

From Labor to Intermediates: Firm Growth, Input Substitution, and Monopsony

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How do firms grow?

- We study this question theoretically and empirically.
 - Firm growth dynamics of (“effects on”) input mix, cost shares, and output elasticities.
 - Main focus: labor vs. intermediate inputs. Account includes capital.
 - Focus on production function properties and input prices (not product markets).
 - Benchmark: input-price-takers w/ Cobb Douglas.
- Main finding: **firms grow by switching from labor (and capital) to intermediates.**
 - Across the board: input mix, output elasticities, cost shares—i.e., labor share shrinks.
 - Analysis permits market power-size gradients.
- **Parsimonious** rationalization based on two (need both!) factors:
 - Labor-intermediates substitution elasticity > 1 .
 - Increasing shadow price of labor—probable source: **monopsony** (or adjustment costs).
- We connect these two literatures and show their joint implications for firm growth.

Preview: firm growth by (i) elasticity of substitution σ (b/w labor and intermediates) and (ii) inverse elasticity of wage to firm $(\varepsilon^L)^{-1}$

| | $\sigma < 1$ | $\sigma = 1$ | $\sigma > 1$ |
|---|--------------|--------------|--|
| L more elastic than M $(\varepsilon^L)^{-1} > (\varepsilon^M)^{-1}$ | | | L_{it}/M_{it} $\theta_{it}^L/\theta_{it}^M$ CS_{it}^L LS_{it} |
| L as elastic as M $(\varepsilon^L)^{-1} = (\varepsilon^M)^{-1}$ | | | L_{it}/M_{it} $\theta_{it}^L/\theta_{it}^M$ CS_{it}^L LS_{it} |
| L less elastic than M $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$ | | | L_{it}/M_{it} $\theta_{it}^L/\theta_{it}^M$ CS_{it}^L LS_{it} |

Notes: Ceteris paribus w.r.t. to non-changing market imperfections with output growth (relaxed in full analysis). θ is output elasticity, LS is labor share, CS is cost share, σ is the elasticity of substitution between labor and intermediate inputs, and ε are the firm-specific elasticities of input prices to input quantities.

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| | = | = | = | L_{it}/M_{it} |
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| | = | = | = | CS_{it}^L LS_{it} |
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| | = | = | = | CS_{it}^L |
| | = | = | = | LS_{it} |
| L less elastic than M $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$ | ↓ ↑ ↑ ↑ | ↓ = | ↓ ↓ ↓ ↓ | L_{it}/M_{it} $\theta_{it}^L/\theta_{it}^M$ CS_{it}^L LS_{it} |

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How do firms grow?

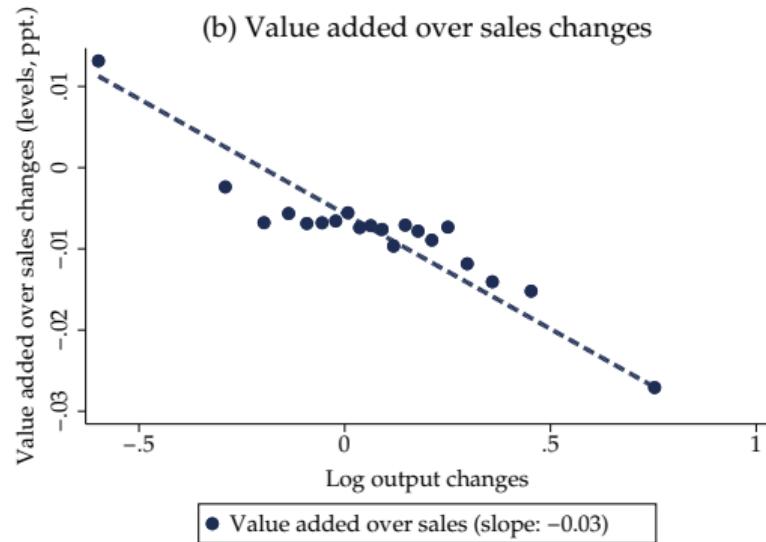
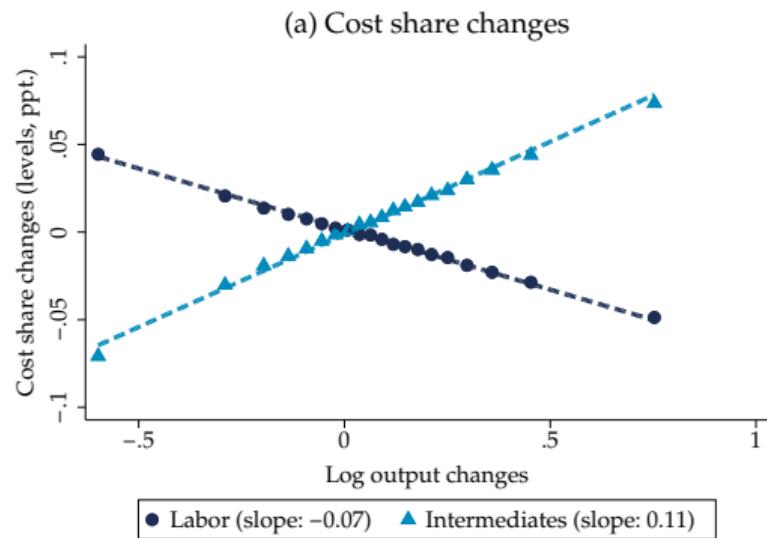
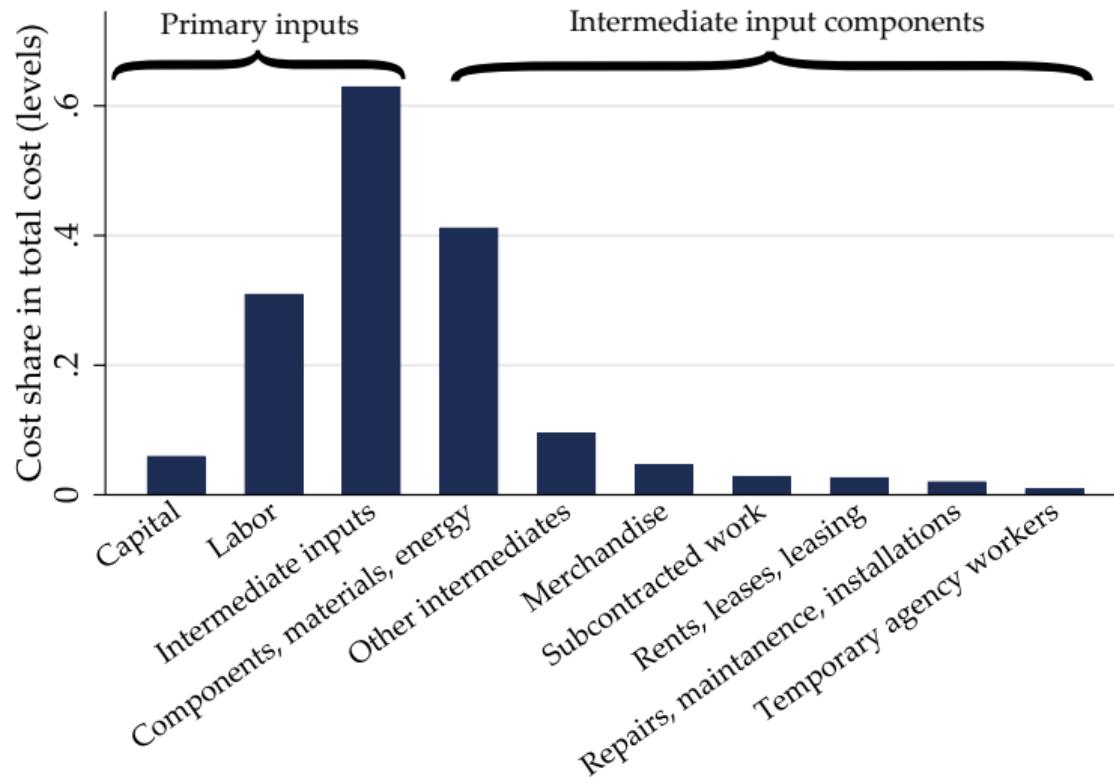


Figure: Change in Labor and Intermediate Cost Shares and Value Added over Sales Ratios (Levels) against Output Growth, 4-Year Changes. German manufacturing micro firm data.

Outline

- Simple model and predictions
- Meta analysis of existing estimates in related literature
- Micro-evidence on firm growth
- Interpretation and implied parameters
- Firm-level labor share implications
- Industry-level results/implications
- Conclusion

Stepping stone and background fact: Cost shares by inputs



German manufacturing firm micro data.

Firm growth in theory: Production function

- Production function:

$$Q_{it} = \Omega_{it} \Lambda_i^K K_{it}^{1-\kappa} \left(\Lambda_i^{LM} \alpha_i^L L_{it}^{\frac{\sigma-1}{\sigma}} + \Lambda_i^{LM} \alpha_i^M M_{it}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\kappa} \quad (1)$$

- where Ω_{it} = TFP and α_i^L , α_i^M , Λ_i^{LM} , and Λ_i^K are distribution parameters. σ is the substitution elasticity between labor and intermediates. κ is a Cobb-Douglas exponent ($1 - \kappa$ equals the capital cost share on perfectly competitive markets).
- Marginal products imply:

$$\frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha_i^L}{\alpha_i^M} \left(\frac{L_{it}}{M_{it}} \right)^{\frac{\sigma-1}{\sigma}} \quad (2)$$

$$\frac{\sigma - 1}{\sigma} = \frac{\Delta \ln(\theta_{it}^L) - \Delta \ln(\theta_{it}^M)}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})} \quad (3)$$

- where $\theta_{it}^X = \frac{\partial Q_{it}}{\partial X_{it}} \frac{X_{it}}{Q_{it}}$ is the output elasticity of input $X = \{L, K, M\}$ and σ is the substitution elasticity between labor and intermediates.

If $\sigma > 1$ ($\sigma < 1$) [$\sigma = 1$]: **Increase** in $\frac{L_{it}}{M_{it}}$ increases (decreases) [doesn't affect] $\frac{\theta_{it}^L}{\theta_{it}^M}$.

Firm growth in theory: Cost minimization

- Firms minimize costs for **given** level of output. Input markets are imperfect (wedges from adjustment costs or monopsony).
- FOC for input $X = \{L, K, M\}$:

$$P_{it}^X \underbrace{\left(1 + \frac{\partial P_{it}^X}{\partial X_{it}} \frac{X_{it}}{P_{it}^X} + \frac{\partial \chi^X}{\partial X_{it}} \right)}_{\gamma_{it}^X} = \lambda_{it} \frac{\partial Q_{it}}{\partial X_{it}} \quad (4)$$

- P^X is price (e.g., wage) for X , χ^X is an adjustment cost function, λ_{it} is marginal costs, γ_{it}^X is the input price wedge, i.e., $P^X \gamma_{it}^X$ is the input **shadow price**.
- Standard input mix–price ratio link guided by substitution elasticity:

$$\Delta \ln \left(\frac{L_{it}}{M_{it}} \right) = -\sigma \cdot \Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M} \right) \Leftrightarrow \Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M} \right) = \frac{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}{-\sigma} \quad (5)$$

- Input mix follows price mix—in turn, effects on output elasticities depends on which side of 1 σ is (previous slide).
- Given σ , any input mix shift implies a price ratio shift (absent other input-biased shifts).

Firm growth in theory: Cost/output shares

- From FOC for L, K, M , the cost share of an input X is:

$$CS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it}^L L_{it} + P_{it}^M M_{it} + P_{it}^K K_{it}} = \frac{\frac{\theta_{it}^X}{\gamma_{it}^X}}{\frac{\theta_{it}^L}{\gamma_{it}^L} + \frac{\theta_{it}^M}{\gamma_{it}^M} + \frac{\theta_{it}^K}{\gamma_{it}^K}} \quad (6)$$

- From FOC for X , the output share of input X (e.g., labor share) is:

$$OS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it} Q_{it}} = \frac{\theta_{it}^X}{\mu_{it} \gamma_{it}^X} \quad (7)$$

- $\mu_{it} = \frac{P_{it}}{\lambda_{it}}$ is the price over marginal cost markup.
⇒ A decline in θ_{it}^X directly lowers the input's shares in output and cost.
- (Analogous property for value added shares.)

Firm growth in theory: Predictions

- To fix ideas: Assume constant markups (relaxed in full analysis). Isoelastic inverse supply function for X : $P_{it}^X = a_{it}^X X_{it}^{\varepsilon^X}$. (Monopsony, but also stands in for quasi-static adj. costs,...)

⇒ Input demand as a function of Q_{it} , parameters, and marginal costs:

$$X_{it} = \left(\frac{\lambda_{it} \alpha_i^X}{(1 + \varepsilon^X) a_{it}^X} \right)^{\frac{1}{\varepsilon^X}} \left(\frac{\partial Q}{\partial X} \right)^{\frac{1}{\varepsilon^X}} \quad (8)$$

$$\Rightarrow \frac{L_{it}}{M_{it}} = \varrho_{it} \lambda_{it}^{\frac{\sigma+\kappa-1}{\kappa} \left(\frac{1}{\sigma\varepsilon^L+1} - \frac{1}{\sigma\varepsilon^M+1} \right)} Q_{it}^{\left(\frac{1}{\sigma\varepsilon^L+1} - \frac{1}{\sigma\varepsilon^M+1} \right)} \quad (9)$$

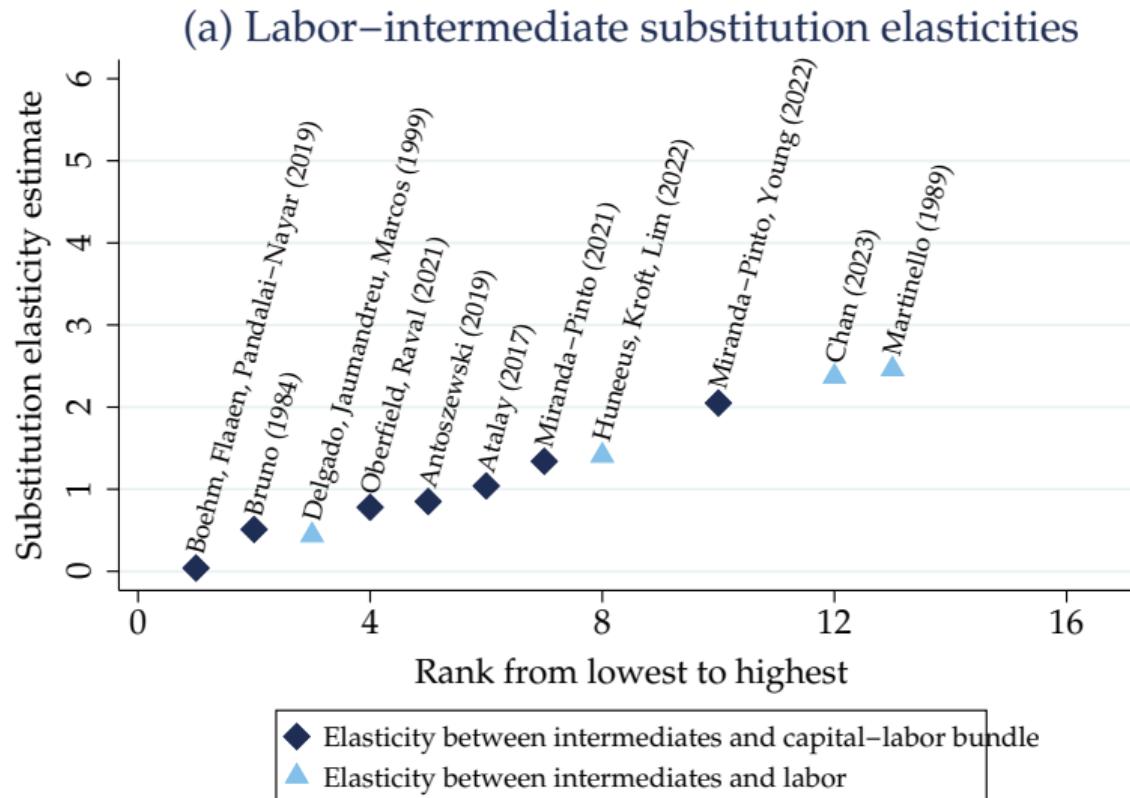
- Response of **input ratio** (and thus **output elasticities** and cost and output shares) to an increase in **output** (i.e., **firm growth**) depends on $\frac{1}{\sigma\varepsilon^L+1} - \frac{1}{\sigma\varepsilon^M+1}$.
- Our estimation: More general, with arbitrarily varying output and input market power/labor adjustment cost.

