

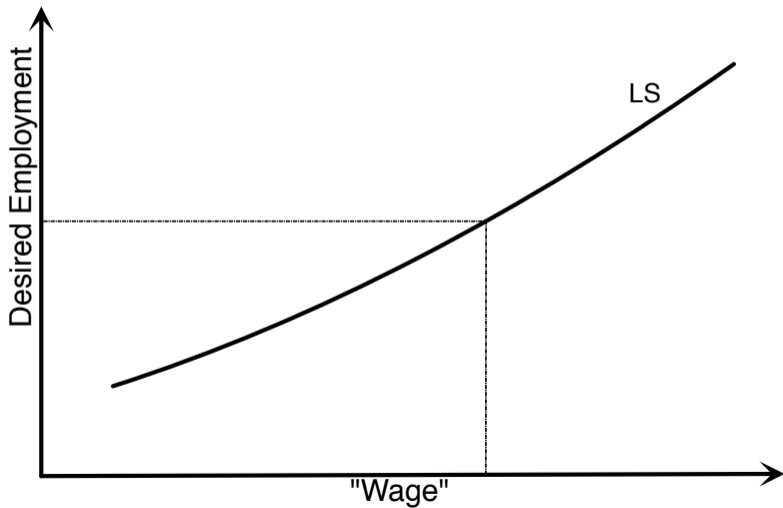
Reservation Raises:
The Aggregate Labor Supply Curve at the Extensive Margin

Preston Mui
UC Berkeley

Benjamin Schoefer
UC Berkeley

LSE Macro Seminar
May 2022

Extensive-Margin Aggregate Labor Supply Curve





Empirical regularity: extensive margin dominates in total-hours fluctuations $H_t = \sum_i h_{it} \cdot e_{it}$

Extensive Margin has Loose Ends

- Empirical elasticity/-ies: no consensus

Chetty, Guren, Manoli, Weber (2012), Rogerson, Keane (2012)

- (Some challenges in modeling practice too)

Either: Iso-elastic ("MaCurdy") case via representative HH " $\Rightarrow L_t^{1/\eta} = \lambda_t W_t$ "

Gali (2015)

Or: "Serious" ext-margin binary choices $e_{it} \in \{0, 1\}$, but at the cost of: Rogerson, Wallenius (2008); Chang, Kim (2006)

- No transparent and easy-to-calibrate ALSC (vs: " η ")
- Complexity (multi-dim. heterogeneity, "hidden calibrations",...)
- Harder to compare labor supply model blocks
- A specific model's labor-supply relevant features calibrated to/estimated off different empirical targets

Empirical Evidence on EM-ALSC

Structural estimation

Heckman and MaCurdy (1980); Chang and Kim (2007); Gourio and Noyal (2009); Blundell, Pistaferri, and Saporta-Eksten (2016); Chang and Kim (2006); Park (2017); Attanasio, Levell, Low, and Sañchez-Marcos (2018); Beffy, Blundell, Bozio, Laroque, and To (2019)

- **Parametric assumptions** on pref's, costs, wage distrib'n (incl. of the nonemployed), tastes

Quasi-experimental evidence—tax holidays

Bianchi, Gudmundsson, and Zoega, 2001; Chetty, Guren, Manoli, and Weber, 2012; Martinez, Saez, and Siegenthaler, 2018; Sigurdsson, 2018)

- **Specific arc** elasticities: large, positive net-of-tax wage changes
 - ⇒ Need not speak to business cycles (small wage changes), especially recessions ($w \downarrow$)
- “Net of frictions”, realized adjustment of employment
 - ⇒ Need not identify labor supply preferences (desired employment)

Survey evidence

Mas and Pallais, 2019; Kneip, Merz, and Storjohann, 2020; Ameriks, Briggs, Caplin, Lee, Shapiro, and Tonetti, 2020

- Selected samples—not representative (e.g., only unemployed seekers, older workers,...)

Our route:

- **Representative** survey of employment pref's ⇒ **nonparametric** estimate **global** EM-ALSC

- ① Propose **BASIC** framework for EM-ALSC in form of **reservation raises**:
 - Which percent increase (or decrease) in your potential wage would render you exactly indifferent between working or not (for a certain time interval)?
 - Related to reservation wages (but is a percent premium over your idiosyncratic potential wage)
- ② CDF of RRs *is* the EM-ALSC
- ③ **Custom reservation-raise surveys of U.S. and German populations**
 - Nonparametrically construct global US EM-ALSC
 - Large local elasticities of 3 and up
 - Non-constant, asymmetric arc-elasticities: smaller elasticities upwards
- ④ **Model meta-analysis**: recast in RR framework to uncover & make comparable ALSCs
 - No existing model provides tight global fit to the empirical curve [constructed in step 3]
- ⑤ **Macro implications of empirical EM-ALSC**
 - Calibrate one model curve globally to the empirical analogue
 - Business cycle accounting: labor wedge considerably less cyclical

Outline

- ① **Basic Framework**
- ② Leading Model Example: Frischian & Spot Labor Market
- ③ Measurement In Survey
- ④ Model Meta-Analysis

Basic Economics of Ext-Margin Labor Supply

- Ext-margin labor supply is a binary choice $e_{it} \in \{0, 1\}$
- Can formulate as **standard** reservation wage (w^r) rule:

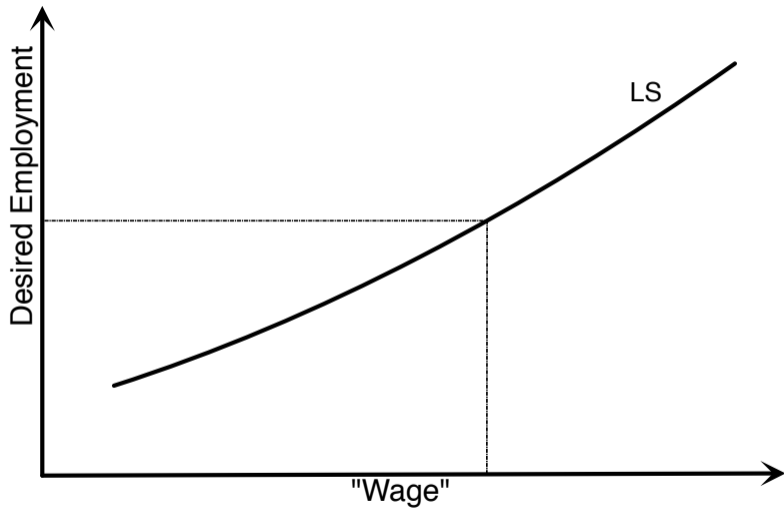
$$e_{it}^* = \mathbb{1}(w_{it} \geq w_{it}^r)$$

- Aggregate extensive-margin labor supply (desired employment rate):

$$\begin{aligned} E_t^* (\text{"Wage"}) &= \int_i e_{it}^* di \\ &= \int_i \mathbb{1}(w_{it} \geq w_{it}^r) di \\ &= \int_{w^r} \int_w \mathbb{1}(w \geq w^r) f(w|w^r) f(w^r) dw dw^r \end{aligned}$$

- Interior employment rate due to heterogeneity in either w_{it} only, w_{it}^r only, or **both**

Pitfalls?



- **Challenge I:** need to know *joint* distribution of w_{it} and w_{it}^r
- **Challenge II:** what is the "wage" argument of the ALC?

Pitfalls?

- Ext-margin labor supply is a binary choice $e_{it} \in \{0, 1\}$
- Can formulate as **standard** reservation wage (w^r) rule:

$$e_{it}^* = \mathbb{1}(w_{it} \geq w_{it}^r)$$

- Aggregate extensive-margin labor supply (desired employment rate):

$$\begin{aligned} E_t^* (\text{"Wage"}) &= \int_i e_{it}^* di \\ &= \int_i \mathbb{1}(w_{it} \geq w_{it}^r) di \\ &= \int_{w^r} \int_w \mathbb{1}(w_{it} \geq w_{it}^r) f(w|w^r) f(w^r) dw dw^r \end{aligned}$$

- Interior employment rate due to heterogeneity in either w_{it} only, w_{it}^r only, **or both**
- **Challenge I:** need to know *joint* distribution of w_{it} and w_{it}^r
- **Challenge II:** what is the "wage" argument of the ALC?

Extensive-Margin Labor Supply: Reservation Raises

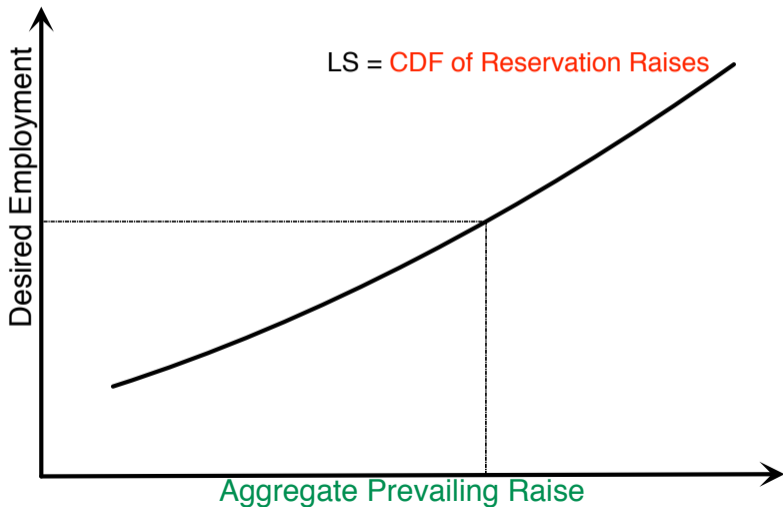
- Ext-margin labor supply is a binary choice $e_{it} \in \{0, 1\}$
- Individual i 's ext-margin labor supply follows **slightly augmented** reservation wage rule:

$$\begin{aligned}
 e_{it}^* &= \mathbb{1}\left(\overbrace{(1 + \Xi_t)}^{\text{Agg. Prevailing Raise}} w_{it} \geq w_{it}^r \right) \\
 &= \mathbb{1}\left(1 + \Xi_t \geq w_{it}^r / w_{it} \right) \\
 &= \mathbb{1}\left(1 + \Xi_t \geq \underbrace{1 + \xi_{it}^*}_{i\text{'s "Reservation Raise"}} \right)
 \end{aligned}$$

- RR combines idiosyncratic reservation wage with idiosyncratic potential wage into scalar sufficient statistic
- ✓ One-dimensional ranking of labor suppliers
- ✓ Aggregate labor supply curve takes as its argument **the aggregate raise**.

$$E_t^*(1 + \Xi_t, F_t) = \int \mathbb{1}(1 + \Xi_t \geq 1 + \xi^*) dF_t(1 + \xi^*) = \underbrace{F_t(1 + \Xi_t)}_{\text{CDF of RRs, evaluated at } 1 + \Xi_t}$$

Our Basic Route: EM ALSC in the Presence of 2-Dim Heterogeneity
in **Both** w and w^r



Properties

- **CDF** of the RR, $F_t(1 + \xi^*)$, fully characterizes the EM-ALSC
- **“Aggregate wage” concept:** shifts in $1 + \Xi_t$
 - Taxes, wage growth (e.g., marginal product shifts,...),...
- **(Desired) Employment Rate:**

$$E_t(1 + \Xi_t) = P(1 + \xi^* \leq 1 + \Xi_t) = F_t(1 + \Xi_t)$$

- **Marginal Individual:** $1 + \xi_{it}^* = 1 + \Xi_t$
- **Employment Adjustment:** Increase in aggregate raise from $(1 + \Xi_t)$ to $(1 + \Xi'_t)$:

$$dE_t = F_t(1 + \Xi'_t) - F_t(1 + \Xi_t)$$

- **Discrete Arc Elasticity:**

$$\frac{F_t(1 + \Xi'_t) - F_t(1 + \Xi_t)}{F_t(1 + \Xi_t)} \bigg/ \frac{(1 + \Xi'_t) - (1 + \Xi_t)}{1 + \Xi_t}$$

Infinitesimal:

$$\frac{(1 + \Xi_t)f_t(1 + \Xi_t)}{F_t(1 + \Xi_t)}$$

Outline

- ① Basic Framework
- ② **Leading Model Example: Frischian & Spot Labor Market**
 - Extensions – not today, see paper
- ③ Measurement In Survey
- ④ Model Meta-Analysis

General Version

- Ext-margin labor supply is a binary choice $e_{it} \in \{0, 1\}$
- Individual i 's ext-margin labor supply follows **slightly augmented** reservation wage rule:

$$\begin{aligned}
 e_{it}^* &= \mathbb{1}\left(\overbrace{(1 + \Xi_t)}^{\text{Agg. Prevailing Raise}} w_{it} \geq w_{it}^r \right) \\
 &= \mathbb{1}\left(1 + \Xi_t \geq w_{it}^r / w_{it} \right) \\
 &= \mathbb{1}\left(1 + \Xi_t \geq \underbrace{1 + \xi_{it}^*}_{i\text{'s "Reservation Raise"}} \right)
 \end{aligned}$$

- RR combines idiosyncratic reservation wage with idiosyncratic potential wage into scalar sufficient statistic
- ✓ One-dimensional ranking of labor suppliers
- ✓ Aggregate labor supply curve takes as its argument **the aggregate raise**.

$$E_t^*(1 + \Xi_t, F_t) = \int \mathbb{1}(1 + \Xi_t \geq 1 + \xi^*) dF_t(1 + \xi^*) = \underbrace{F_t(1 + \Xi_t)}_{\substack{\text{CDF of RRs,} \\ \text{evaluated at } 1 + \Xi_t}}$$

Leading Specific Model Class: Frischian & Spot Labor Supply

$$\max_{a_{it}, h_{it}, c_{it}} \mathbb{E}_t \sum_{s \geq t} \beta^{s-t} u_i(h_{is}, c_{is}) \quad \text{s.t.} \quad a_{is} + c_{is} \leq a_{i,s-1}(1 + r_{i,s-1}) + (1 + \Xi_s)y_{is}(h_{is}) + T_{is}(\cdot) \quad \forall s \geq t$$

- Prevailing aggregate labor income raise $1 + \Xi_t$
- Indivisible labor: $h_{it} \in \{0, \tilde{h}_{it}\}$ permit int-margin hours choice in paper
- Earnings $y_{it}(h_{it}) = w_{it}h_{it}$ (*potential* earnings $y_{it} = w_{it}\tilde{h}_{it}$)
- Labor disutility $v_{it} = u_i(c_{it}^{h=0, \lambda_{it}}, 0) - u_i(c_{it}^{h=\tilde{h}_{it}, \lambda_{it}}, \tilde{h}_{it})$

Labor supply is binary (employment) choice:

$$\Rightarrow h_{it}^* = \begin{cases} 0 & \text{if } v_{it} > (1 + \Xi_t)w_{it}\tilde{h}_{it}\lambda_{it} \\ \tilde{h}_{it} & \text{if } v_{it} \leq (1 + \Xi_t)w_{it}\tilde{h}_{it}\lambda_{it} \end{cases} \Leftrightarrow e_{it}^* = \begin{cases} 0 & \text{if } v_{it} > (1 + \Xi_t)y_{it}\lambda_{it} \\ 1 & \text{if } v_{it} \leq (1 + \Xi_t)y_{it}\lambda_{it} \end{cases}$$

Reservation raise $1 + \xi_{it}^*$: hypothetical $1 + \Xi_t$ rendering i indifferent between working and not:

$$v_{it} \stackrel{\square}{=} (1 + \xi_{it}^*)y_{it}\lambda_{it} \quad \Leftrightarrow \quad 1 + \xi_{it}^* = \frac{v_{it}}{y_{it}\lambda_{it}} = \frac{y_{it}^r}{y_{it}}$$

Sufficient statistic for individual i 's Frischian employment preferences:

$$\Rightarrow e_{it}^* = \begin{cases} 0 & \text{if } 1 + \Xi_t < 1 + \xi_{it}^* \\ 1 & \text{if } 1 + \Xi_t \geq 1 + \xi_{it}^* \end{cases}$$

Outline

- ① Basic Framework
- ② Leading Model Example: Frischian & Spot Labor Market
 - Extensions – not today, see paper
- ③ Measurement In Survey
- ④ Model Meta-Analysis

Measurement of the EM-ALSC

1 Our approach: survey to elicit preferences (reservation raise)

- Nonparametric & sufficient statistic

Caveat: Response quality

2 Quasi-experimental: response of *realized* employment to shifts in monetary benefit to working

- Canonical example: Icelandic tax holiday

Bianchi, Gudmundsson, Zoega (2001)

Caveat: Identify **one specific arc elasticity** of global EM-ALSC:

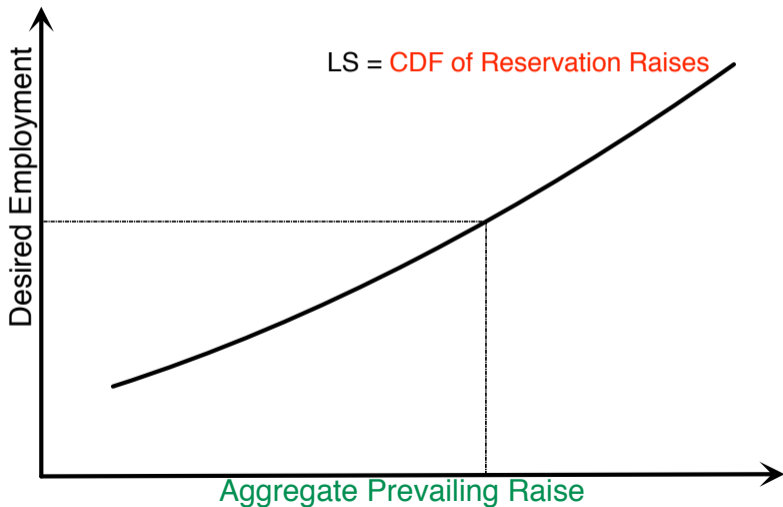
$$\epsilon_{E_t, (1+\Xi_t) \rightarrow (1+\Xi'_t)} = \frac{F_t(1+\Xi'_t) - F_t(1+\Xi_t)}{F(1+\Xi_t)} \bigg/ \frac{(1+\Xi'_t) - (1+\Xi_t)}{1+\Xi_t}$$

Caveat: Realized ("net of frictions") vs. desired labor supply

3 Structural estimation

Caveat: Functional form and parametric assumptions, relies on realized employment too

Our Basic Route: EM ALSC in the Presence of 2-Dim Heterogeneity
in **Both** w and w^r



Comparison of Surveys: Reservation **Raise** vs. Reservation **Wage**

Our attempt at a reservation **raise** survey

- Ask full cross section across all labor forces statuses (incl. out of labor force as well as the currently employed)
- Percent change compared to respondents' idiosyncratic potential earnings
- Frischian, neoclassical context

Vs. standard reservation **wage** surveys

- Ask unemployed job seekers
- Wage level only – no reference to potential earnings
- McCall, search-frictional context

Measurement of Reservation Raise

- **Ideal Measure** (Frischian temporary tax raise in a spot labor market):

You are currently [non-]employed. Suppose the following thought experiment: you (and only you) receive a temporary linear incremental tax [or subsidy] on your take-home earnings (at whichever hours or job you may choose to work). At what incremental tax [or subsidy] rate would you be indifferent between not working for this period and working (at whichever job would be your best choice at that given tax [subsidy] rate)?

- **Feedback from MTurk pilots:**
 - "The wording of the question was confusing"
 - "bizarre scenario"
 - ...

Custom Reservation Raise Survey

- **In practice:**
 - We translate this ideal questions into three variants, **routed by labor force status**.
 - Specify specific "Frischian" time horizon to "one month"
 - Piloting: evoke "job-constant" scenario
- Nationally representative U.S. survey of 2,000 respondents.
- Fielded by NORC (University of Chicago): AmeriSpeak Omnibus program
- March and April, 2019.
- Additional survey (painting very similar picture): we integrated similar questionnaire into fall 2019 wave of German Socio-Economic Panel

Survey Question: Employed

Suppose, for reasons unrelated to you, your employer offers you the following choice: Either you take unpaid time off from work for one month, or you stay in your job for that month and only receive a fraction of your regular salary. No matter what choice you take, after the month is over, your salary will return to normal.

In this hypothetical scenario, you cannot take an additional job to make up for the lost income during that month.

Assume this choice is real and you have to make it. At what point would the cut in your salary be just large enough that you would choose the unpaid month of time off over working for the month at that lower salary?

For example, an answer of 5% means that a 5% wage cut would be the point where you would choose to take unpaid time off for the month instead of working for 5% lower pay during that month. But if the wage cut was less than 5%, you would instead choose to work for that than take unpaid time off. Choose any percentage between 1% to 100%, where the cut wage cut is just large enough that you would prefer to not work at all for no pay than work at reduced pay for that month.

Survey Question: Out of Labor Force

Think of the range of jobs that you would realistically be offered if you searched for jobs (even if you currently are not looking for a job and may not accept any of these potential jobs).

Suppose you had such job offers in hand. Currently you would likely not take such jobs, at least not at the usual salary. However, suppose the employer were nevertheless trying hard to recruit you, specifically by offering an additional sign-up bonus. The requirement to receive the bonus is that you will work for at least one month. The bonus comes as a raise of the first month's salary. This sign-up bonus will only be paid in the first month (on top of the regular salary that month), afterwards the salary returns to the regular salary.

...

5% means you would take the job if your employer paid a bonus of just 5% of the regular salary in the first month. 100% means you would require a bonus as large as the regular salary. 500% would mean you require a bonus equal to five times as large as the regular salary.

Survey Question: Unemployed

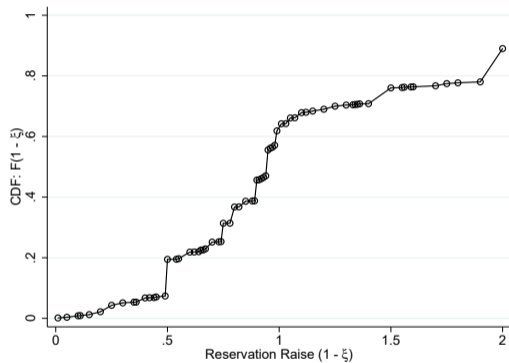
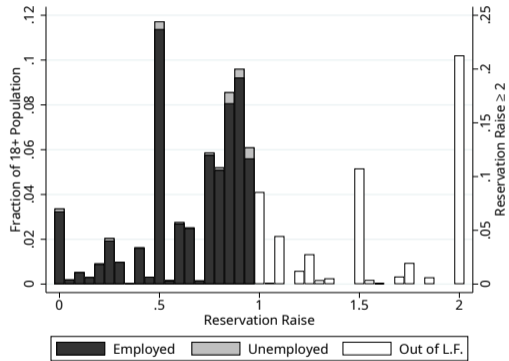
Suppose you have found the kind of job you are looking for and the employer would like to hire you. The regular start date for the job is one month away. As an alternative, your employer offers you the option to start working immediately, rather than waiting a month.

*However, if you chose to start work immediately, for that first month, you will **only receive a fraction of the regular salary**. The job is otherwise exactly the same. No matter what choice you take, after the month is over, the salary will then resume at the regular salary. In this hypothetical scenario, you cannot take an additional job to make up for the lost income during that month.*

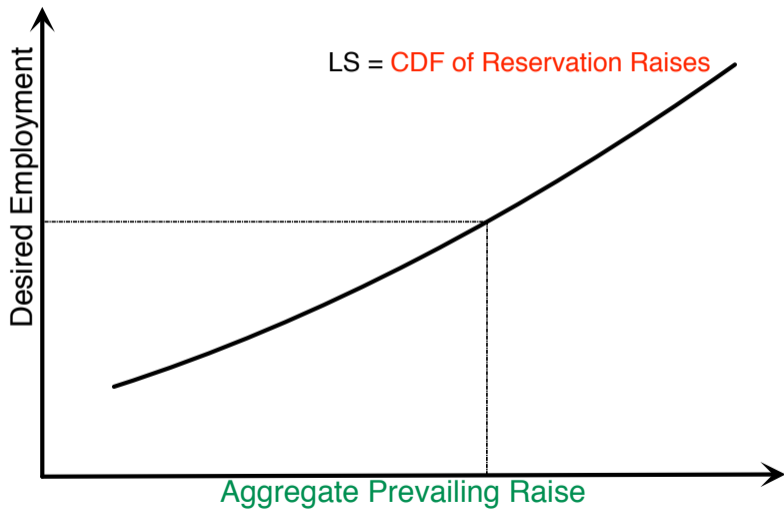
.... At what point would the cut in your salary be just large enough that you would choose the waiting a month without working and without the salary over starting the job immediately for the first month at that lower salary?

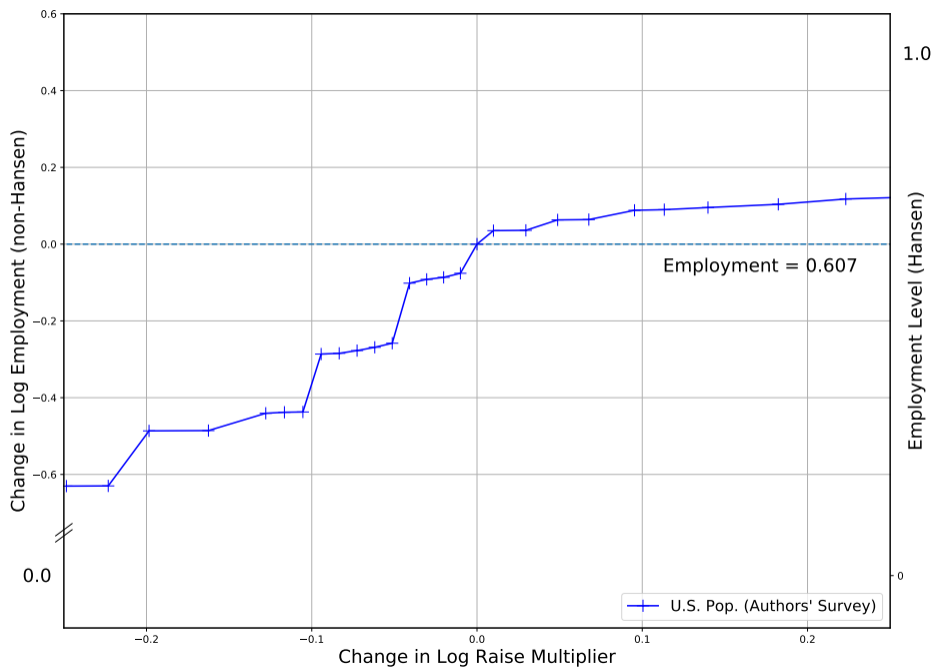
For example, an answer of 5% means that a 5% wage cut...

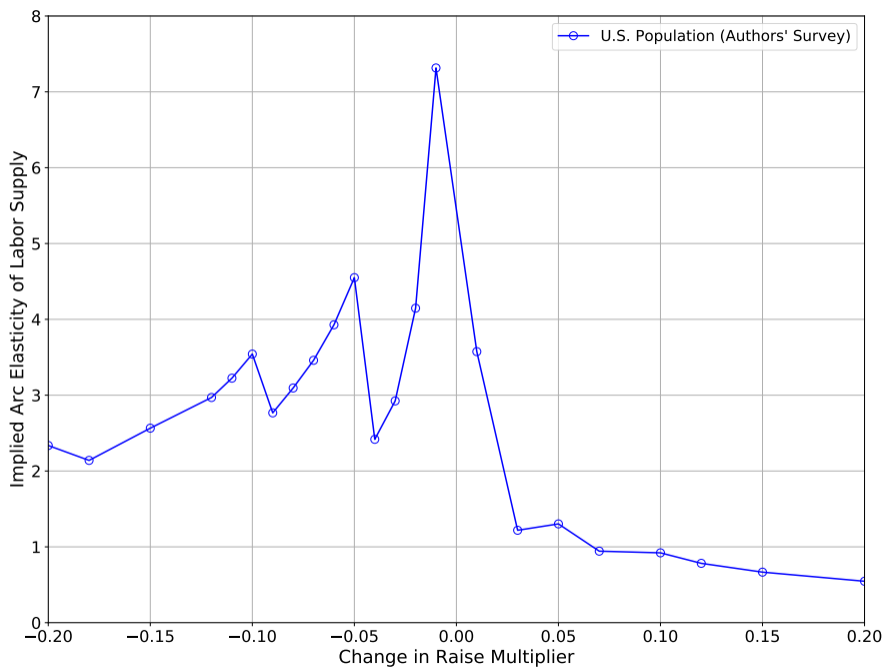
The Empirical Reservation Raise Distribution: US



EM-ALSC



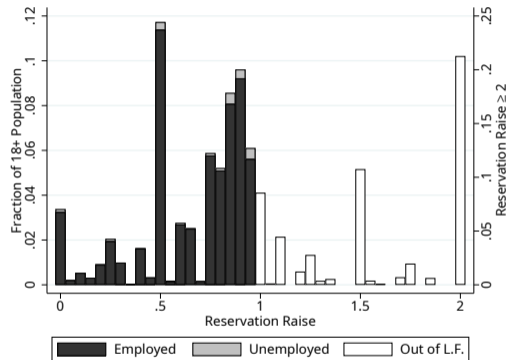




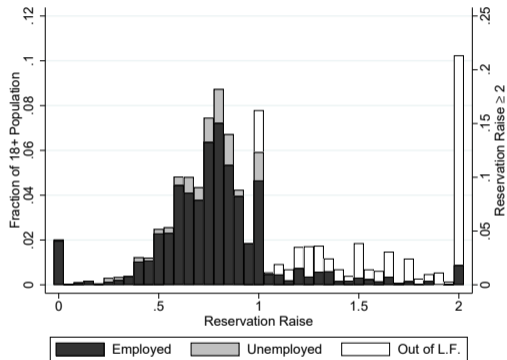
Second Survey: GSOEP

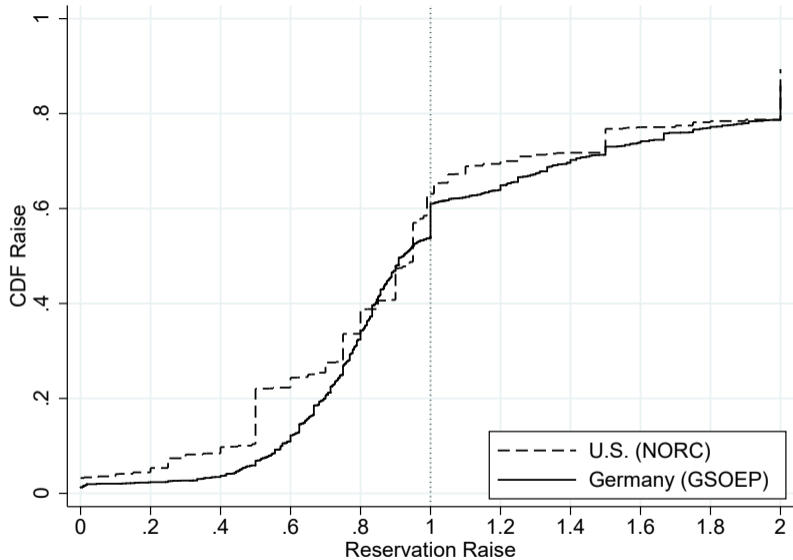
- Integrated survey into German Socioeconomic Panel, was fielded in the fall of 2019, became available in 2020
- Maximally representative
- Elicit one-stop raise vs. separately the reservation wage and the potential wage (and we will take the ratio)
- Two treatments: with and without “vacation” scenario for the employment and unemployed
- Ask for levels of potential and reservation wage separately

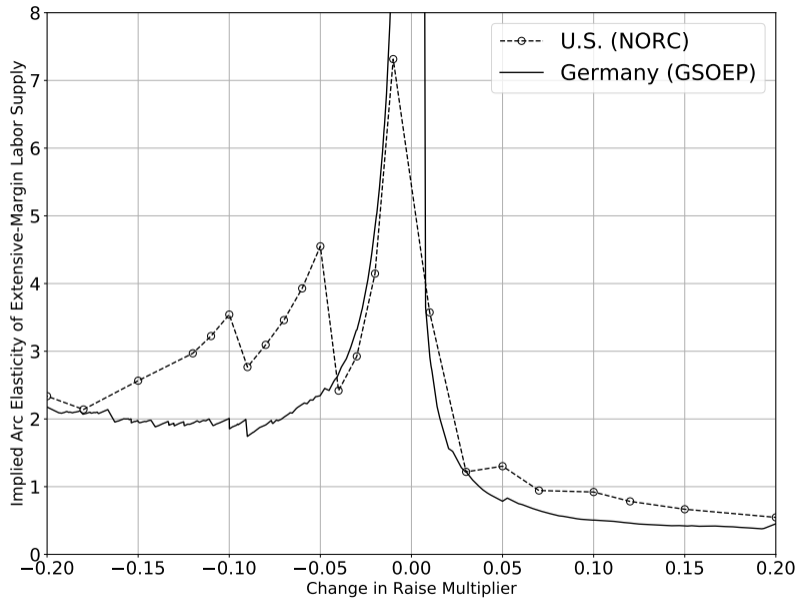
US (NORC)



New: Germany (GSOEP)





