.973***	-0.224**	0.022	-0.205***	0.125***	-0.208***	-0.272***	-0.409***	5.13	2.49	76%
Obs.: 236	(0.053)	(0.053)	(0.090)	(0.0140)	(0.054)	(0.031)	(0.058)	(0.062)	(0.126)			
Netherlands (2007-22)	0.439***	1.107***	-0.251***	0.152***	-0.308***	0.126***	-0.313***	-0.293***	-0.141***	2.85	2.52	107%
Obs.: 1,774	(0.030)	(0.028)	(0.032)	(0.024)	(0.024)	(0.012)	(0.024)	(0.033)	(0.035)			
Panel B: mature firms	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
France (1995-21)	0.498***	1.029***	-0.0824***	0.330***	-0.131***	0.199***	-0.122***	-0.382***	-0.357***	2.64	4.48	32%
Obs.: 253,635	(0.004)	(0.004)	(0.003)	(0.004)	(0.002)	(0.002)	(0.002)	(0.003)	(0.004)			
Hungary (2003-22)	0.454***	1.060***	-0.326***	0.168***	-0.343***	0.124***	-0.356***	-0.434***	-0.354***	4.36	5.41	82%
Obs.: 31,916	(0.008)	(0.007)	(0.009)	(0.006)	(0.007)	(0.003)	(0.008)	(0.009)	(0.011)			
Slovakia (2000-23)	0.491***	1.018***	-0.272***	0.176***	-0.283***	0.108***	-0.274***	-0.428***	-0.391***	3.88	6.67	64%
Obs.: 8,399	(0.018)	(0.022)	(0.015)	(0.012)	(0.015)	(0.008)	(0.016)	(0.018)	(0.028)			
Slovenia (2002-23)	0.533***	1.097***	-0.306***	0.179***	-0.376***	0.137***	-0.371***	-0.387***	-0.244***	11.06	7.14	96%
Obs.: 6,656	(0.017)	(0.014)	(0.016)	(0.012)	(0.019)	(0.008)	(0.019)	(0.016)	(0.016)			
Latvia (2005-21)	0.479***	0.938***	-0.245***	0.0305***	-0.269***	0.127***	-0.267***	-0.338***	-0.424***	7.29	2.50	80%
Obs.: 2,482	(0.026)	(0.024)	(0.029)	(0.006)	(0.024)	(0.010)	(0.025)	(0.024)	(0.042)			
Netherlands (2007-22)	0.363***	1.117***	-0.308***	0.171***	-0.364***	0.145***	-0.369***	-0.359***	-0.188***	3.14	2.74	103%
Obs.: 28,411	(0.010)	(0.008)	(0.012)	(0.007)	(0.008)	(0.004)	(0.009)	(0.012)	(0.008)			

Notes: The table reports OLS regressions from estimating the specification in Eq. (11) for different countries (Columns (1)-(9)) using 4-year differences. The table reports coefficients on log output changes in Columns (1)-(9). The dependent variables in Columns (1)-(9) are log changes in labor and intermediate quantities, cost shares, and output elasticities divided by returns to scale, labor output elasticities, labor shares in sales, and labor shares in value added, respectively. All columns report regressions of those dependent variables on changes in log output for 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data. Columns (10) and (11) report estimates of the substitution elasticity between labor and intermediates based on Eq. (15) using output elasticity coefficients (Column (10)) and cost share coefficients (Column (11)). Column (12) reports the contribution of changes in labor output elasticities (Column (7)) to changes in labor shares (Column (8)). Panel A reports results for firms not older than five years. Panel B reports results for firms older than five years. Firm-level micro data from CompNet data providers for a subset of CompNet countries with information on registration years.

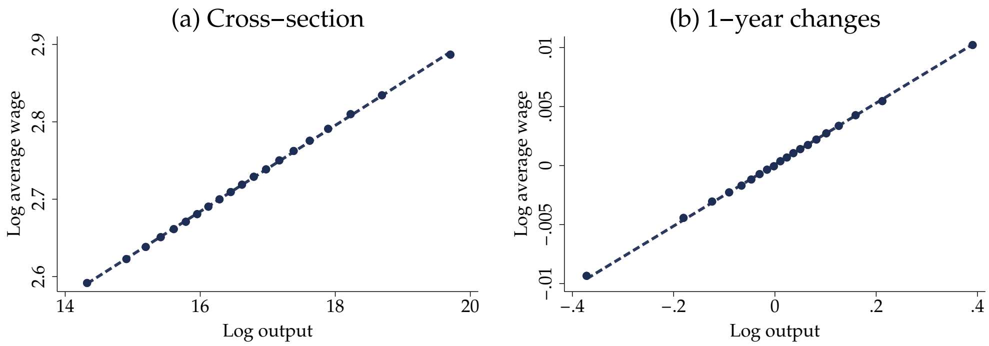
Table A.14: Implied industry-level substitution elasticities and changes in inputs, output elasticities, and cost share ratios based on 4-year differences and by countries.

	$\Delta \ln \left(\frac{L_{cjt}}{M_{cjt}}\right)$	$\Delta \ln \left(\frac{\theta^L_{cjt}}{\theta^M_{cjt}}\right)$	$\Delta \ln \left(\frac{CS^L_{cjt}}{CS^M_{cjt}}\right)$	σ^{OE}	σ^{CS}
	(1)	(2)	(3)	(4)	(5)
Belgium (2000-2020)	-0.23	-0.10	-0.16	1.80	3.36
Croatia (2002-2021)	-0.70	-0.16	-0.64	1.31	12.71
Czech Republic (2005-2020)	-0.40	-0.01	-0.31	1.03	4.67
Denmark (2001-2020)	-0.13	-0.01	-0.09	1.12	3.13
Finland (1999-2020)	-0.09	-0.08	-0.13	5.72	-2.45
France (2004-2020)	-0.40	-0.05	-0.22	1.13	2.25
Germany (2001-2018)	-0.06	0.01	-0.05	0.85	6.59
Hungary (2003-2020)	-0.51	-0.09	-0.38	1.21	4.03
Italy (2006-2020)	-0.18	-0.09	-0.12	2.11	2.89
Latvia (2007-2019)	-0.43	-0.11	-0.25	1.33	2.46
Lithuania (2000-2020)	-0.15	-0.08	-0.11	2.04	3.59
Netherlands (2007-2019)	-0.11	0.03	-0.06	0.81	2.51
Poland (2002-2020)	-0.13	-0.01	-0.09	1.09	2.98
Portugal (2010-2020)	-0.68	0.02	-0.48	0.98	3.42
Romania (2005-2020)	-0.22	-0.03	-0.13	1.17	2.58
Slovakia (2000-2020)	-0.18	-0.09	-0.16	1.98	12.73
Slovenia (2002-2021)	-0.09	0.01	-0.13	0.92	-2.49
Spain (2008-2020)	-0.44	-0.23	-0.16	2.11	1.60
Sweden (2003-2020)	-0.03	-0.04	0.02	-5.77	0.64
Switzerland (2009-2020)	-0.31	-0.19	-0.24	2.63	4.58
Europe (1999-2021, manufac.)	-0.27	-0.07	-0.20	1.34	4.01
USA (1958-2016, manufac.)	-0.31		-0.23		3.88
Europe (1999-2021, non-manufac.)	-0.23	-0.08	-0.15	1.48	2.72

Notes: The table reports implied changes in labor-intermediate input ratios (Column (1)), output elasticity ratios (Column (2)), cost shares (Column (3)), and substitution elasticities based on output elasticity coefficients (Column (4)) and cost share coefficients (Column (5)) as estimated from country-specific versions of the country-industry-level regressions reported in Figure 8 (4-year changes). All results are based on manufacturing industries, except for the last row, which uses non-manufacturing industries. Industries are 2-digit NACE rev. 2 industries for Europe and 6-digit NAICS industries for the U.S. . For U.S. cost shares, capital costs are approximated as 8% of the real capital stock, as nominal values are not reported; labor and material expenditures are reported in nominal terms. CompNet data and NBER-CES Manufacturing Industry Database.