Firm growth in theory: Predictions

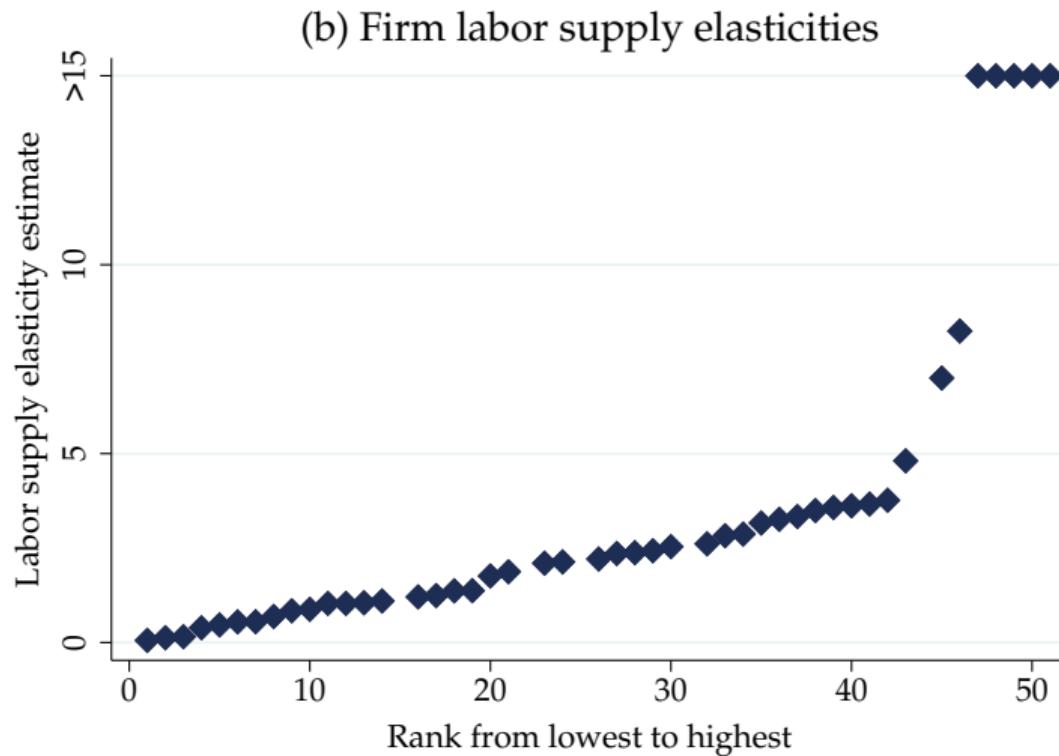
| | $\sigma < 1$ | $\sigma = 1$ | $\sigma > 1$ | |
|--|--|-----------------------------------|--|--|
| L more elastic than M $(\varepsilon^L)^{-1} > (\varepsilon^M)^{-1}$ | \uparrow \downarrow \downarrow \downarrow | \uparrow $=$ $=$ $=$ | \uparrow \uparrow \uparrow \uparrow | L_{it}/M_{it} $\theta_{it}^L/\theta_{it}^M$ CS_{it}^L LS_{it} |
| L as elastic as M $(\varepsilon^L)^{-1} = (\varepsilon^M)^{-1}$ | $=$ $=$ $=$ $=$ | $=$ $=$ $=$ $=$ | $=$ $=$ $=$ $=$ | L_{it}/M_{it} $\theta_{it}^L/\theta_{it}^M$ CS_{it}^L LS_{it} |
| L less elastic than M $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$ | \downarrow \uparrow \uparrow \uparrow | \downarrow $=$ $=$ $=$ | \downarrow \downarrow \downarrow \downarrow | L_{it}/M_{it} $\theta_{it}^L/\theta_{it}^M$ CS_{it}^L LS_{it} |

Notes: Ceteris paribus means constant returns to scale and non-changing market imperfections with output growth.

Benchmarks: Elasticity of substitution estimates in literature



Benchmarks: Labor supply elasticity estimates in literature



Underlying source: meta analysis in Sokolova and Sorensen (2021)

Evidence: Data

- PART 1: **German manufacturing sector firm-level data, 1995-2017**
 - Firms with at least 20 employees, representative 40% sample.
 - Contains quantities and prices of products (allows to account for "price bias" when estimating output elasticities).
 - Contains information on input quantities, costs, capital stock (perpetual inventory method), sales.
 - Data recently used in Mertens (2020), Mertens et al. (2022), Haelbig et al. (2023).
- PART 2: **CompNet country-industry data for 20 European countries, 1999-2021**
 - Harmonized industry-level data for 20 European countries (manufacturing and non-manufacturing).
 - Firms with at least 20 employees (for 14 countries also smaller firms).
 - Yearly coverage varies by country.
 - Contains industry-information on output elasticities, input quantities, costs, labor cost and output shares.
 - Data recently used in Autor et al. (2020), Biondi (2022), Bighelli et al. (2023).
 - Supplement with NBER-CES US data.

Evidence: Main regression & identification

- Idea: “ $\Delta \ln(Y_{it})$ against $\Delta \ln(Q_{it})$ (or $\Delta \ln(P_{it} Q_{it})$)”
- Firm-level regression for variables Y_{it} (input quantities, cost shares, output elasticities):

$$\Delta^h \ln(Y_{it}) = \beta^h \Delta^h \ln Q_{it} + v_{jt} + \epsilon_{it} \quad (10)$$

- h captures different differences horizons (1-10 years). v_{jt} are (4-digit) industry-year FE.
- Visualize with binned scatter plots (residuals).
- Later: also effects on markups and wage markdowns.
- Ideally: exogenous output shifter (input-neutral: TFP or product demand shift).
- Instrument: Bartik demand shock from world export demand in other countries:

$$INS_{it} = \sum_g s_{git=0} \ln \left(\sum_c ex_{gct}^{n \rightarrow world} \right) \quad (11)$$

- g denotes (10-digit) products, $s_{git=0}$ are product sales share in firm i and baseline period 0.
- c are countries $\{AUS, NOR, SWE, SGP, NZL, GBR, CAN, JPN, USA\}$, following Dauth, Findeisen, Suedekum JEEA 2014).

Evidence: Estimation of output elasticities

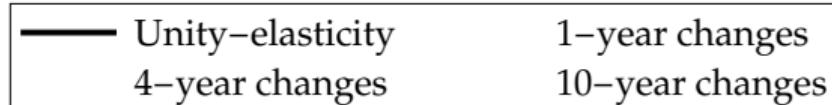
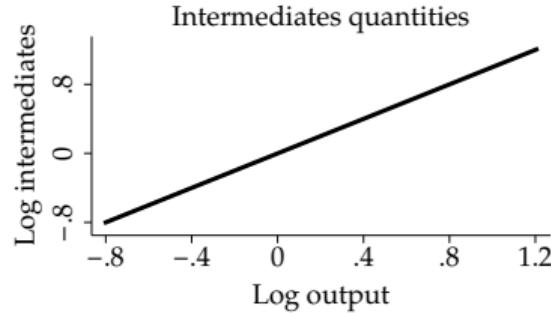
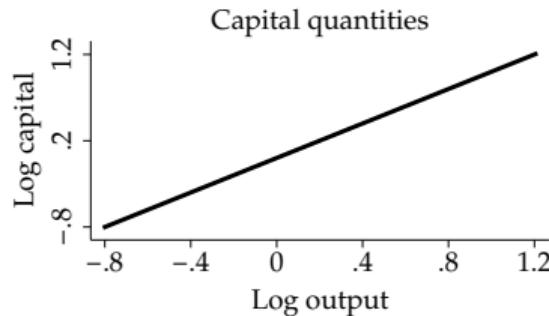
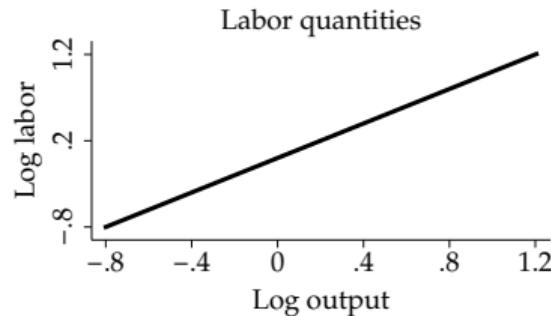
- Dual approach:
 - Translog production function estimation - allows for non-constant returns, monopsony, & input adjustment costs; imposes Hicks-neutrality

$$q_{it} = \beta_I l_{it} + \beta_m m_{it} + \beta_k k_{it} + \beta_{II} l_{it}^2 + \beta_{mm} m_{it}^2 + \beta_{kk} k_{it}^2 + \beta_{Ik} l_{it} k_{it} + \beta_{Im} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \beta_{Ikm} l_{it} k_{it} m_{it} + \omega_{it} + \epsilon_{it} \quad (12)$$

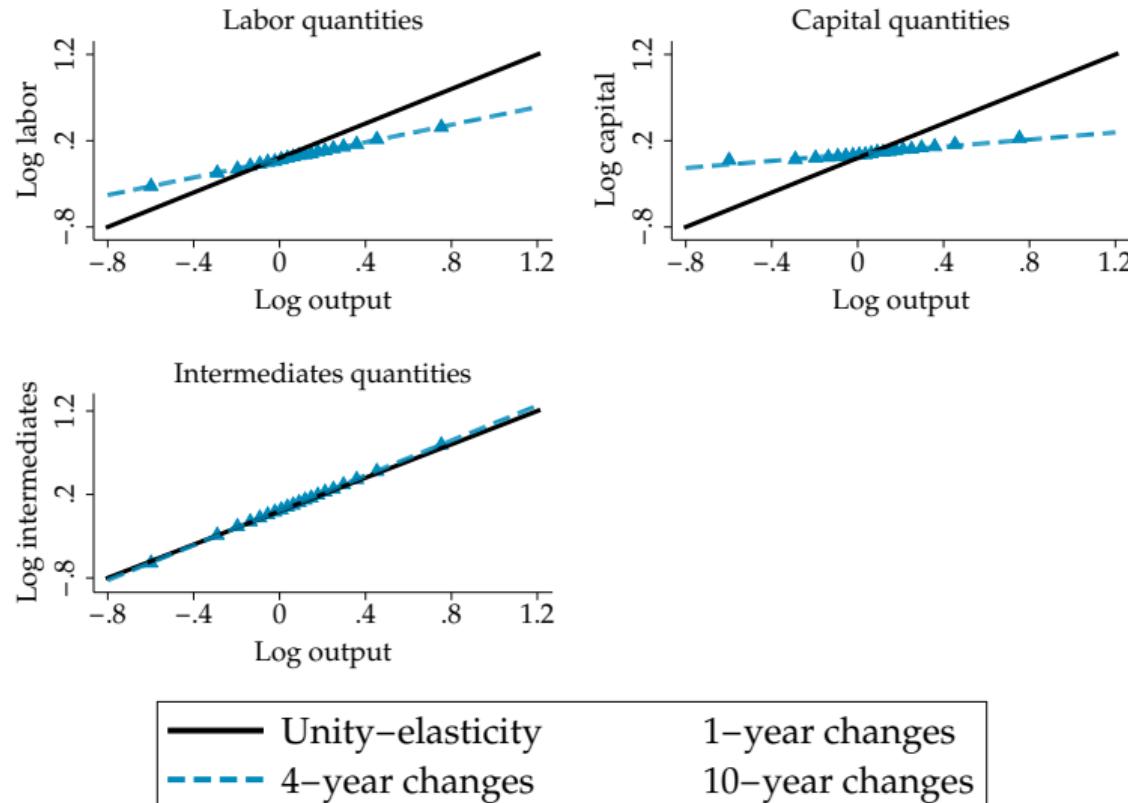
- Lower-case letters denote logs. Estimation via control function approach similar to Wooldridge (2009). Controlling for price biases using a firm-level adaption of De Loecker et al. (2016).
- Cost shares as estimates for output elasticities - allows for non-Hicks-neutrality; imposes **non-changing** returns to scale, monopsony, & input adjustment costs.

$$CS_{it}^L = \frac{P_{it}^L L_{it}}{P_{it}^L L_{it} + P_{it}^M M_{it} + P_{it}^K K_{it}} = \frac{\frac{\theta_{it}^L}{\gamma_{it}^L}}{\frac{\theta_{it}^L}{\gamma_{it}^L} + \frac{\theta_{it}^M}{\gamma_{it}^M} + \frac{\theta_{it}^K}{\gamma_{it}^K}} \quad (13)$$

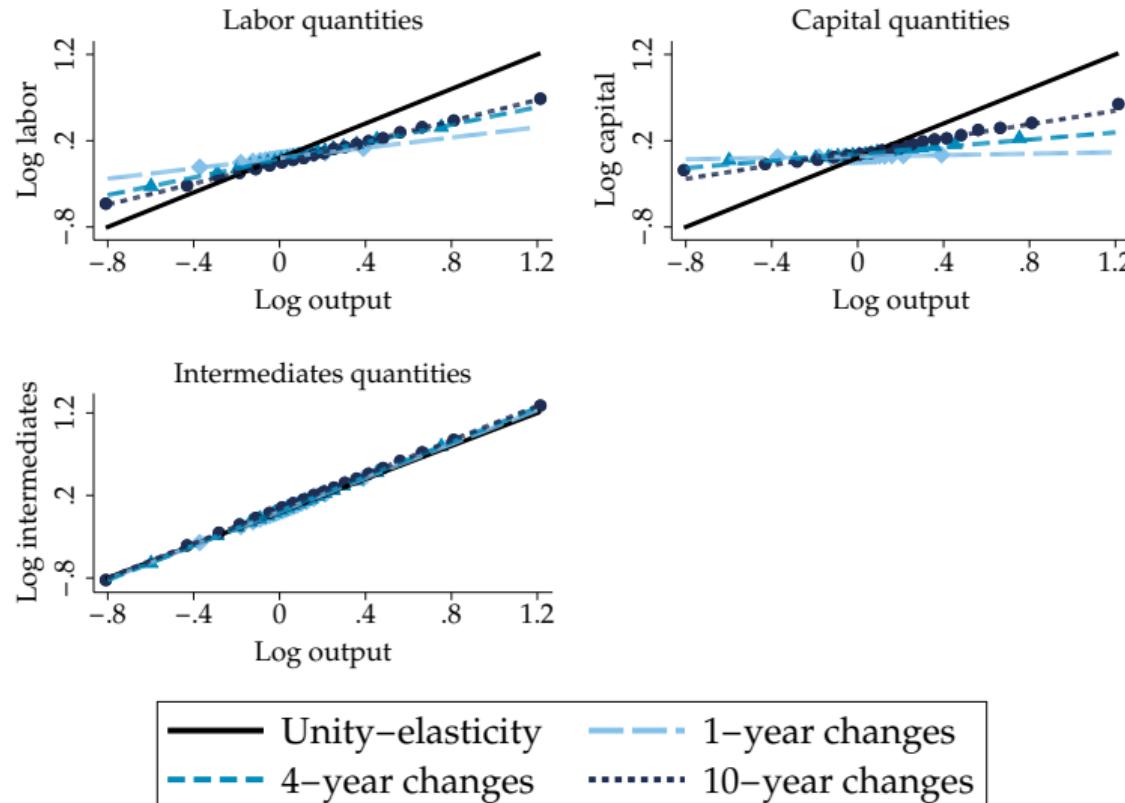
Evidence (OLS): input quantities



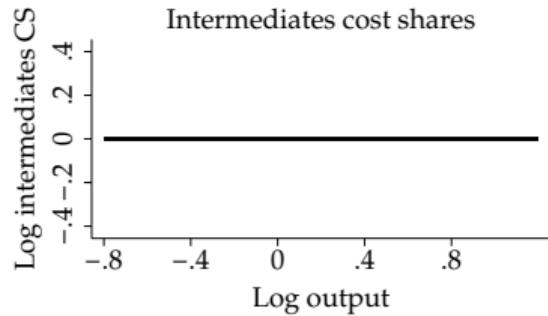
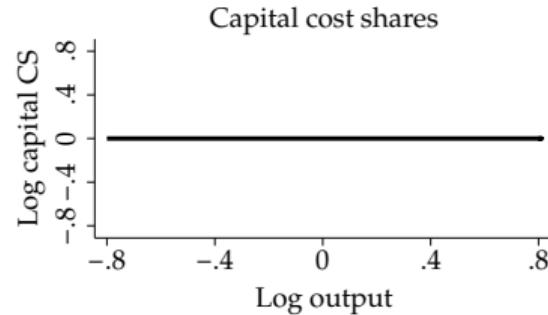
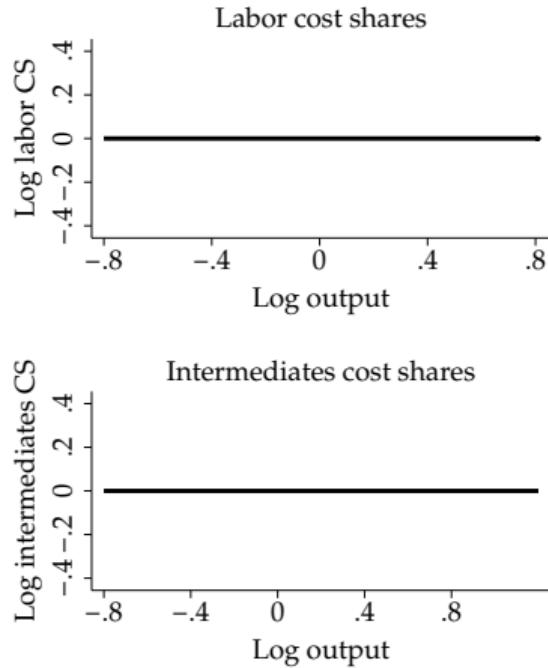
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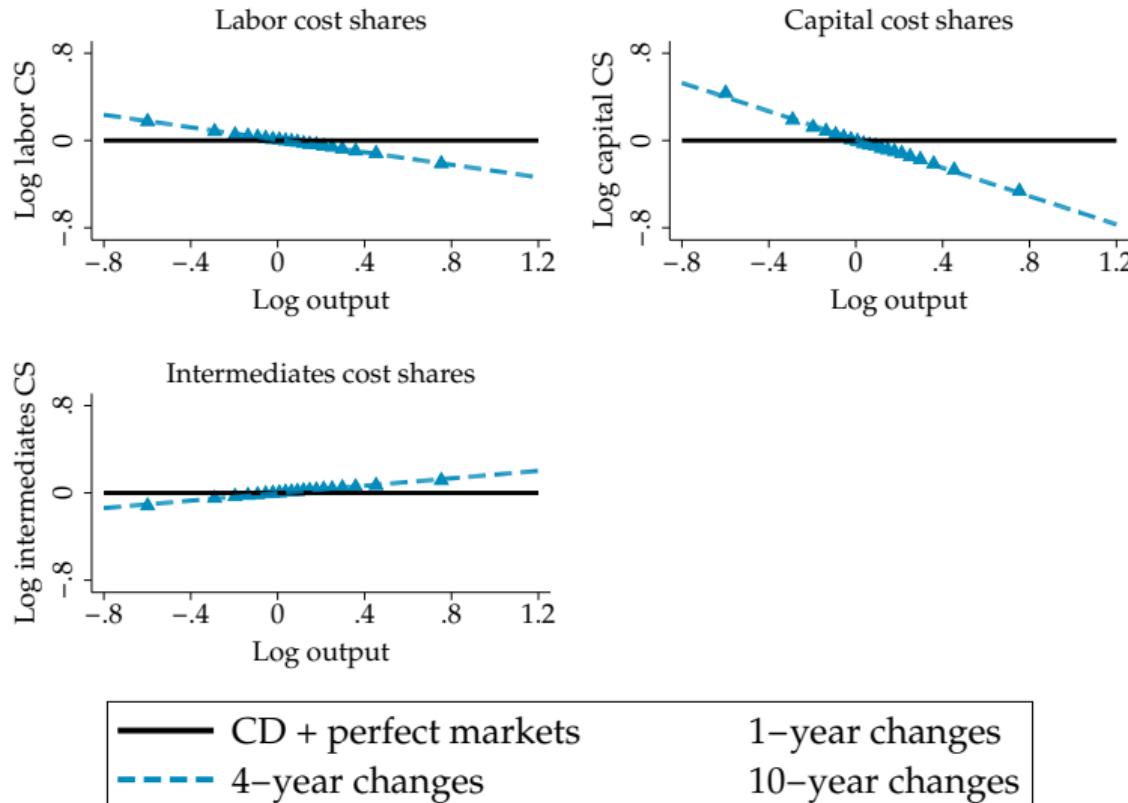