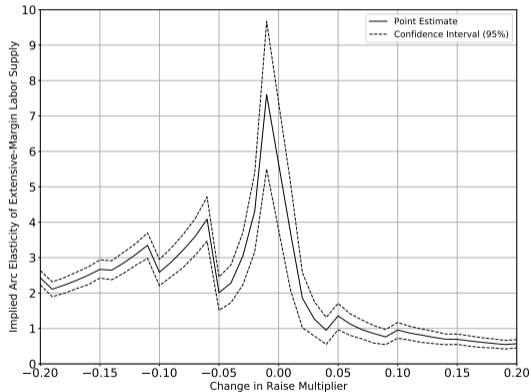


Robustness and Extensions

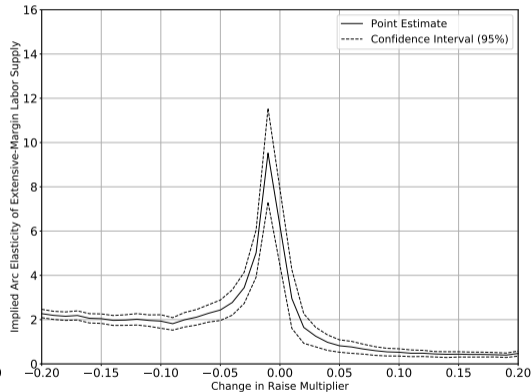
- Confidence intervals around elasticities (bootstrapped)
- Adjustment frictions
- Allocative consequences? Validation
- Correlates

Bootstrapped Confidence Intervals of Arc Elasticities

NORC



GSOEP



Robustness and Extensions

- Confidence intervals around elasticities (bootstrapped)
- **Adjustment frictions**
- Allocative consequences? Validation
- Correlates

Robustness to Frictions etc.: “Non-Vacation” Treatment

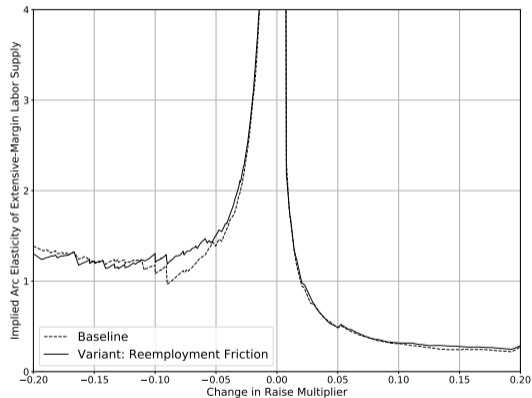
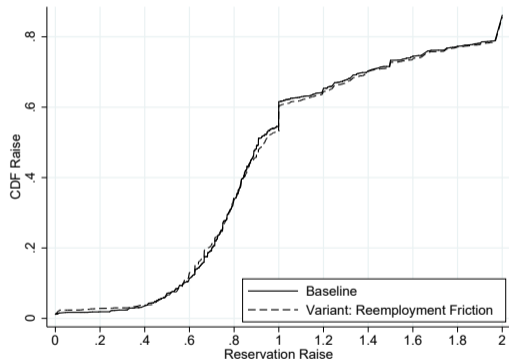
Baseline in NORC (and GSOEP):

Suppose, for reasons unrelated to you, your employer offers you the following choice: Either you take unpaid time off from work for one month, or you stay in your job for that month and only receive a fraction of your regular salary. No matter what choice you take, after the month is over, your salary will return to normal. ...

“Non-vacation” treatment:

- Instead: the salary falls for that month, and we do not guarantee “return option” (“vacation” scenario), but point to potentially required job separation (quit)
- Same for unemployed.
- Randomize (un-)employed into either vacation treatment or not (in GSOEP)—50/50.

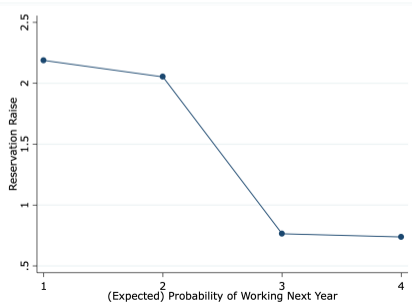
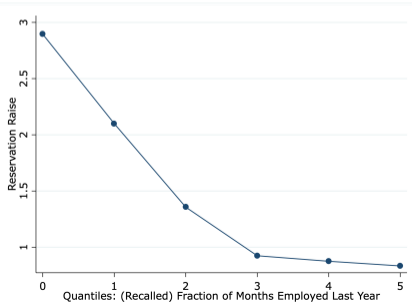
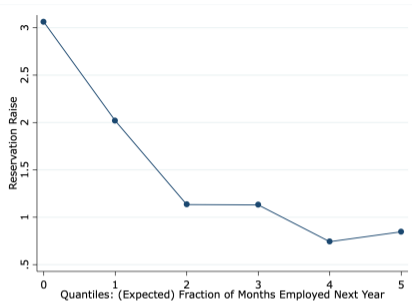
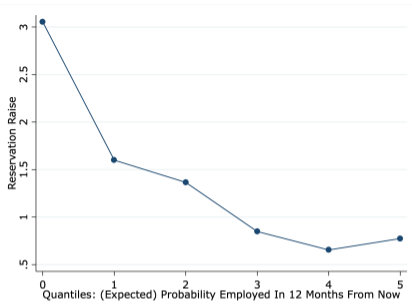
Robustness to Adjustment Frictions



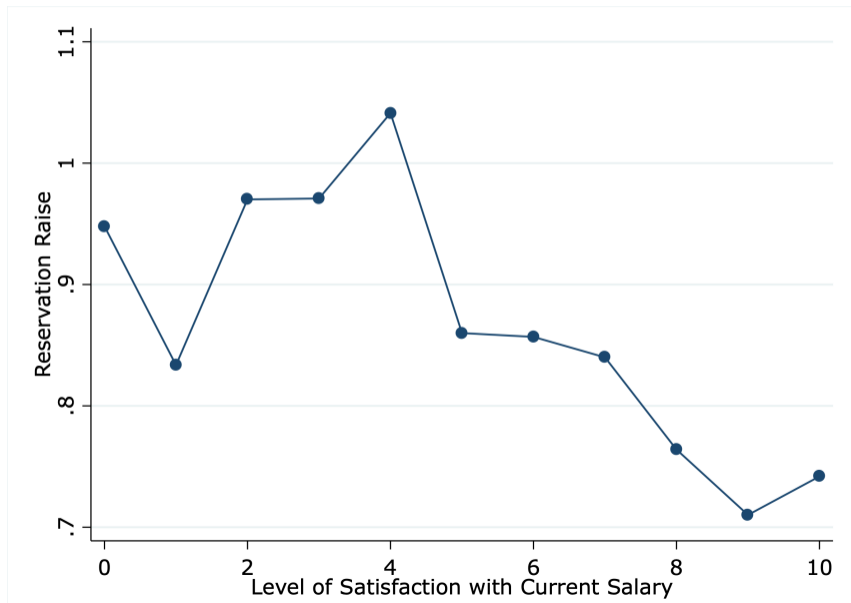
Robustness and Extensions

- Confidence intervals around elasticities (bootstrapped)
- Adjustment frictions
- Allocative consequences? Validation
- Correlates

Validation: Accordance with Respondents' "Subjective" Turnover



Validation: Satisfaction with Salary (GSOEP)



Validation: Macro Evidence

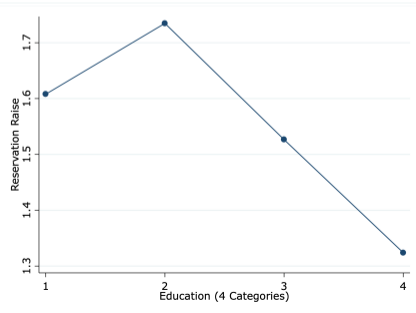
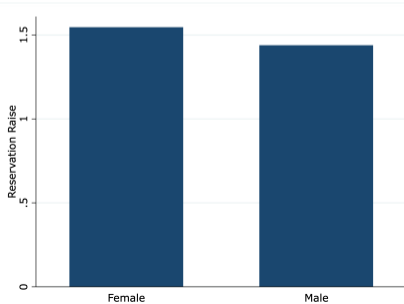
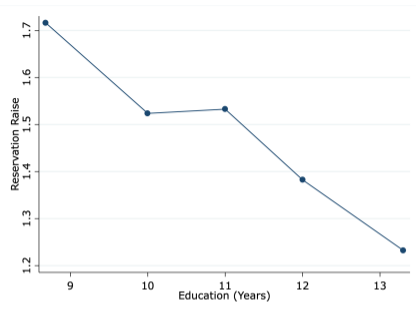
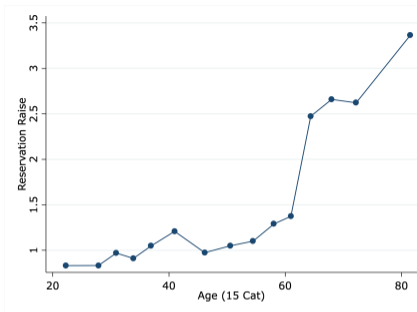
Goal:

- Check whether realized employment outcomes are related to reservation raises

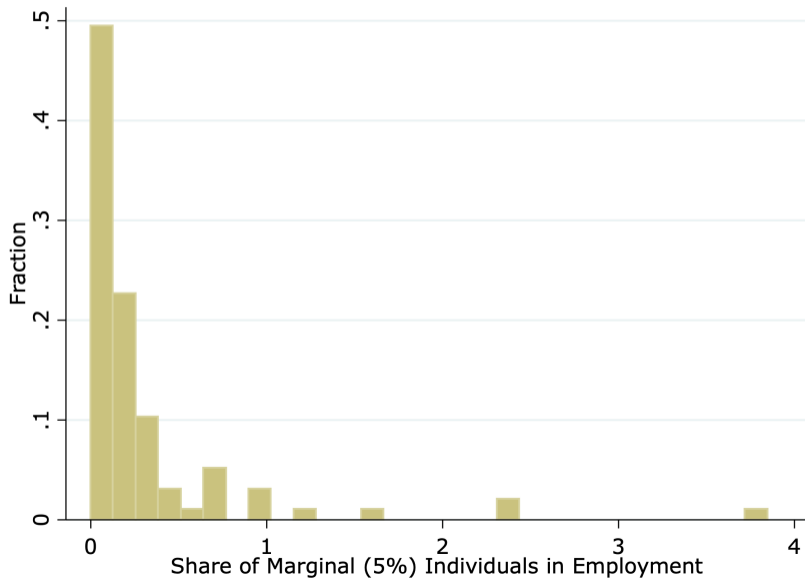
Strategy:

- Split up NORC sample into “Mincer” cells (education, age, gender)
- Compute “share of marginal agents” in this cell
 - Respondents with $|\xi_{it}^*| \leq x$, where $x = 0.01, \dots, 0.1$
- Check, in CPS, realized turnover and employment fluctuations by cell

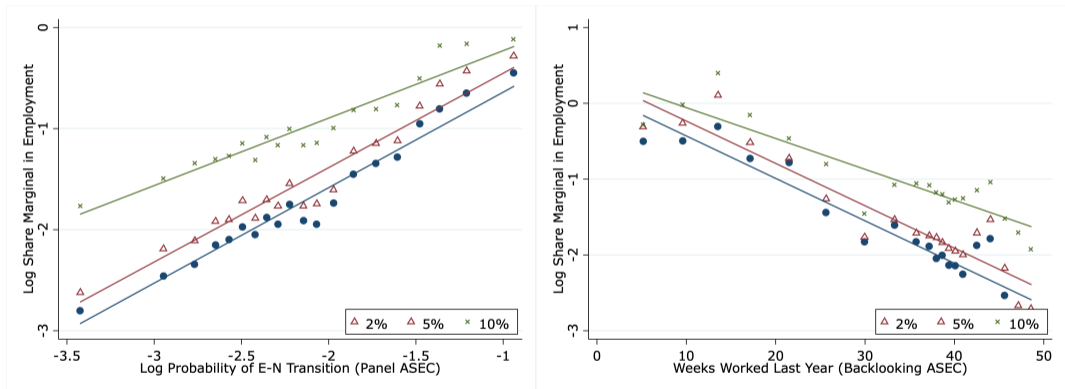
Correlates: Mincer Variables



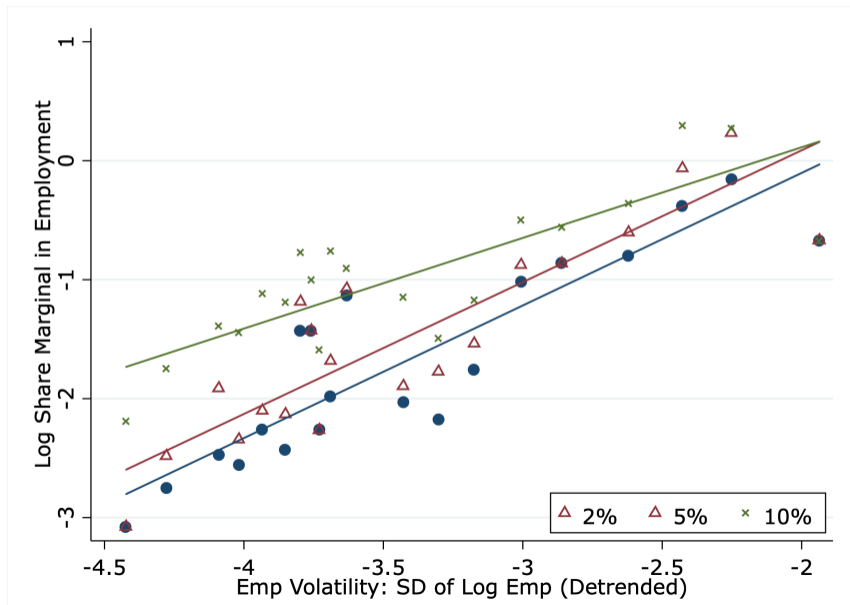
Variation in Share Marginal Across Mincer Cells ($N = 113$, Mean and Median ~ 15)



Validation: Realized Turnover and Share Marginal

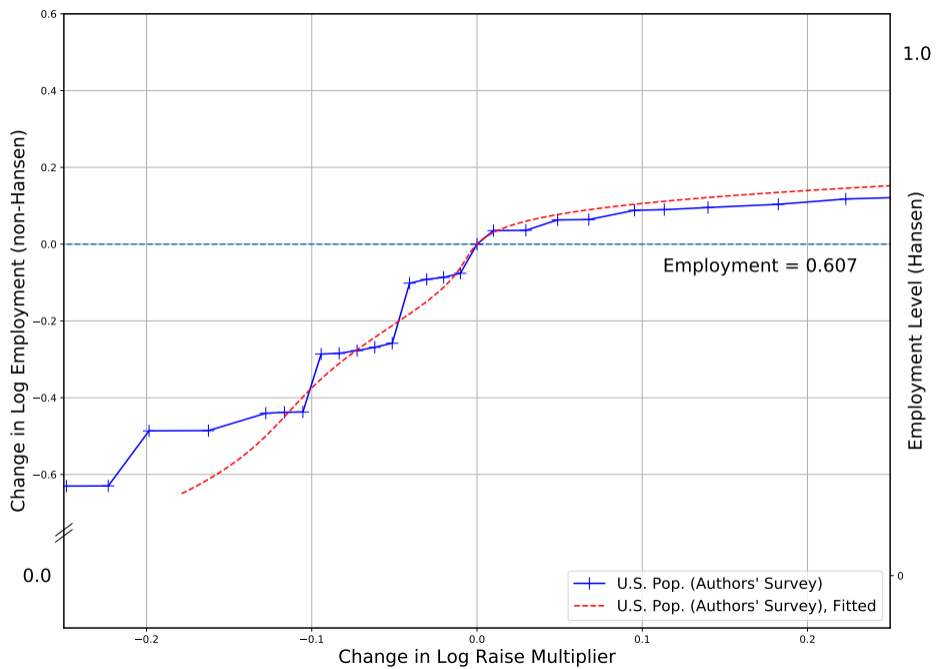


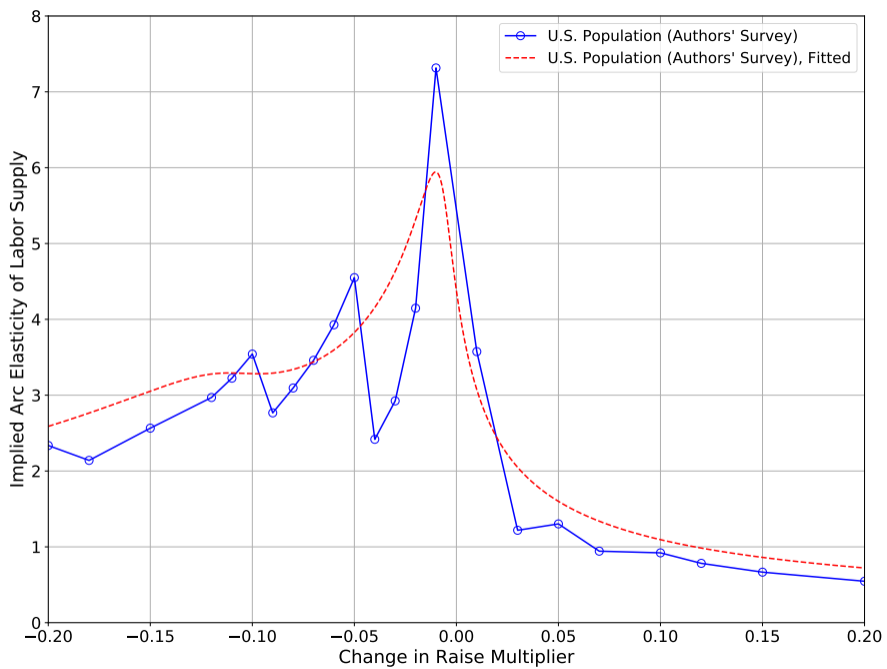
Validation: Employment Fluctuations and Share Marginal



