The Aggregate Labor Supply Curve at the Extensive Margin: A Reservation Wedge Approach

Preston Mui  
UC Berkeley

Benjamin Schoefer  
UC Berkeley

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**Empirical regularity:** extensive margin dominates in fluctuations in $H_t = \sum_i h_{it} \cdot e_{it}$
Motivation: Labor Supply in Macro

- **Intensive margin relatively uncontroversial:**
  - E.g., transparent “MaCurdy” constant-Frisch-elasticity modeling approach:
    \[ h^{1/\eta}_{it} = \lambda_{it} w_{it} \]
  - Empirical estimates implying \( 0.0 < \eta < 1.0 \) (e.g. Chetty, Guren, Manoli, Weber 2012)

- **Extensive margin has “loose ends”:**
  - No empirical consensus on e.g. elasticity (e.g. Chetty, Guren, Manoli, Weber 2012, Rogerson and Keane 2012)
  - In modeling practice,
    
    Either: Impose or “reverse-engineer” iso-elastic MaCurdy case:
    \[ L^{1/\eta}_{t} = \lambda_{t} w_{t} \] (e.g., micro-founded in Gali 2015)

    Or: “Serious” ext-margin discrete choices \( e^* \in \{0, 1\} \), but: (e.g., Rogerson-Wallenius 2008; Het Agents...)
  - No transparent and easy-to-calibrate ALSC (vs: “\( \eta \)”) 
  - Overwhelming complexity (multi-dim. heterogeneity, “hidden calibrations”,...) 
  - Hard to (i) interrelate and (ii) adjudicate between empirically
This Paper

1. Propose basic framework for EM-ALSC
   1 − Ξ_t: Prevailing aggregate wedge: linear homogeneous mark-up/down on potential labor earnings
   1 − ξ_{it}^*: Micro reservation wedge: hypothetical 1 − Ξ_t rendering i indifferent b/w working and not:
         - Frischian i.e. λ_{it}-constant pertubation in 1 − Ξ_t
   \[ F_t(1 − ξ^*) \]: Aggregation of individual RW’s ⇒ short-run EM-ALSC:
     \[ E_t = P(1 − ξ_{it}^* ≤ 1 − Ξ_t) = F_t(1 − Ξ_t) \]

2. Model meta-analysis: recast in RW framework to uncover & make comparable ALSCs

       - Large local elasticities of 3 and up
       - Non-constant, asymmetric arc-elasticities: smaller elasticities upwards

4. Macro implications of empirical EM-ALSC used as calibration target

5. No existing model provides tight fit
       - Fit one model’s \( F_t(1 − ξ^*) \) tightly to the empirical analogue
       - Labor wedge exercise: considerably less cyclical
Outline

1. **Reservation Wedge Framework**
   - Individual-Level EM Choice
   - Extensions
   - Aggregation into EM ALSC

2. Measurement In Survey

3. Model Meta-Analysis

4. Macro Implications
Micro EM Labor Supply

\[
\max_{a_{it}, h_{it}, c_{it}} \mathbb{E}_t \sum_{s \geq t} \beta^{s-t} u_i(h_{is}, c_{is}) \quad \text{s.t.} \quad a_{is} + c_{is} \leq a_{i,s-1}(1 + r_{i,s-1}) + (1 - \Xi_s)y_{is}(h_{is}) + T_{is}(.) \quad \forall s \geq t
\]

- Prevailing aggregate labor wedge \(1 - \Xi_t\)
- Indivisible labor: \(h_{it} \in \{0, \tilde{h}_{it}\}\)
- Potential earnings \(y_{it} = w_{it}h_{it}\)
- Labor disutility \(v_{it} = u_i(c_{it}^e, h) - u_i(c_{it}^n, 0)\)

Labor supply is discrete (employment) choice:

\[
\Rightarrow h_{it}^* = \begin{cases} 
0 & \text{if } v_{it} > (1 - \Xi_t)w_{it}\tilde{h}_{it}\lambda_{it} \\
\tilde{h}_{it} & \text{if } v_{it} \leq (1 - \Xi_t)w_{it}\tilde{h}_{it}\lambda_{it}
\end{cases} \quad \leftrightarrow \quad e_{it}^* = \begin{cases} 
0 & \text{if } v_{it} > (1 - \Xi_t)y_{it}\lambda_{it} \\
1 & \text{if } v_{it} \leq (1 - \Xi_t)y_{it}\lambda_{it}
\end{cases}
\]