Evidence (OLS): cost shares

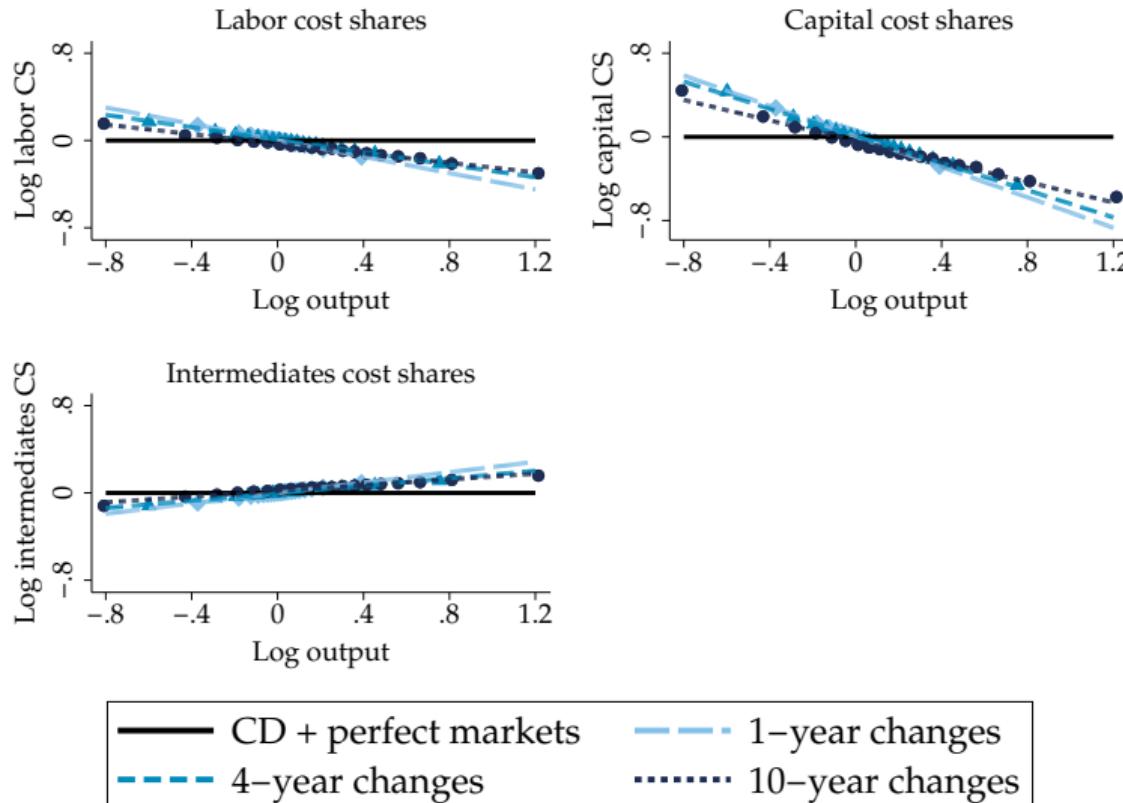


| | |
|----------------------|-----------------|
| CD + perfect markets | 1-year changes |
| 4-year changes | 10-year changes |

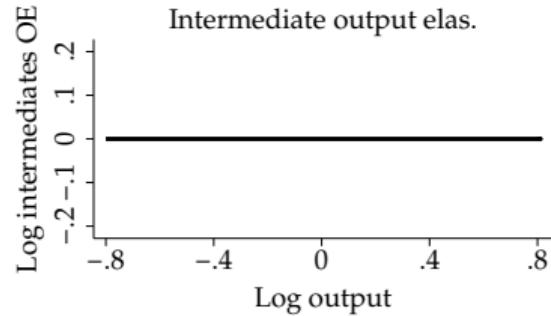
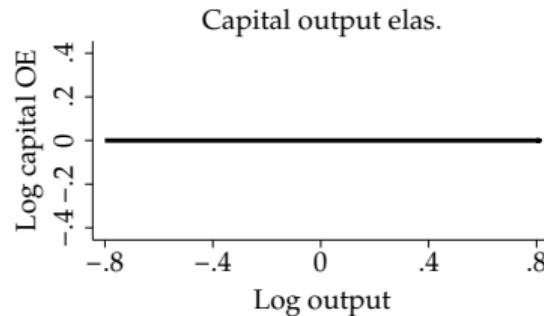
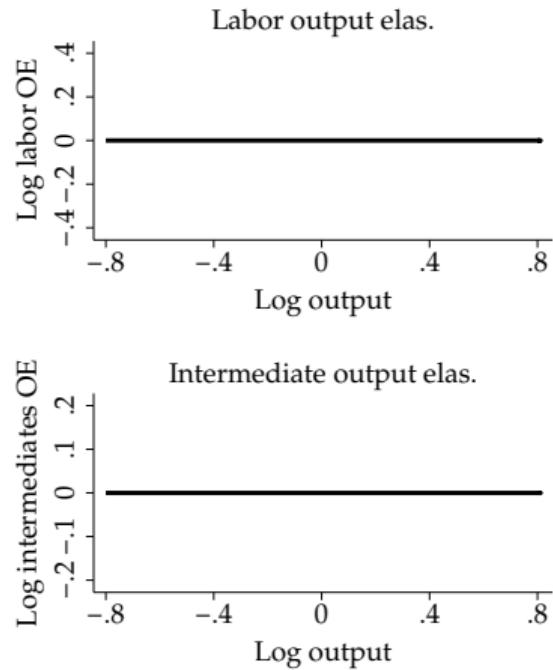
Evidence (OLS): cost shares



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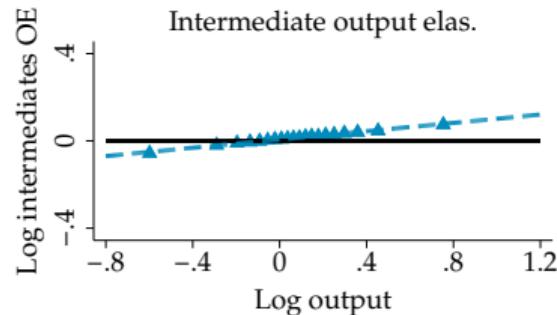
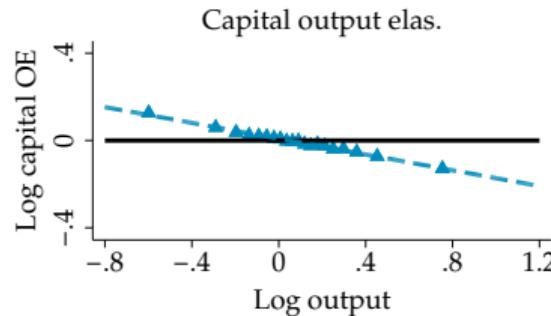
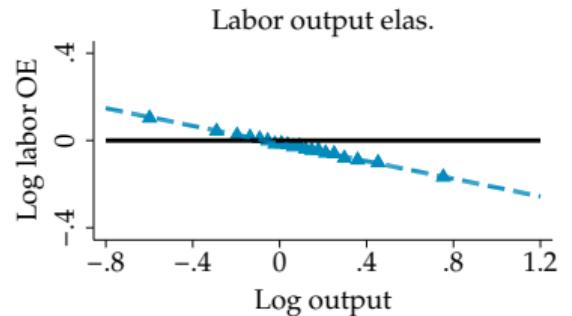


Evidence (OLS): output elasticities



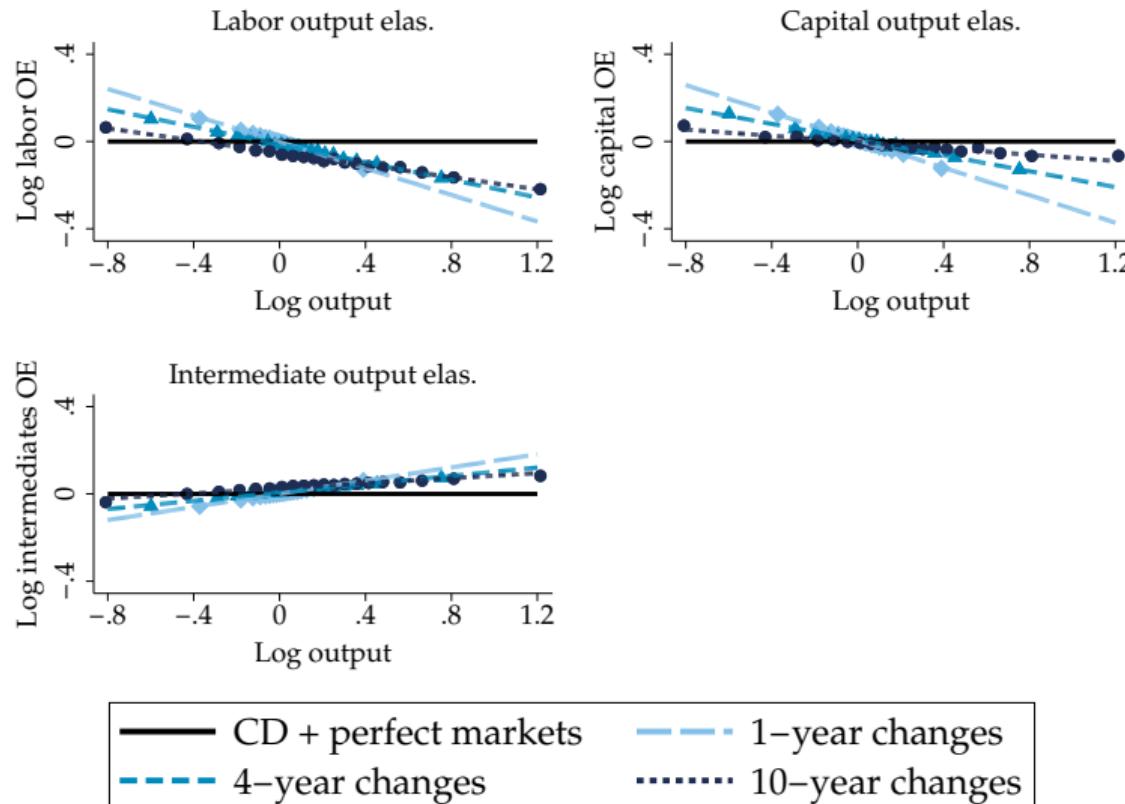
| | |
|------------------------|-----------------|
| — CD + perfect markets | 1-year changes |
| 4-year changes | 10-year changes |

Evidence (OLS): output elasticities



| | |
|----------------------|-----------------|
| CD + perfect markets | 1-year changes |
| 4-year changes | 10-year changes |

Evidence (OLS): output elasticities



Evidence: Regression tables (OLS)

| | $\ln(L_{it})$ | $\ln(K_{it})$ | $\ln(M_{it})$ | $\ln(CS_{it}^L)$ | $\ln(CS_{it}^K)$ | $\ln(CS_{it}^M)$ | $\ln(\frac{\theta_{it}^L}{RTS_{it}})$ | $\ln(\frac{\theta_{it}^K}{RTS_{it}})$ | $\ln(\frac{\theta_{it}^M}{RTS_{it}})$ |
|------------------------|-----------------------------|-----------------------|----------------------|------------------------|-----------------------|----------------------|---------------------------------------|---------------------------------------|---------------------------------------|
| Panel A: 1-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Log output growth | 0.298*** (0.0040) | 0.0405*** (0.0022) | 1.025*** (0.0033) | -0.376*** (0.00393) | -0.728*** (0.0033) | 0.238*** (0.0022) | -0.304*** (0.004) | -0.315*** (0.0063) | 0.151*** (0.0015) |
| Observations | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 |
| R ² | 0.215 | 0.043 | 0.741 | 0.343 | 0.556 | 0.340 | 0.230 | 0.137 | 0.325 |
| Panel B: 4-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Log output growth | 0.510*** (0.0064) | 0.206*** (0.0067) | 1.045*** (0.0045) | -0.286*** (0.0062) | -0.649*** (0.0071) | 0.171*** (0.0034) | -0.202*** (0.0060) | -0.181*** (0.0074) | 0.0951*** (0.0025) |
| Observations | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 |
| R ² | 0.471 | 0.144 | 0.860 | 0.348 | 0.476 | 0.355 | 0.220 | 0.170 | 0.268 |
| Panel C: 10-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Log output growth | 0.606*** (0.0066) | 0.395*** (0.0082) | 1.041*** (0.0043) | -0.219*** (0.0061) | -0.490*** (0.0083) | 0.131*** (0.0033) | -0.140*** (0.0053) | -0.0720*** (0.0069) | 0.0582*** (0.0024) |
| Observations | 49,915 | 49,915 | 49,915 | 49,915 | 49,915 | 49,915 | 49,915 | 49,915 | 49,915 |
| R ² | 0.594 | 0.256 | 0.901 | 0.296 | 0.329 | 0.303 | 0.188 | 0.149 | 0.200 |

Evidence: Regression tables (IV)

| | 1st stage | $\ln(L_{it})$ | $\ln(K_{it})$ | $\ln(M_{it})$ | $\ln(CS_{it}^L)$ | $\ln(CS_{it}^K)$ | $\ln(CS_{it}^M)$ | $\ln(\frac{\theta_{it}^L}{RTS_{it}})$ | $\ln(\frac{\theta_{it}^K}{RTS_{it}})$ | $\ln(\frac{\theta_{it}^M}{RTS_{it}})$ |
|-------------------------|-----------------------|----------------------|----------------------|----------------------|-----------------------|-----------------------|----------------------|---------------------------------------|---------------------------------------|---------------------------------------|
| Panel A: 1-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| Export demand shock | 0.0451*** (0.0045) | | | | | | | | | |
| Log output growth | | 0.324*** (0.0543) | 0.0612 (0.0461) | 0.929*** (0.0508) | -0.444*** (0.0521) | -0.684*** (0.0565) | 0.171*** (0.0292) | -0.327*** (0.0623) | -0.229** (0.0980) | 0.119*** (0.0198) |
| Observations | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 |
| First-stage F-Statistic | | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 |
| R ² | 0.205 | 0.214 | 0.042 | 0.736 | 0.336 | 0.554 | 0.321 | 0.229 | 0.132 | 0.314 |
| Panel B: 4-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| Export demand shock | 0.0635*** (0.0091) | | | | | | | | | |
| Log output growth | | 0.536*** (0.0737) | 0.262*** (0.0973) | 0.963*** (0.0542) | -0.269*** (0.0664) | -0.570*** (0.0969) | 0.124*** (0.0369) | -0.242*** (0.0706) | -0.147 (0.0938) | 0.0823*** (0.0268) |
| Observations | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 |
| First-stage F-Statistic | | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 |
| R ² | 0.183 | 0.470 | 0.139 | 0.855 | 0.347 | 0.471 | 0.339 | 0.217 | 0.169 | 0.266 |

Taking stock

Table: Growth predictions for different substitution and supply elasticities (ceteris paribus)

| | $\sigma < 1$ | $\sigma = 1$ | $\sigma > 1$ | |
|---|--------------|--------------|--------------|-------------------------------|
| $(\varepsilon^L)^{-1} > (\varepsilon^M)^{-1}$ | \uparrow | \uparrow | \uparrow | L_{it}/M_{it} |
| | \downarrow | $=$ | \uparrow | $\theta_{it}^L/\theta_{it}^M$ |
| | \downarrow | $=$ | \uparrow | CS_{it}^L |
| | \downarrow | $=$ | \uparrow | LS_{it} |
| $(\varepsilon^L)^{-1} = (\varepsilon^M)^{-1}$ | $=$ | $=$ | $=$ | L_{it}/M_{it} |
| | $=$ | $=$ | $=$ | $\theta_{it}^L/\theta_{it}^M$ |
| | $=$ | $=$ | $=$ | CS_{it}^L |
| | $=$ | $=$ | $=$ | LS_{it} |
| $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$ | \downarrow | \downarrow | \downarrow | L_{it}/M_{it} |
| | \uparrow | $=$ | \downarrow | $\theta_{it}^L/\theta_{it}^M$ |
| | \uparrow | $=$ | \downarrow | CS_{it}^L |
| | \uparrow | $=$ | \downarrow | LS_{it} |

Notes: Ceteris paribus means constant returns to scale and non-changing market imperfections with output growth.