Outline

- ① **Basic Framework**
- ② **Leading Model Example: Frischian & Spot Labor Market**
 - Extensions – not today, see paper
- ③ **Measurement In Survey**
- ④ **Model Meta-Analysis** – much more in paper (and appendix)





Leading Model Example: Frischian & Spot Labor Supply

$$\max_{a_{it}, h_{it}, c_{it}} \mathbb{E}_t \sum_{s \geq t} \beta^{s-t} u_i(h_{is}, c_{is}) \quad \text{s.t.} \quad a_{is} + c_{is} \leq a_{i,s-1}(1 + r_{i,s-1}) + (1 + \Xi_s)y_{is}(h_{is}) + T_{is}(\cdot) \quad \forall s \geq t$$

- o Prevailing aggregate labor income raise $1 + \Xi_t$
- o Indivisible labor: $h_{it} \in \{0, \tilde{h}_{it}\}$ permit int-margin hours choice in paper
- o Earnings $y_{it}(h_{it}) = w_{it} h_{it}$ (*potential* earnings $y_{it} = w_{it} \tilde{h}_{it}$)
- o Labor disutility $v_{it} = u_i(c_{it}^{h=0, \lambda_{it}}, 0) - u_i(c_{it}^{h=\tilde{h}_{it}, \lambda_{it}}, \tilde{h}_{it})$

Labor supply is binary (employment) choice:

$$\Rightarrow h_{it}^* = \begin{cases} 0 & \text{if } v_{it} > (1 + \Xi_t)w_{it}\tilde{h}_{it}\lambda_{it} \\ \tilde{h}_{it} & \text{if } v_{it} \leq (1 + \Xi_t)w_{it}\tilde{h}_{it}\lambda_{it} \end{cases} \Leftrightarrow e_{it}^* = \begin{cases} 0 & \text{if } v_{it} > (1 + \Xi_t)y_{it}\lambda_{it} \\ 1 & \text{if } v_{it} \leq (1 + \Xi_t)y_{it}\lambda_{it} \end{cases}$$

Reservation raise $1 + \xi_{it}^*$: hypothetical $1 + \Xi_t$ rendering i indifferent between working and not:

$$v_{it} \stackrel{\square}{=} (1 + \xi_{it}^*)y_{it}\lambda_{it} \quad \Leftrightarrow \quad 1 + \xi_{it}^* = \frac{v_{it}}{y_{it}\lambda_{it}} = \frac{y_{it}^r}{y_{it}}$$

Sufficient statistic for individual i 's Frischian employment preferences:

$$\Rightarrow e_{it}^* = \begin{cases} 0 & \text{if } 1 + \Xi_t < 1 + \xi_{it}^* \\ 1 & \text{if } 1 + \Xi_t \geq 1 + \xi_{it}^* \end{cases}$$

Model Meta-Analysis: EM-ALSCs as RRs

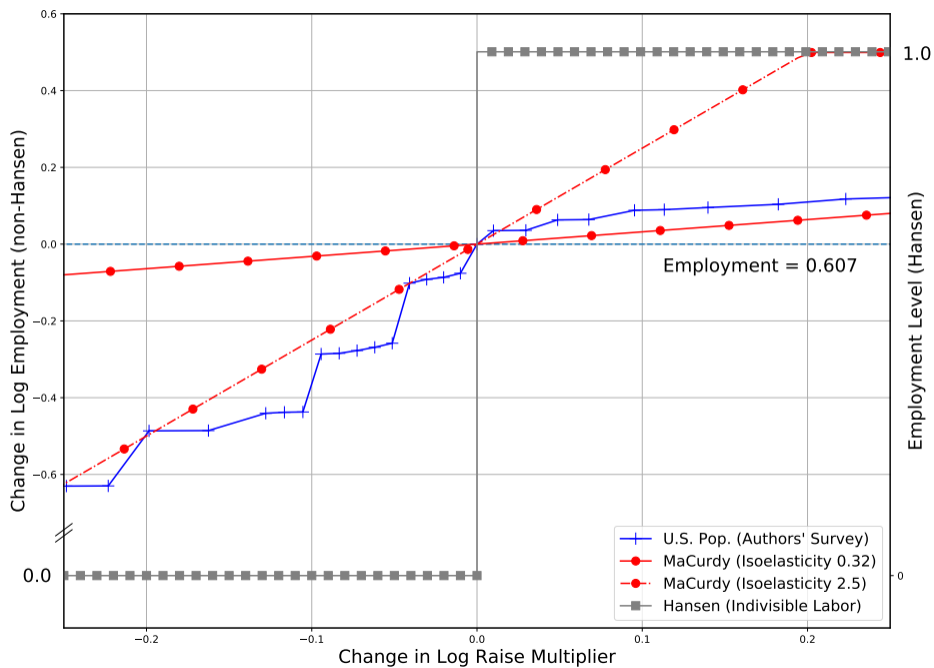
- Make tangible \Rightarrow recast respective detailed model into RR framework
- Make comparable \Rightarrow unifying bridge across models

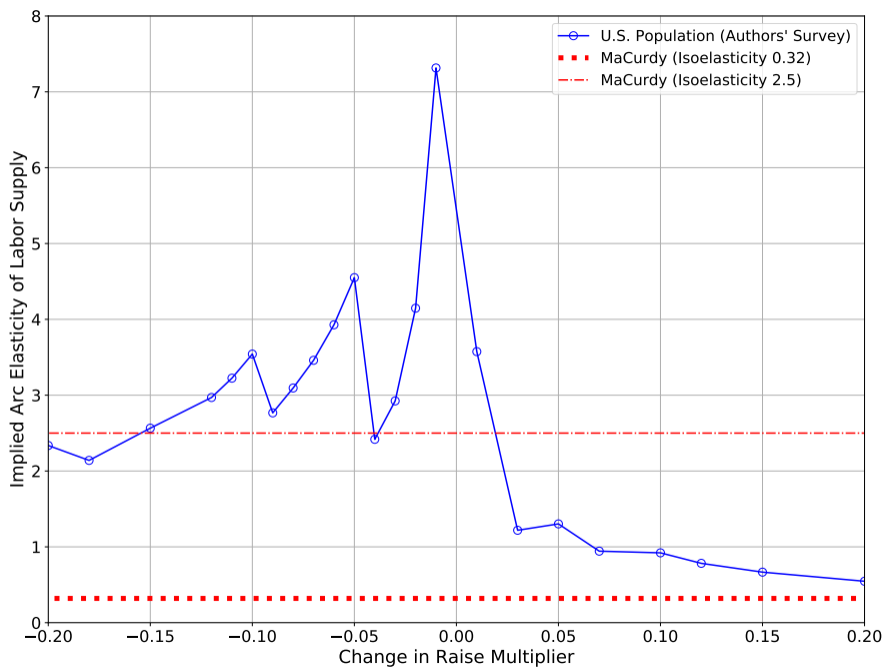
Three-step "recipe" for each model:

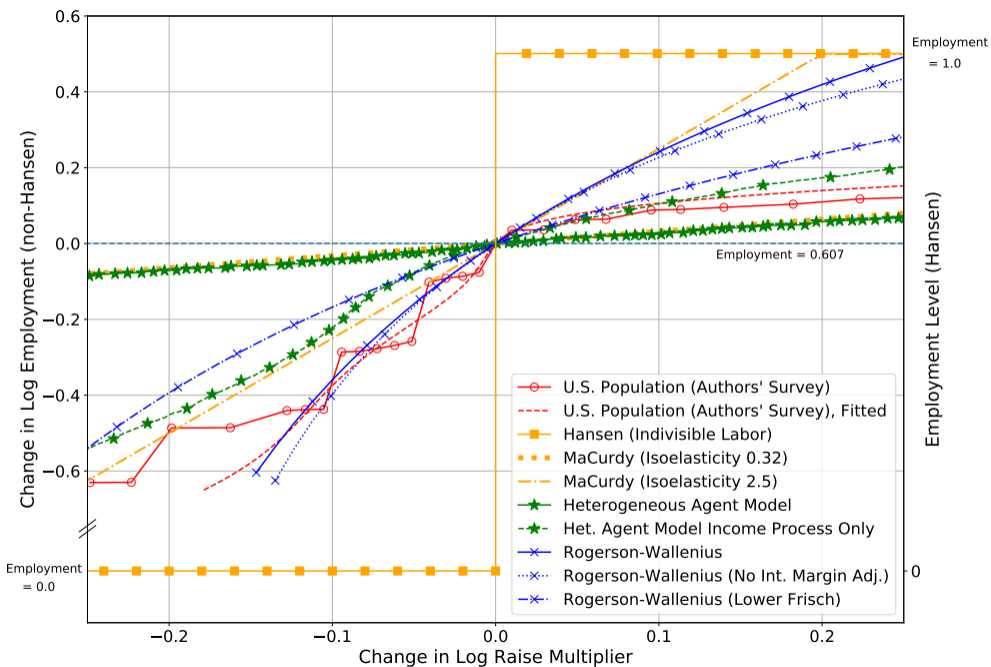
- ① Define individual-level RR $1 + \xi_{it}^*$
- ② Construct ALSC from CDF $F(1 + \xi_{it}^*)$ (reflecting joint equilibrium distribution of all RR-relevant factors)
- ③ Study properties – e.g. (local) elasticity

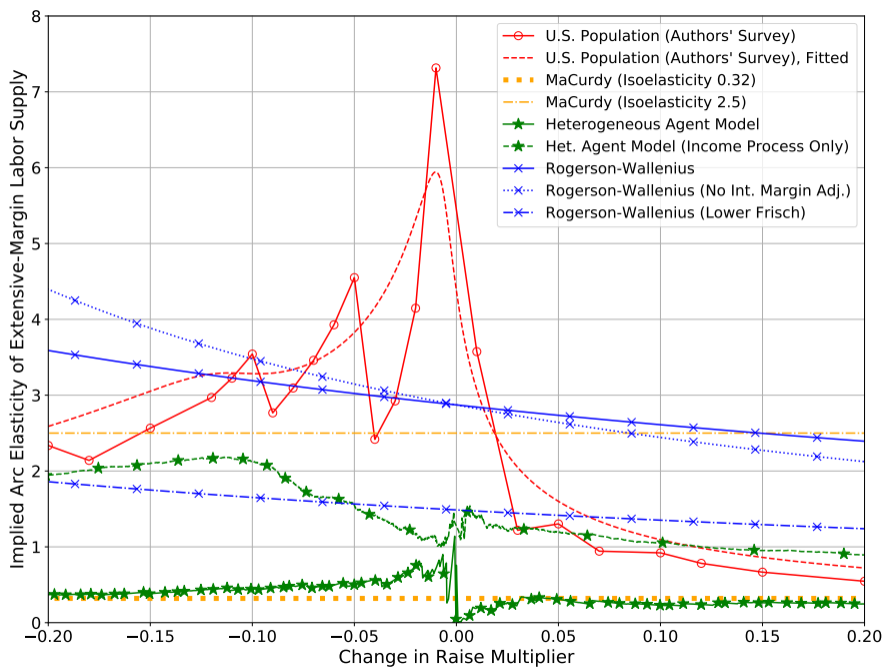
Models:

- ① Representative “command” household with consumption insurance
- ② Heterogeneous agent models with extensive margin
- ③ Lifecycle, intensive margins, and nonconvexities (Rogerson and Wallenius, 2008)

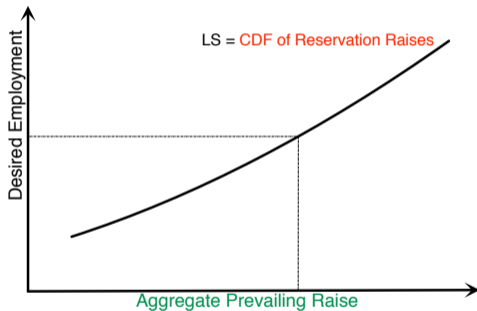
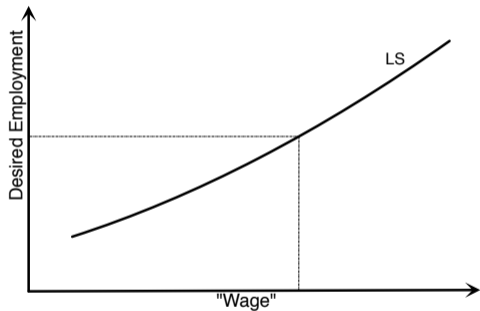








Summary: Reservation Raises and Aggregate Labor Supply



Summary: Reservation Raises and Aggregate Labor Supply

① Put to use **BASIC** framework for EM-ALSC

$1 + \Xi_t$: **Prevailing aggregate raise**: linear, homogeneous shifter of labor income

$1 + \xi_{it}^*$: **Micro reservation raise**: hypothetical $1 + \Xi_t$ rendering i indifferent b/w working and not:

$F_t(1 + \xi^*)$: **Aggregation** of individual RR's \Rightarrow short-run EM-ALSC:

$$E_t = P(1 + \xi_{it}^* \leq 1 + \Xi_t) = F_t(1 + \Xi_t)$$

② Custom survey of U.S. + German **pop'ns**: directly measure RR distribution (EM-ALSC)

- Large local elasticities of 3 and up
- Non-constant, asymmetric arc elasticities: smaller arc elasticities upwards

③ Model meta-analysis: recast in RR framework to uncover & make comparable ALSCs

- No existing model provides good global fit to empirical curve

④ Macro implications of empirical EM-ALSC used as calibration target

- Fit one model's $F_t(1 + \xi^*)$ tightly to the empirical analogue
- BCA labor wedge considerably less cyclical

Rep HH w/ Insurance and "Command" Labor Supply

$$\begin{aligned} & \max_{\{c_{it}, e_{it}\}_i, A_t} \mathbb{E}_t \sum_{s \geq t} \beta^{s-t} \int_0^1 [u_i(c_{is}) - e_{is} v_{is}] di \\ \text{s.t. } & A_s + \int_0^1 c_{is} di \leq A_{s-1}(1 + r_{s-1}) + \int_0^1 (1 + \Xi_s) y_{is} e_{is} di + T_s \quad \forall s \geq t \end{aligned}$$

- o Pooled budget constraint & full "insurance" $\Rightarrow \bar{\lambda}_t = \frac{\partial u_i(c_{it})}{\partial c_{it}} \quad \forall i$

1 Define micro RR:

$$1 + \xi_{it}^* = \frac{v_{it}}{\bar{\lambda}_t y_{it}}$$

2 CDF i.e. EM-ALSC:

$$\begin{aligned} E_t &= F_t(1 + \Xi_t) = P(1 + \xi_{it} \leq 1 + \Xi) = P\left(\frac{v_{it}}{y_{it} \bar{\lambda}_t} \leq 1 + \Xi_t\right) = P\left(\frac{v_{it}}{y_{it}} \leq (1 + \Xi_t) \bar{\lambda}_t\right) \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbb{1}\left[\frac{v}{y} \leq (1 + \Xi_t) \bar{\lambda}_t\right] dG(v, y) \Rightarrow \text{Properties of EM-ALSC follow } G(v, y) \end{aligned}$$

Rep HH w/ Insurance and "Command" Labor Supply

- o Pooled budget constraint & full "insurance" $\Rightarrow \bar{\lambda}_t = \frac{\partial u_i(c_{it})}{\partial c_{it}} \quad \forall i$