Reservation wedge \(1 - \xi_{it}^*\): hypothetical \(1 - \Xi_t\) rendering \(i\) indifferent between working and not:

\[
v_{it}(1 - \xi_{it}^*)y_{it}\lambda_{it} \quad \leftrightarrow \quad 1 - \xi_{it}^* = \frac{v_{it}}{y_{it}\lambda_{it}}
\]

Sufficient statistic for \(i\)'s Frischian employment preferences:

\[
\Rightarrow e_{it}^* = \begin{cases} 
0 & \text{if } 1 - \Xi_t < 1 - \xi_{it}^* \\
1 & \text{if } 1 - \Xi_t \geq 1 - \xi_{it}^*
\end{cases}
\]
Extensions

- Nonemployment subsidies/opportunity cost of working
  Chodorow-Reich and Karabounis (2016)
- Non-Frischian contexts, income/wealth effects
- Intensive margin / hours choices / job menus – now
- Long-term jobs
  Mui and Schoefer (2019)
- Human-capital accumulation
- Adjustment costs
  Chetty (2012)
- Amenities
  Hall and Mueller (2018)
- Specific models – later in meta-analysis
Aggregation: Labor Supply Curve at Extensive-Margin

- **CDF** of the RWs, $F_t(1 - \xi^*)$, fully characterizes the EM-ALSC
- **“Aggregate wage” concept**: shifts in $1 - \Xi_t$
  - Taxes, wage growth (e.g., marginal product shifts,...),...
  - Frischian, $\lambda$-constant variation
- **(Desired) Employment Rate**: 
  \[
  E_t(1 - \Xi_t) = P(1 - \xi^* \leq 1 - \Xi_t) = F_t(1 - \Xi_t)
  \]
- **Marginal Individual**: $1 - \xi^*_i = 1 - \Xi_t$
- **Employment Adjustment**: Increase in aggregate wedge from $(1 - \Xi_t)$ to $(1 - \Xi'_t)$:
  \[
  dE_t = F_t(1 - \Xi'_t) - F_t(1 - \Xi_t)
  \]
- **Discrete Arc Elasticity**:
  \[
  \frac{F_t(1 - \Xi'_t) - F_t(1 - \Xi_t)}{F_t(1 - \Xi_t)} \left/ \frac{(1 - \Xi'_t) - (1 - \Xi_t)}{1 - \Xi_t} \right.
  \]
- **Infinitesimal**:
  \[
  \frac{(1 - \Xi_t)f_t(1 - \Xi_t)}{F_t(1 - \Xi_t)}
  \]
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Measurement of the EM-ALSC

1 Our approach: survey to elicit preferences (reservation wedge)

Caveat: response quality

2 Quasi-experimental: response of realized employment to shifts in return to working

- Canonical example: Icelandic tax holiday
  Bianchi, Gudmundsson, Zoega (2001)
- Identify one specific arc elasticity of global EM-ALSC:

\[
\epsilon_{E_t,(1-\Xi_t)\rightarrow(1-\Xi'_t)} = \frac{F_t (1 - \Xi'_t) - F_t (1 - \Xi_t)}{F (1 - \Xi_t)} / \left( \frac{(1 - \Xi'_t) - (1 - \Xi_t)}{1 - \Xi_t} \right)
\]

Caveat: Realized ("net of frictions") vs. desired labor supply

3 Structural estimation

Caveat: Distributional and functional form assumptions, relies on realized employment too
Measurement of Reservation Wedge

- **Ideal Measure** (Frischian temporary tax wedge in a spot labor market):

  You are currently [non-]employed. Suppose the following thought experiment: you (and only you) receive a temporary linear incremental tax [or subsidy] on your take-home earnings (at whichever hours or job you may choose to work). At what incremental tax [or subsidy] rate would you be indifferent between not working for this period and working (at whichever job would be your best choice at that given tax [subsidy] rate)?

- **Feedback from MTurk pilots:**
  - "The wording of the question was confusing"
  - "bizarre scenario"
  - ...