Quantitative interpretation: Substitution and supply elasticities

- Implied substitution elasticities:

$$\frac{\sigma - 1}{\sigma} = \frac{\Delta \ln(\theta_{it}^L) - \Delta \ln(\theta_{it}^M)}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})} \quad (14)$$

- σ in hand and using cost minimization, compute the shadow price ratio gradient:

$$\Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M} \right) = \frac{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}{-\sigma} \quad (15)$$

- Back out firm-specific labor supply elasticity (assuming constant shadow prices for intermediates, consistent with results (i.e., $\Delta \ln(P_{it}^M \gamma_{it}^M) = 0$)—and here (but not later) assume $\Delta \ln(\gamma^L) = 0$):

$$\varepsilon^{L^{-1}} = \frac{\Delta \ln(L_{it})}{\Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M} \right)} \quad (16)$$

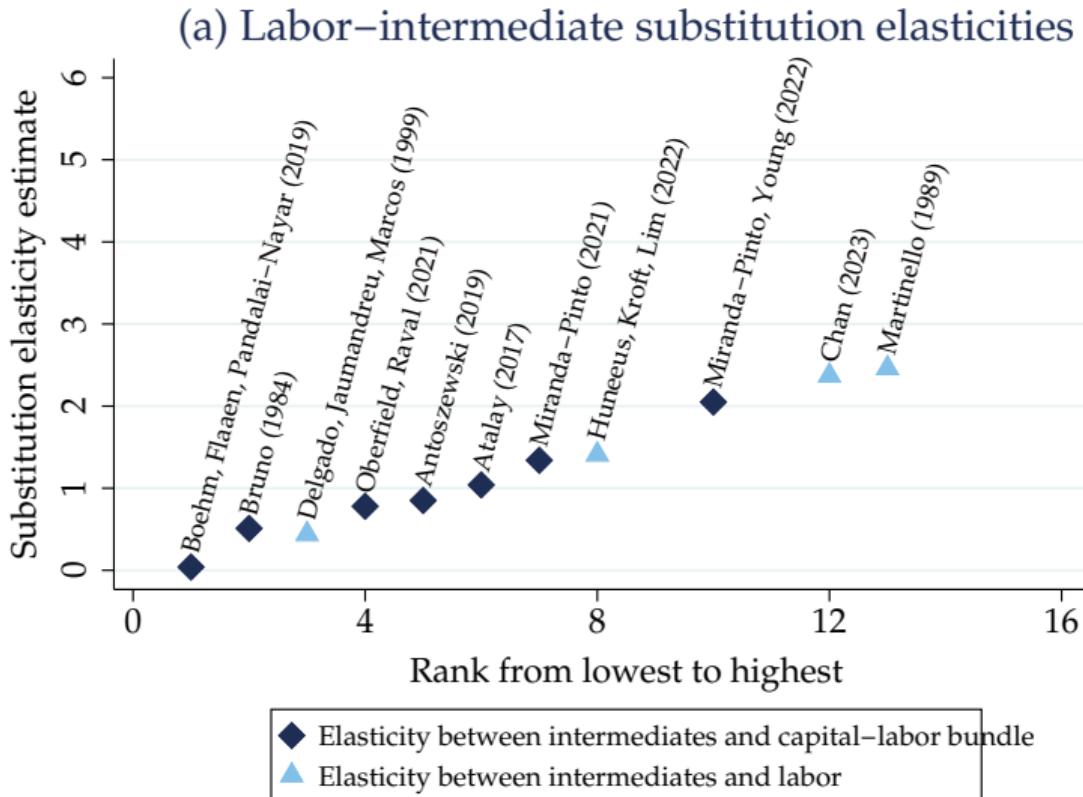
Estimates of substitution and supply elasticities

Table: Quantitative interpretation: Substitution and supply elasticities

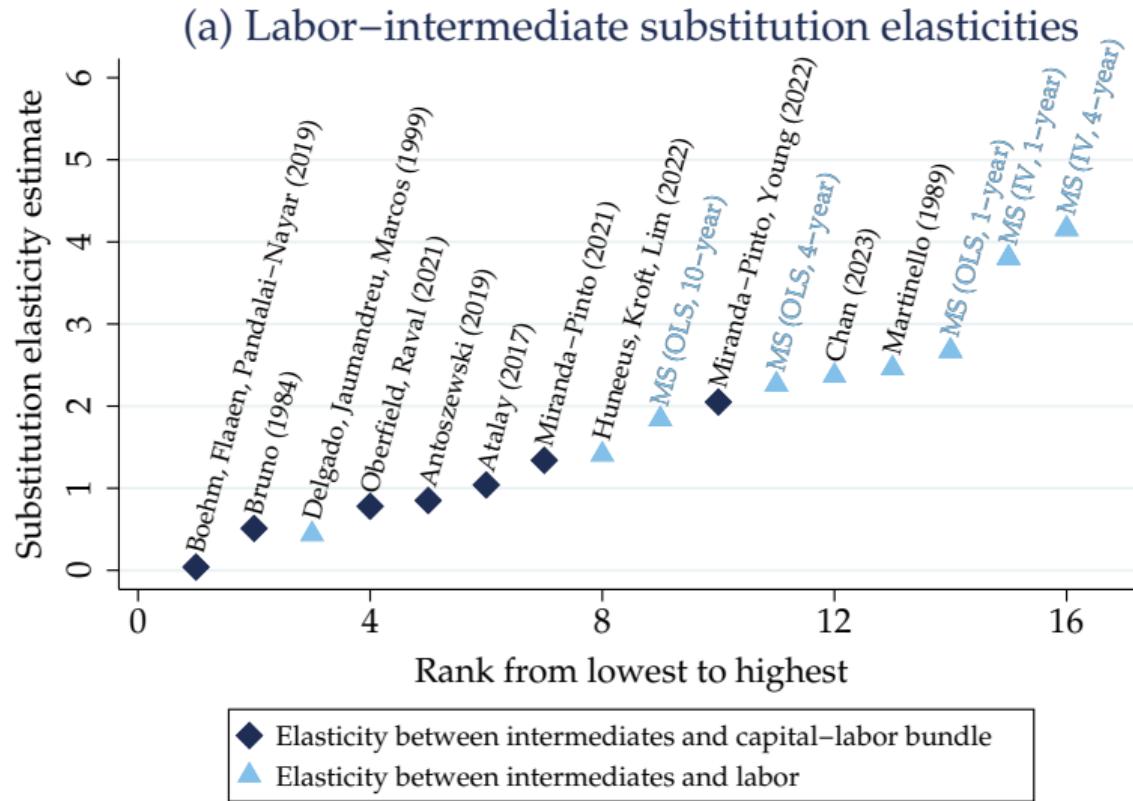
| | OLS | | | | IV | | | |
|---------------|----------|---|---|---------------------|----------|---|---|---------------------|
| | σ | $\Delta \ln \left(\frac{L_{it}}{M_{it}} \right)$ | $\Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M} \right)$ | ε^{L-1} | σ | $\Delta \ln \left(\frac{L_{it}}{M_{it}} \right)$ | $\Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M} \right)$ | ε^{L-1} |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 1-year diff. | 2.673 | -0.727 | 0.272 | 1.10 | 3.805 | -0.605 | 0.159 | 2.04 |
| 4-year diff. | 2.259 | -0.535 | 0.236 | 2.16 | 4.158 | -0.427 | 0.103 | 5.20 |
| 10-year diff. | 1.837 | -0.435 | 0.237 | 2.56 | | | | |

Notes: The table reports substitution elasticities (Columns (1) and (5)), changes in input factor ratios (Columns (2) and (6)), changes in effective factor price ratios (Columns (3) and (7)), and implied labor supply elasticities, assuming perfectly elastic intermediate input supply (Columns (4) and (8)), based on our OLS (Columns (1)-(4)) and IV (Columns (5)-(8)) regressions.

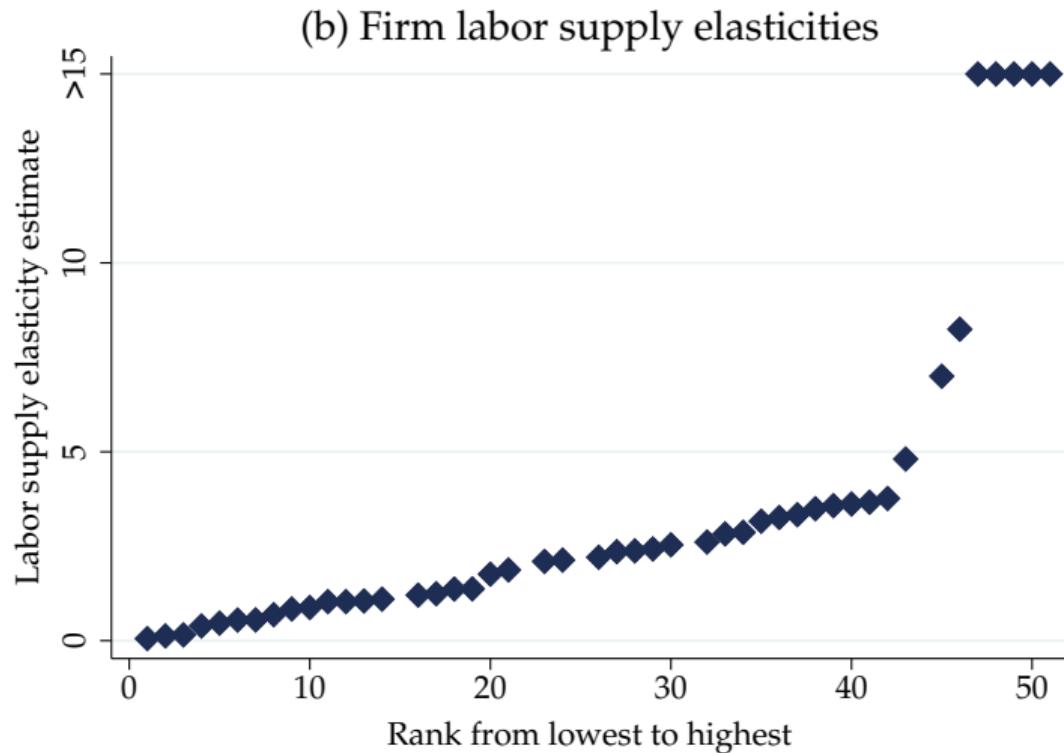
Quantitative interpretation: Elasticity of substitution vs. literature



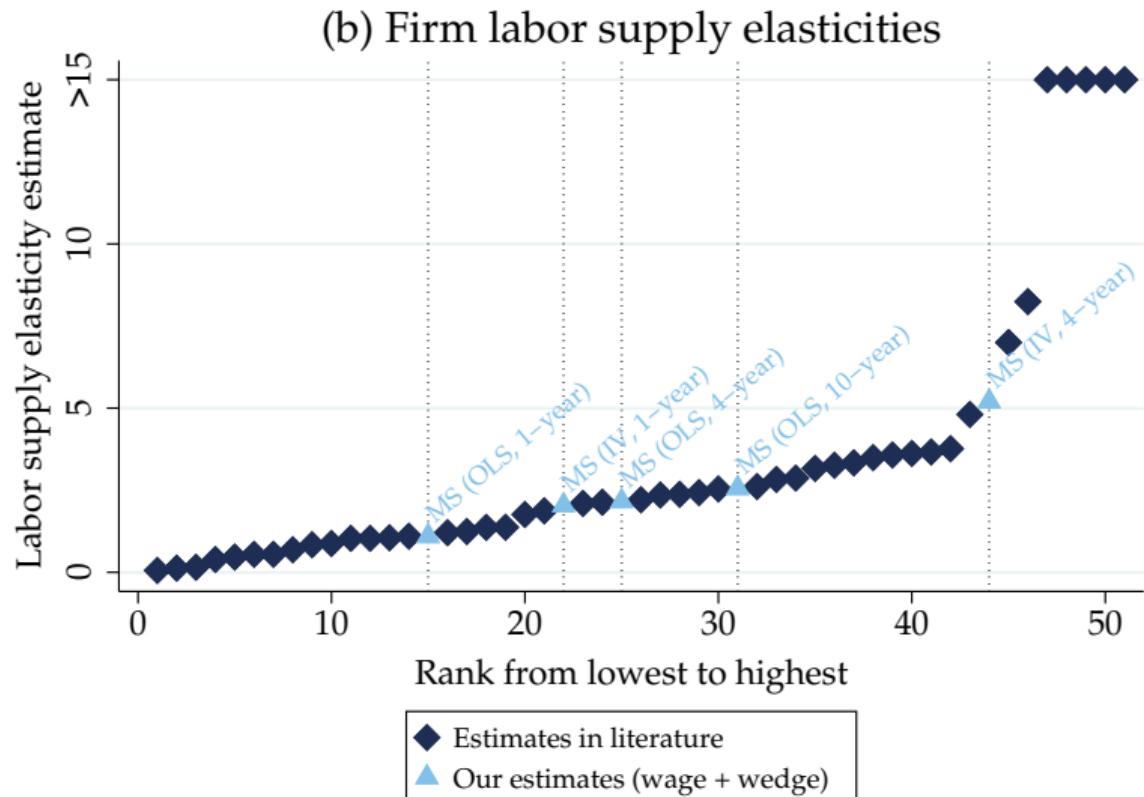
Quantitative interpretation: Elasticity of substitution vs. literature



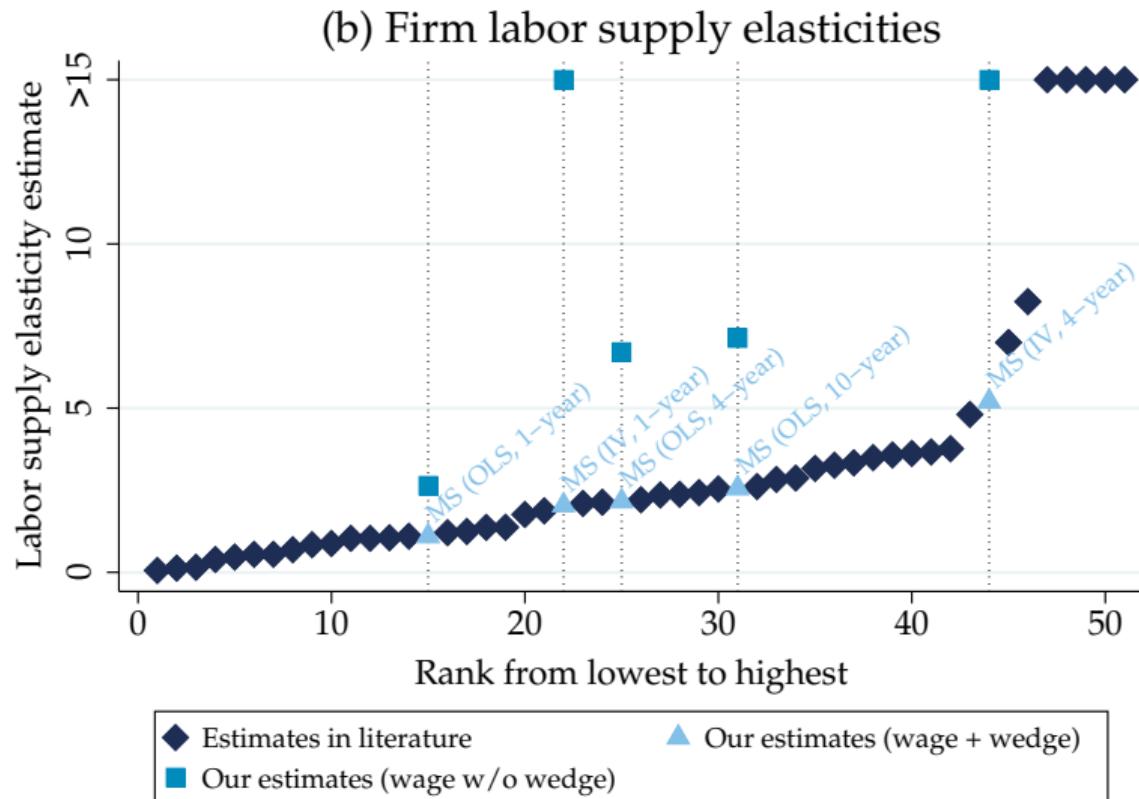
Quantitative interpretation: Labor supply elasticity vs. literature