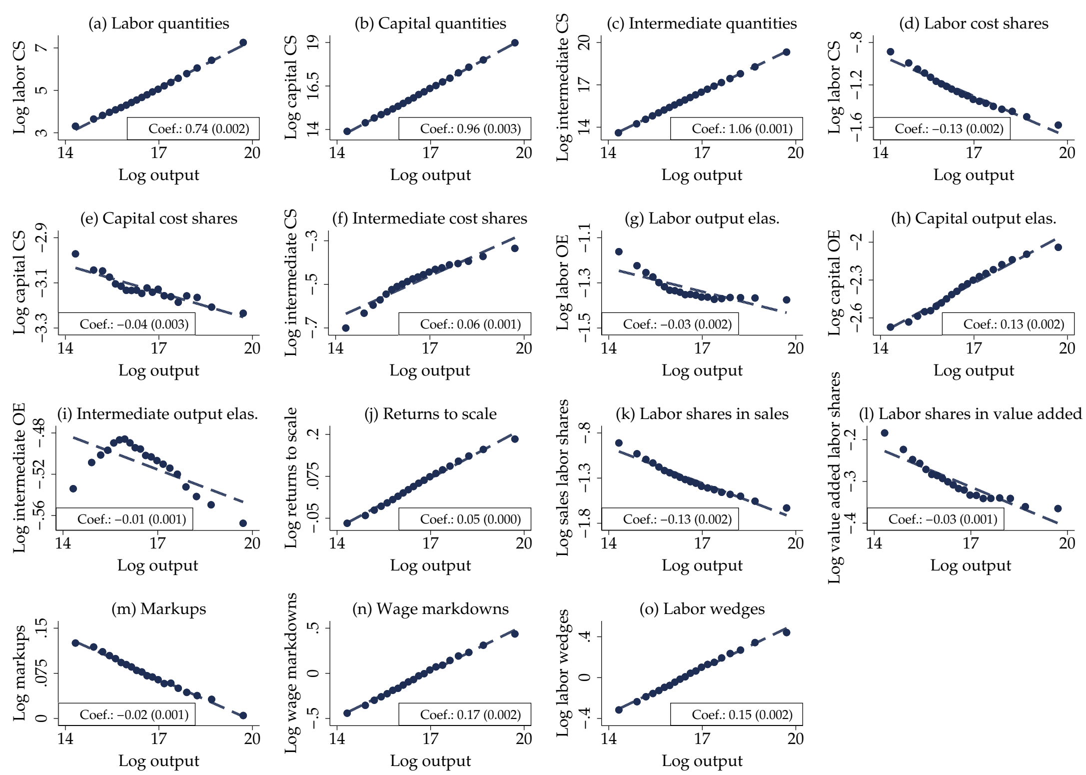
B Additional Figures

Figure A.1: Wage-size gradient (OLS).



Notes: The figures report binned scatter plots from regressing log wages on log firm size (output). Panel (a) reports cross-sectional results in levels; Panel (b) reports first-difference results (1-year changes). All regressions control for industry-year fixed effects. German firm-level data.

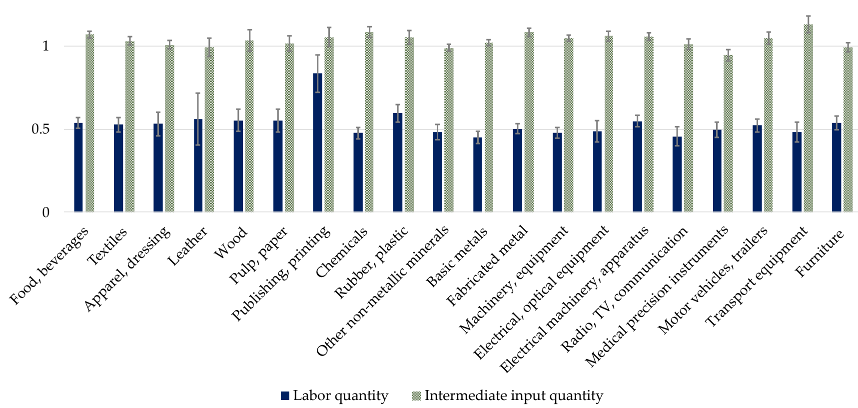
Figure A.2: The cross-sectional relationships.



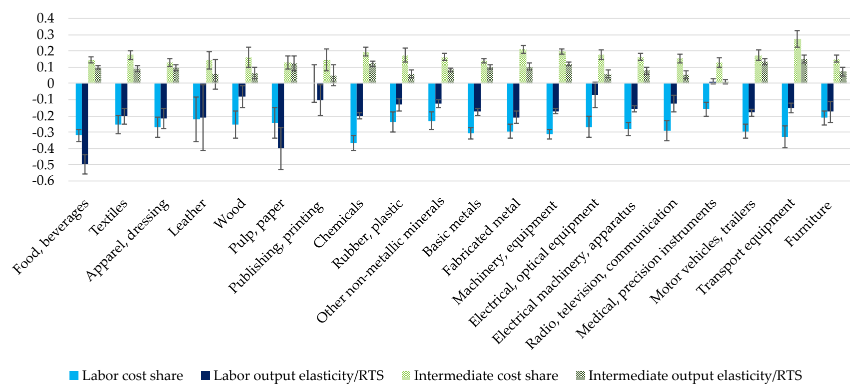
Notes: The figure reports binned scatter plots estimating the specification in Eq. (11) in levels with OLS. Panels (a)-(o) report on regressions of the logs of labor, capital, and intermediate quantities, labor, capital, and intermediate cost shares, and labor, capital, and intermediate output elasticities over returns to scale, returns to scale, labor shares in sales and value-added, markups, wage markdowns, and labor wedges on log output, respectively. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Figure A.3: Firm-level adjustments separately by industry, 4-year changes, OLS.

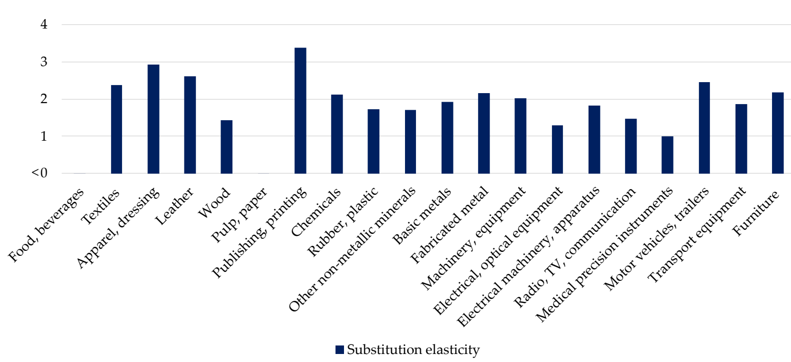
(a) Input quantities



(b) Output elasticities and cost shares



(c) Substitution elasticities



Notes: The figure reports OLS regressions from estimating Eq. (11) in 4-year changes by two-digit NACE manufacturing industries. The dependent variables are logged labor and intermediate input quantities (Panel (a)), cost shares and labor and intermediate output elasticities divided by returns to scale ((Panel (b)), and implied substitution elasticities derived as described in Section 5 (Panel (c)). In Panel (c), values for "Food, beverages" and "Pulp, paper" are truncated at zero as these two estimates are negative (-8.25 and -7.61, respectively) and should not be interpreted, as they result from output elasticity responses exceeding input quantity responses (likely due to measurement errors or small sample sizes). All regressions control for industry-year fixed effects. Standard errors are clustered at the firm level. The error bars indicate 95% confidence intervals. Results for 1- and 10-year changes are similar. German firm-level data.