Custom Reservation Wedge Survey

- **In practice:**
  - We translate this ideal questions into three variants, **routed by labor force status**.
  - Specify specific "Frischian" time horizon to "one month"
  - Piloting: evoke "job-constant" scenario

- Nationally representative U.S. survey of 2,000 respondents.
- Fielded by NORC (University of Chicago): AmeriSpeak Omnibus program
- March and April, 2019.
- To come (Fall 2019): Integration into German Socio-Economic Panel (additional covariates, etc.)
Survey Question: Out of Labor Force

Think of the range of jobs that you would realistically be offered if you searched for jobs (even if you currently are not looking for a job and may not accept any of these potential jobs).

Suppose you had such job offers in hand. Currently you would likely not take such jobs, at least not at the usual salary. However, suppose the employer were nevertheless trying hard to recruit you, specifically by offering an additional sign-up bonus. The requirement to receive the bonus is that you will work for at least one month. The bonus comes as a raise of the first month’s salary. This sign-up bonus will only be paid in the first month (on top of the regular salary that month), afterwards the salary returns to the regular salary.

...

5% means you would take the job if your employer paid a bonus of just 5% of the regular salary in the first month. 100% means you would require a bonus as large as the regular salary. 500% would mean you require a bonus equal to five times as large as the regular salary.
Survey Question: Employed

Suppose, for reasons unrelated to you, your employer offers you the following choice: Either you take unpaid time off from work for one month, or you stay in your job for that month and only receive a fraction of your regular salary. No matter what choice you take, after the month is over, your salary will return to normal.

In this hypothetical scenario, you cannot take an additional job to make up for the lost income during that month.

Assume this choice is real and you have to make it. At what point would the cut in your salary be just large enough that you would choose the unpaid month of time off over working for the month at that lower salary?

For example, an answer of 5% means that a 5% wage cut would be the point where you would choose to take unpaid time off for the month instead of working for 5% lower pay during that month. But if the wage cut was less than 5%, you would instead choose to work for that than take unpaid time off. Choose any percentage between 1% to 100%, where the cut wage cut is just large enough that you would prefer to not work at all for no pay than work at reduced pay for that month.
Survey Question: Unemployed

Suppose you have found the kind of job you are looking for and the employer would like to hire you. The regular start date for the job is one month away. As an alternative, your employer offers you the option to start working immediately, rather than waiting a month.

However, if you chose to start work immediately, for that first month, you will only receive a fraction of the regular salary. The job is otherwise exactly the same. No matter what choice you take, after the month is over, the salary will then resume at the regular salary. In this hypothetical scenario, you cannot take an additional job to make up for the lost income during that month.

.... At what point would the cut in your salary be just large enough that you would choose the waiting a month without working and without the salary over starting the job immediately for the first month at that lower salary?

For example, an answer of 5% means that a 5% wage cut...
The Empirical Reservation Wedge Distribution
The Empirical Reservation Wedge Distribution
Employment = 0.607
Outline

1. Reservation Wedge Framework
2. Measurement In Survey
3. Model Meta-Analysis
4. Macro Implications
Model Meta-Analysis: EM-ALSCs as RWs

- Make tangible ⇒ recast respective detailed model into RW framework
- Make comparable ⇒ unifying bridge across models

Three-step "recipe" for each model:

1. Define individual-level RW $1 - \xi_{it}^*$
2. Construct ALSC from CDF $F(1 - \xi_{it}^*)$ (reflecting joint equilibrium distribution of all RW-relevant factors)
3. Study properties – e.g. (local) elasticity

Models:
1. Representative “command” household with consumption insurance
2. Heterogeneous agent models with extensive margin
3. Lifecycle, intensive margins, and nonconvexities (Rogerson and Wallenius, 2008)
Rep HH w/ Insurance and "Command" Labor Supply

\[
\max_{\{c_{it}, e_{it}\}_i, A_t} \mathbb{E}_t \sum_{s \geq t}^{\infty} \beta^{s-t} \int_0^1 \left[ u_i(c_{is}) - e_{is}v_{is} \right] di \\
\text{s.t. } A_s + \int_0^1 c_{is} di \leq A_{s-1}(1 + r_{s-1}) + \int_0^1 (1 - \Xi_s) y_{is} e_{is} di + T_s \quad \forall s \geq t
\]

- Pooled budget constraint & full "insurance" ⇒ \( \overline{\lambda}_t = \frac{\partial u_i(c_{it})}{\partial c_{it}} \) \( \forall i \)

1 Define micro RW:

\[1 - \xi^*_it = \frac{v_{it}}{\overline{\lambda}_t y_{it}}\]