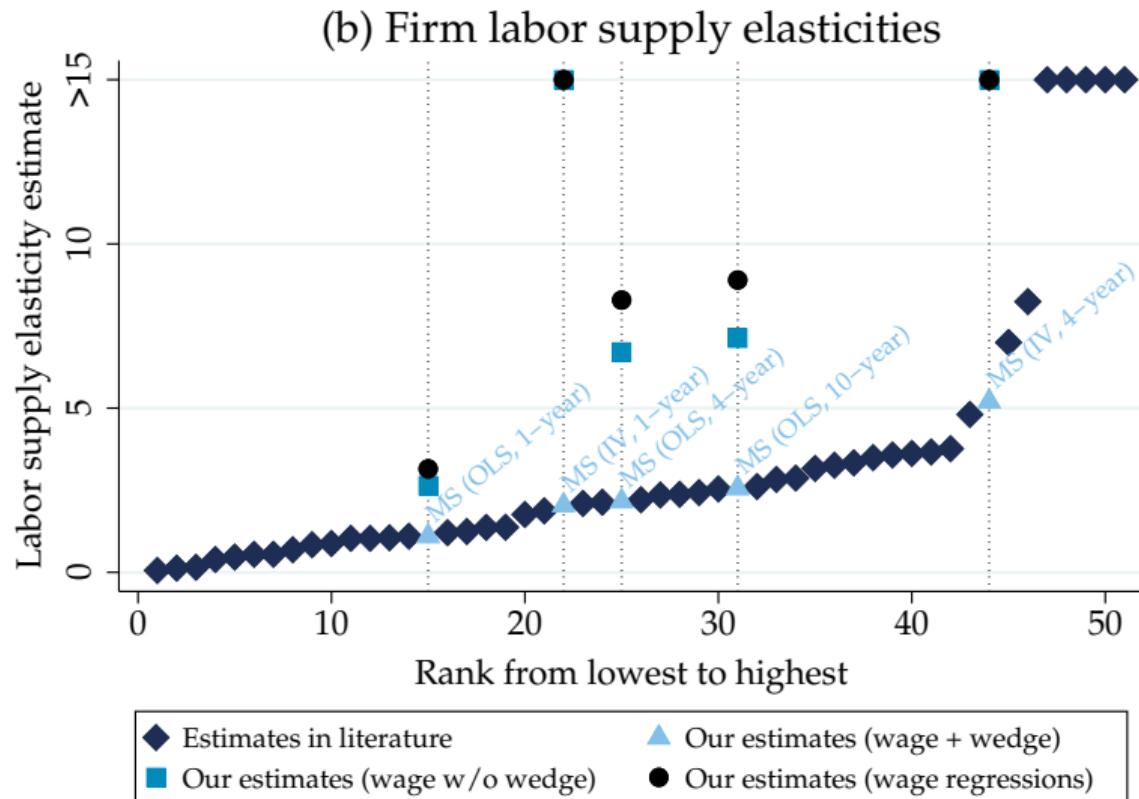
Quantitative interpretation: Labor supply elasticity vs. literature



Quantitative interpretation: Labor supply elasticity vs. literature



Quantitative interpretation: Labor supply elasticity vs. literature



Firm growth in theory: Predictions

| | $\sigma < 1$ | $\sigma = 1$ | $\sigma > 1$ | |
|--|------------------|------------------|------------------|--|
| L more elastic than M $(\varepsilon^L)^{-1} > (\varepsilon^M)^{-1}$ | ↑ ↓ ↓ ↓ | ↑ = = = | ↑ ↑ ↑ ↑ | L_{it}/M_{it} $\theta_{it}^L/\theta_{it}^M$ CS_{it}^L LS_{it} |
| L as elastic as M $(\varepsilon^L)^{-1} = (\varepsilon^M)^{-1}$ | = = = = | = = = = | = = = = | L_{it}/M_{it} $\theta_{it}^L/\theta_{it}^M$ CS_{it}^L LS_{it} |
| L less elastic than M $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$ | ↓ ↑ ↑ ↑ | ↓ = = | ↓ ↓ ↓ | L_{it}/M_{it} $\theta_{it}^L/\theta_{it}^M$ CS_{it}^L LS_{it} |

Notes: Ceteris paribus means constant returns to scale and non-changing market imperfections with output growth.

Additional results and robustness

- Wage regressions. ●
- Estimates based on cost shares. ●
- Non-homotheticity: fixed costs and size dependence - not for labor vs. intermediates (some evidence for capital). ●
- Non-homotheticity: direct effect of scale on substitution elasticity can't explain results. ●
- Aggregate prices and factor demands consistent with labor-intermediate substitution. ●
- Effects are similar across industries. ●
- Reduced form regressions. ●
- Cross-section. ●
- Components of intermediates. ●
- Results without logs/without dividing by RTS. ●
- Firm-level results for other countries. ●
- Young and mature firms. ●

Interim conclusion

Have organized firm growth patterns into straightforward formal account:

- As firms grow, shadow price of labor goes up—monopsony as a plausible and natural source.
- As a response, firms substitute from labor to intermediate inputs.
- Due to $\sigma > 1$, this substitution is strong—and entails a reduction in output elasticities of labor towards intermediates, and hence cost and output shares.
- Parsimonious CES production function captures this pattern w/ $\sigma > 1$ (2-4).
- Need both monopsony and substitution elasticity > 1 for realistic account of firm growth.
 - Without monopsony over labor, no reason to move away from labor even with $\sigma \gg 1$.
 - If $\sigma = 1$, wouldn't shift output elasticities or cost shares even with monopsony.

Implications: firm-level labor share dynamics

- Recap formula for firm-level labor share:

$$\ln(LS_{it}) = \ln(\theta_{it}^L) - \ln(\mu_{it}) - \ln(\gamma_{it}^L) \quad (17)$$

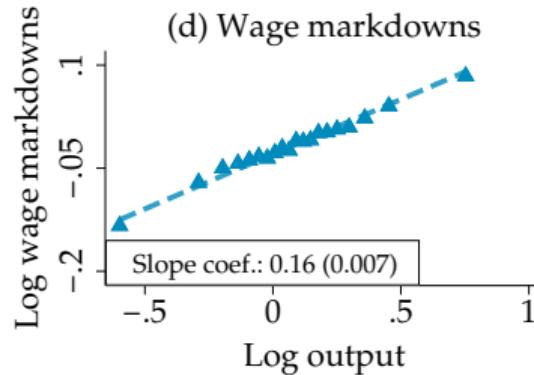
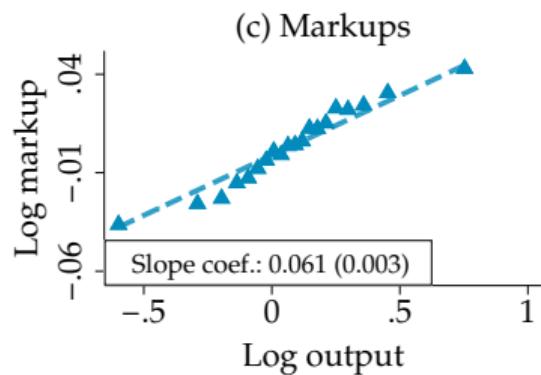
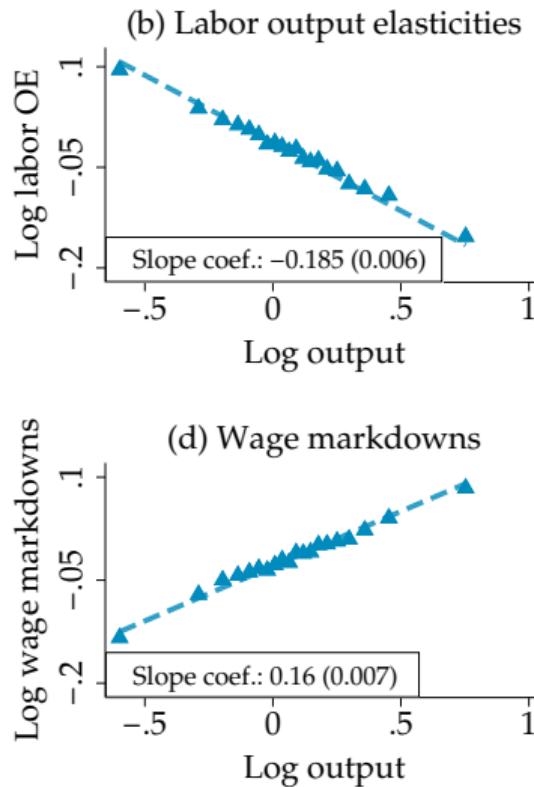
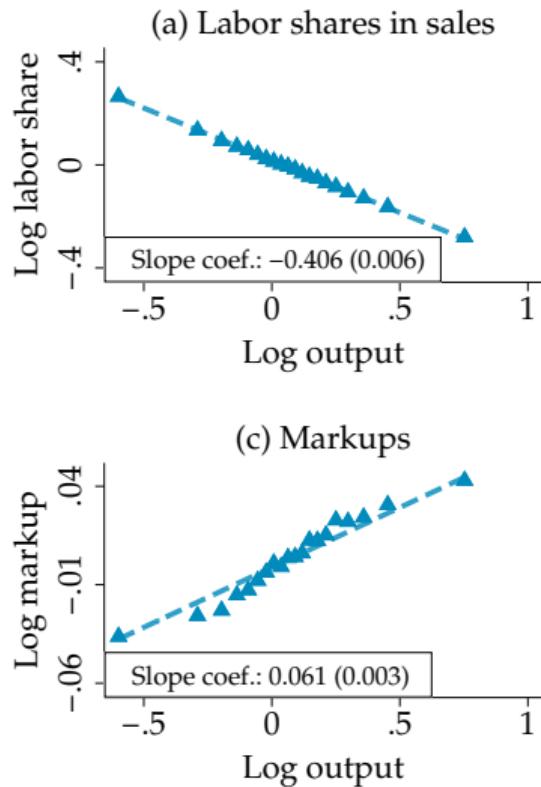
(In words (logs): labor share is equal to output elasticity of labor minus markup and minus markdown.)

- Estimate markups as in De Loecker and Warzynski (2012) and others:

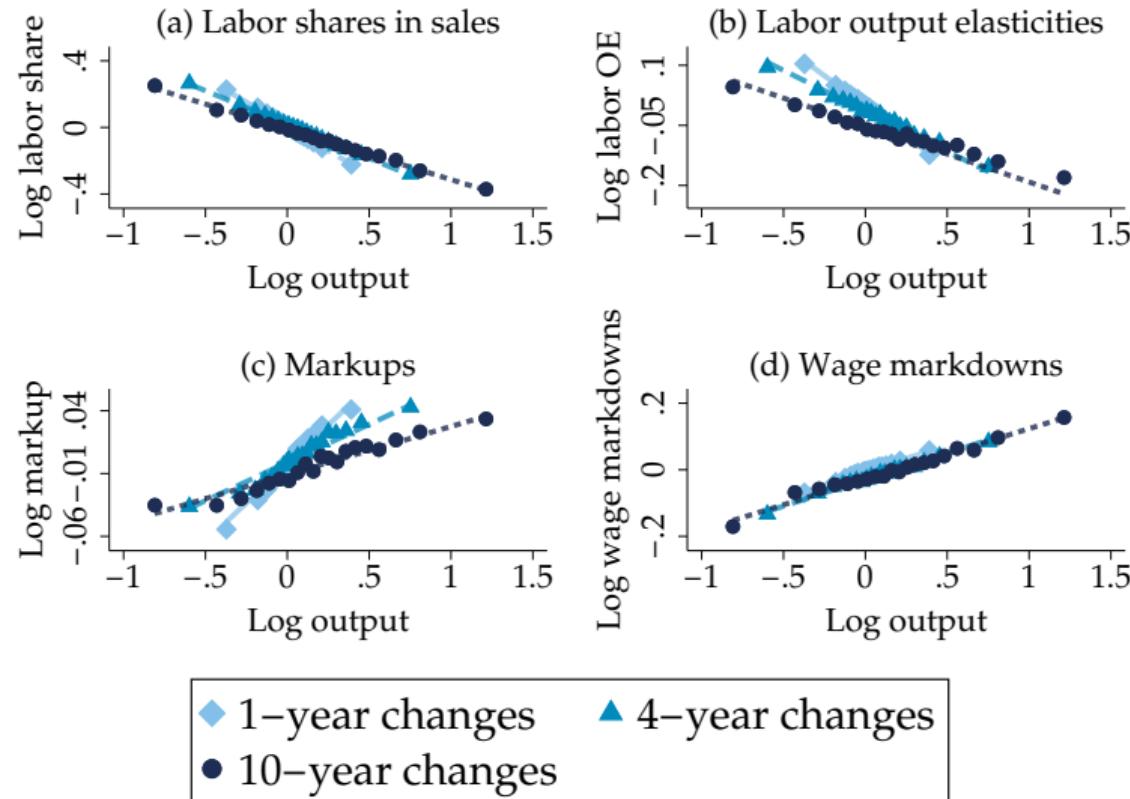
$$\mu_{it} = \frac{P_{it}}{MC_{it}} = \theta_{it}^M \frac{P_{it} Q_{it}}{P_{it}^M M_{it}} \quad (18)$$

$$\tilde{\gamma}_{it}^L = \left(\frac{\gamma_{it}^L}{\gamma_{it}^M} \right) = \frac{\theta_{it}^L}{\theta_{it}^M} \frac{P_{it}^M M_{it}}{P_{it}^L L_{it}} \quad (19)$$

Results: Firm-level labor share dynamics (4-year changes, OLS)



Results: Firm-level labor share dynamics (1-10 year changes, OLS)



Results: Firm-level labor share dynamics (1-10 year changes, OLS)

| | $\ln(LS_{it})$ | $\ln(\mu_{it})$ | $\ln(\gamma_{it})$ | $\ln(\mu_{it}\gamma_{it})$ | $\ln\left(\frac{\theta_{it}^L}{RTS_{it}}\right)$ | $\ln(\theta_{it}^L)$ | $\ln\left(\frac{w_{it}L_{it}}{VA_{it}}\right)$ |
|------------------------|-----------------------|-----------------------|----------------------|----------------------------|--|-----------------------|--|
| Panel A: 1-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Log output growth | -0.581*** (0.0040) | 0.124*** (0.0025) | 0.159*** (0.0048) | 0.283*** (0.0044) | -0.304*** (0.005) | -0.298*** (0.0046) | -0.561*** (0.0076) |
| Observations | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 |
| R ² | 0.534 | 0.096 | 0.115 | 0.191 | 0.230 | 0.215 | 0.190 |
| Panel B: 4-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Log output growth | -0.406*** (0.0061) | 0.0608*** (0.0032) | 0.160*** (0.0065) | 0.220*** (0.0055) | -0.202*** (0.0060) | -0.185*** (0.0063) | -0.334*** (0.0081) |
| Observations | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 |
| R ² | 0.461 | 0.126 | 0.193 | 0.239 | 0.220 | 0.201 | 0.188 |
| Panel C: 10-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Log output growth | -0.303*** (0.0062) | 0.0383*** (0.0030) | 0.152*** (0.0060) | 0.191*** (0.0048) | -0.140*** (0.0053) | -0.112*** (0.0057) | -0.231*** (0.0068) |
| Observations | 49,915 | 49,915 | 49,915 | 49,915 | 49,915 | 49,915 | 49,915 |
| R ² | 0.378 | 0.125 | 0.213 | 0.253 | 0.188 | 0.161 | 0.178 |

Results: Firm-level labor share dynamics (1-4 year changes, IV)

| | 1st stage | $\ln(LS_{it})$ | $\ln(\mu_{it})$ | $\ln(\gamma_{it})$ | $\ln(\mu_{it}\gamma_{it})$ | $\ln\left(\frac{\theta_{it}^L}{RTS_{it}}\right)$ | $\ln(\theta_{it}^L)$ | $\ln\left(\frac{w_{it}L_{it}}{VA_{it}}\right)$ |
|-------------------------|-----------------------|-----------------------|----------------------|---------------------|----------------------------|--|-----------------------|--|
| Panel A: 1-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Export demand shock | 0.0451*** (0.0045) | | | | | | | |
| Log output growth | | -0.674*** (0.0521) | 0.186*** (0.0418) | 0.169** (0.0759) | 0.355*** (0.0697) | -0.327*** (0.0623) | -0.318*** (0.0645) | -0.842*** (0.120) |
| Observations | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 |
| First-stage F-Statistic | | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 |
| R ² | 0.205 | 0.524 | 0.084 | 0.115 | 0.185 | 0.229 | 0.215 | 0.160 |
| Panel B: 4-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Export demand shock | 0.0635*** (0.0091) | | | | | | | |
| Log output growth | | -0.443*** (0.0726) | 0.154*** (0.0444) | 0.0688 (0.0800) | 0.223*** (0.0713) | -0.242*** (0.0706) | -0.220*** (0.0742) | -0.603*** (0.119) |
| Observations | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 |
| First-stage F-Statistic | | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 |
| R ² | 0.183 | 0.459 | 0.072 | 0.180 | 0.239 | 0.217 | 0.199 | 0.135 |

Aggregate Implications: Industry-level dynamics

- PART 1: **German manufacturing sector firm-level data, 1995-2017**
 - Firms with at least 20 employees, representative 40% sample.
 - Contains quantities and prices of products (allows to account for "price bias" when estimating output elasticities).
 - Contains information on input quantities, costs, capital stock (perpetual inventory method), sales.
 - Data recently used in Mertens (2020), Mertens et al. (2022), Haelbig et al. (2023).
- PART 2: **CompNet country-industry data for 20 European countries, 1999-2021**
 - Harmonized industry-level data for 20 European countries (manufacturing and non-manufacturing).
 - Firms with at least 20 employees (for 14 countries also smaller firms).
 - Yearly coverage varies by country.
 - Contains industry-information on output elasticities, input quantities, costs, labor cost and output shares.
 - Data recently used in Autor et al. (2020), Biondi (2022), Bighelli et al. (2023).
 - Supplement with NBER-CES US data.
 - Also aggregate German micro data to industry level.