C Details on Meta-Analysis (Figure 4 Panel (a))

We distinguish between studies estimating substitution between intermediates and a capital–labor bundle and those estimating substitution between intermediates and labor. The former impose identical substitution between intermediates and capital and labor and typically do not identify the elasticity from variation in labor-intermediate input (or price) ratios. This restriction could explain why these studies report lower elasticities compared to studies estimating intermediate–labor elasticities directly (as we do). We exclude studies focusing on energy–labor substitution (e.g., Bretschger and Jo, 2024), as energy constitutes only a small share of intermediates. The definition of intermediates also matters: substitution with raw materials or energy is likely weaker than substitution with labor-intensive components (particularly relevant for our case) or intermediate services.

Table B.1: Studies used in the substitution–elasticity meta-analysis.

Substitution elasticities between intermediates and a capital–labor bundle	
Peter and Ruane (2022)	Plant-level data, India, manufacturing. Identification from input-price variation. Mean of OLS (0.479) and IV (0.618) estimates from Table 6: **0.5485**.
Boehm et al. (2019)	Firm-level data, US, manufacturing and non-manufacturing. Identification from short-run variation in output and inputs under constant input prices. Value for all firms in Table 2: **0.037**.
Bruno (1984)	Country-level data (US, UK, Germany, Japan), manufacturing and non-manufacturing. Identification from input-price variation. Mean of non-negative values in Table 3 (0.196, 0.57, 0.337, 0.46, 0.35, 0.132, 0.472, 0.649, 0.812, 0.766, 0.91): **0.514**.
Oberfield and Raval (2021)	Firm-level data, US, manufacturing. Identification from input-price variation. Mean of Table 3 values (1.03, 0.83, 0.69, 0.78, 0.57): **0.78**.
Antoszewski (2019)	Industry-level WIOD data, OECD countries, manufacturing and non-manufacturing. Identification from input-price variation. Mean of all Table 8 values: **0.8211**.
Atalay (2017)	Industry-level data, US, manufacturing and non-manufacturing. Identification from input-price variation. Mean of Table 8 values (1.18, 1.27, 0.84, 0.88): **1.043**.
Rotemberg and Woodford (1996)	Industry-level data, US, manufacturing. Identification from input-price variation. Table 2 value: **0.69**.
Miranda-Pinto (2021)	Industry-level data, multiple countries, manufacturing and non-manufacturing. Identification from input-price variation. Table 4 value for all firms: **1.34**.
Miranda-Pinto and Youngs (2022)	Industry-level data, US, manufacturing and non-manufacturing. Identification from input-price variation. Mean of FE (0.85) and IV (2.14) specifications in Table 2: **1.495**.
Substitution elasticities between intermediates and labor	
Delgado et al. (1999)	Firm-level data, Spain, manufacturing. Identification from input-price variation. Mean of all $1-\beta$ estimates in Table 2: **0.414**.
Doraszelski and Jaumandreu (2018)	Firm-level data, Spain, manufacturing. Identification from input-price variation. Mean of all values in Tables 3–5: **0.691**.
Huneeus et al. (2022)	Firm-level data, Chile, manufacturing and non-manufacturing. Identification from input-price variation. Mean of Table 5 values (1.55, 1.62, 1.05): **1.401**. Authors emphasize that improved input-price measures yield the higher estimates (1.55, 1.62).
Chan (2023)	Firm-task-level data, Denmark, manufacturing and non-manufacturing. Identification from input-price variation. Mean of Table 6 values: **2.368**.
Martinello (1989)	Industry-level data, Canada and US, wood-products industry. Identification from input-price variation and a structural labor-market model. Mean of Table 4 values (2.662, 2.257): **2.46**.

Notes: The table summarizes all studies used in our meta-analysis of substitution elasticities, distinguishing whether elasticities are estimated between intermediates and a capital–labor bundle or directly between intermediates and labor.

D Theoretical Appendix

Throughout this section, the notation follows the main text.

D.1 Derivations for Section 2.1

Production function-based substitution elasticity. We first derive Eq. (3). To that end, we only rely on the specific production function (Eq. (1)). The partial derivatives with respect to labor and intermediate yield Eq. (2), i.e., the stepping stone towards Eq. (3):

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} (\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}} \Lambda_i^{LM} \alpha^L \kappa L_{it}^{\frac{-1}{\sigma}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}}{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} (\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}} \Lambda_i^{LM} \alpha^M \kappa M_{it}^{\frac{-1}{\sigma}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}} = \frac{\alpha^L}{\alpha^M} \left(\frac{L_{it}}{M_{it}}\right)^{\frac{-1}{\sigma_{it}}}. \tag{C.1}$$

Multiplying this expression by $\frac{L_{it}}{Q_{it}}/\frac{M_{it}}{Q_{it}}$ and taking logs and differences (which eliminates the firm fixed effect, $\frac{\alpha^L}{\alpha^M}$) yields Eq. (3):

$$\frac{\sigma-1}{\sigma} = \frac{\Delta \ln(\theta_{it}^L) - \Delta \ln(\theta_{it}^M)}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}. \tag{C.2}$$

Results assuming cost minimization. For the following results, we can define the production function in general terms, $Q_{it}(K_{it}, L_{it}, M_{it}, \Omega_{it}) = Q_{it}(.)$ and write firms' cost minimization as a Lagrangian function:

$$\begin{aligned}\mathcal{L}_{it} = P_{it}^L(L_{it})L_{it} + \chi^L(L_{it})P_{it}^L + P_{it}^M(M_{it})M_{it} + \chi^M(M_{it})P_{it}^M \\ +P_{it}^K(K_{it})K_{it} + \chi^K(K_{it})P_{it}^K - \lambda_{it}(Q_{it} - Q_{it}(.)),\end{aligned} \tag{C.3}$$

where, for simplicity, we model input adjustment costs through convex functions $\chi^X(X_{it})$ with $X = \{L, M, K\}$ and write the optimization in a quasi-static way as in Bond et al. (2021) (this avoids writing out the dynamic problem). One interpretation of this setting is that each input is associated with a baseline quantity, $\bar{X}_{it}$, and that firms incur adjustment costs when choosing an input quantity that differs from $\bar{X}_{it}$. The first-order conditions for each production input lead to Eq. (4) from the main text:

$$\frac{\partial \mathcal{L}_{it}}{\partial X_{it}} = 0 \quad \Rightarrow \quad P_{it}^X \underbrace{\left(1 + \frac{\partial P_{it}^X}{\partial X_{it}} \frac{X_{it}}{P_{it}^X} + \frac{\partial \chi^X}{\partial L_{it}}\right)}_{\gamma_{it}^X} = \lambda_{it} \frac{\partial Q_{it}}{\partial X_{it}}. \tag{C.4}$$

Multiplying Eq. (C.4) by $\frac{X_{it}}{Q_{it}}$ using the definition of the output elasticity, $\theta^X = \frac{\partial Q}{\partial X}\frac{X}{Q}$, and noting that $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$ (price over markup) leads to:

$$X_{it}P_{it}^X = P_{it}Q_{it}\frac{\theta_{it}^X}{\mu_{it}\gamma_{it}^X}. \tag{C.5}$$

Rearranging yields Eq. (7) of the main text:

$$OS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it}Q_{it}} = \frac{\theta_{it}^X}{\mu_{it}\gamma_{it}^X}. \tag{C.6}$$

Similarly, we can define Eq. (C.5) for each input and recover Eq. (6) of the main text:

$$CS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it}^X L_{it} + P_{it}^M M_{it} + P_{it}^K K_{it}} = \frac{\frac{\theta_{it}^X}{\gamma_{it}^X}}{\frac{\theta_{it}^L}{\gamma_{it}^L} + \frac{\theta_{it}^M}{\gamma_{it}^M} + \frac{\theta_{it}^K}{\gamma_{it}^K}}. \tag{C.7}$$