1 Micro RR:

$$1 + \xi_{it}^* = \frac{v_{it}}{\bar{\lambda}_t y_{it}}$$

2 CDF i.e. EM-ALSC... depends on $G(v, y)$

3 Same for its properties (arc elasticities)

RHH Example: Heterogeneity in Disutility Only

- Heterogeneity in v only: $v_{it} \sim G^v(v)$

- 1 Micro RRs characterized by disutility type $v(i)$:

$$1 + \xi_{it}^* = \frac{v_{it}}{\bar{y}_t \bar{\lambda}_t} = 1 - \xi_{vt}^*$$

- 2 Raise distribution – and hence CDF and ALSC – inherits shape of v_{it} -distr'n:

$$E_t = F_t(1 + \Xi_t) = P(1 + \xi_{it}^* \leq 1 - \Xi_t) = P\left(v_{it} \leq \frac{1 + \Xi_t}{\bar{y}_t \bar{\lambda}_t}\right) = G^v\left(\frac{1 + \Xi_t}{\bar{y}_t \bar{\lambda}_t}\right)$$

- 3 Elasticity (local at $1 + \Xi_t$) is given by $\left[(1 + \Xi_t) g^v\left(\frac{1 + \Xi_t}{\bar{y}_t \bar{\lambda}_t}\right) \right] / \left[1 - G^v\left(\frac{1 + \Xi_t}{\bar{y}_t \bar{\lambda}_t}\right) \right]$

Rep HH Specific Example 1: Hansen (1985); Rogerson (1988)

- $G(v, w)$ for homogeneous households:

$$y_{it} = \bar{y}_t \quad \forall i$$
$$v_{it} = \bar{v} = A \ln(1 - h_{it}) \quad \forall i$$

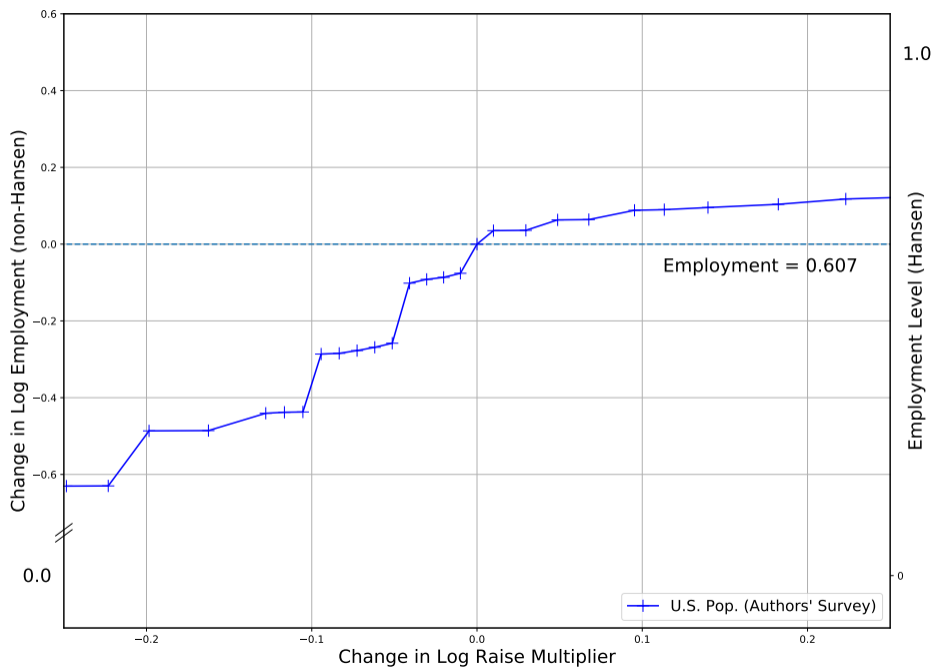
- 1 Homogeneous micro RRs:

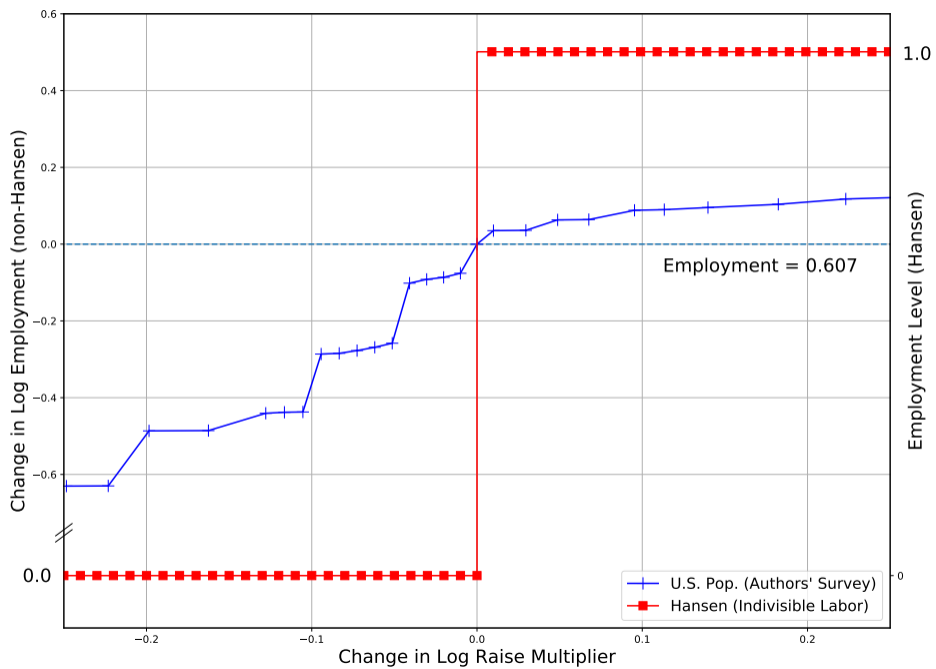
$$1 + \xi_{it}^* = 1 + \bar{\xi}_t^* = \frac{\bar{v}}{\lambda_t \bar{y}_t}$$

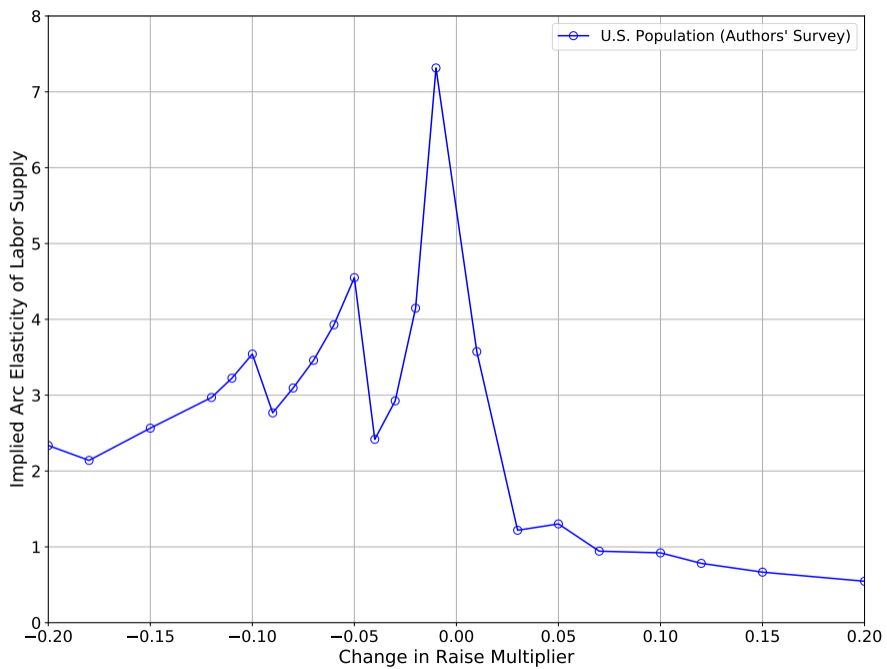
- 2 Degenerate raise distribution

$$F_t(1 + \Xi_t) = \begin{cases} 0 & \text{if } 1 + \Xi_t < \frac{\bar{v}}{\lambda_t \bar{y}_t} \\ 1 & \text{if } 1 + \Xi_t > \frac{\bar{v}}{\lambda_t \bar{y}_t} \end{cases}$$

- 3 Infinite Frisch elasticity





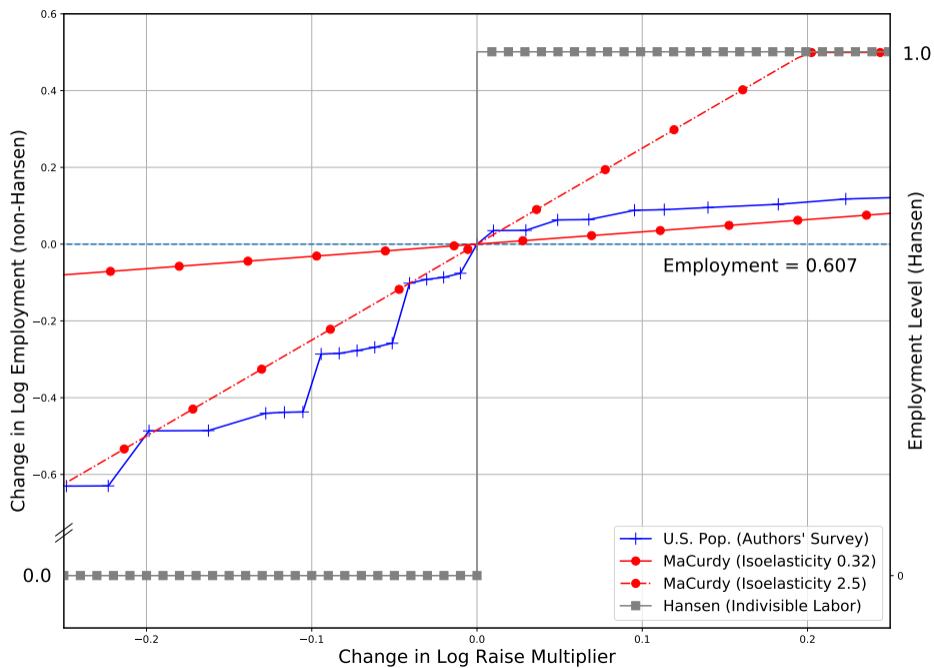


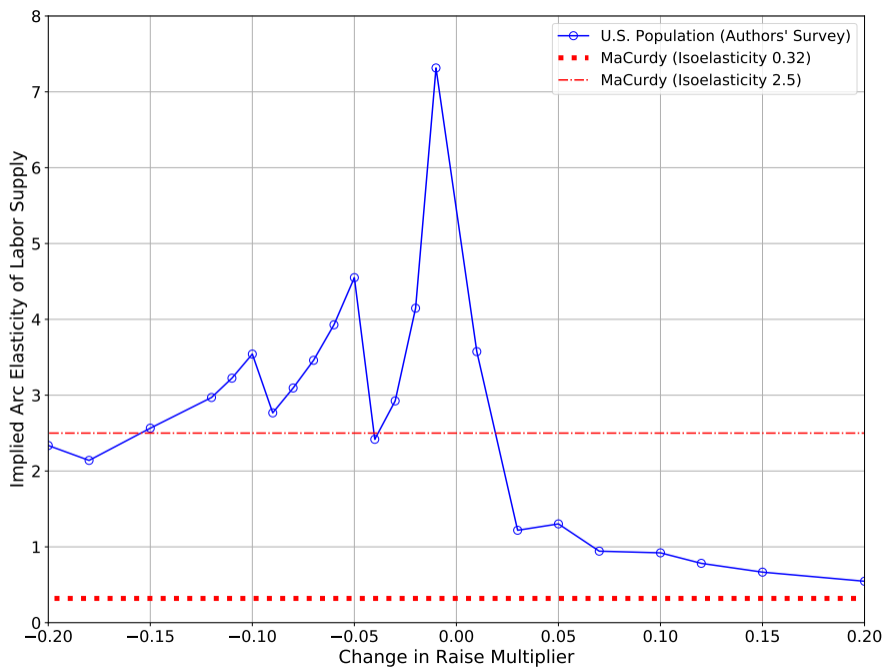
RHH Example: Constant Elasticity

Ex: If v follows power distribution, \Rightarrow iso-elasticity ("MaCurdy", Gali):

$$\epsilon_{E_t, 1+\Xi_t} = \frac{(1 + \Xi_t)F_t(1 + \Xi_t)}{F_t(1 + \Xi_t)} = \frac{(1 + \Xi_t)\alpha_v(1 + \Xi_t)^{-1} \left(\frac{(1+\Xi_t)\bar{y}_t\bar{\lambda}_t}{v_{\max}} \right)^{\alpha_v}}{\left((1 + \Xi_t)\bar{y}_t\bar{\lambda}_t/v_{\max} \right)^{\alpha_v}} = \boxed{\alpha_v}$$

- General conditions on RR dist in paper





Model Meta-Analysis: EM-ALSCs as RRs

- Make tangible \Rightarrow recast respective detailed model into RR framework
- Make comparable \Rightarrow unifying bridge across models

Three-step "recipe" for each model:

- ① Define individual-level RR $1 + \xi_{it}^*$
- ② Construct ALSC from CDF $F(1 + \xi_{it}^*)$ (reflecting joint equilibrium distribution of all RR-relevant factors)
- ③ Study properties – e.g. (local) elasticity

Models:

- ① Representative “command” household with consumption insurance
- ② Heterogeneous agent models with extensive margin
- ③ Lifecycle, intensive margins, and nonconvexities (Rogerson and Wallenius, 2008)

Heterogeneous Agent Models

- Huggett (1993) model (one asset plus borrowing constraint) + extensive margin
 - Stochastic potential earnings (productivity)
 - Incomplete markets \Rightarrow imperfect insurance

$$\begin{aligned} & \max_{c_{it}, e_{it} \in \{0,1\}, a_{it}} \mathbb{E}_t \sum_{s \geq t} \beta^{s-t} \left[\frac{c_{is}^{1-\sigma}}{1-\sigma} - \bar{v} e_{is} \right] \\ \text{s.t. } & a_{i,s+1} = (1 + \bar{\Xi}_s) y_{is} e_{is} + (1 + r_s) a_{is} - c_{is} \quad \forall s \geq t \\ & a_{is} \geq \underline{a} \quad \forall s \geq t \end{aligned}$$

Heterogeneous Agent Models

- No insurance \implies heterogeneity in λ_{it}

1 Micro RR (can be indexed by assets a and potential earnings/productivity y)

$$1 + \xi_{ay}^* = \frac{\bar{v}}{\lambda_{ay}y}$$

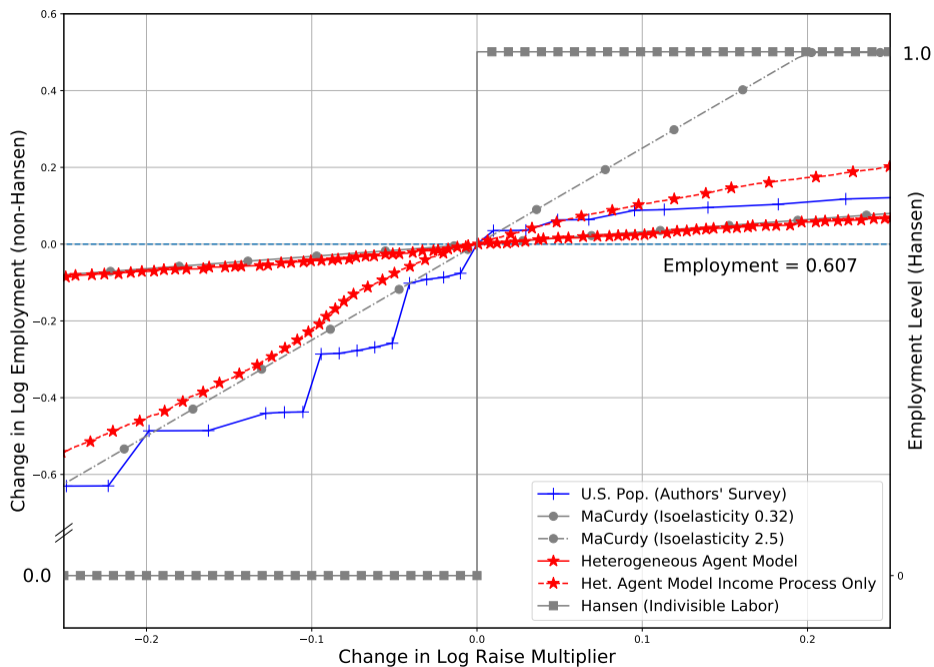
2 CDF i.e. EM-ALSC:

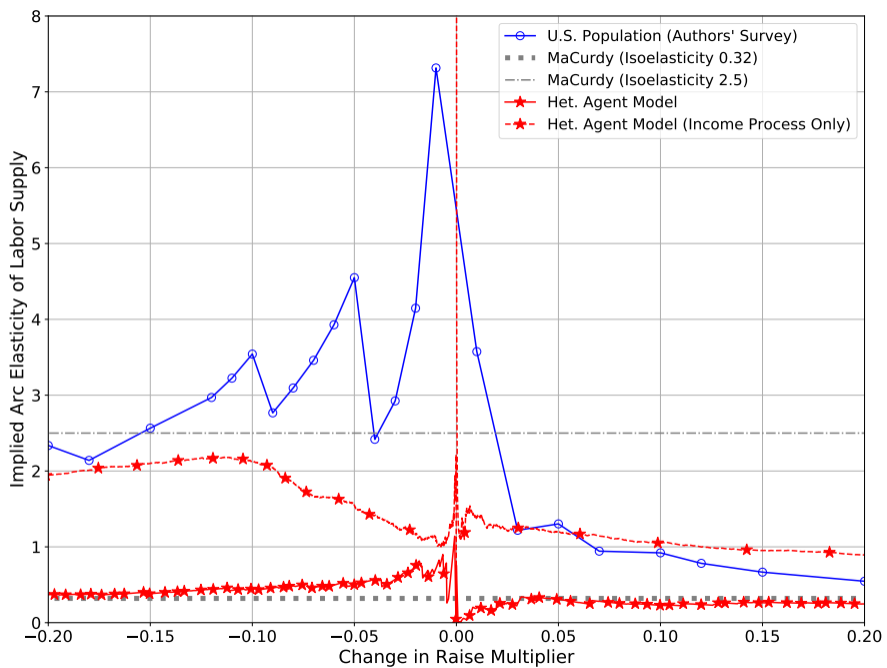
$$E_t = F_t(1 + \Xi_t) = \sum_y \int_{\underline{a}}^{\infty} \mathbb{1}[1 + \xi_{a,y}^* \leq 1 + \Xi_t] \underbrace{g(a, y)} da$$

Complicated object given by earnings process and consumption/savings decisions!