2 CDF i.e. EM-ALSC:

\[E_t = F_t(1 - \Xi_t) = P(1 - \xi_{it} \leq 1 - \Xi) = P \left( \frac{v_{it}}{y_{it} \overline{\lambda}_t} \leq 1 - \Xi_t \right) = P \left( \frac{v_{it}}{y_{it}} \leq (1 - \Xi_t) \overline{\lambda}_t \right)\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} 1 \left[ \frac{\nu}{\mu} \leq (1 - \Xi_t) \overline{\lambda}_t \right] dG(\nu, \mu) \Rightarrow \text{Properties of EM-ALSC follow } G(\nu, \mu)
\]
Rep HH w/ Insurance and "Command" Labor Supply

- Pooled budget constraint & full "insurance" \( \Rightarrow \lambda_t = \frac{\partial u_i(c_{it})}{\partial c_{it}} \forall i \)

1. Micro RW:

\[
1 - \xi^*_it = \frac{v_{it}}{\lambda_t y_{it}}
\]

2. CDF i.e. EM-ALSC... depends on \( G(v, y) \)

3. Same for its properties (arc elasticities)
RHH Example: Heterogeneity in Disutility Only

- Heterogeneity in $v$ only: $v_{it} \sim G^v(v)$

1. Micro RWs characterized by disutility type $v(i)$:

$$1 - \xi_{it}^* = \frac{v_{it}}{y_t \lambda_t} = 1 - \xi_{vt}$$

2. Wedge distribution – and hence CDF and ALSC – inherits shape of $v_{it}$-distr’n:

$$E_t = F_t(1 - \Xi_t) = P(1 - \xi_{it}^* \leq 1 - \Xi_t) = P\left(v_{it} \leq \frac{1 - \Xi_t}{y_t \lambda_t}\right) = G^v\left(\frac{1 - \Xi_t}{y_t \lambda_t}\right)$$

3. Elasticity (local at $1 - \Xi_t$) is given by

$$\left[(1 - \Xi_t)g^v\left(\frac{1 - \Xi_t}{y_t \lambda_t}\right)\right] / \left[1 - G^v\left(\frac{1 - \Xi_t}{y_t \lambda_t}\right)\right]$$
Rep HH Specific Example 1: Hansen (1985)

- $G(v, w)$ for homogeneous households:

$$y_{it} = \bar{y}_t \quad \forall i$$
$$v_{it} = \bar{v} = A \ln(1 - h_{it}) \forall i$$

1. Homogeneous micro RWs:

$$1 - \xi^*_t = 1 - \bar{\xi}^*_t = \frac{\bar{V}}{\lambda_t \bar{y}_t}$$

2. Degenerate wedge distribution

$$F_t(1 - \Xi_t) = \begin{cases} 
0 & \text{if } 1 - \Xi_t < \frac{\bar{V}}{\lambda_t \bar{y}_t} \\
1 & \text{if } 1 - \Xi_t > \frac{\bar{V}}{\lambda_t \bar{y}_t}
\end{cases}$$

3. Infinite Frisch elasticity
Implied Arc Elasticity of Labor Supply

U.S. Population (Authors' Survey)
RHH Example: Constant Elasticity

Ex: If $v$ follows power distribution, $\Rightarrow$ iso-elasticity ("MaCurdy", Gali):

\[
\epsilon_{E_t,1-\Xi_t} = \frac{(1 - \Xi_t)F_t(1 - \Xi_t)}{F_t(1 - \Xi_t)} = \frac{(1 - \Xi_t)\alpha_v(1 - \Xi_t)^{-1} \left( \frac{(1-\Xi_t)\bar{y}_t\lambda_t}{\nu_{max}} \right)^{\alpha_v}}{\left( (1 - \Xi_t)\bar{y}_t\lambda_t/\nu_{max} \right)^{\alpha_v}} = \alpha_v
\]

- General conditions on RW dist in paper
Change in Log Wedge

Change in Log Employment (non-Hansen)

U.S. Pop. (Authors' Survey)

MaCurdy (Isoelasticity 0.32)

MaCurdy (Isoelasticity 2.5)

Hansen (Indivisible Labor)

Employment Level (Hansen)

Employment = 0.607
Change in Wedge

Implied Arc Elasticity of Labor Supply
U.S. Population (Authors' Survey)
MaCurdy (Isoelasticity 0.32)
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3. Lifecycle, intensive margins, and nonconvexities (Rogerson and Wallenius, 2008)
Heterogeneous Agent Models