D.2 Derivations for Section 2.2

To recover Eq. (8) from the main text, we impose more structure by defining inverse input supply functions as $P_{it}^X = a_{it}^X X_{it}^{\varepsilon^X}$ for $X = \{L, M\}$ and abstract from adjustment costs for labor and intermediates. For simplicity, we also assume that markups and capital shadow costs do not depend on firm scale. Inserting the input supply functions into Eq. (C.3) and dropping adjustment cost terms (except for capital) yields:

$$\mathcal{L}_{it} = a_{it}^L L_{it}^{1+\varepsilon^L} + a_{it}^M M_{it}^{1+\varepsilon^M} + P_{it}^K(K_{it})K_{it} + \chi^K(K_{it})P_{it}^K - \lambda_{it}(Q_{it} - Q_{it}(.)). \tag{C.8}$$

The first-order conditions for $X = \{L, M\}$ yield Eq. (8) from the main text:

$$\frac{\partial \mathcal{L}_{it}}{\partial X_{it}} = 0 \quad \Rightarrow \quad \left(1+\varepsilon^X\right) a_{it}^X X_{it}^{\varepsilon^X} = \lambda_{it}\frac{\partial Q_{it}}{\partial X_{it}} \quad \Rightarrow \quad X_{it} = \left(\frac{\lambda_{it}}{\left(1+\varepsilon^X\right) a_{it}^X}\frac{\partial Q_{it}}{\partial X_{it}}\right)^{\frac{1}{\varepsilon^X}}. \tag{C.9}$$

Inserting the production function yields:

$$X_{it} = \left(\frac{\lambda_{it}(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha^X\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{\left(1+\varepsilon^X\right) a_{it}^X} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^X+1)}}. \tag{C.10}$$

Combining this expression for labor and intermediates yields:

$$\frac{L_{it}}{M_{it}} = \frac{\left(\frac{\lambda_{it}(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha^L\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^L)a_{it}^L}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(\frac{\lambda_{it}(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha^M\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^M)a_{it}^M}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}} \frac{\left(K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}}. \tag{C.11}$$

Combining the FOC for capital and using the production function yields capital demand:

$$K_{it} = \left(\frac{\lambda_{it}(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)Q_{it}. \tag{C.12}$$

Inserting this capital demand equation into Eq. (C.11) yields Eq. (9) from the main text:

$$\frac{L_{it}}{M_{it}} = \varrho_{it}\lambda_{it}^{\left(\frac{\sigma+\kappa-1}{\kappa(\sigma\varepsilon^L+1)}-\frac{\sigma+\kappa-1}{\kappa(\sigma\varepsilon^M+1)}\right)}Q_{it}^{\left(\frac{1}{\sigma\varepsilon^L+1}-\frac{1}{\sigma\varepsilon^M+1}\right)}, \tag{C.13}$$

where $\varrho_{it} = \frac{\left(\frac{(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha^L\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^L)a_{it}^L}\left(\frac{(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)^{\frac{(1-\kappa)(\sigma-1)}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(\frac{(\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^{LM}\alpha^M\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^M)a_{it}^M}\left(\frac{(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)^{\frac{(1-\kappa)(\sigma-1)}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}}$ is a function of parameters and markdowns only.

D.3 Clarification: Substitution Elasticities Beyond CES

We focus on how firm growth induces a shift from labor to intermediates. In our setting, growing firms move along labor and intermediate input supply curves, which affects relative input (shadow) prices. Our main text reads the substitution patterns through a CES setup, where there is a single well-defined substitution elasticity. Away from CES, naturally and by definition, there exists no single substitution elasticity that describes substitution patterns. Nevertheless, we can define a tightly related notion of local substitution elasticities even in a more general production function context, and derive the counterparts of our main results. Specifically, the shift from labor to intermediates implies an (arc) elasticity of relative input quantities with respect to relative shadow price changes (a substitution elasticity), although it may be local to the baseline input quantity/price levels and the specific input or price change under consideration. Formally, that more general elasticity can be written as:

$$\sigma_{it}^{Gen} = \frac{\Delta \ln(\frac{M_{it}}{L_{it}})}{\Delta \ln(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M})}. \tag{C.14}$$

Assuming that firms minimize cost, it holds that $P_{it}^X = \frac{P_{it}Q_{it}}{X_{it}} \frac{\theta_{it}^X}{\mu_{it}\gamma_{it}^X}$, where γ_{it}^X, μ_{it}, and $P_{it}Q_{it}$ are the input wedge, markup, and sales. Using this, we can rewrite Eq. (C.14) as:

$$\sigma_{it}^{Gen} = \frac{\Delta \ln(\frac{M_{it}}{L_{it}})}{\Delta \ln\left(\frac{\theta_{it}^L}{L_{it}} / \frac{\theta_{it}^M}{M_{it}}\right)} \quad \Leftrightarrow \quad \frac{\sigma_{it}^{Gen} - 1}{\sigma_{it}^{Gen}} = \frac{\Delta \ln(\theta_{it}^L) - \Delta \ln(\theta_{it}^M)}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}. \tag{C.15}$$

Eq. (C.15) is identical to the substitution elasticity that we use in the main part of the paper (Eq. (3)) with the exception that a CES production function will yield a constant substitution elasticity that is identical across firms and does not depend on the constellation of particular baseline input levels/prices or input quantity/price changes. Eq. (C.15) gives the elasticity of substitution as the responsiveness of input ratios to input shadow price ratios, for a given scenario.

D.4 OLS Bias under Factor-Augmenting Productivity

We now characterize how labor-augmenting productivity (LAP) biases the identification of σ from firm-growth regressions. To ease notation, this appendix omits industry-year fixed effects, so that all variables are implicitly residualized accordingly. Let $\mathcal{X}_{it} \equiv \Delta \ln\left(\frac{\alpha_{it}^L}{\alpha_{it}^M}\right)$ denote the *unobserved* relative, firm-year-specific (relative) LAP shift. Intuitively, our OLS design omits this term while the IV model draws on growth variation orthogonal to it. We consider a situation where the data generating process does feature a correlation between firm growth and LAP. Hence, the true versions of Eq. (11) under *observed* LAP are:

$$\Delta \ln Y_{it} = \rho_{\Delta \ln(Y), \Delta \ln(Q)}^{True} \Delta \ln Q_{it} + \beta_Y \mathcal{X}_{it} + u_{it}^Y, \tag{C.16}$$

where Y denotes labor and intermediates quantities and output elasticities. The ρ^{True}s and βs capture how these variables respond to firm growth and LAP, conditional on each other (and fixed effects). Our actual regressions omit $\mathcal{X}_{it}^{LM}$, and the OLS coefficient estimates are subject to omitted variable bias (OVB):

$$\rho_{\Delta \ln(Y), \Delta \ln(Q)}^{OLS} = \rho_{\Delta \ln(Y), \Delta \ln(Q)}^{True} + \beta_Y \psi, \tag{C.17}$$

where $\psi \equiv \frac{\mathrm{Cov}(\Delta \ln Q_{it}, \mathcal{X}_{it})}{\mathrm{Var}(\Delta \ln Q_{it})} = \rho^{OLS}_{\mathcal{X}_{it}, \Delta \ln(Q)}$. Define $D^{\mathcal{M}} \equiv \rho^{\mathcal{M}}_{\Delta \ln(L), \Delta \ln(Q)} - \rho^{\mathcal{M}}_{\Delta \ln(M), \Delta \ln(Q)}$ and $N^{\mathcal{M}} \equiv \rho^{\mathcal{M}}_{\Delta \ln(\theta^L), \Delta \ln(Q)} - \rho^{\mathcal{M}}_{\Delta \ln(\theta^M), \Delta \ln(Q)}$, where $\mathcal{M} = \{OLS, True\}$. To evaluate the OVB for our identification of σ, we characterize the coefficients in the structural model equation (Eq. (2)), which, under LAP, becomes:[52]

$$\Delta \ln \left(\frac{\theta^L_{it}}{\theta^M_{it}} \right) = \frac{\sigma - 1}{\sigma} \left[\Delta \ln \left(\frac{L_{it}}{M_{it}} \right) + \mathcal{X}_{it} \right]. \tag{C.18}$$