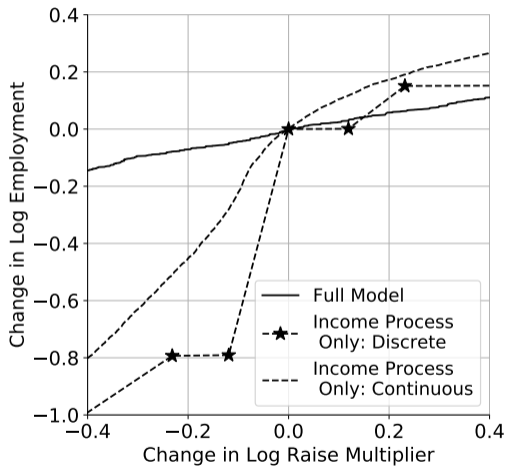
3 Elasticities etc.?

- Calibrate $\sigma = 2$, $r = 0.03$, $\beta = 0.97$, earnings process Markovian (33 states) from HANK (Kaplan et. al 2018), \bar{v} to match 0.607 (BLS E-Pop Feb. 2019)





The “Stabilizing” Role of Incomplete Markets



Model Meta-Analysis: EM-ALSCs as RRs

- Make tangible \Rightarrow recast respective detailed model into RR framework
- Make comparable \Rightarrow unifying bridge across models

Three-step "recipe" for each model:

- ① Define individual-level RR $1 + \xi_{it}^*$
- ② Construct ALSC from CDF $F(1 + \xi_{it}^*)$ (reflecting joint equilibrium distribution of all RR-relevant factors)
- ③ Study properties – e.g. (local) elasticity

Models:

- ① Representative “command” household with consumption insurance
- ② Heterogeneous agent models with extensive margin
- ③ Lifecycle, intensive margins, and nonconvexities (Rogerson and Wallenius, 2008)

Model Meta-Analysis: Rogerson-Wallenius (2008)

Model with **heterogeneous wages** and **intensive margin hours choices**

- OLG: unit mass of individuals born at every instant; alive between age $a \in [0, 1]$.
- a -specific wages w_a – triangle \Rightarrow lifecycle labor supply

$$\begin{aligned} \max_{c_a, h_a} \int_{a=0}^1 e^{-\rho a} [u(c_a) - v(h_a)] da \\ \text{s.t. } \int_{a=0}^1 e^{-ra} c_a = \int_0^1 e^{-ra} y_a(h) da \end{aligned}$$

- Intensive-margin choice: pick optimal hours with MaCurdy disutility $v(h_a) = \Gamma \frac{h_a^{1+1/\gamma}}{1+1/\gamma}$.
- Nonconvexity in form of fixed hours cost of working: $y_a(h_a) = w_a \max\{h_a - \underline{h}, 0\}$

Model Meta-Analysis: Rogerson-Wallenius (2008)

❶ Define micro RR:

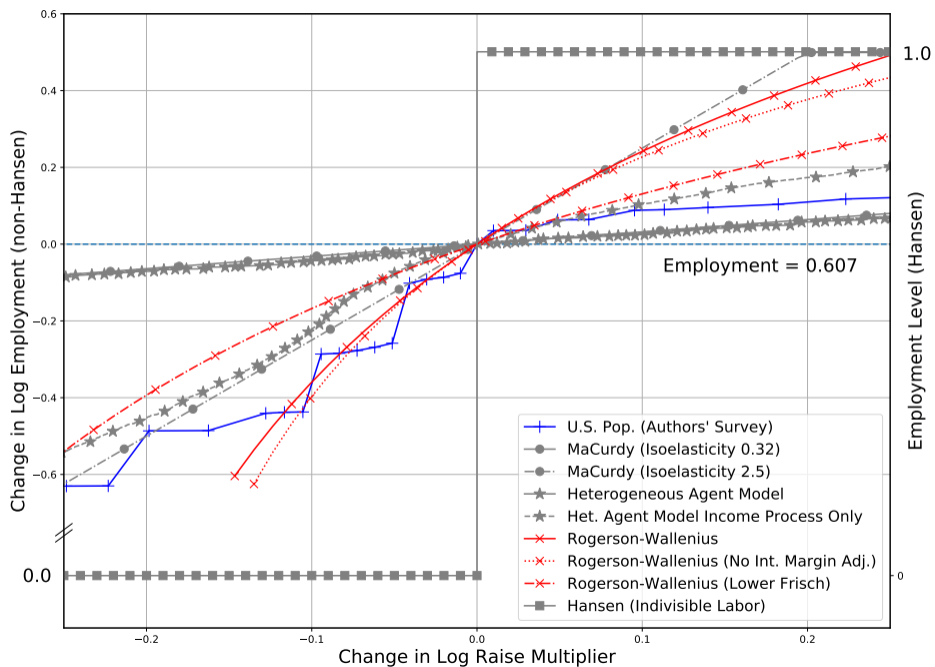
$$1 + \xi_a^* = \frac{v(h(a, 1 + \xi_a^*))}{w_a[h(a, 1 - \xi_a^*) - \bar{h}]\lambda} = \frac{\Gamma(\underline{h}(1/\gamma + 1))^{1/\gamma}}{w_a\lambda}$$

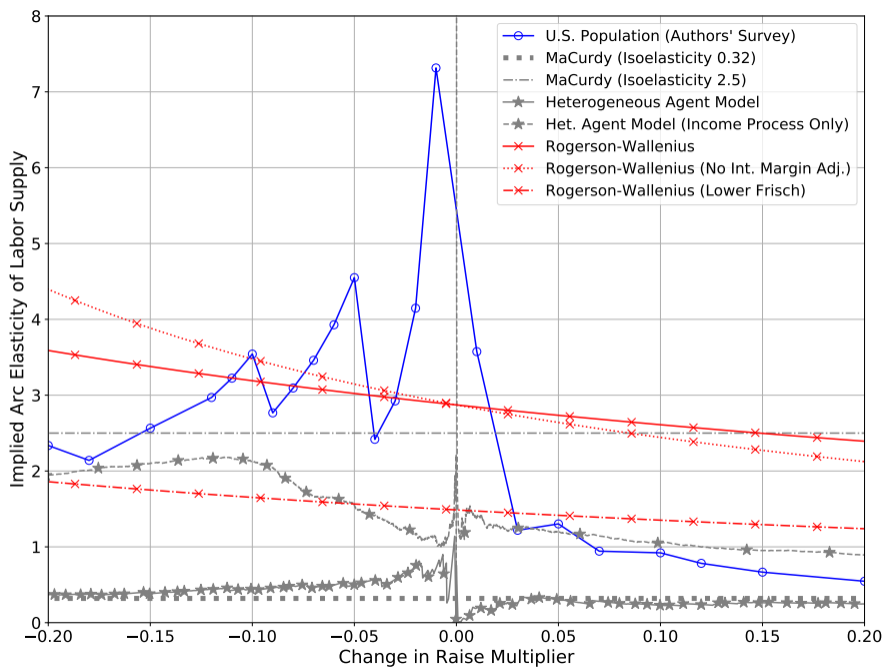
❷ CDF:

$$F(1 + \Xi) = P\left(\frac{\Gamma(\underline{h}(1/\gamma + 1))^{1/\gamma}}{w_a\lambda} \leq 1 + \Xi\right) = P\left(\frac{1}{w_a} \leq \frac{(1 + \Xi)\lambda}{\Gamma(\underline{h}(1/\gamma + 1))^{1/\gamma}}\right)$$

❸ Shape? Elasticity?

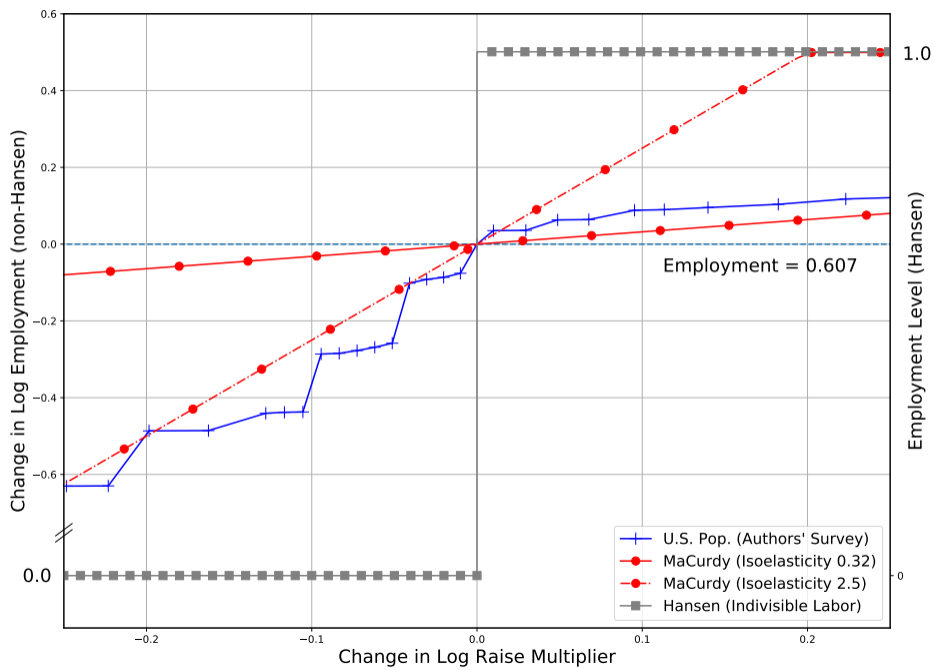
Determined by age (i) distribution and (ii) wage-age profile

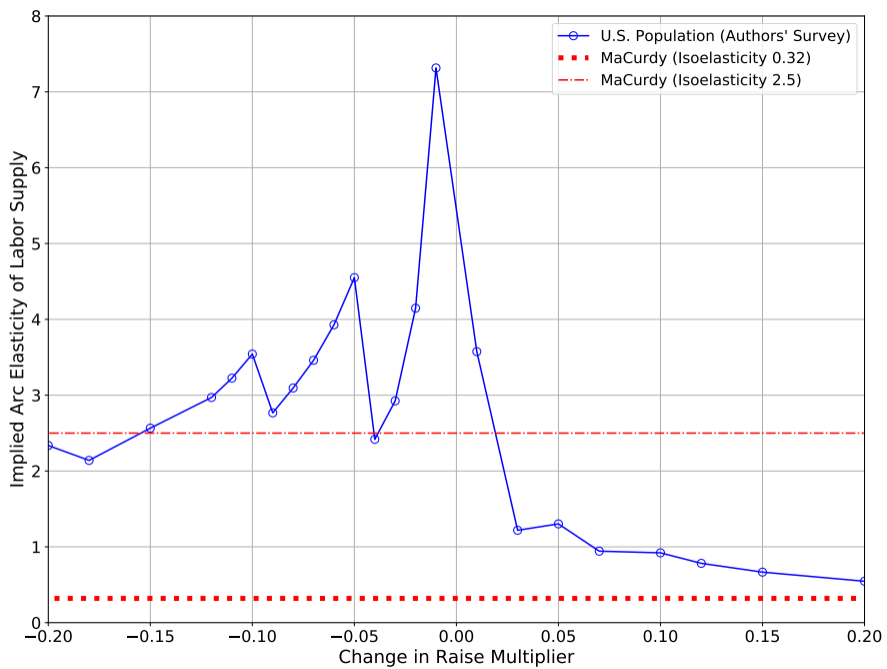




Outline

- ① **Basic Framework**
- ② **Leading Model Example: Frischian & Spot Labor Market**
 - Extensions – see paper
- ③ **Measurement In Survey**
- ④ **Model Meta-Analysis** – much more in paper (and appendix)





Outline

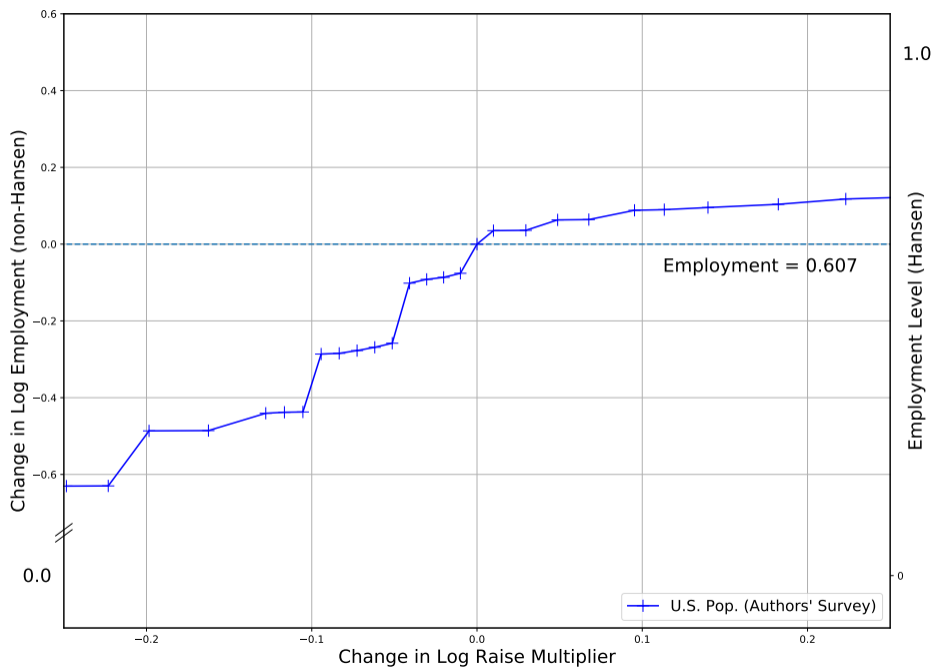
- ① **Basic Framework**
- ② **Leading Model Example: Frischian & Spot Labor Market**
 - Extensions – not today, see paper
- ③ **Measurement In Survey**
- ④ **Model Meta-Analysis** – much more in paper (and appendix)
- ⑤ **Macro Implications**
 - Calibration target: global ALSC
 - **BCA labor wedge with data-consistent ALSC**