- Huggett (1993) model (one asset plus borrowing constraint) + extensive margin
  - Stochastic potential earnings (productivity)
  - Incomplete markets ⇒ imperfect insurance

\[
\max_{c_{it}, e_{it} \in \{0, 1\}, a_{it}} \mathbb{E}_t \sum_{s \geq t} \beta^{s-t} \left( \frac{c_{is}^{1-\sigma}}{1-\sigma} - \bar{\nu} e_{is} \right)
\]

s.t. 
\[
a_{i,s+1} = (1 - \Xi_s) y_{is} e_{is} + (1 + r_s) a_{is} - c_{is} \quad \forall \ s \geq t
\]
\[
a_{is} \geq a \quad \forall \ s \geq t
\]
Heterogeneous Agent Models

- No insurance $\implies$ heterogeneity in $\lambda_{it}$

1. Micro RW (can be indexed by assets $a$ and potential earnings/productivity $y$)

   $1 - \xi_{ay} = \frac{\bar{V}}{\lambda_{ay} y}$

2. CDF i.e. EM-ALSC:

   $E_t = F_t(1 - \Xi_t) = \sum_y \int_a^\infty \mathbb{1}[1 - \xi_{a,y} \leq 1 - \Xi_t] g(a, y) \, da$

   Complicated object given by earnings process and consumption/savings decisions!

3. Elasticities etc.?

   - Calibrate $\sigma = 2$, $r = 0.03$, $\beta = 0.97$, earnings process Markovian (33 states) from HANK (Kaplan et. al 2018), $\bar{V}$ to match 0.607 (BLS E-Pop Feb. 2019)
Change in Wedge

Implied Arc Elasticity of Labor Supply

- U.S. Population (Authors' Survey)
- MaCurdy (Isoelasticity 0.32)
- MaCurdy (Isoelasticity 2.5)
- Het. Agent Model
- Het. Agent Model (Income Process Only)
The “Stabilizing” Role of Incomplete Markets
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Model with heterogeneous wages and intensive margin hours choices
- OLG: unit mass of individuals born at every instant; alive between age $a \in [0, 1]$.
- $a$-specific wages $w_a$ – triangle $\Rightarrow$ lifecycle labor supply

$$\max_{c_a, h_a} \int_{a=0}^{1} e^{-\rho a} \left[ u(c_a) - v(h_a) \right] da$$

s.t. $\int_{a=0}^{1} e^{-ra} c_a = \int_{0}^{1} e^{-ra} y_a(h) da$

- Intensive-margin choice: pick optimal hours with MaCurdy disutility $v(h_a) = \Gamma \frac{h_a^{1+1/\gamma}}{1+1/\gamma}$.
- Nonconvexity in form of fixed hours cost of working: $y_a(h_a) = w_a \max\{h_a - \underline{h}, 0\}$

1. Define micro RW:

\[ 1 - \xi_a^* = \frac{v(h(a, 1 - \xi_a^*))}{w_a[h(a, 1 - \xi_a^*) - \bar{h}]\lambda} = \frac{\Gamma\left(h(1/\gamma + 1)\right)^{1/\gamma}}{w_a \lambda} \]

2. CDF:

\[ F(1 - \Xi) = P\left(\frac{\Gamma\left(h(1/\gamma + 1)\right)^{1/\gamma}}{w_a \lambda} \leq 1 - \Xi\right) = P\left(\frac{1}{w_a} \leq \frac{(1 - \Xi)\lambda}{\Gamma\left(h(1/\gamma + 1)\right)^{1/\gamma}}\right) \]

3. Shape? Elasticity? **Determined by age (i) distribution and (ii) wage-age profile**
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1. Reservation Wedge Framework
2. Measurement In Survey
3. Model Meta-Analysis
4. Macro Implications
   - Calibration target: global AGLSC \( (V(E)) \)
   - Labor wedge with data-consistent \( V(E) \)
Calibrating the Labor Supply Curve

- None of the models capture the global empirical wedge distribution
- Next: one example matching it nearly perfectly
- Rep. HH with heterogeneity in disutility of labor $v \sim G^v(v)$
- Reverse-engineer $v \sim G^v(v)$ to yield a RHH-level disutility of labor $V(E)$ consistent with the empirical wedge distribution

$$U(C_t) - V(E_t)$$ (1)