Projecting both sides on $\Delta \ln Q_{it}$ (i.e., inserting the versions of Eq. (C.16) for the variables) yields for the true population and the OLS models:

$$N^{True} = \frac{\sigma - 1}{\sigma} D^{True} \qquad N^{OLS} = \frac{\sigma - 1}{\sigma} \left(D^{OLS} + \psi \right). \tag{C.19}$$

Hence, using Eq. (C.19), we obtain:

$$\sigma = \frac{1}{1 - \frac{N^{True}}{D^{True}}} = \frac{1}{1 - \frac{N^{OLS}}{D^{OLS} + \psi}}. \tag{C.20}$$

The naive OLS estimator subject to OVB due to unobserved LAP is:

$$\sigma^{OLS} = \frac{1}{1 - \frac{N^{OLS}}{D^{OLS}}} = \sigma \cdot \frac{1}{1 + \boxed{(\sigma - 1)\psi} \underbrace{\frac{-1}{D^{OLS}}}_{>0}}. \tag{C.21}$$

The sign of $\boxed{(\sigma - 1)\psi}$ inversely determines the direction of the OLS bias (as $D^{OLS} < 0$ and $-1/D^{OLS} > 0$). Under LAP (the canonical concern in the literature and consistent with our findings), we have $\psi > 0$.[53] Then, OLS underestimates $\sigma^{OLS} < \sigma$ if the true $\sigma > 1$. If instead $\sigma < 1$, then $\sigma^{OLS} > \sigma$.[54] Hence, $\sigma^{OLS} < \sigma^{IV}$ (see next paragraph as a LAP-cleansed estimate) is fully consistent with $\sigma > 1$ and some moderate LAP.

IV estimates recover σ. We now make further progress in gauging the role of LAP with our IV estimates (see Table 3). By drawing on firm growth variation orthogonal to LAP, the IV estimates feature $\psi = 0$, so that $\sigma^{IV} = \sigma^{OLS}\big|_{\mathcal{X}=0} = \frac{1}{1 - \frac{N^{IV}}{D^{IV}}}$ recovers the true σ. Being well above 1, σ^{IV} is consistent with the downward bias of σ^{OLS} derived above.

[52] The corresponding version of our CES production Eq. (1) would be: $Q_{it} = \Omega_{it} \Lambda^K K_{it}^{1-\kappa} \left(\Lambda^{LM} (\alpha^L_{it} L_{it})^{\frac{\sigma-1}{\sigma}} + \Lambda^{LM} (\alpha^M_{it} M_{it})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\kappa}$.

[53] The below results are reversed for $\psi < 0$, which has not received attention or support in the literature. Our IV-based σ^{IV} (addressing OVB) exceeds σ^{OLS}, consistent with $\psi > 0$.

[54] The exception is the extreme case that $-D^{OLS} < \psi < \frac{-D^{OLS}}{1-\sigma}$ and true $\sigma < 1$, for which $\sigma^{OLS} < 0$ (contrary to our OLS finding).

Implied LAP ψ. Given our σ^{IV} estimates, we can constructively back out LAP, $\psi = \frac{D^{OLS}}{\sigma-1}\left(1 - \frac{\sigma}{\sigma^{OLS}}\right)$, using estimates for D^{OLS}, σ^{OLS} and σ^{IV}. Drawing on the 4-year changes in Table 3, we find only moderate LAP of $\psi = 0.15$.[55]

D.5 Non-homothetic CES Production Function: Derivations

The ratio of marginal products from the non-homothetic production function in Eq. (20) from the main text is written:

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} (\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}} \Lambda_i^{LM} \alpha_i^L \kappa L_{it}^{\frac{-1}{\sigma}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}}{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} (\Lambda_i^K)^{\frac{\sigma-1}{\sigma\kappa}} \Lambda_i^{LM} \alpha_i^M \kappa \left(\frac{M_{it}}{Q_{it}^{\eta}}\right)^{\frac{-1}{\sigma}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{-1}{\sigma_{it}}} Q_{it}^{\frac{-\eta}{\sigma}}. \quad \text{(C.22)}$$

Multiplying this expression by $\frac{L_{it}}{Q_{it}}/\frac{M_{it}}{Q_{it}}$ yields Eq. (21) of the main text:

$$\frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}} Q_{it}^{\frac{-\eta}{\sigma}}. \quad \text{(C.23)}$$

D.6 Markups and Wage Markdowns

We now derive the equations for firms' markups and wage markdowns in terms of observables and output elasticities following De Loecker and Warzynski (2012) and Dobbelaere and Mairesse (2013). As discussed in the main text, we assume that intermediates are supplied perfectly elastically and that there are no adjustment costs (as standard in the literature for this derivation). *Strictly speaking, for our firm-level analysis in changes, a weaker assumption is sufficient: we can permit intermediate input market imperfections γ_{it}^M but those must be constant over time within a firm, i.e., $\gamma_{it}^M = \gamma_i^M$* (indeed, our results provide suggestive evidence for that property). This implies the following cost-minimization problem:

$$\mathcal{L}_{it} = P_{it}^L(L_{it})L_{it} + P^M M_{it} + P^K K_{it} - \lambda_{it}(Q_{it} - Q_{it}(.)), \quad \text{(C.24)}$$

where, for simplicity, we also abstract from capital market imperfections, and where $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$. The first-order conditions for labor and intermediates are:

$$P_{it}^M = \frac{P_{it}}{\mu_{it}}\frac{\partial Q_{it}}{\partial M_{it}}, \qquad P_{it}^L \gamma_{it}^L = \frac{P_{it}}{\mu_{it}}\frac{\partial Q_{it}}{\partial M_{it}}. \quad \text{(C.25)}$$

[55] This calculation ignores that LAP biases translog output elasticities. Corresponding cost share-based estimates of σ^{OLS} and σ^{IV} using 4-year changes (Table A.8) imply $\psi = 0.04$. Jaumandreu and Mullens (2026) show for the U.S. that LAP played only a limited role in manufacturing productivity growth after 2005.

Rearranging the FOC for intermediates yields an expression for the markup:

$$\mu_{it} = \frac{P_{it}}{MC_{it}} = \theta_{it}^{M} \frac{P_{it} Q_{it}}{P_{it}^{M} M_{it}}. \tag{C.26}$$

Combining the FOCs for labor and intermediates recovers the wage markdown:

$$\gamma_{it}^{L} = \frac{\theta_{it}^{L}}{\theta_{it}^{M}} \frac{P_{it}^{M} M_{it}}{P_{it}^{L} L_{it}}. \tag{C.27}$$

Rearranging the FOC for labor yields a measure of combined distortions from markup and labor market imperfections:

$$\mu_{it} \gamma_{it}^{L} = \theta_{it}^{L} \frac{P_{it} Q_{it}}{P_{it}^{L} L_{it}}. \tag{C.28}$$

D.7 Fixed-Cost Model

We now provide a simple assessment of how fixed costs could account for our findings. We do so in a simple model in which a portion of the labor is fixed. To assess the capacity of fixed costs to explain our findings, we compute the latent fixed labor cost share that would be required to explain our results without labor-intermediate substitution and imperfect input markets. To eliminate the role of intermediate-labor input substitutability, we impose a Cobb-Douglas production function.

Firms produce output using labor, intermediates, and capital. Now, total labor L_{it} consists of fixed overhead labor L_{it}^{F} (where the time index in principle permits non-constant amounts, but the key point is that it is not a choice variable) and variable production labor is $L_{it} - L_{it}^{F}$. The production function is:

$$Q_{it} = (L_{it} - L_{it}^{F})^{\Phi^{L}} M_{it}^{\Phi^{M}} K_{it}^{\Phi^{K}}. \tag{C.29}$$

Differentiating Q_{it} with respect to L_{it} yields the output elasticity of *total* labor:

$$\underbrace{\frac{dQ_{it}}{Q_{it}} \frac{L_{it}}{dL_{it}}}_{=\theta_{it}^{L}} = \Phi^{L} \frac{L_{it}}{L_{it} - L_{it}^{F}}. \tag{C.30}$$

Hence, the output elasticity of total labor—the object we study in the main text—can vary due to the presence of fixed labor costs, even though the structural output elasticity of variable production labor is constant at Φ^{L}.