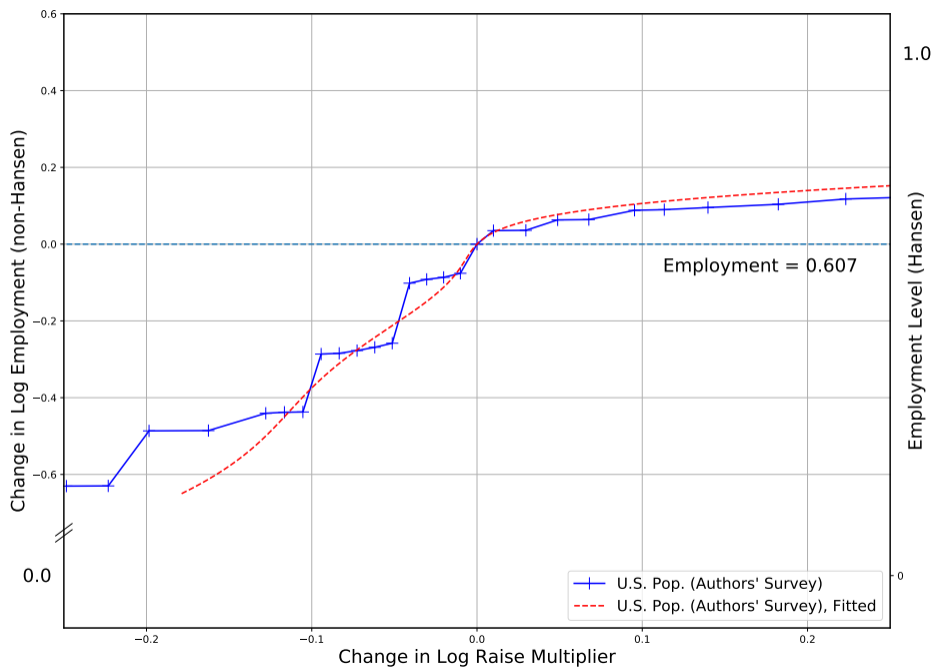
Calibrating the Labor Supply Curve

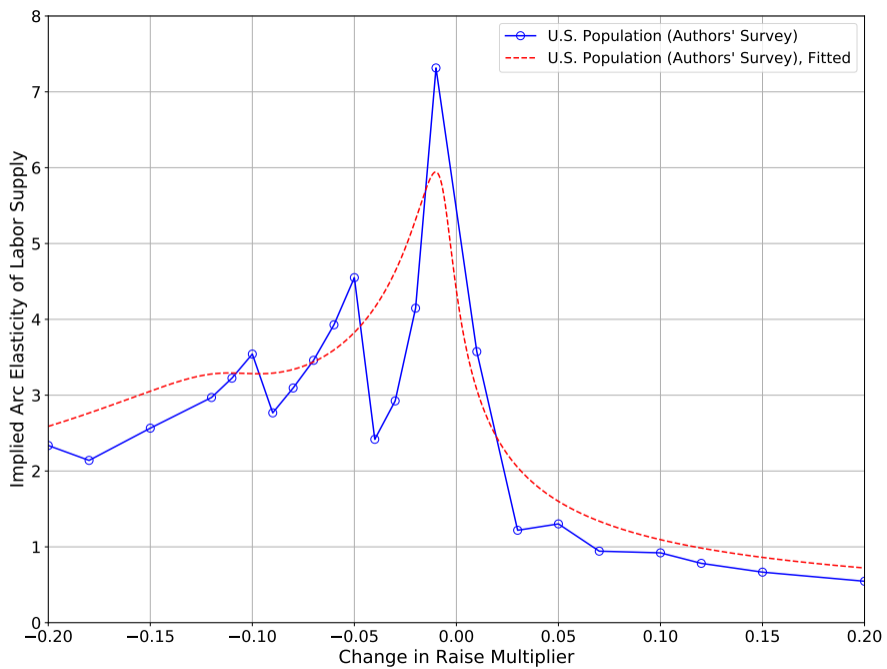
- None of the models capture the **global** empirical raise distribution
- Next: **one** example matching it nearly perfectly
- Full insurance (e.g., rep HH) with heterogeneity in disutility of labor $v \sim G^v(v)$
- Reverse-engineer $v \sim G^v(v)$ to yield a RHH-level disutility of labor $V(E)$ consistent with the empirical raise distribution

$$U(C_t) - V(E_t) \tag{1}$$

- Fit polynomial of model $V'(E)$ function to empirical res-raise CDF (and take anti-derivative; details in paper)







Calibrating the Labor Supply Curve

- None of the models capture the **global** empirical raise distribution
- Next: **one** example matching it perfectly
- RHH $v \sim G^v(v)$

$$\max_{\bar{c}_t, \{e_{vt}\}, A_t} \mathbb{E}_t \sum_{s \geq t}^{\infty} \beta^{s-t} \left[u(\bar{c}_s) - \int e_{vs} v dG_s^v(v) \right]$$

$$\text{s.t. } A_s + \bar{c}_s \leq A_{s-1}(1 + r_{s-1}) + (1 + \Xi_s) y_s \int e_{vs} dG_s^v(v) + T_s \quad \forall s \geq t$$

Data individual of type $1 + \hat{\xi}_{vt}$ has peer $v = (1 + \hat{\xi}_{vt}) \bar{y}_t \bar{\lambda}_t$ in the model

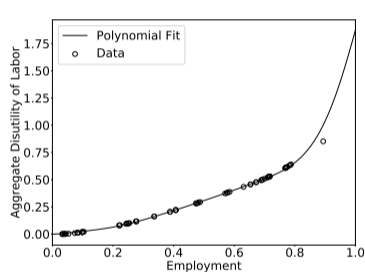
$$V(E) \equiv \int e_v v dG^v(v) = \int_{-\infty}^{\mu(E)} v dG^v(v)$$

$$V'(E) = \mu(E)$$

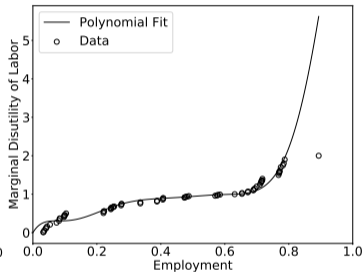
where $\mu(E) \equiv (G^v)^{-1}(E)$ is the quantile function of the disutility distribution.

Fitting Polynomial to $V(E)$

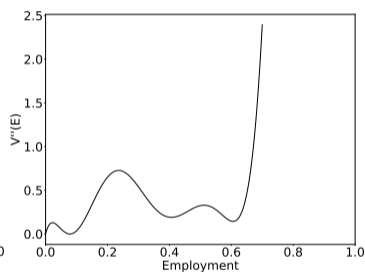
$V(E)$



Fit: $V'(E)$

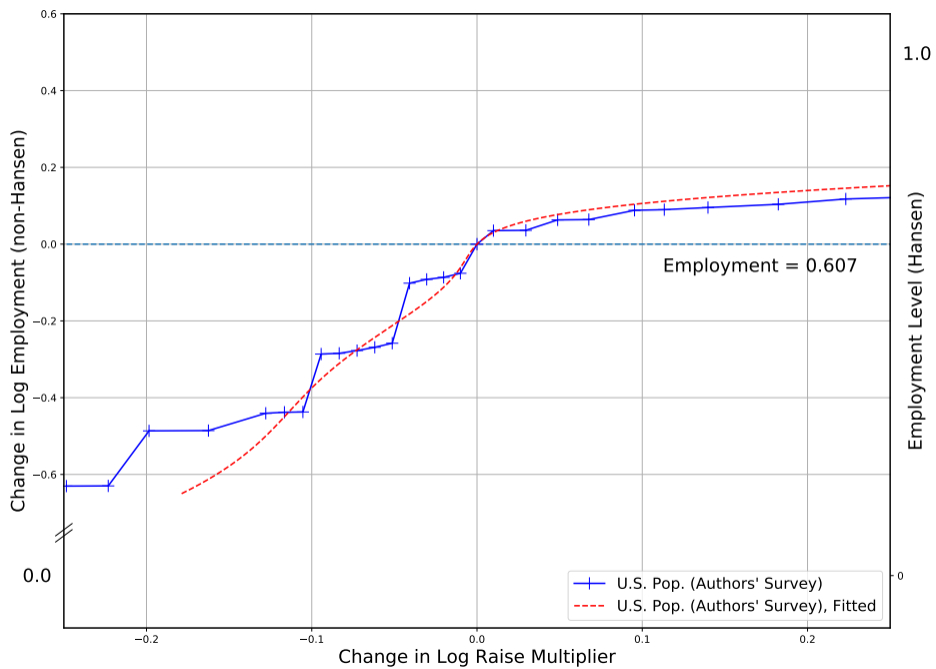


$V''(E)$



Fit a (7th-order) polynomial to function of $V'(E) = \mu(E)$ (raise at point E to E (CDF) – get analytical antiderivative $V(E)$ and derivative $V''(E)$. Weight more around unit raise.

$E = G(\mu(E))$. $V'(E) = \mu(E)g(\mu(E))\mu'(E) = \mu(E) > 0$, as $\mu'(E) = \frac{1}{g(\mu(E))}$. So, $V''(E) = \frac{1}{g(\mu(E))} > 0$ over the support.



Business Cycle Accounting: The Labor Wedge $(1 - \theta_t)$

$$\ln C_t - V(E_t) \quad (2)$$

Three variants for $V(E_t)$:

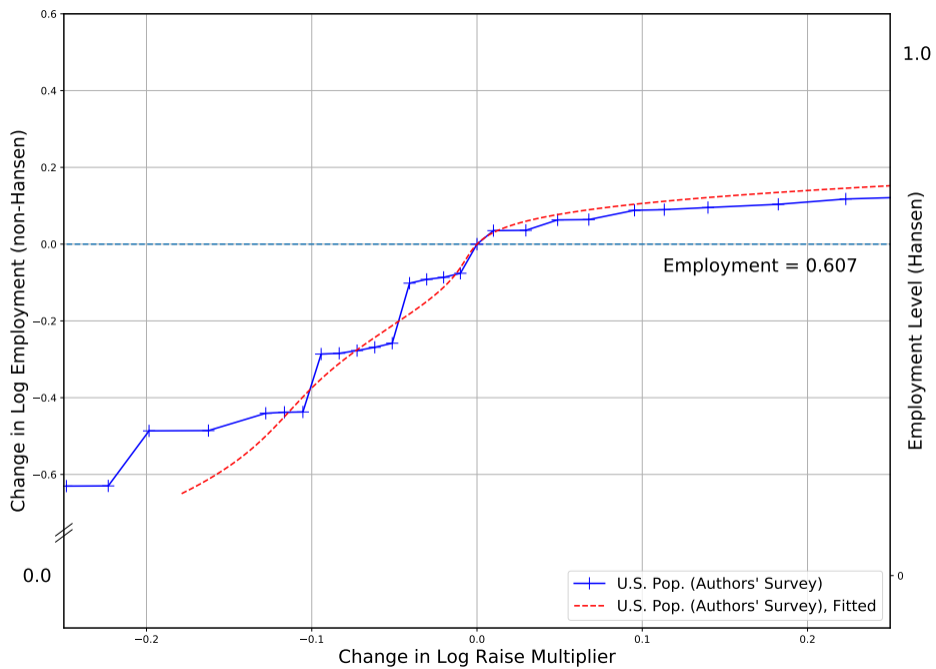
- Our fitted polynomial approximating the empirical global curve
- Iso-elasticity 0.32
- Iso-elasticity 2.5

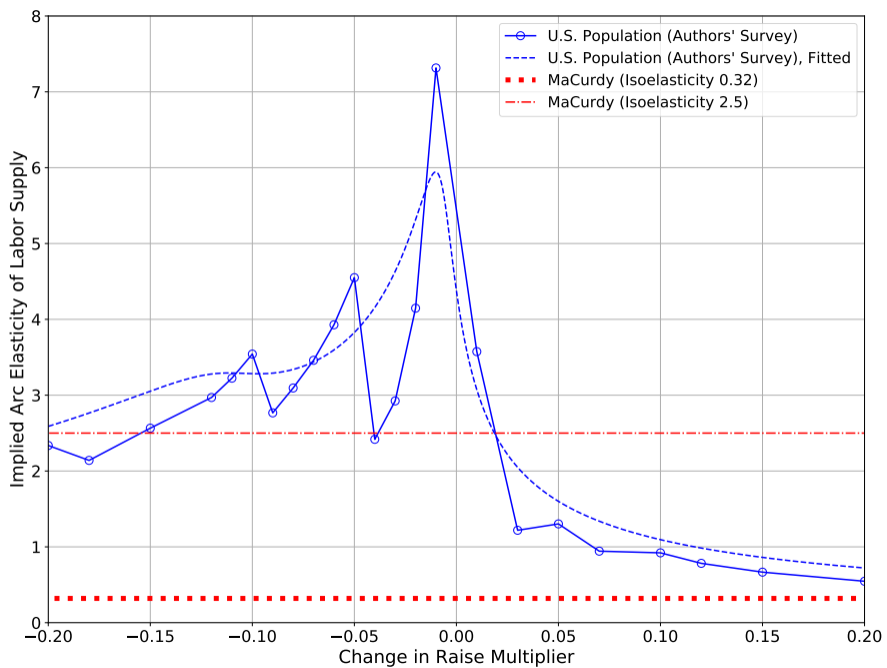
$$(1 - \theta_t) F_L(L_t, K_{t-1}) = \frac{-U_L(C_t, L_t)}{U_C(C_t, L_t)} \quad (3)$$

$$= V'(E_t) \cdot C_t \quad (4)$$

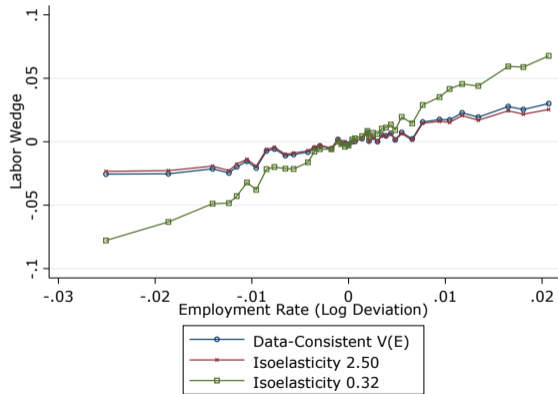
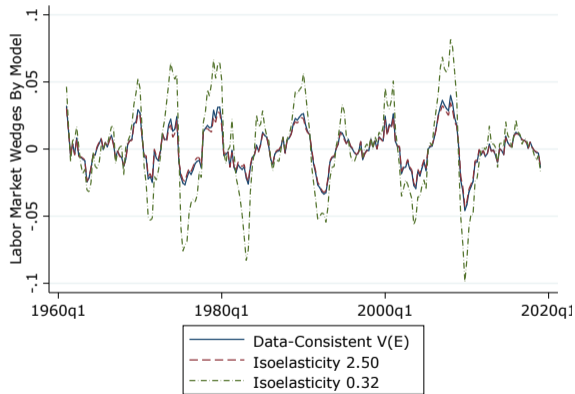
$(1 - \theta_t)$: measure of disequilibrium (symptom of frictions), mismeasurement or model misspecification

- See Chari, Kehoe, McGrattan (2007), Shimer (2009)
- Apply to US business cycles post-1960

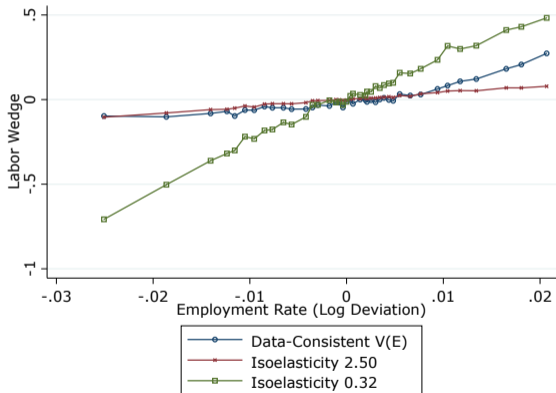
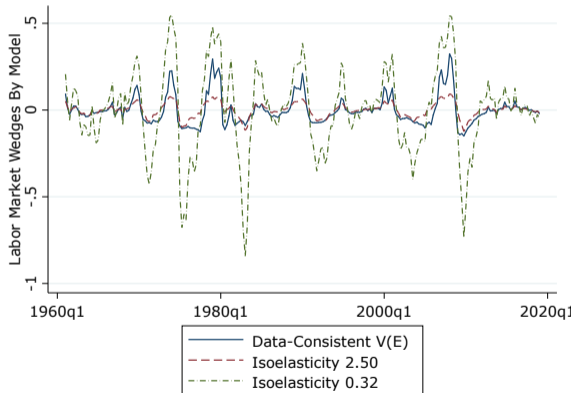




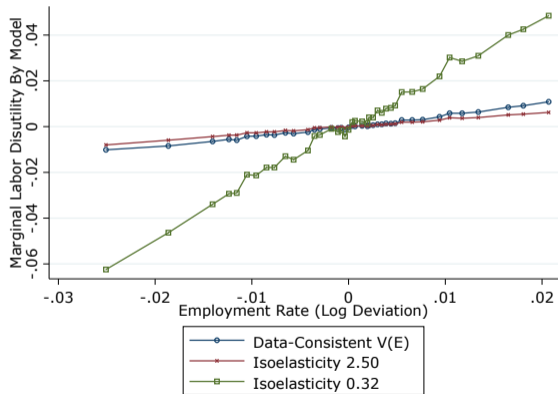
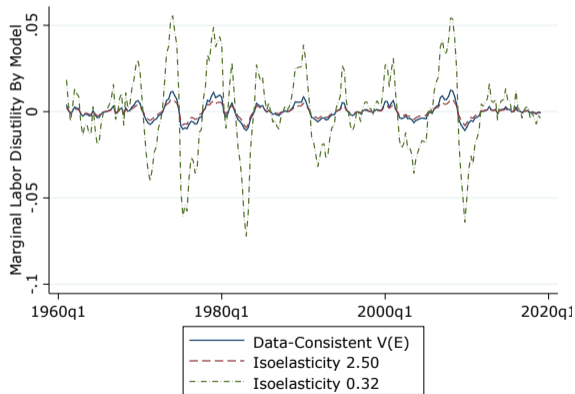
Labor Wedges: Time Series and Binned Scatter Plot