- Fit polynomial of model $V'(E)$ function to empirical res-wedge CDF (and take anti-derivative; details in paper)
Employment Level (Hansen) = 0.607

Change in Log Employment (non-Hansen)
U.S. Pop. (Authors' Survey)
U.S. Pop. (Authors' Survey), Fitted
Outline

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   ▪ Calibration target: global AGLSC $V(E)$
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Revisiting the Labor Wedge \((1 - \theta_t)\)

\[
\ln C_t - V(E_t) \tag{2}
\]

Three variants for \(V(E_t)\):
- Our fitted polynomial approximating the empirical global curve
- Iso-elasticity 0.32
- Iso-elasticity 2.5

\[
(1 - \theta_t)F_L(L_t, K_{t-1}) = \frac{-U_L(C_t, L_t)}{U_C(C_t, L_t)} = V'(E_t) \cdot C_t \tag{3}
\]

\((1 - \theta_t)\): measure of disequilibrium (symptom of frictions), mismeasurement or model misspecification
- Apply to US business cycles post-1960
Change in Log Wedge

Change in Log Employment (non-Hansen)

U.S. Pop. (Authors' Survey)

U.S. Pop. (Authors' Survey), Fitted

Employment = 0.607
Calibrating the Labor Supply Curve

- None of the models capture the global empirical wedge distribution
- Next: one example matching it perfectly
- \( \text{RHH } v \sim G^v(v) \)

\[
\max_{\bar{c}_t, \{e_{vt}\}, A_t} \mathbb{E}_t \sum_{s \geq t} \beta^{s-t} \left[ u(\bar{c}_s) - \int e_{vs} vdG^v(v) \right]
\]

s.t. \( A_s + \bar{c}_s \leq A_{s-1}(1 + r_{s-1}) + (1 - \Xi_s)y_s \int e_{vs} dG^v(v) + T_s \ \forall s \geq t \)

Data individual of type \( 1 - \hat{\xi}_{vt} \) has peer \( v = (1 - \hat{\xi}_{vt}) \bar{y}_t \bar{\lambda}_t \) in the model

\[
V(E) \equiv \int e_{v} vdG^v(v) = \int_{-\infty}^{\mu(E)} vdG^v(v)
\]

\[
V'(E) = \mu(E)
\]

where \( \mu(E) \equiv (G^v)^{-1}(E) \) is the quantile function of the disutility distribution.
Fitting Polynomial to $V(E)$

Fit a (7th-order) polynomial to function of $V'(E) = \mu(E)$ (wedge at point $E$) to $E$ (CDF) – get analytical antiderivative $V(E)$ and derivative $V''(E)$. Weight more around unit wedge.

$E = G(\mu(E))$. $V'(E) = \mu(E)g(\mu(E))\mu'(E) = \mu(E) > 0$, as $\mu'(E) = \frac{1}{g(\mu(E))}$. So, $V''(E) = \frac{1}{g(\mu(E))} > 0$ over the support.
Employment = 0.607
Implied Arc Elasticity of Labor Supply
U.S. Population (Authors' Survey)

Change in Wedge

Diagram showing the relationship between change in wedge and implied arc elasticity of labor supply.
Labor Wedges: Time Series and Binned Scatter Plot

- Labor Market Wedges By Model
- Data-Consistent V(E)
- Isoelasticity 2.50
- Isoelasticity 0.32

- Employment Rate (Log Deviation)
- Data-Consistent V(E)
- Isoelasticity 2.50
- Isoelasticity 0.32
Labor Wedges: Amplify x10 Emp-Fluct Entering $V'(E)$
Marginal Disutility of Labor

Data-Consistent V(E)

<table>
<thead>
<tr>
<th>Model</th>
<th>1960q1</th>
<th>1980q1</th>
<th>2000q1</th>
<th>2020q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isoelasticity 2.50</td>
<td>-0.06</td>
<td>-0.04</td>
<td>-0.02</td>
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</tr>
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Employment Rate (Log Deviation)

Data-Consistent V(E)

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Marginal Dis’y of Labor: Amplify x10 Emp-Fluct Entering $V'(E)$
Implied Arc Elasticity of Labor Supply