In the fixed-cost model, the fixed portion of labor governs the relationship between firm growth and the output elasticity of total labor:

$$\frac{d\theta_{it}^L}{\theta_{it}^L} = -\underbrace{\frac{L_{it}^F}{L_{it} - L_{it}^F}}_{\substack{\text{Fixed labor to} \\ \text{variable labor ratio}}} \frac{dL_{it}}{L_{it}} \Leftrightarrow \underbrace{\frac{L_{it}^F}{L_{it}}}_{\substack{\text{Share of fixed} \\ \text{labor in total labor}}} = \frac{-\frac{d\theta_{it}^L}{\theta_{it}^L}\frac{dL_{it}}{L_{it}}}{1 - \frac{d\theta_{it}^L}{\theta_{it}^L} \Big/ \frac{dL_{it}}{L_{it}}}. \tag{C.31}$$

We now insert the coefficients from Table 2 for $\frac{d\theta_{it}^L}{\theta_{it}^L}$ and $\frac{dL_{it}}{L_{it}}$, as they capture the average change in labor output elasticities (θ_{it}^L) and labor quantities in response to output growth, into Eqs. (C.31). This allows us to infer the fixed labor share required to fully explain our findings without relying on factor substitution and imperfect input markets. For 1-year changes, the OLS (IV) estimates imply a portion of fixed labor in total labor of 0.51 (0.50) for the one-year horizon and 0.28 (0.31) at the four-year horizon.

In addition, the nature of fixed input requirements—strictly speaking, not adjusting in the short run and remaining constant across firm sizes—suggest that the shrinking output elasticity of labor with firm growth should be strongly sized-based.[56] However, we do not find evidence supporting this notion; instead, we document stable effects of firm growth on inputs and output elasticities by firm size classes (Table A.6).

Finally, fixed labor costs cannot explain the shift from the output elasticity of labor to the output elasticity of intermediates. In the above model, the latter is fixed at Φ^M:

$$\theta_{it}^M = \Phi^M \quad \Rightarrow \quad \frac{d\theta_{it}^M}{\theta_{it}^M} = 0. \tag{C.32}$$

Our findings that intermediate input output elasticities increase and absorb the labor output elasticity declines contradict this prediction of a fixed-cost model, whereas our proposed substitution mechanism can fully account for our findings.[57] (And if anything, fixed costs in intermediates, too, would predict a decline rather than, as we find, an increase in output elasticities of intermediates.) We caveat that our simple fixed-cost setup may not capture richer versions of fixed costs, including those that may scale with firm size, multiple fixed costs across products or business activities/markets/export status (although we do not find striking heterogeneity across industries in Figure A.3) or fixed costs in other factors, but we believe that our findings above suggest that fixed costs are unlikely to be a parsimonious explanation of the full set of our findings. However, we do not aim to definitively adjudicate between our mechanism and the fixed-cost view.

[56] To see this, fix L^F and let L grow, such that the output elasticity becomes constant and converges to Φ^L.

[57] Of course, *cost shares* would shift towards intermediates, but we directly study output elasticities as well.

E Further Details on the German Firm-Product Level data

Table E.1: Variable definition in the German micro data.

Variable	Definition
L_{it}	Labor in headcounts.
P_{it}^L	Firm wage (firm average), defined as gross salary before taxes (including mandatory social costs) + "other social expenses" (including expenditures for company outings, advanced training, and similar costs) divided by the number of employees.
K_{it}	Capital (including tangible and intangible assets) derived by a perpetual inventory method precisely as in Bräuer et al. (2023) (who provided us with their codes). Intangibles include, software, patents, licenses, brand/trademark value, and similar items.
M_{it}	Deflated total intermediate input expenditures, defined as expenditures for raw materials, energy, intermediate services, goods for resale, renting, repairs, and contracted work conducted by other firms.
E_{it}	Deflated expenditures for raw, auxiliary, and operating materials and energy inputs (includes external product components). E_{it} is part of M_{it}.
$Merch_{it}$	Deflated expenditures for merchandise. $Merch_{it}$ is part of M_{it}.
Sub_{it}	Deflated expenditures for subcontracted production performed by other companies. Sub_{it} is part of M_{it}.
Rep_{it}	Deflated expenditures for repairs, maintenance, installation, and assembly. Rep_{it} is part of M_{it}.
$Temp_{it}$	Deflated expenditures for temporary agency workers. $Temp_{it}$ is part of M_{it}.
$Rent_{it}$	Deflated expenditures for rent, leases, leasing. Rep_{it} is part of M_{it}.
$Other_{it}$	Deflated expenditures for Other intermediate costs (insurance, postage, transport, etc.). $Other_{it}$ is part of M_{it}.
$P_{it}^M M_{it}$	Nominal values of total intermediate input expenditures.
$P_{it}Q_{it}$	Nominal total revenue, defined as total gross output, including, among others, sales from own products, sales from intermediate goods, revenue from offered services, and revenue from commissions/brokerage.
Q_{it}	Quasi-quantity measure of physical output, i.e., $P_{it}Q_{it}$ deflated by a firm-specific price index (denoted by PI_{it}).
PI_{it}	Firm-specific Törnqvist price index, derived as in Eslava et al., 2004. See Appendix F.1 for its construction.
P_{igt}	Price of a product g.
$share_{igt}$	Revenue share of a product g in total firm revenue.
ms_{it}	Weighted average of firms' product market shares in terms of revenues. The weights are the sales of each product in firms' total product market sales.
G_{it}	Headquarter location of the firm. 90% of firms in our sample are single-plant firms.
D_{it}	A four-digit industry indicator variable. The industry of each firm is defined as the industry in which the firm generates most of its sales.
Exp_{it}	Dummy-variable being one, if firms generate export market sales.
$NumP_{it}$	The number of products a firm produces.

Data access. The data can be accessed at the Research Data Centres of the Federal Statistical Office and those of the Länder (states). DOIs: 10.21242/42131.2017.00.03.1.1.0, 10.21242/42221.2018.00.01.1.1.0, and 10.21242/42111.2018.00.01.1.1.0.

Variable definitions. Table E.1 summarizes variable definitions.

Outlier cleaning. We exclude the top and bottom two percent outliers with respect to value added over revenue and revenue over labor, capital, intermediate input expenditures, and labor costs. We replace quantity and price information with missing values for products displaying a price deviation from the average price in the top and bottom one percent. We drop the sectors 16 (tobacco), 23 (mineral oil and coke), and 37 (recycling) due to insufficient observation counts.

Capital stock estimation. We calculate firm-specific time series of capital stocks following

Bräuer et al. (2023) and consider the law of motion:

$$K_{it} = K_{it-1}(1 - depr_{jt-1}) + I_{it-1}. \tag{E.1}$$

K_{it}, $depr_{jt-1}$, and I_{it-1} denote firm i's capital stock, the depreciation rate of capital, and total investment (buildings, equipment, machines, and other investment goods, including, among others, software, patents, licenses). Nominal values are deflated by two-digit industry-level deflators supplied by the German Statistical Office. The industry- and year-specific depreciation rate is derived from official information on the expected lifetime of capital goods (supplied by the statistical offices, by years, separately for building and equipment capital).[58] The initial capital stock for the perpetual inventory method is derived from reported tax depreciation.[59]

Deriving a time-consistent industry classification. The NACE classification of industries (and thus firms into industries) changed in 2002 and 2008. To recover a time-consistent industry classification (required for the production function estimation), we follow Mertens (2022) and classify firms into NACE Rev 1.1 industries based on their main production activities based on revenue.[60] Mertens (2022) shows that this classification matches closely the one of the statistical offices for 2002-2008 (when industries are already reported in NACE Rev 1.1) in 95% (two digit) and 86% (four digit) of all cases.