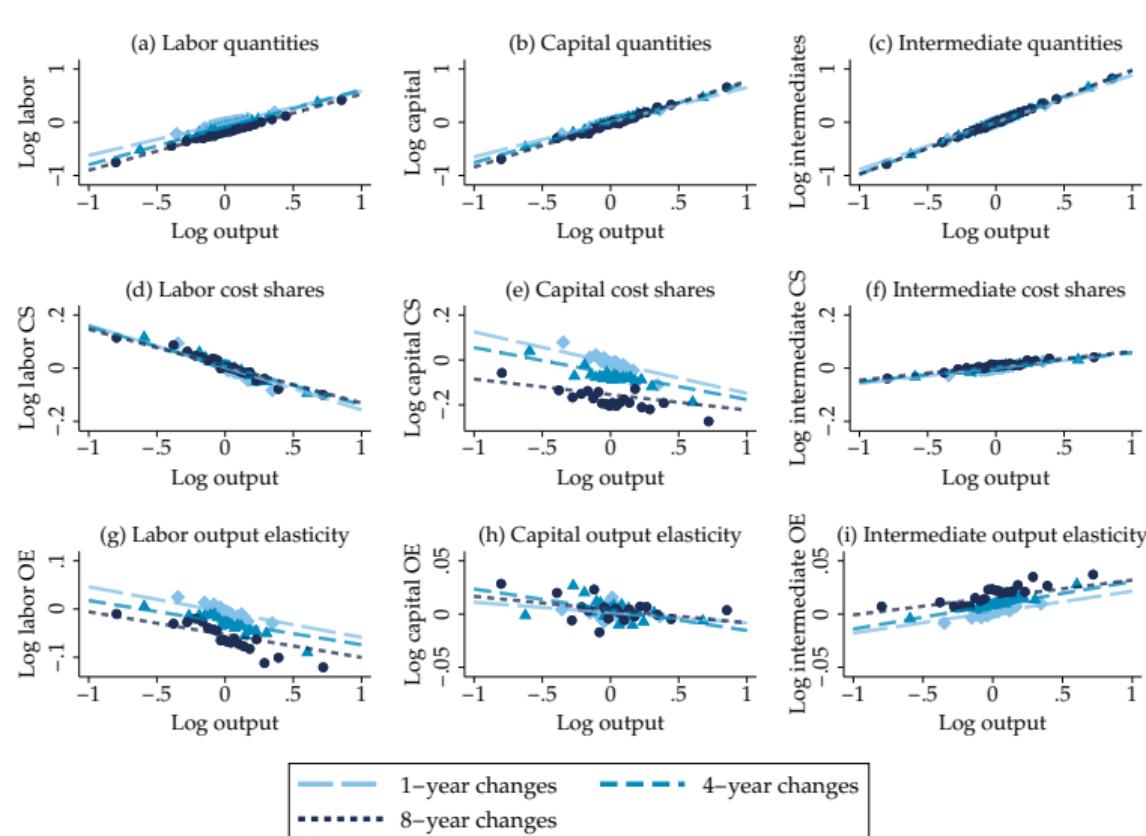
Aggregate Implications: Industry-level dynamics

Country-industry-level version of growth regressions:

$$\Delta^h \ln(Y_{cjt}) = \beta^h \Delta^h \ln Q_{cjt} + v_{ct} + v_{jt} + \epsilon_{cjt} \quad (20)$$

- Industry-level - focus on 4-year changes. Europe: 22 2-digit NACE industries in manufacturing.
- Aggregation: Denominator weights - sales weights for labor shares and output elasticities, intermediate input weights for markups, wage bill weights for labor wedges (all results robust to using sales weights everywhere).
- Aggregation bias: Jensen's inequality, entry/exit, intra-industry trade, market level shifts in labor demand and supply curves.

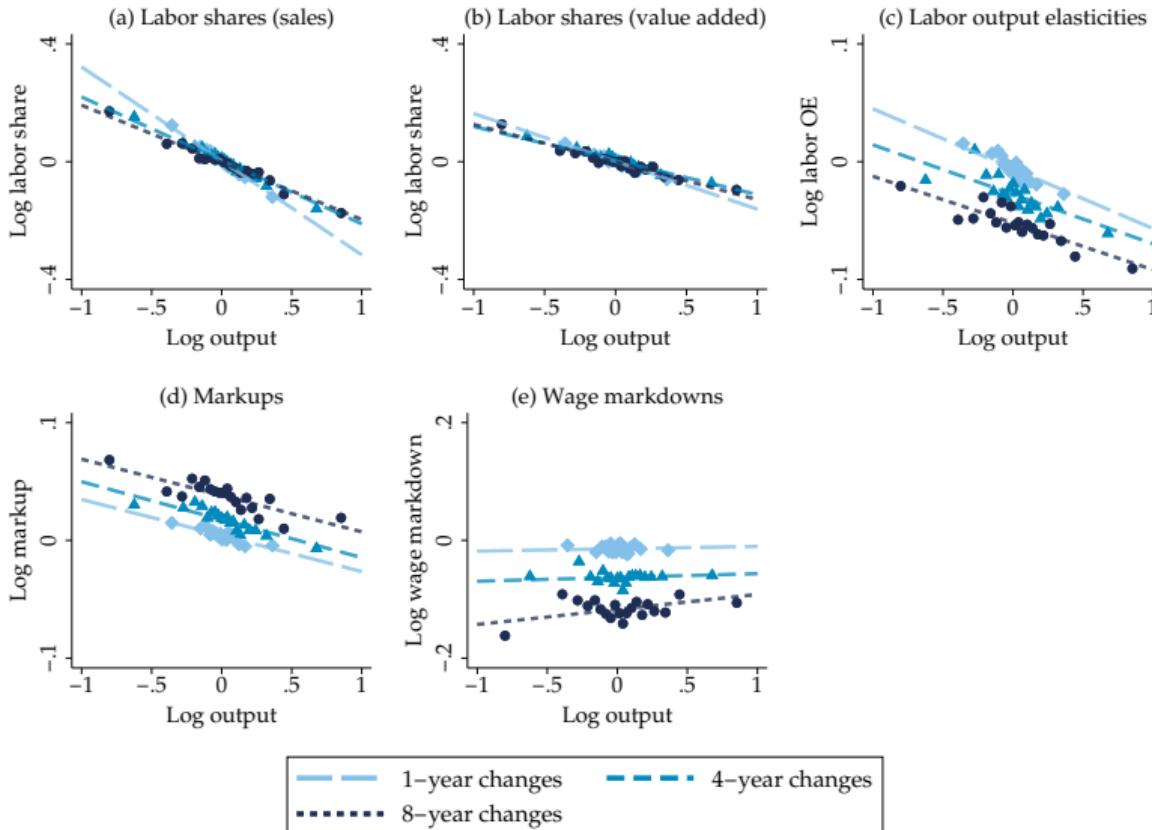
Results: Industry-level results. Europe (20 countries).



Results: Industry-level results. Europe (20 countries).

| | $\Delta \ln \left(\frac{L_{cjt}}{M_{cjt}} \right)$ (1) | $\Delta \ln \left(\frac{\theta_{cjt}^L}{\theta_{cjt}^M} \right)$ (2) | $\Delta \ln \left(\frac{CS_{cjt}^L}{CS_{cjt}^M} \right)$ (3) | σ^{OE} (4) | σ^{CS} (5) |
|-----------------------|--|--|--|----------------------|----------------------|
| Belgium | -0.23 | -0.10 | -0.16 | 1.80 | 3.36 |
| Croatia | -0.70 | -0.16 | -0.64 | 1.31 | 12.71 |
| Czech Republic | -0.40 | -0.01 | -0.31 | 1.03 | 4.67 |
| Denmark | -0.13 | -0.01 | -0.09 | 1.12 | 3.13 |
| Finland | -0.09 | -0.08 | -0.13 | 5.72 | -2.45 |
| France | -0.40 | -0.05 | -0.22 | 1.13 | 2.25 |
| Germany | -0.06 | 0.01 | -0.05 | 0.85 | 6.59 |
| Hungary | -0.51 | -0.09 | -0.38 | 1.21 | 4.03 |
| Italy | -0.18 | -0.09 | -0.12 | 2.11 | 2.89 |
| Latvia | -0.43 | -0.11 | -0.25 | 1.33 | 2.46 |
| Lithuania | -0.15 | -0.08 | -0.11 | 2.04 | 3.59 |
| Netherlands | -0.11 | 0.03 | -0.06 | 0.81 | 2.51 |
| Poland | -0.13 | -0.01 | -0.09 | 1.09 | 2.98 |
| Portugal | -0.68 | 0.02 | -0.48 | 0.98 | 3.42 |
| Romania | -0.22 | -0.03 | -0.13 | 1.17 | 2.58 |
| Slovakia | -0.18 | -0.09 | -0.16 | 1.98 | 12.73 |
| Slovenia | -0.09 | 0.01 | -0.13 | 0.92 | -2.49 |
| Spain | -0.44 | -0.23 | -0.16 | 2.11 | 1.60 |
| Sweden | -0.03 | -0.04 | 0.02 | -5.77 | 0.64 |
| Switzerland | -0.31 | -0.19 | -0.24 | 2.63 | 4.58 |
| Europe (manufac.) | -0.27 | -0.07 | -0.20 | 1.34 | 4.01 |
| USA (manufac.) | -0.31 | | -0.23 | | 3.88 |
| Europe (non-manufac.) | -0.23 | -0.08 | -0.15 | 1.48 | 2.72 |

Results: Industry-level results. Europe (20 countries).



Results: Industry-level results. Europe (20 countries).

Table: Industry labor share, market imperfection, and output elasticity changes in response to industry growth (Europe). OLS regressions.

| | $\ln(LS_{cjt})$ | $\ln(\mu_{cjt})$ | $\ln(\gamma_{cjt})$ | $\ln(\frac{\theta_{cjt}^L}{RTS_{cjt}})$ | $\ln(\theta_{cjt}^L)$ | $\ln(\frac{P_{cjt}^L L_{cjt}}{VA_{cjt}})$ |
|-----------------------|----------------------|----------------------|---------------------|---|-----------------------|---|
| Panel A: 1-year diff. | (1) | (2) | (3) | (4) | (5) | (6) |
| Log output growth | -0.319*** (0.041) | -0.031*** (0.009) | 0.004 (0.013) | -0.053*** (0.013) | -0.051*** (0.014) | -0.162*** (0.029) |
| Observations | 5,233 | 5,233 | 5,233 | 5,233 | 5,233 | 5,233 |
| R ² | 0.544 | 0.257 | 0.332 | 0.217 | 0.214 | 0.330 |
| Panel B: 4-year diff. | (1) | (2) | (3) | (4) | (5) | (6) |
| Log output growth | -0.216*** (0.029) | -0.032*** (0.007) | 0.006 (0.015) | -0.046*** (0.013) | -0.042*** (0.014) | -0.116*** (0.023) |
| Observations | 4,207 | 4,207 | 4,207 | 4,207 | 4,207 | 4,207 |
| R ² | 0.547 | 0.341 | 0.438 | 0.252 | 0.243 | 0.383 |
| Panel C: 8-year diff. | (1) | (2) | (3) | (4) | (5) | (6) |
| Log output growth | -0.194*** (0.024) | -0.031*** (0.009) | 0.026 (0.021) | -0.047*** (0.0048) | -0.040** (0.017) | -0.128*** (0.021) |
| Observations | 2,904 | 2,904 | 2,904 | 2,904 | 2,904 | 2,904 |
| R ² | 0.516 | 0.400 | 0.412 | 0.298 | 0.282 | 0.406 |

Results: Industry-level labor share dynamics. Growing vs. shrinking industries (Europe)

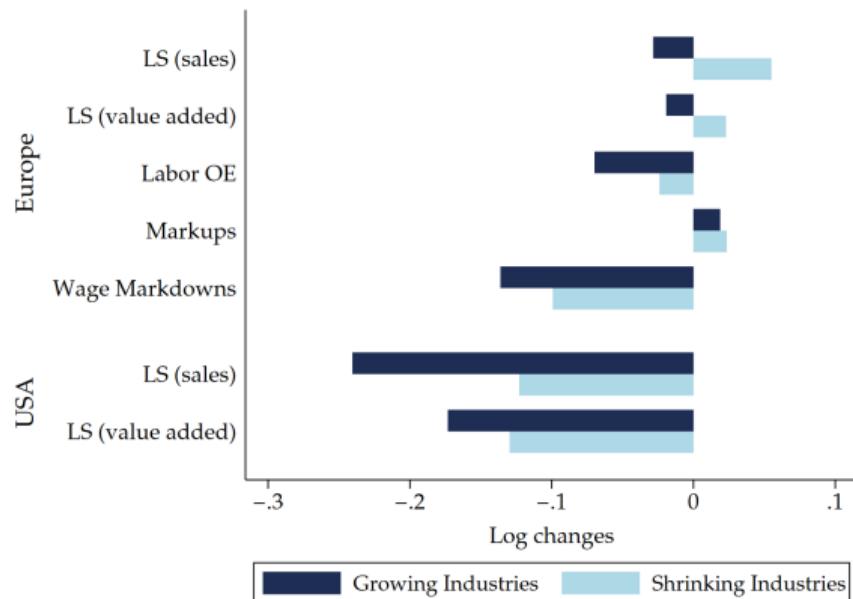


Figure: Changes in labor shares and components, growing vs. shrinking industries

Results: Industry-level results, Europe, further results

- Non-manufacturing industries. ●
- In paper: firm-level micro replication in 11 other European countries besides Germany

Conclusion

- How do firms grow?

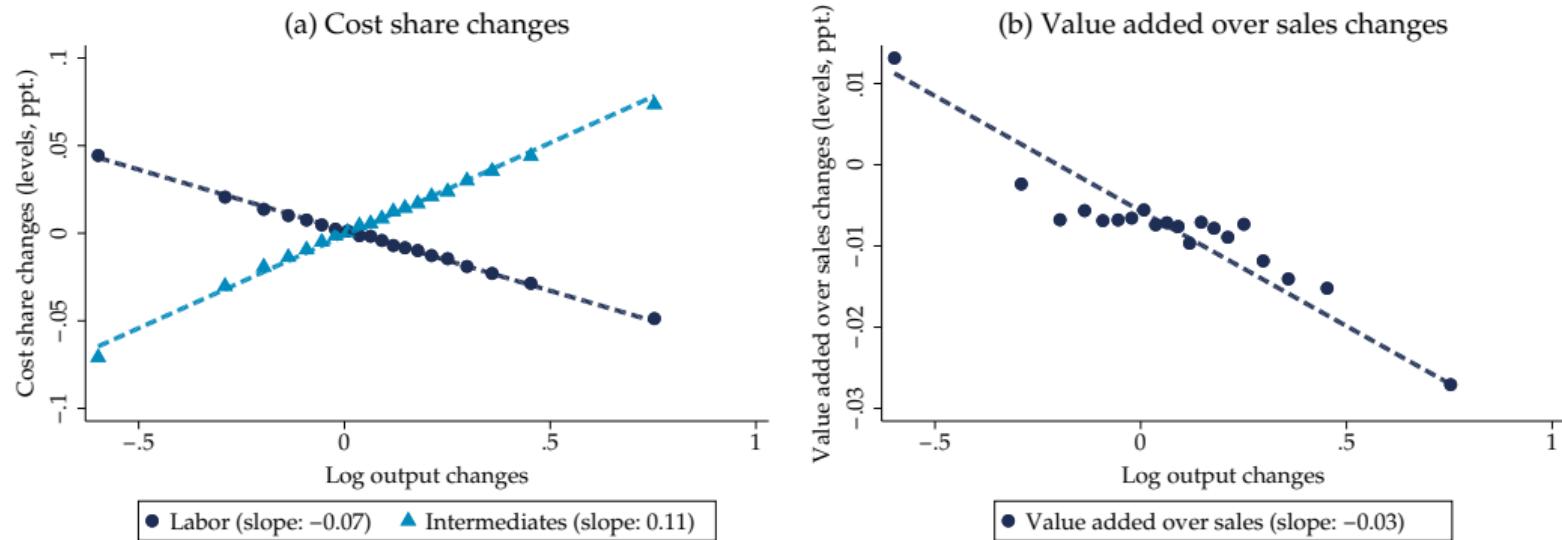


Figure: Change in Labor and Intermediate Cost Shares and Value Added over Sales Ratios (Levels) against Output Growth, 4-Year Changes

Conclusion

- Firms growth entails substitution from labor and capital to intermediate inputs.
- Increase in intermediates-labor ratio lowers output elasticity of labor and labor share.
- Parsimonious model accounts for findings:
 - Labor and intermediates are substitutes.
 - Lower labor supply elasticity (in terms of shadow prices) relative to intermediates (**monopsony**, adjustment costs).
- Our findings provide an technological explanation why growing firms experience declining labor shares (concentrated in growing firms and industries) .
- Findings translate to industry level
 - Industry labor shares in output and costs and industry labor output elasticities decline with industry growth. Intermediate cost shares and output elasticities increase.
 - Only growing industries in Europe see declining labor shares.
- Potentially relevant to studies of: firm responses to even input-unbiased growth-relevant shocks, productivity/misallocation measurement, firm-level effects from productivity shocks, trade, competition, or subsidies,...

Changes in wages in response to output changes

| | OLS | IV |
|--------------------------|---------------------|--------------------|
| | Real wage | Real wage |
| Panel A: 1-year changes | (1) | (2) |
| Log output growth | 0.095*** (0.003) | -0.024 (0.0562) |
| R ² | 0.062 | 0.035 |
| Panel B: 4-year changes | (1) | (2) |
| Log output growth | 0.062*** (0.003) | 0.031 (0.046) |
| R ² | 0.126 | 0.122 |
| Panel C: 10-year changes | (1) | |
| Log output growth | 0.068*** (0.003) | |
| R ² | 0.156 | |

Substitution and labor supply elasticities based on cost shares

Table: Implied substitution elasticities and changes in factor cost ratios based on cost shares as output elasticity measures

| | OLS | | | IV | | |
|---------------|----------|---|---------------------|-------------|---|---------------------|
| | σ | $\Delta \ln \left(\frac{w_{it} \gamma_{it}^L}{z_{it} \gamma_{it}^M} \right)$ | ε^{L-1} | σ | $\Delta \ln \left(\frac{w_{it} \gamma_{it}^L}{z_{it} \gamma_{it}^M} \right)$ | ε^{L-1} |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| 1-year diff. | 6.434 | 0.113 | 2.64 | 2.28 - inf. | 0.00 - 0.35 | 0.62 - inf. |
| 4-year diff. | 6.859 | 0.078 | 6.54 | 12.559 | 0.034 | 15.765 |
| 10-year diff. | 5.118 | 0.085 | 7.13 | | | |

Back.