- U.S. Population (Authors' Survey)
- U.S. Population (Authors' Survey), Fitted
- MaCurdy (Isoelasticity 0.32)
- MaCurdy (Isoelasticity 2.5)
- Heterogeneous Agent Model
- Het. Agent Model (Income Process Only)
- Rogerson-Wallenius
- Rogerson-Wallenius (No Int. Margin Adj.)
- Rogerson-Wallenius (Lower Frisch)
Summary: Reservation Labor Wedges and Aggregate Labor Supply

1. **Propose basic framework for EM-ALSC**
   1. $1 - \Xi_t$: *Prevailing aggregate wedge*: linear, homogeneous mark-up/down on labor income
   2. $1 - \xi^{*}_{it}$: *Micro reservation wedge*: hypothetical $1 - \Xi_t$ rendering $i$ indifferent b/w working and not:
      - Frischian i.e. $\lambda_{it}$-constant pertubation in $1 - \Xi_t$

   $F_t(1 - \xi^{*})$: *Aggregation* of individual RW’s ⇒ short-run EM-ALSC:
   \[
   E_t = P(1 - \xi^{*}_{it} \leq 1 - \Xi_t) = F_t(1 - \Xi_t)
   \]

2. **Model meta-analysis**: recast in RW framework to uncover & make comparable ALSCs
3. **Custom survey of U.S. pop’n**: directly measure RW distribution (US EM-ALSC)
   - Large local elasticities of 3 and up
   - Non-constant, asymmetric arc elasticities: smaller arc elasticities upwards
4. No existing model provides good global fit
5. **Macro implications of empirical EM-ALSC** used as calibration target
   - Fit one model’s $F_t(1 - \xi^{*})$ tightly to the empirical analogue
   - Labor wedge exercise: considerably less cyclical
Job Menus: Allowing for Intensive Margins

- Choice $j$ from a menu $J_{it} \in \{(y_{it,j}, v_{it,j})\}_j$: $j$-specific earnings and disutility/amenities
  - Nests $j$-specific heterogeneity in hours $\tilde{h}_{it}$
    
    \[
    \max_{a_{it}, j \in J_{it}, c_{it}} \mathbb{E}_t \sum_t u(j, c_{it}) \quad \text{s.t.} \quad a_{it} + c_{it} \leq a_{i,t-1}(1 + r_{t-1}) + (1 - \Xi_t)y_{it,j} + T_{it}(.)
    \]

- For any given wedge $1 - \Xi_t$, intensive-margin job choice (ignoring participation):
  \[
  \Rightarrow j^*(1 - \Xi_t) = \arg\max_{j \in J_{it}} \{\mathbb{E}_t \sum_t u(j, c_{it}) \text{ s.t. } (BC) \mid 1 - \Xi_t\}
  \]

- Extensive-margin choice respecting intensive margin choice:
  \[
  \Rightarrow e^*_{it} = \begin{cases} 
  0 & \text{if } (1 - \Xi_t)y_{it}^{j^*(1-\Xi_t)}\lambda_{it} < v_{it}^{j^*(1-\Xi_t)} \\
  1 & \text{if } (1 - \Xi_t)y_{it}^{j^*(1-\Xi_t)}\lambda_{it} \geq v_{it}^{j^*(1-\Xi_t)}
  \end{cases}
  \]

- Implicitly defined RW conditional on having (re-)optimized job choice:
  \[
  1 - \xi_{it}^* = \frac{v_{it}^{j^*(1-\xi_{it}^*)}}{y_{it}^{j^*(1-\xi_{it}^*)}\lambda_{it}}
  \]
Suppose MaCurdy preferences, with flexible hours choice $h^i \in [0, \infty)$. FOC for hours is:

**Appendix: Specific Intensive-Margin: Hours Choice**

\[
\Psi h_{it}^{1/\eta} = (1 - \Xi_t) w_{it} \lambda_{it}
\]

- Without reoptimization of hours:

\[
\frac{v_{it}}{1+1/\eta} = \frac{\psi h_{it}^{1+1/\eta}}{1+1/\eta} = (1 - \xi_{it}^*) w_{it} h_{it}^* \lambda_{it}
\]

\[
\Leftrightarrow 1 - \xi_{it}^* = \frac{\psi h_{it}^{1+1/\eta}}{(1+1/\eta) w_{it} \lambda_{it}} = \frac{1}{1+1/\eta} > 0
\]

- With reoptimization, note that it holds that:

\[
\Psi h_{it}^{1/\eta} = (1 - \Xi_t) w_{it} h_{it}^*
\]