[58]Bräuer et al. (2023) define the lifetime of a capital good LT as a function of its depreciation rate: $LT = depr \int_0^\infty (1 - depr)^t t dt$, where $0 < depr < 1$ is the depreciation rate. Partial integration yields: $LT = \frac{depr}{\ln(1-depr) \times \ln(1-depr)}$, which can be solved numerically for $depr$.

[59]We do not use the reported tax depreciation when calculating capital stock series as tax depreciation may vary due to state-induced tax incentives and might therefore not reliably reflect the true amount of depreciated capital. As firms likely report too high depreciation levels due to such tax incentives, our first capital values within a capital series are likely overestimated. However, over time, observed investment decisions gradually receive a larger weight in estimated capital stocks, mitigating the impact of the first capital stock. As we estimate very reasonable output elasticities (see Table A.1), we are confident that our capital variables reliably reflect firms' true capital stocks.

[60]This procedure exploits that the first four digits of the ten-digit GP product classification reported in the German data equal the NACE classification (product's industry). Applying this method requires a consistent reclassification of all products into the NACE Rev 1.1-equivalent GP2002 scheme. In the few ambiguous cases, we follow the firms' product mix over the reclassification periods and unambiguously reclassify most products (we observe production before and after reclassification).

F Production Function Estimation in the German Data

We follow Mertens (2022) in estimating the production function. This approach yields firm- and time-specific output elasticities by assuming the following translog production function (throughout, lower case letter denote logs):

$$\tilde{q}_{it} = \boldsymbol{\phi}'_{it}\,\boldsymbol{\beta} + \omega_{it} + \epsilon_{it}\,. \tag{F.1}$$

$\tilde{q}_{it}$ denotes the log of produced quantities measured as sales deflated by a firm-specific input price index as described below and $\boldsymbol{\phi}'_{it}$ captures the inputs capital (k_{it}), labor (l_{it}), and intermediates (m_{it}) and its interactions. The industry-specific production function that we will estimate for each two-digit NACE Rev. 1.1 industry is specified in logs as:

$$\begin{aligned} \tilde{q}_{it} =& \beta_l l_{it} + \beta_m m_{it} + \beta_k k_{it} + \beta_{ll} l_{it}^2 + \beta_{mm} m_{it}^2 + \beta_{kk} k_{it}^2 \\ &+ \beta_{lk} l_{it} k_{it} + \beta_{lm} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \beta_{lkm} l_{it} k_{it} m_{it} + \omega_{it} + \epsilon_{it}. \end{aligned} \tag{F.2}$$

The output elasticity of labor is:

$$\frac{\partial \tilde{q}_{it}}{\partial l_{it}} = \beta_l + 2\beta_{ll} l_{it} + \beta_{lm} m_{it} + \beta_{lk} k_{it} + \beta_{lkm} k_{it} m_{it}\,. \tag{F.3}$$

ϵ_{it} is an i.i.d. error term and ω_{it} denotes Hicks-neutral productivity and follows a Markov process. ω_{it} is unobserved to the econometrician, yet firms know ω_{it} before making input decisions for flexible inputs (intermediates in our case). We assume that only firms' input decision for intermediates depends on productivity shocks. Labor and capital do not respond to contemporary productivity shocks. However, our results are similar when allowing labor to respond to productivity innovations. In fact, the CompNet routine for the production function estimation models labor as flexible and we find consistent results in both datasets. In Eq. (F.2), we explicitly include labor quantities rather than labor expenditure, which is preferable in case of imperfect labor markets. Intermediates and capital enter as deflated monetary values, and we account for input price variation through the input price control function as described below.

There are three issues preventing us from estimating the production function in Eq. (F.1) using OLS. *First*, we need to estimate a physical production model to recover the relevant output elasticities. Although we observe product quantities, quantities cannot be aggregated across the various products of multi-product firms. Relying on the standard practice to apply sector-specific output deflators does not solve this issue if output prices vary within industries. While we address this bias below we note that this bias does not affect ratios of output elasticities, as the elasticities themselves are biased to the same extent. *Second*, we do not observe firm-specific input prices for capital and intermediate inputs. If input prices are correlated with input decisions and output levels, an endogeneity issue arises. Unlike the previous identification issue, this bias affects output elasticities differentially. *Third*, the fact that productivity is unobserved and that firms' flexible input decisions depend on productivity shocks, creates another endogeneity problem. We now discuss how we solve these three identification problems.

F.1 Challenge 1: Deriving a Firm-specific Output Price Index

As we cannot aggregate output quantities across different products of a firm (a common problem), we follow Eslava et al. (2004) and construct a firm-specific price index from observed output prices. We use this index to purge firm revenue from price variation by deflating firm revenues with it. We construct firm-specific Törnqvist price indices for each firm's composite revenue from its various products in the following way:

$$PI_{it} = \prod_{g=1}^{n} \frac{p_{igt}}{p_{igt-1}}^{1/2(share_{igt}+share_{igt-1})} PI_{it-1}\,. \tag{F.4}$$

PI_{it} is the price index, p_{igt} is the firm-specific price of good, g, that we observe in the data, and $share_{igt}$ is the share of this good in total product market sales of firm i in period t. The growth of the index value is the product of the individual products' price growths, weighted with the average sales share of that product over the current and the last year. The first year available in the data is the base year (i.e., $PI_{it=1995} = 100$). If firms enter after 1995, we follow Eslava et al. (2004) and use an industry average of the computed firm price indices as a starting value. Similarly, we impute missing product price growth information in other cases with an average of product price changes within the same industry.[61] After deflating firm revenue with this price index, we have a quasi-quantity measure of output, which we highlight with a tilde: $\tilde{q}_{it}$.[62]

[61]For roughly 30% of all product observations in the data, firms do not have to report quantities as the statistical office views them as not being meaningful.

[62]Note that, as discussed in Bond et al. (2021), using an output price index does not fully purge firm-specific price variation. There remains a base year difference in prices. Yet, using a firm-specific price index follows the usual practice of using price indices to deflate nominal values, we are thus following the best practice. Moreover, it is the only available approach when pooling multi- and single-product firms. Estimating the production function separately by single-plant firms requires other strong assumptions like perfect input divisibility of all inputs across all products. Finally, our results are also robust to using cost-share approaches to estimate the production function, which requires other assumptions (constant returns to scale, competitive input markets, and the absence of adjustment costs).

F.2 Challenge 2: Accounting for Unobserved Input Price Variation

To control for input price variation across firms, we use a firm-level analog of De Loecker et al. (2016) and define a price-control function from firm-product-level output price information that we add to the production function in Eq. (F.1):

$$\tilde{q}_{it} = \boldsymbol{\phi}'_{it}\boldsymbol{\beta} + B_{it}((pi_{it}, ms_{it}, G_{it}, Ind_{it}) \times \boldsymbol{\phi}^c_{it}) + \omega_{it} + \epsilon_{it}\,. \tag{F.5}$$

$B_{it}(.) = B_{it}((pi_{it}, ms_{it}, G_{it}, Ind_{it}) \times \boldsymbol{\phi}^c_{it})$ is the price control function. It consists of our logged firm-specific output price index (pi_{it}), a logged sales-weighted average of firms' product market sales shares (ms_{it}), a headquarter location dummy (G_{it}) and a four-digit industry dummy (Ind_{it}). $\boldsymbol{\phi}^c_{it} = [1; \boldsymbol{\phi}_{it}]$, where $\boldsymbol{\phi}_{it}$ includes the production function input terms as specified in Eq. (F.2). These are either in monetary terms and deflated by an industry-level deflator (capital and intermediates) or already reported in quantities (labor).[63] Conditional on elements in $B(.)$, we assume that there are no remaining input price differences across firms. This assumption is more general than the ones employed in most other studies estimating production functions without access to firm-specific price data and which implicitly assume identical input and output prices within industries.