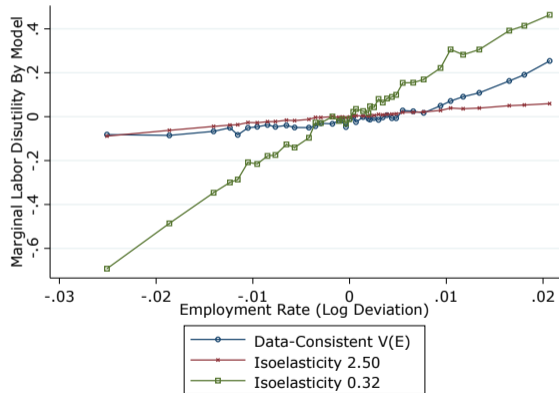
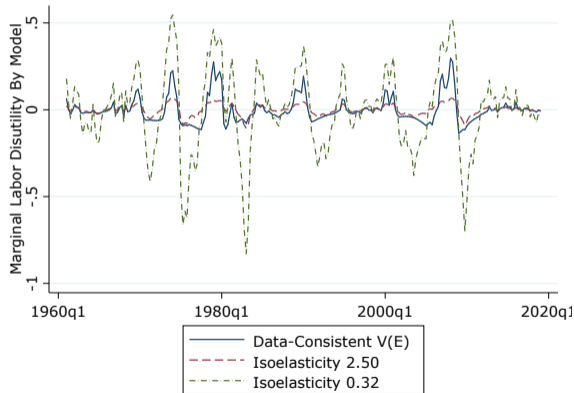
Labor Wedges: Amplify x10 Emp-Fluct Entering $V'(E)$

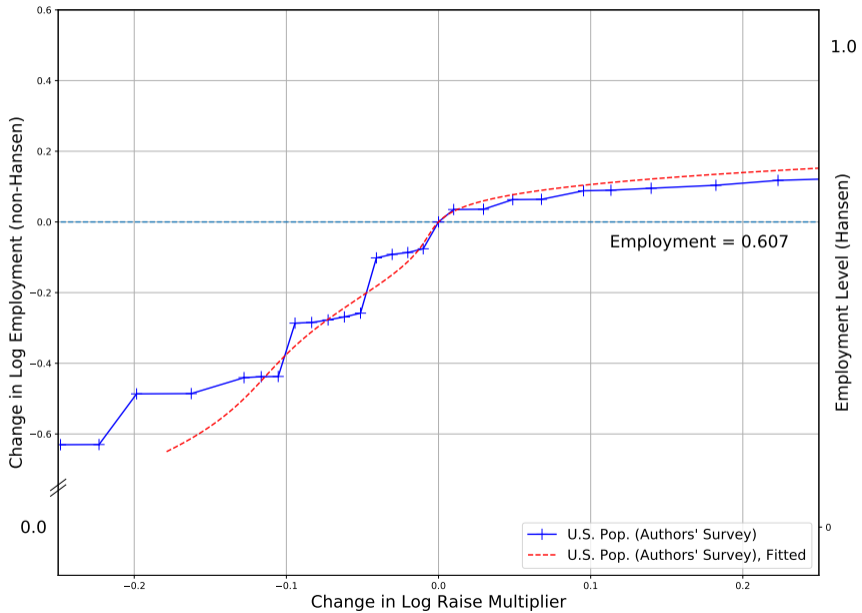


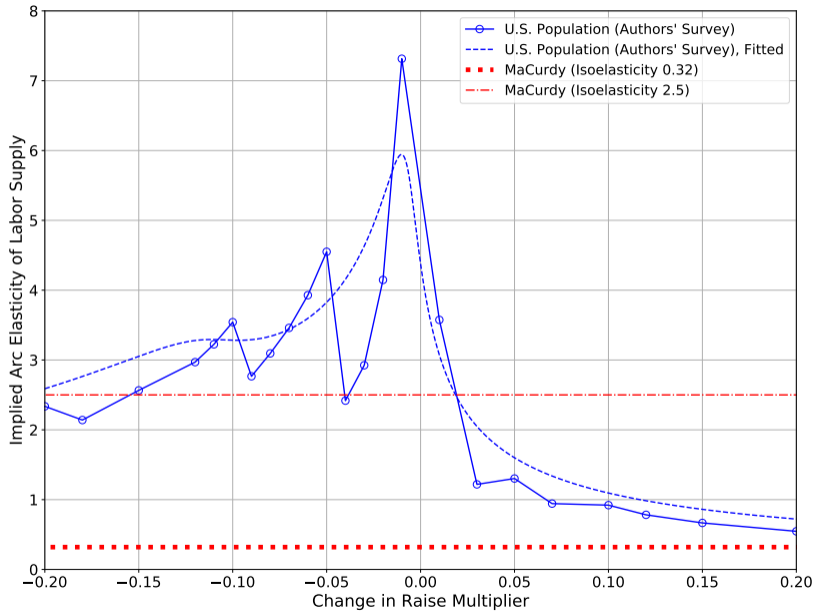
Marginal Disutility of Labor

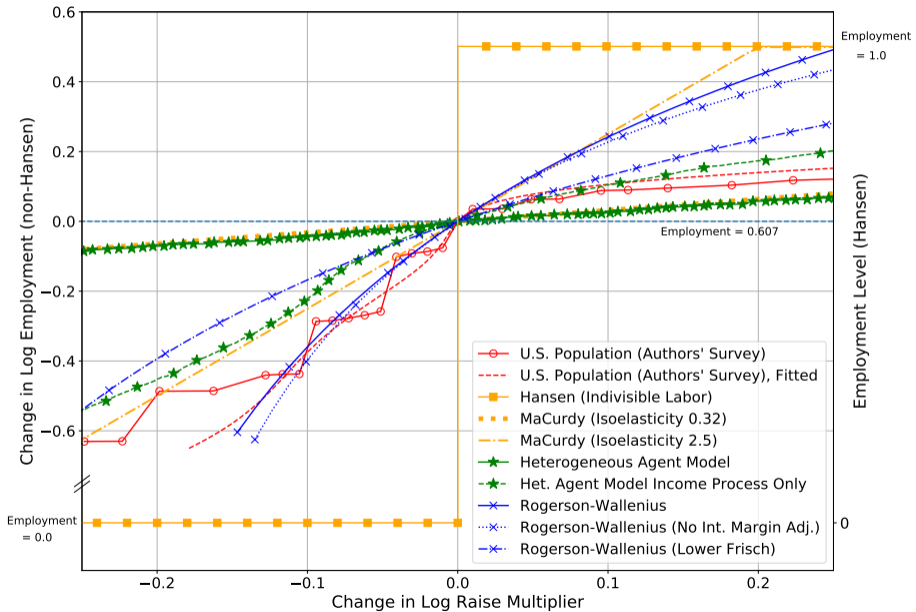


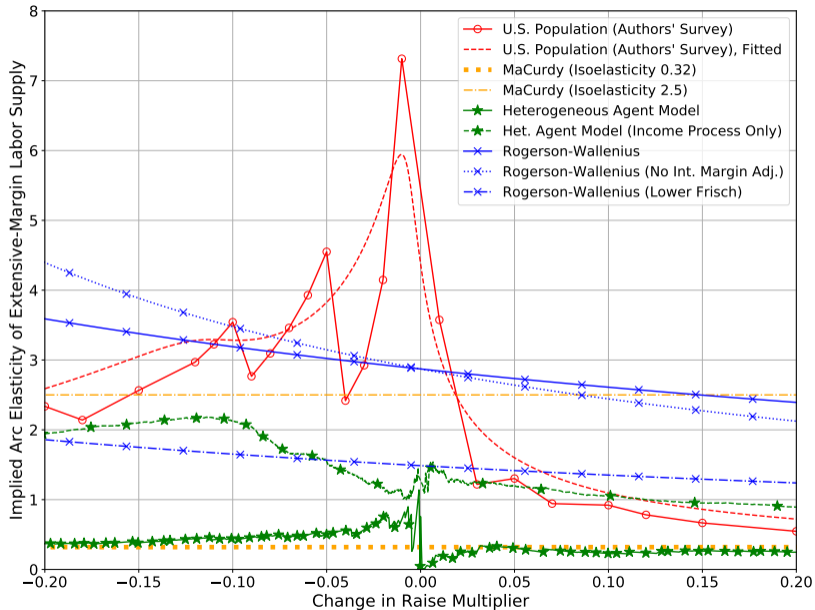
Marginal Dis'y of Labor: Amplify x10 Emp-Fluct Entering $V'(E)$











Summary: Reservation Raises and Aggregate Labor Supply

① Put to use **BASIC** framework for EM-ALSC

$1 + \Xi_t$: **Prevailing aggregate raise**: linear, homogeneous shifter of labor income

$1 + \xi_{it}^*$: **Micro reservation raise**: hypothetical $1 + \Xi_t$ rendering i indifferent b/w working and not:

$F_t(1 + \xi^*)$: **Aggregation** of individual RR's \Rightarrow short-run EM-ALSC:

$$E_t = P(1 + \xi_{it}^* \leq 1 + \Xi_t) = F_t(1 + \Xi_t)$$

② Custom survey of U.S. + German **pop'ns**: directly measure RR distribution (EM-ALSC)

- Large local elasticities of 3 and up
- Non-constant, asymmetric arc elasticities: smaller arc elasticities upwards

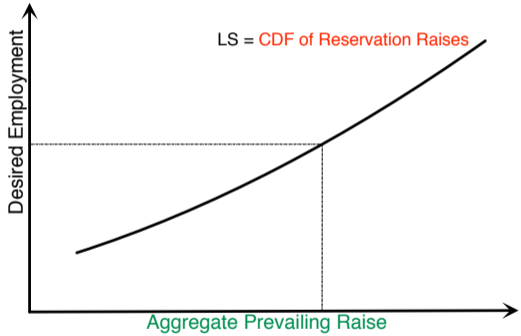
③ Model meta-analysis: recast in RR framework to uncover & make comparable ALSCs

- No existing model provides good global fit to empirical curve

④ Macro implications of empirical EM-ALSC used as calibration target

- Fit one model's $F_t(1 + \xi^*)$ tightly to the empirical analogue
- BCA labor wedge considerably less cyclical

Summary: Reservation Raises and Aggregate Labor Supply



APPENDIX

Job Menus: Allowing for Intensive Margins

- Choice j from a menu $J_{it} \in \{(y_{it,j}, v_{it,j})\}_j$: j -specific earnings and disutility/amenities
 - Nests j -specific heterogeneity in hours \tilde{h}_{it}^j

$$\max_{a_{it}, J_{it} \in J_{it}, c_{it}} \mathbb{E}_t \sum_t u(j, c_{it}) \quad \text{s.t.} \quad a_{it} + c_{it} \leq a_{i,t-1}(1 + r_{t-1}) + (1 + \Xi_t)y_{it,j} + T_{it}(\cdot)$$

- For any given raise $1 + \Xi_t$, intensive-margin job choice (ignoring participation):

$$\Rightarrow j^*(1 + \Xi_t) = \operatorname{argmax}_{j \in J_{it}} \{ \mathbb{E}_t \sum_t u(j, c_{it}) \quad \text{s.t. (BC)} \mid 1 + \Xi_t \}$$

- Extensive-margin choice respecting intensive margin choice:

$$\Rightarrow e_{it}^* = \begin{cases} 0 & \text{if } (1 + \Xi_t)y_{it}^{j^*(1+\Xi_t)} \lambda_{it} < v_{it}^{j^*(1+\Xi_t)} \\ 1 & \text{if } (1 + \Xi_t)y_{it}^{j^*(1+\Xi_t)} \lambda_{it} \geq v_{it}^{j^*(1+\Xi_t)} \end{cases}$$

- Implicitly defined RR conditional on having (re-)optimized job choice:

$$1 + \xi_{it}^* = \frac{v_{it}^{j^*(1+\xi_{it}^*)}}{y_{it}^{j^*(1+\xi_{it}^*)} \lambda_{it}}$$

Appendix: Specific Intensive-Margin: Hours Choice

Suppose MaCurdy preferences, with flexible hours choice $h^i \in [0, \infty)$. FOC for hours is:

$$\Psi h_{it}^{1/\eta} = (1 + \Xi_t) w_{it} \lambda_{it}$$

- Without reoptimization of hours:

$$\frac{\overbrace{\Psi h_{it}^{*1+1/\eta}}^{v_{it}}}{1 + 1/\eta} = (1 + \xi_{it}^*) \overbrace{w_{it} h_{it}^*}^{y_{it}} \lambda_{it} \Leftrightarrow 1 - \xi_{it}^* = \frac{\Psi_{it} h_{it}^{*1+1/\eta}}{(1 + 1/\eta) w_{it} \lambda_{it}} = \frac{1}{1 + 1/\eta} > 0$$

- With reoptimization, note that it holds that:

$$\Psi h_{it}^{*1/\eta} = (1 + \Xi_t) \lambda_{it} w_{it} h_{it}^*$$

...and so the RR is trivial – no meaningful extensive margin:

$$h_{it}^* = 0 \Leftrightarrow 1 + \xi_{it}^* = 0 \quad \forall i$$

⇒ Need non-convexity/fixed cost (e.g. Rogerson and Wallenius, 2008) or limited job menu.

Application: Conditions for Constant Elasticity

Specifically, the distributional assumptions for the property in power-law terms specify a standard power law distribution $F(X) = P(x < X) = a \cdot \left(\frac{x}{X_{\min}}\right)^{-\gamma+1}$ with shape parameter $\gamma > 0$. A comparison with our raise-based power-law-like distribution (??) and a rearrangement clarify that we require the *inverse* of our raise to follow a power distribution:

$$G_{1+\xi^*}(1+\xi^*) = P(X < 1+\xi^*) = \left(\frac{1+\xi^*}{(1+\xi^*)_{\max}}\right)^{\alpha_{1+\xi^*}} \quad (5)$$

$$\Leftrightarrow P\left(\frac{1}{1+\xi^*} < \frac{1}{X}\right) = \left(\frac{\frac{1}{1+\xi^*}}{\frac{1}{(1+\xi^*)_{\max}}}\right)^{-\alpha_{1+\xi^*}} \quad (6)$$

which is a power-law distribution of $\frac{1}{1+\xi^*}$ with minimum $\frac{1}{(1+\xi^*)_{\max}}$, and shape parameter $\gamma = \alpha_{1+\xi^*} + 1$.

Parameter	Symbol	Value (by Variant)	
Panel A: Hansen (Indivisible Labor)			
Ext. Margin Labor supply disutility	\bar{v}	1.0	
Potential earnings	\bar{y}	1.0	
Marginal utility of consumption	$\bar{\lambda}$	1.0	
Panel B: MaCurdy (Isoelasticity)			
		Low Frisch (0.32)	High Frisch (2.50)
CRRA consumption parameter	σ	1.00	"
Potential earnings	\bar{y}	1.00	"
Shape parameter of labor disutility dist.	α_v	0.32	2.50
Max. labor disutility	v_{\max}	4.759	1.221
Panel C: Heterogeneous Agent Model			
		Toy Model	HANK Earnings Process
Potential-earnings states		$[y_1, y_2] = [0.0797, 0.15]$	33-State process from
Transition probabilities		$[\lambda_{12}, \lambda_{21}] = [0.1, 0.2]$	HANK
CRRA consumption parameter	γ	2.0	2.0
Interest rate	r	0.03	0.03
Discount rate	β	0.95	0.97
Labor disutility	\bar{v}	3.0	2.083×10^{-5}
Unemployment insurance	b	0.06	0.00
Min. assets	a_{\min}	-0.02	-1.775
Max. assets	a_{\max}	0.75	5,000,000

Parameter	Symbol	Value (by Variant)	
Panel D: Rogerson-Wallenius			
		Baseline	Low-Frisch Variant
Interest rate	r	0.0	"
CRRA consumption parameter	γ	1.0	"
Labor disutility shifter	α	42.492	40.000
Minimum hours	\bar{h}	0.258	0.272
Maximum labor productivity	e_0	1.000	1.112
Slope of labor productivity	e_1	0.851	1.320
Intensive-margin Frisch elasticity	η	0.5	"
Tax rate	t	26.0%	"