...and so the RW is trivial – no meaningful extensive margin:

\[
h_{it}^* = 0 \Leftrightarrow 1 - \xi_{it}^* = 0 \ \forall i
\]

\[\Rightarrow\] Need non-convexity/fixed cost (e.g. Rogerson and Wallenius, 2008) or limited job menu.
### B. Intertemporal Substitution (Frisch) Elasticities

<table>
<thead>
<tr>
<th>Study</th>
<th>Elasticity</th>
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</thead>
<tbody>
<tr>
<td>Carrington (1996)</td>
<td>0.43</td>
</tr>
<tr>
<td>Gruber and Wise (1999)</td>
<td>0.23</td>
</tr>
<tr>
<td>Bianchi, Gudmunndsson, and Zoega (2001)</td>
<td>0.42</td>
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<tr>
<td>Card and Hyslop (2005)</td>
<td>0.38</td>
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<tr>
<td>Brown (2009)</td>
<td>0.18</td>
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<tr>
<td>Manoli and Weber (2011)</td>
<td>0.25</td>
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<tr>
<td>Unweighted Mean</td>
<td>0.32</td>
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Chetty, Guren, Manoli, Weber (2012)
Application: Conditions for Constant Elasticity

Specifically, the distributional assumptions for the property in power-law terms specify a standard power law distribution $F(X) = P(x < X) = a \cdot \left( \frac{x}{X_{\text{min}}} \right)^{-\gamma+1}$ with shape parameter $\gamma > 0$. A comparison with our wedge-based power-law-like distribution ($??$) and a rearrangement clarify that we require the inverse of our wedge to follow a power distribution:

$$G_{1-\xi^*} (1 - \xi^*) = P(X < 1 - \xi^*) = \left( \frac{1 - \xi^*}{(1 - \xi^*)_{\text{max}}} \right)^{\alpha_{1-\xi^*}}$$

$$\Leftrightarrow P \left( \frac{1}{1 - \xi^*} < \frac{1}{X} \right) = \left( \frac{\frac{1}{1-\xi^*}}{\frac{1}{(1-\xi^*)_{\text{max}}}} \right)^{-\alpha_{1-\xi^*}}$$

which is a power-law distribution of $\frac{1}{1-\xi^*}$ with minimum $\frac{1}{(1-\xi^*)_{\text{max}}}$, and shape parameter $\gamma = \alpha_{1-\xi^*} + 1$. 
Appendix: Two-State Heterogeneous Agent
Appendix: 3 states from Heterogeneous Agent Model
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value (by Variant)</th>
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<tbody>
<tr>
<td><strong>Panel A: Hansen (Indivisible Labor)</strong></td>
<td></td>
<td></td>
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<tr>
<td>Ext. Margin Labor supply disutility</td>
<td>$\bar{v}$</td>
<td>1.0</td>
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<tr>
<td>Potential earnings</td>
<td>$\bar{y}$</td>
<td>1.0</td>
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<tr>
<td>Marginal utility of consumption</td>
<td>$\bar{\lambda}$</td>
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<td><strong>Panel B: MaCurdy (Isolesticity)</strong></td>
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<tr>
<td>CRRA consumption parameter</td>
<td>$\sigma$</td>
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<td>Potential earnings</td>
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<td>Shape parameter of labor disutility dist.</td>
<td>$\alpha_v$</td>
<td>0.32, 2.50</td>
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<td>Max. labor disutility</td>
<td>$v_{\text{max}}$</td>
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<td><strong>Panel C: Heterogeneous Agent Model</strong></td>
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<td>Toy Model</td>
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<td>Transition probabilities</td>
<td>$[\gamma_1, \gamma_2] = [0.0797, 0.15]$</td>
<td>33-State process from HANK</td>
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<td>CRRA consumption parameter</td>
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<td>HANK Earnings Process</td>
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<td><strong>Panel D: Rogerson-Wallenius</strong></td>
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<td>Minimum hours</td>
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<td>Slope of labor productivity</td>
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<td>Intensive-margin Frisch elasticity</td>
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<td>Baseline</td>
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<td>Low-Frisch Variant</td>
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Appendix Figures

- Mean Log Res. Wedge vs Age
- Log Res. Wedge / Employment vs Log Years Education

Legend:
- NORC
- GSOEP
- PASS
- Employment
- Log Reservation Wedge
Appendix Figures