A difference with De Loecker et al. (2016) is that we transfer their product-level framework to the firm level. For that, we aggregate firm-product sales shares in firms' total sales to the firm-level. This assumes that i) such firm aggregates of product quality increase in firm aggregates of product prices and input quality, ii) firm-level costs for inputs entering as deflated expenditures increase in firm-level input quality, and iii) product price elasticities are equal across the firms' products. These or even stricter assumptions are always implicitly invoked when estimating firm-level production functions.

Finally, we note that if some of the above assumptions do not hold, including the price control function is still preferable to omitting it as it can nevertheless absorb some of the unobserved price variations and does not require that input prices vary between firms with respect to all (or any) elements of $B_{it}(.)$. The attractiveness of a price control function lies in its agnostic view about the existence and degree of input price variation.

[63] The constant entering $\boldsymbol{\phi}^c_{it}$ highlights that elements of $B(.)$ enter the price control function linearly and interacted with $\boldsymbol{\phi}_{it}$ (a consequence of the translog production function). The idea behind the price-control function $B(.)$ is that output prices, product market shares, and firms' location and industry are informative about firms' input prices. In particular, we assume that product prices and market shares contain information about product quality and that producing high-quality products requires expensive high-quality inputs. De Loecker et al. (2016) discuss that this motivates adding a control function containing output price and market share information to the right-hand side of the production function to control for unobserved input price variation emerging from input quality differences across firms. We also include year, location, and four-digit industry dummies into $B(.)$ to further absorb the remaining differences in local and four-digit industry-specific input prices.

F.3 Challenge 3: Controlling for Unobserved Productivity

To address the dependence of firms' intermediate input decisions on unobserved productivity, we employ a control function approach (i.e., Wooldridge, 2009). We base our control function on firms' consumption of energy and materials, which we denote by e_{it} and which are components of total intermediate inputs. Inverting the demand function for e_{it} defines an expression for productivity:

$$\omega_{it} \equiv g_{it}(.) = g_{it}(ene_{it}, k_{it}, l_{it}, \mathbf{\Gamma}_{it}). \tag{F.6}$$

$\mathbf{\Gamma}_{it}$ captures state variables of the firm, that in addition to k_{it} and l_{it} affect firms' demand for ene_{it}. Ideally, $\mathbf{\Gamma}_{it}$ should include a wide set of variables affecting productivity and demand for e_{it}. We include dummy variables for export (EX_{it}) activities, the log of the number of products a firm produces ($NumP_{it}$), and the average wage a firm pays (P^L_{it}) into $\mathbf{\Gamma}_{it}$. The latter absorbs unobserved quality and price differences that shift input demand for e_{it}. Including wages also helps to absorb variation in marginal costs and wage markdowns, which addresses some of the critique raised by Doraszelski and Jaumandreu (2021) because wages are a serially correlated input price that is also correlated with other input prices. Remember that productivity follows a first-order Markov process. Firms can shift this Markov process as described in De Loecker (2013), giving rise to the following law of motion for productivity: $\omega_{it} = h_{it}(\omega_{it-1}, \mathbf{T}_{it-1}) + \xi_{it} = h_{it}(.) + \xi_{it}$, where ξ_{it} denotes the innovation in productivity and $\mathbf{T}_{it} = (EX_{it}, NumP_{it})$ reflects that we allow for learning effects from export market participation and (dis)economies of scope through adding and dropping products to influence firm productivity.[64] Plugging Eq. (F.6) and the law of motion for productivity into Eq. (F.5) yields the equation that we estimate:

$$\tilde{q}_{it} = \boldsymbol{\phi}'_{it}\boldsymbol{\beta} + B_{it}(.) + h_{it}(.) + \epsilon_{it} + \xi_{it}. \tag{F.7}$$

F.4 Identifying Moments and Results

We estimate Eq. (F.7) separately by two-digit NACE Rev. 1.1 industries with a one-step estimator as in Wooldridge (2009).[65] Our estimator instruments for current flexible

[64]We would also like to add information on R&D investment to the productivity model but do not observe R&D expenditures for the early years in our data.

[65]We approximate $h_{it}(.)$ by a third-order polynomial in all of its elements, except for the variables in $\mathbf{\Gamma}_{it}$. Those we add linearly. $B_{it}(.)$ is approximated by a flexible polynomial where we interact the output price index with elements in $\boldsymbol{\phi}_{it}$ and add the vector of market shares, the output price index, and the location and industry dummies linearly. Interacting further elements of $B_{it}(.)$ with $\boldsymbol{\phi}_{it}$ creates too many parameters to be estimated. This implementation is similar to De Loecker et al. (2016).

inputs (i.e., intermediates) with lagged values to address the dependence of firms' flexible input decisions on realizations of ξ_{it}—similarly we instrument firms' market share and output price index with their lags as we consider them as flexible variables.[66] We define identifying moments jointly on ϵ_{it} and ξ_{it}:

$$E[(\epsilon_{it} + \xi_{it})\mathbf{Y}_{it}] = 0. \tag{F.8}$$

$\mathbf{Y}_{it}$ includes lagged interactions of intermediate inputs with labor and capital, contemporaneous interactions of labor and capital, contemporaneous location and industry dummies, the lagged output price index, lagged market shares, lagged elements of $h_{it}(.)$, and lagged interactions of the output price index with production inputs. Formally, this implies:

$$\mathbf{Y}'_{it} = (J_{it}(.), A_{it-1}(.), T_{it-1}(.), \Psi_{it}(.), \boldsymbol{\vartheta}_{it-1}) \ , \tag{F.9}$$

where we defined: $J_{it}(.) = (l_{it},\, k_{it},\, l_{it}^2,\, k_{it}^2,\, l_{it}k_{it},\, G_{it},\, Ind_{it})$; $A_{it}(.) = (m_{it},\, m_{it}^2,\, l_{it}m_{it},\, k_{it}m_{it},\, l_{it}k_{it}m_{it},\, ms_{it},\, \pi_{it})$; $T_{it}(.) = ((l_{it},\, k_{it},\, l_{it}^2,\, k_{it}^2,\, l_{it}k_{it},\, m_{it},\, m_{it}^2,\, l_{it}m_{it},\, k_{it}m_{it},\, l_{it}k_{it}m_{it}) \times \pi_{it})$; $\Psi_{it}(.) = \sum_{n=0}^{3}\sum_{w=0}^{3-b}\sum_{h=0}^{3-n-b} l_{it-1}^{n}\, k_{it-1}^{b}\, ene_{it-1}^{h}$; and $\boldsymbol{\vartheta}_{it-1} = (Exp_{it-1},\, NumP_{it-1},\, P_{it-1}^{L})$, with P_{it}^{L} denoting average wages. Table A.1 reports the resulting output elasticities.

F.5 Limitations

Our approach addresses biases stemming from input and output prices, which affect existing production function estimates. There remain inherent limitations in the general methodology, which are subjects of active research. For example, Bond et al. (2021) highlight potential biases in output elasticities if unobserved input sub-components directly influence demand (e.g., intermediates used in marketing). Similarly, aggregating labor into a single measure can bias output elasticities (arguably less critical in manufacturing contexts). Furthermore, incomplete specification of the productivity control function may violate the monotonicity assumption (Doraszelski and Jaumandreu, 2021).[67] Although we extend traditional methods by incorporating wages, export status, and the number of products into our productivity control function, important unobserved factors may still exist that could cause the monotonicity assumption to fail. We therefore additionally draw on the non-parametric identification of output elasticities based on cost shares.

[66]These timing assumptions also address any simultaneity concerns with respect to the price variables entering the right-hand side of our estimation.

[67]Gandhi et al. (2020) also critique the control function approach. An alternative is to estimate a dynamic panel model, as in Blundell and Bond (2000), which assumes an AR(1) productivity process.

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