Checking for direct effect of size (1)

- A version of the NH-CES in Lashkari et al. (2024) adapted to our case:

$$Q_{it} = \Omega_{it} \Lambda_i^K K_{it}^{1-\kappa} \left(\Lambda_i^{LM} \alpha_i^L L_{it}^{\frac{\sigma-1}{\sigma}} + \Lambda_i^{LM} \alpha_i^M \left(\frac{M_{it}}{Q_{it}^\eta} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1} \kappa} \quad (21)$$

- implies a direct effect on the output elasticity ratio through output (in addition to "our" effect through input substitution):

$$\frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha_i^L}{\alpha_i^M} \left(\frac{L_{it}}{M_{it}} \right)^{\frac{\sigma-1}{\sigma}} Q_{it}^{\eta(\frac{\sigma-1}{\sigma})} \quad (22)$$

- Test: Horse race regressions projecting output elasticities on output changes while controlling for input ratios.
- If non-homotheticity in output matters: coefficients on output changes should be unchanged. If "our" input substitution channel matters: most of the effect should be captured by input ratio changes.

Checking for direct effect of size (2)

Table: Testing the role of non-homotheticity in the production function, OLS results

| | $\ln\left(\frac{\theta_{it}^L}{RTS_{it}}\right)$ | $\ln\left(\frac{\theta_{it}^L}{RTS_{it}}\right)$ | $\ln(\theta_{it}^L)$ | $\ln\left(\frac{\theta_{it}^L}{\theta_{it}^M}\right)$ | $\ln\left(\frac{\theta_{it}^L}{\theta_{it}^M}\right)$ | $\ln\left(\frac{\theta_{it}^L}{\theta_{it}^M}\right)$ |
|-------------------------------|--|--|----------------------|---|---|---|
| Panel A: 1-year diff. | (1) | (2) | (3) | (4) | (5) | (6) |
| Log output growth | -0.304*** (0.004) | | 0.006* (0.003) | -0.455*** (0.00549) | | 0.0206*** (0.00390) |
| Log labor-interm. input ratio | | 0.424*** (0.004) | 0.427*** (0.004) | | 0.644*** (0.00373) | 0.655*** (0.00396) |
| R ² | 0.230 | 0.466 | 0.466 | 0.276 | 0.618 | 0.618 |
| Panel B: 4-year diff. | (1) | (2) | (3) | (4) | (5) | (6) |
| Log output growth | -0.202*** (0.006) | | 0.027*** (0.005) | -0.297*** (0.00785) | | 0.0512*** (0.00555) |
| Log labor-interm. input ratio | | 0.412*** (0.006) | 0.427*** (0.007) | | 0.622*** (0.00656) | 0.651*** (0.00726) |
| R ² | 0.189 | 0.492 | 0.497 | 0.199 | 0.612 | 0.623 |

Firm-level regressions by size quintiles (1)

Table: Firm-level adjustments in response to firm growth, by size quintiles (4-year changes).

| | $\ln(L_{it})$ | $\ln(K_{it})$ | $\ln(M_{it})$ | $\ln(\frac{\theta_{it}^L}{RTS_{it}})$ | $\ln(\frac{\theta_{it}^K}{RTS_{it}})$ | $\ln(\frac{\theta_{it}^M}{RTS_{it}})$ | $\ln(\theta_{it}^{RTS})$ |
|--|---------------------|---------------------|---------------------|---------------------------------------|---------------------------------------|---------------------------------------|--------------------------|
| Panel A: 1 st size quintile | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Log output growth | 0.505*** (0.011) | 0.259*** (0.016) | 1.051*** (0.009) | -0.179*** (0.010) | -0.183*** (0.017) | 0.095*** (0.005) | 0.024*** (0.001) |
| Panel B: 2 nd size quintile | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Log output growth | 0.484*** (0.012) | 0.217*** (0.015) | 1.034*** (0.009) | -0.206*** (0.011) | -0.175*** (0.014) | 0.098*** (0.004) | 0.018*** (0.001) |
| Panel C: 3 rd size quintile | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Log output growth | 0.492*** (0.012) | 0.208*** (0.013) | 1.032*** (0.010) | -0.200*** (0.011) | -0.164*** (0.013) | 0.093*** (0.005) | 0.017*** (0.001) |
| Panel D: 4 th size quintile | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Log output growth | 0.528*** (0.015) | 0.178*** (0.012) | 1.047*** (0.009) | -0.202*** (0.013) | -0.169*** (0.010) | 0.094*** (0.005) | 0.014*** (0.001) |
| Panel E: 5 th size quintile | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Log output growth | 0.529*** (0.016) | 0.150*** (0.012) | 1.055*** (0.012) | -0.196*** (0.015) | -0.206*** (0.021) | 0.099*** (0.006) | 0.008*** (0.002) |

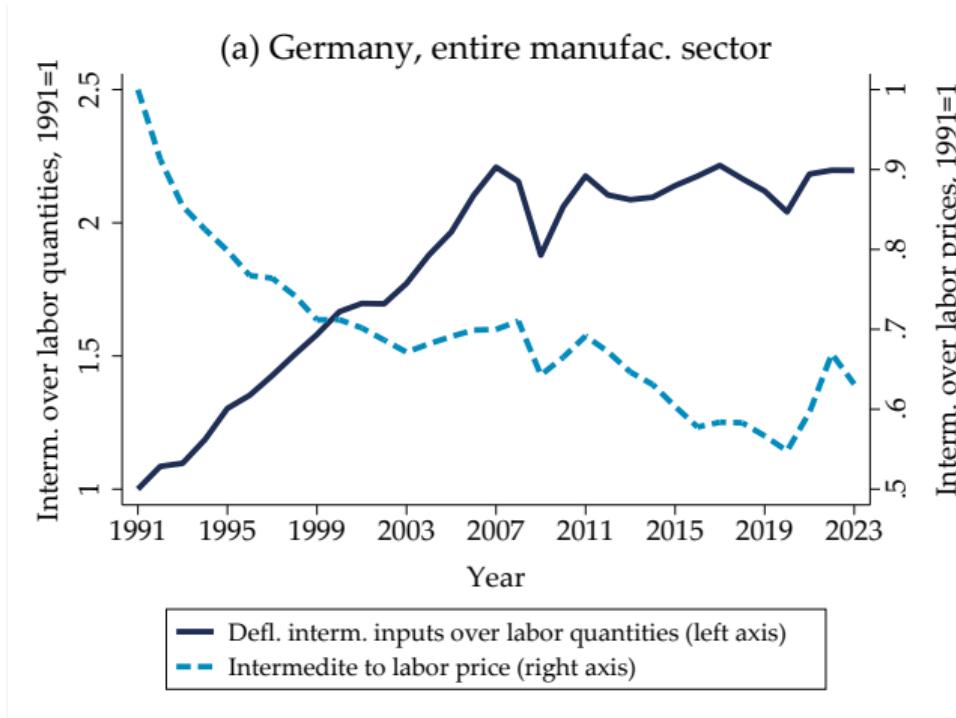
Firm-level regressions by size quintiles (2)

Table: Implied substitution and supply elasticities, 4-year changes (OLS)

| | OLS | | | |
|--------------------------|----------|---|---|---------------------|
| | σ | $\Delta \ln \left(\frac{L_{it}}{M_{it}} \right)$ | $\Delta \ln \left(\frac{w_{it} \gamma_{it}^L}{z_{it} \gamma_{it}^M} \right)$ | ε^{L-1} |
| | (1) | (2) | (3) | (4) |
| Overall | 2.259 | -0.535 | 0.236 | 2.16 |
| 1 st quintile | 2.007 | -0.546 | 0.272 | 1.86 |
| 2 nd quintile | 2.236 | -0.550 | 0.246 | 1.97 |
| 3 rd quintile | 2.186 | -0.540 | 0.247 | 1.99 |
| 4 th quintile | 2.327 | -0.519 | 0.223 | 2.37 |
| 5 th quintile | 2.277 | -0.526 | 0.231 | 2.29 |

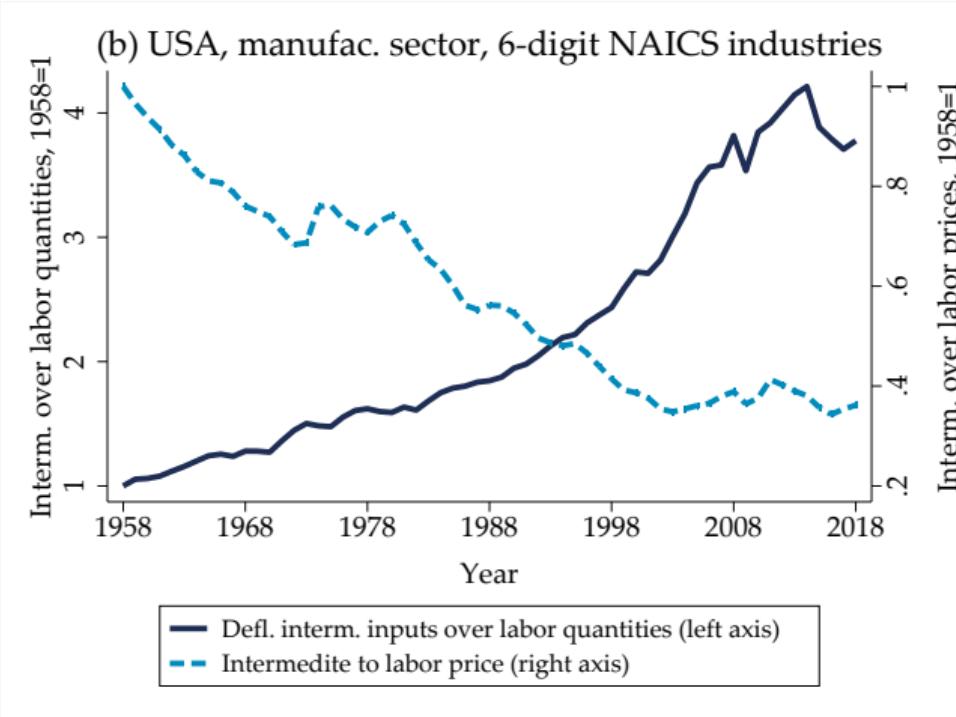
Manufacturing intermediate-labor quantities and prices (Germany)

Figure: Changes in aggregate intermediate to labor price and quantities (Germany).



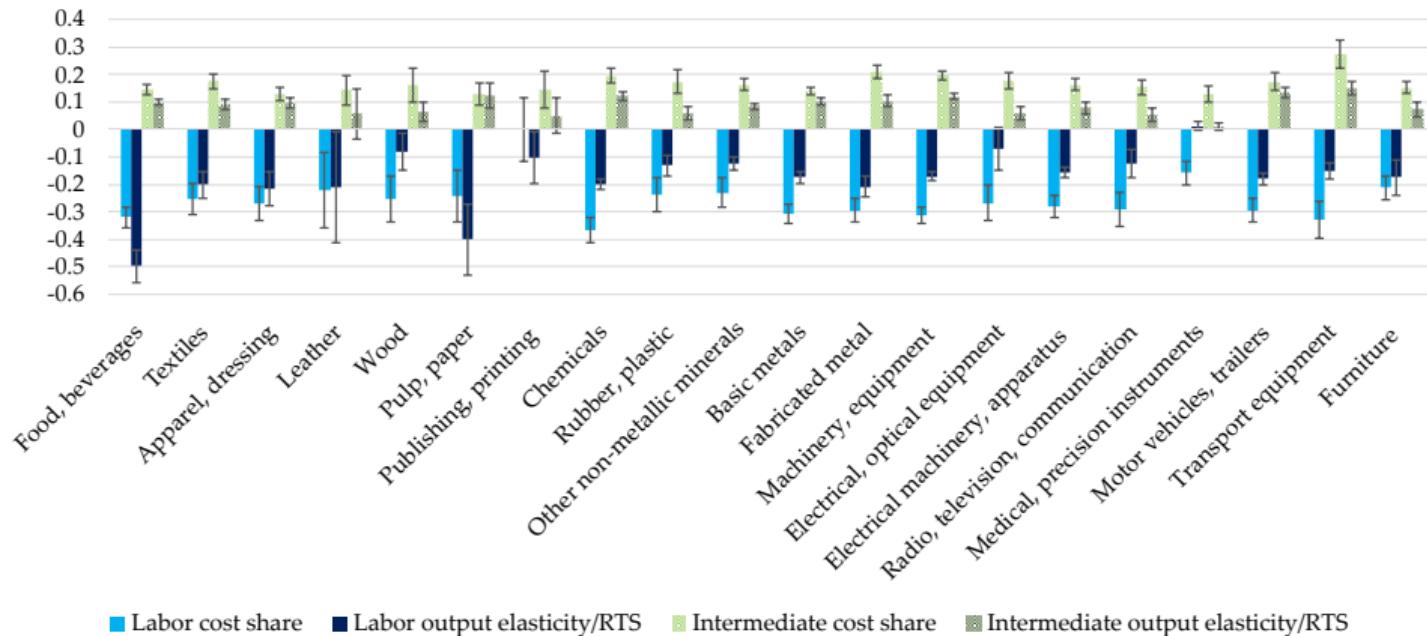
Manufacturing intermediate-labor quantities and prices (USA)

Figure: Changes in aggregate intermediate to labor price and quantities (USA)



Firm-level regressions by industries

Figure: Firm-level adjustments in labor and intermediate input output elasticities and cost shares by industries, 4-year changes, OLS



■ Labor cost share ■ Labor output elasticity/RTS ■ Intermediate cost share ■ Intermediate output elasticity/RTS

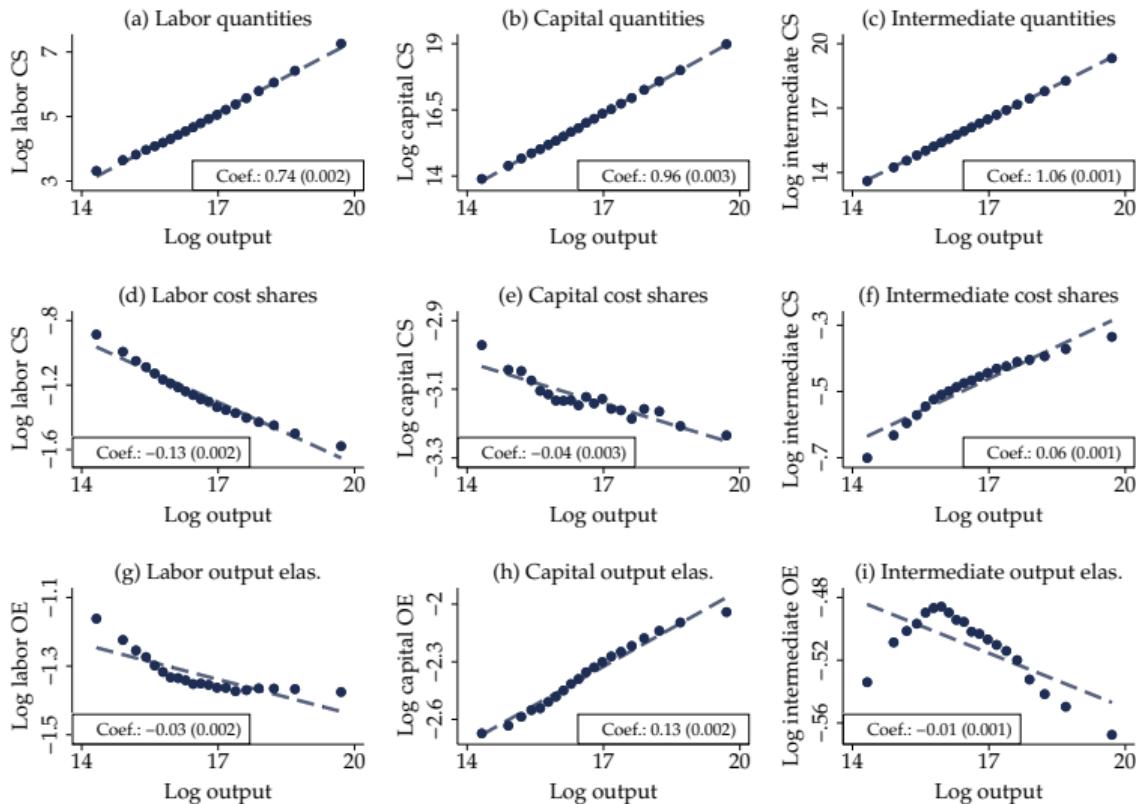
Reduced form regressions

Table: Reduced form regression.

| | $\ln(L_{it})$ | $\ln(K_{it})$ | $\ln(M_{it})$ | $\ln(CS_{it}^L)$ | $\ln(CS_{it}^K)$ | $\ln(CS_{it}^M)$ | $\ln(\frac{\theta_{it}^L}{RTS_{it}})$ | $\ln(\frac{\theta_{it}^K}{RTS_{it}})$ | $\ln(\frac{\theta_{it}^M}{RTS_{it}})$ |
|-----------------------|------------------------|-----------------------|-----------------------|------------------------|------------------------|-----------------------|---------------------------------------|---------------------------------------|---------------------------------------|
| Panel A: 1-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Export demand shock | 0.0146*** (0.0027) | 0.0028 (0.0021) | 0.0419*** (0.0049) | -0.0200*** (0.0028) | -0.0308*** (0.0041) | 0.0077*** (0.0016) | -0.0147*** (0.0031) | -0.0092* (0.0048) | 0.0054*** (0.0011) |
| Observations | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 | 183,813 |
| R ² | 0.064 | 0.039 | 0.142 | 0.118 | 0.160 | 0.109 | 0.104 | 0.258 | 0.095 |
| Panel B: 4-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Export demand shock | 0.0341*** (0.00729) | 0.0167** (0.00689) | 0.0612*** (0.0101) | -0.0171*** (0.0046) | -0.0362*** (0.0080) | 0.0079*** (0.0026) | -0.0154*** (0.0045) | -0.0093 (0.0061) | 0.0052*** (0.0018) |
| Observations | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 | 70,936 |
| R ² | 0.608 | 0.084 | 0.155 | 0.175 | 0.136 | 0.145 | 0.140 | 0.130 | 0.133 |

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Cross-section



Results without logs (1)

Table: Firm-level adjustments in cost share and output elasticities to firm growth. Different specification (no logs, not dividing by RTS). OLS regressions.

| | $\frac{w_{it}L_{it}}{C_{it}}$ | $\frac{r_{it}K_{it}}{C_{it}}$ | $\frac{x_{it}M_{it}}{C_{it}}$ | $\frac{\theta_{it}^L}{RTS_{it}}$ | $\frac{\theta_{it}^K}{RTS_{it}}$ | $\frac{\theta_{it}^M}{RTS_{it}}$ | $\ln(\theta_{it}^L)$ | $\ln(\theta_{it}^K)$ | $\ln(\theta_{it}^M)$ | θ_{it}^L | θ_{it}^K | θ_{it}^M | RTS_{it} |
|------------------------|-------------------------------|-------------------------------|-------------------------------|----------------------------------|----------------------------------|----------------------------------|------------------------|------------------------|-----------------------|-------------------------|------------------------|-----------------------|------------------------|
| Panel A: 1-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| Log output growth | -0.103*** (0.0011) | -0.0380*** (0.0003) | 0.141*** (0.0011) | -0.0710*** (0.0007) | -0.0201*** (0.0003) | 0.0911*** (0.0009) | -0.298*** (0.0046) | -0.309*** (0.0064) | 0.157*** (0.00129) | -0.0723*** (0.0008) | -0.0203*** (0.0004) | 0.0991*** (0.0007) | 0.00647*** (0.0004) |
| R ² | 0.303 | 0.419 | 0.413 | 0.266 | 0.191 | 0.277 | 0.215 | 0.132 | 0.397 | 0.278 | 0.183 | 0.425 | 0.157 |
| Panel B: 4-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| Log output growth | -0.0692*** (0.00157) | -0.0366*** (0.0006) | 0.106*** (0.00179) | -0.0438*** (0.00108) | -0.0136*** (0.000525) | 0.0574*** (0.0014) | -0.185*** (0.0063) | -0.164*** (0.0077) | 0.112*** (0.0021) | -0.0420*** (0.0014) | -0.0124*** (0.0007) | 0.0729*** (0.0013) | 0.0185*** (0.0008) |
| R ² | 0.303 | 0.419 | 0.413 | 0.266 | 0.191 | 0.277 | 0.201 | 0.160 | 0.369 | 0.220 | 0.154 | 0.388 | 0.157 |
| Panel C: 10-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| Log output growth | -0.0535*** (0.0016) | -0.0275*** (0.0006) | 0.0810*** (0.0018) | -0.0305*** (0.0011) | -0.0051*** (0.0005) | 0.0356*** (0.0014) | -0.112*** (0.00569) | -0.0441*** (0.0073) | 0.0860*** (0.0021) | -0.0236*** (0.00137) | -0.0018*** (0.0007) | 0.0559*** (0.0013) | 0.0304*** (0.0008) |
| R ² | 0.259 | 0.318 | 0.347 | 0.222 | 0.155 | 0.201 | 0.161 | 0.141 | 0.322 | 0.174 | 0.142 | 0.334 | 0.262 |

Results without logs (2)

Table: Firm-level adjustments in cost share and output elasticities to firm growth. Different specification (no logs, not dividing by RTS). IV regressions.

| | 1st stage | $\frac{w_{it}L_{it}}{C_{it}}$ | $\frac{r_{it}K_{it}}{C_{it}}$ | $\frac{z_{it}M_{it}}{C_{it}}$ | $\frac{\theta_{it}^L}{RTS_{it}}$ | $\frac{\theta_{it}^K}{RTS_{it}}$ | $\frac{\theta_{it}^M}{RTS_{it}}$ | $\ln(\theta_{it}^L)$ | $\ln(\theta_{it}^K)$ | $\ln(\theta_{it}^M)$ | θ_{it}^L | θ_{it}^K | θ_{it}^M | RTS_{it} |
|-------------------------|-----------------------|-------------------------------|-------------------------------|-------------------------------|----------------------------------|----------------------------------|----------------------------------|-----------------------|----------------------|----------------------|------------------------|-------------------------|-----------------------|------------------------|
| Panel A: 1-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) |
| Export demand shock | 0.0451*** (0.0045) | | | | | | | | | | | | | |
| Log output growth | | -0.0933*** (0.0144) | -0.0283*** (0.0042) | 0.122*** (0.0153) | -0.0626*** (0.0098) | -0.0133*** (0.0039) | 0.0759*** (0.0116) | -0.318*** (0.0645) | -0.220** (0.0995) | 0.127*** (0.0170) | -0.0672*** (0.0113) | -0.0135*** (0.00455) | 0.0914*** (0.0102) | 0.0107* (0.00575) |
| First-stage F-Statistic | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 | 102.6 |
| R ² | 0.205 | 0.294 | 0.427 | 0.400 | 0.309 | 0.208 | 0.329 | 0.215 | 0.127 | 0.387 | 0.277 | 0.171 | 0.424 | 0.049 |
| Panel B: 4-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) |
| Export demand shock | 0.0635*** (0.0091) | | | | | | | | | | | | | |
| Log output growth | | -0.0555*** (0.0177) | -0.0280*** (0.0062) | 0.0836*** (0.0201) | -0.0465*** (0.0127) | -0.0070 (0.0058) | 0.0536*** (0.0157) | -0.220*** (0.0742) | -0.125 (0.0973) | 0.105*** (0.0227) | -0.0445*** (0.0153) | -0.0005 (0.0076) | 0.0739*** (0.0143) | 0.0289*** (0.00876) |
| First-stage F-Statistic | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 | 48.57 |
| R ² | 0.297 | 0.403 | 0.402 | 0.266 | 0.175 | 0.276 | 0.183 | 0.199 | 0.158 | 0.368 | 0.220 | 0.121 | 0.388 | 0.139 |

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Regressions by intermediate components (1)

Table: Intermediate input components definitions in the German microdata.

| Variable | Avg. cost share | Definition |
|--------------|-----------------|--|
| E_{it} | 0.41 | Expenditures for raw, auxiliary, and operating materials and energy inputs (includes external product components). |
| $Merch_{it}$ | 0.05 | Expenditures for merchandise. |
| Sub_{it} | 0.03 | Expenditures for subcontracted work performed by other companies. |
| $Temp_{it}$ | 0.01 | Deflated expenditures for temporary agency workers. |
| Rep_{it} | 0.02 | Expenditures for repairs, maintenance, installation, and assembly. |
| $Rent_{it}$ | 0.03 | Rents, leases, leasing. |
| $Other_{it}$ | 0.10 | Expenditures for Other intermediate costs (insurance, postage, transport, etc.). |

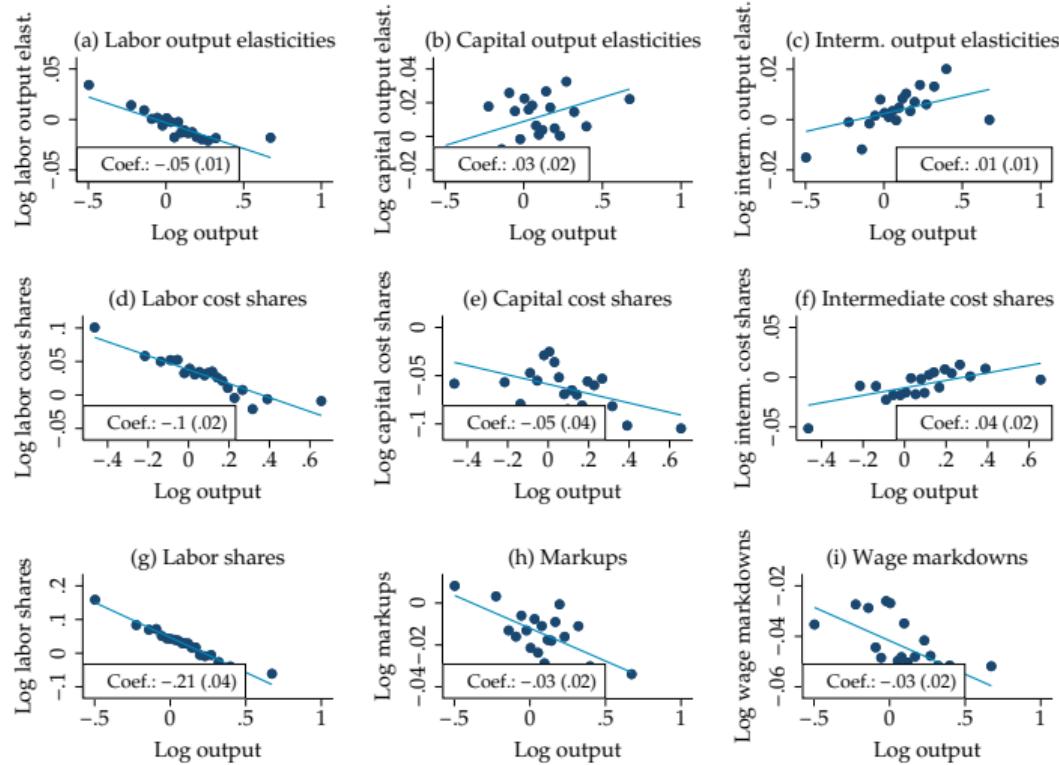
Regressions by intermediate components (2)

Table: Firm-level adjustments in intermediate cost shares by intermediate type (OLS).

| | $\ln(E_{it}^{CS})$ | $\ln(Merch_{it}^{CS})$ | $\ln(Sub_{it}^{CS})$ | $\ln(Rep_{it}^{CS})$ | $\ln(Rent_{it}^{CS})$ | $\ln(Temp_{it}^{CS})$ | $\ln(Other_{it}^{CS})$ |
|------------------------|---------------------|------------------------|----------------------|----------------------|-----------------------|-----------------------|------------------------|
| Panel A: 1-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Log output growth | 0.297*** (0.001) | 0.408*** (0.029) | 0.388*** (0.022) | -0.104*** (0.011) | -0.417*** (0.011) | 1.932*** (0.0323) | -0.107*** (0.008) |
| R ² | 0.108 | 0.069 | 0.061 | 0.029 | 0.055 | 0.205 | 0.030 |
| Panel B: 4-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Log output growth | 0.187*** (0.001) | 0.429*** (0.044) | 0.218*** (0.035) | -0.113*** (0.014) | -0.281*** (0.018) | 1.427*** (0.0419) | -0.054*** (0.011) |
| R ² | 0.128 | 0.117 | 0.115 | 0.072 | 0.079 | 0.232 | 0.065 |
| Panel C: 10-year diff. | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Log output growth | 0.137*** (0.008) | 0.217*** (0.051) | 0.148*** (0.038) | -0.072*** (0.015) | -0.218*** (0.021) | 0.434*** (0.0416) | 0.011 (0.012) |
| R ² | 0.130 | 0.126 | 0.120 | 0.074 | 0.080 | 0.124 | 0.068 |

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Results: Industry-level results, Europe, non-manufac. industries (1)



Firm-level results for other countries

Table: Firm-level results for other countries: Coefficients on log output growth, substitution elasticities, and contribution of labor output elasticity changes to labor share changes (4-year changes, OLS).

| | $\ln(L_{it})$ | $\ln(M_{it})$ | $\ln(CS_{it}^L)$ | $\ln(CS_{it}^M)$ | $\ln(\frac{\theta_{it}^L}{RTS_{it}})$ | $\ln(\frac{\theta_{it}^M}{RTS_{it}})$ | $\ln(\theta_{it}^L)$ | $\ln(LS_{it})$ | $\ln(\frac{P_{it}^L L_{it}}{VA_{it}})$ | σ^{OE} |
|----------------------|---------------|---------------|------------------|------------------|---------------------------------------|---------------------------------------|----------------------|----------------|--|---------------|
| Panel B: all sectors | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| France (1995-21) | 0.511*** | 0.975*** | -0.0486*** | 0.268*** | -0.0951*** | 0.183*** | -0.0874*** | -0.340*** | -0.333*** | 2.50 |
| Obs.: 589,669 | (0.003) | (0.004) | (0.002) | (0.004) | (0.001) | (0.001) | (0.001) | (0.002) | (0.002) | |
| Hungary (2003-22) | 0.497*** | 1.039*** | -0.270*** | 0.125*** | -0.276*** | 0.0942*** | -0.292*** | -0.356*** | -0.297*** | 3.15 |
| Obs.: 82,062 | (0.005) | (0.004) | (0.005) | (0.004) | (0.005) | (0.002) | (0.005) | (0.005) | (0.007) | |
| Poland (2002-22) | 0.535*** | 1.064*** | -0.273*** | 0.128*** | -0.276*** | 0.0900*** | -0.278*** | -0.336*** | -0.231*** | 3.25 |
| Obs.: 229,205 | (0.003) | (0.002) | (0.003) | (0.002) | (0.003) | (0.001) | (0.003) | (0.003) | (0.003) | |
| Portugal (2004-22) | 0.501*** | 1.044*** | -0.240*** | 0.165*** | -0.293*** | 0.149*** | -0.301*** | -0.361*** | -0.305*** | 5.38 |
| Obs.: 74,869 | (0.005) | (0.005) | (0.005) | (0.005) | (0.005) | (0.003) | (0.005) | (0.005) | (0.006) | |
| Slovakia (2000-23) | 0.493*** | 0.916*** | -0.200*** | 0.130*** | -0.202*** | 0.0764*** | -0.211*** | -0.412*** | -0.420*** | 2.93 |
| Obs.: 25,395 | (0.010) | (0.012) | (0.009) | (0.006) | (0.009) | (0.005) | (0.009) | (0.009) | (0.014) | |
| Slovenia (2002-23) | 0.591*** | 1.080*** | -0.259*** | 0.142*** | -0.294*** | 0.116*** | -0.298*** | -0.321*** | -0.191*** | 6.19 |
| Obs.: 17,696 | (0.014) | (0.008) | (0.011) | (0.007) | (0.012) | (0.006) | (0.012) | (0.011) | (0.012) | |

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Firm-level results: young and mature firms

| | $\ln(L_{it})$ | $\ln(M_{it})$ | $\ln(CS_{it}^L)$ | $\ln(CS_{it}^M)$ | $\ln(\frac{\theta_{it}^L}{RTS_{it}})$ | $\ln(\frac{\theta_{it}^M}{RTS_{it}})$ | $\ln(\theta_{it}^L)$ | $\ln(LS_{it})$ | $\ln(\frac{P_{it}^L L_{it}}{VA_{it}})$ | σ^{OE} |
|-----------------------|---------------|---------------|------------------|------------------|---------------------------------------|---------------------------------------|----------------------|----------------|--|---------------|
| Panel A: young firms | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| France (1995-21) | 0.477*** | 1.018*** | -0.109*** | 0.227*** | -0.131*** | 0.195*** | -0.122*** | -0.318*** | -0.305*** | 2.52 |
| Obs.: 21,988 | (0.008) | (0.010) | (0.007) | (0.008) | (0.004) | (0.006) | (0.005) | (0.007) | (0.008) | |
| Hungary (2003-22) | 0.488*** | 1.052*** | -0.249*** | 0.134*** | -0.287*** | 0.116*** | -0.299*** | -0.330*** | -0.274*** | 3.50 |
| Obs.: 3,966 | (0.016) | (0.013) | (0.015) | (0.011) | (0.015) | (0.006) | (0.015) | (0.015) | (0.019) | |
| Slovakia (2000-23) | 0.570*** | 1.014*** | -0.255*** | 0.134*** | -0.248*** | 0.0830*** | -0.242*** | -0.375*** | -0.370*** | 3.93 |
| Obs.: 1,157 | (0.031) | (0.031) | (0.031) | (0.021) | (0.028) | (0.013) | (0.029) | (0.031) | (0.051) | |
| Slovenia (2002-23) | 0.590*** | 1.097*** | -0.220*** | 0.140*** | -0.279*** | 0.140*** | -0.273*** | -0.266*** | -0.163*** | 5.76 |
| Obs.: 719 | (0.036) | (0.026) | (0.032) | (0.023) | (0.038) | (0.017) | (0.038) | (0.033) | (0.029) | |
| Panel B: mature firms | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| France (1995-21) | 0.498*** | 1.029*** | -0.0824*** | 0.330*** | -0.131*** | 0.199*** | -0.122*** | -0.382*** | -0.357*** | 2.64 |
| Obs.: 253,635 | (0.004) | (0.004) | (0.003) | (0.004) | (0.002) | (0.002) | (0.002) | (0.003) | (0.004) | |
| Hungary (2003-22) | 0.454*** | 1.060*** | -0.326*** | 0.168*** | -0.343*** | 0.124*** | -0.356*** | -0.434*** | -0.354*** | 4.36 |
| Obs.: 31,916 | (0.008) | (0.007) | (0.009) | (0.006) | (0.007) | (0.003) | (0.008) | (0.009) | (0.011) | |
| Slovakia (2000-23) | 0.491*** | 1.018*** | -0.272*** | 0.176*** | -0.283*** | 0.108*** | -0.274*** | -0.428*** | -0.391*** | 3.88 |
| Obs.: 8,399 | (0.018) | (0.022) | (0.015) | (0.012) | (0.015) | (0.008) | (0.016) | (0.018) | (0.028) | |
| Slovenia (2002-23) | 0.533*** | 1.097*** | -0.306*** | 0.179*** | -0.376*** | 0.137*** | -0.371*** | -0.387*** | -0.244*** | 11.06 |
| Obs.: 6,656 | (0.017) | (0.014) | (0.016) | (0.012) | (0.019) | (0.008) | (0.019) | (0.016) | (0.016) | |

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