Department of Economics
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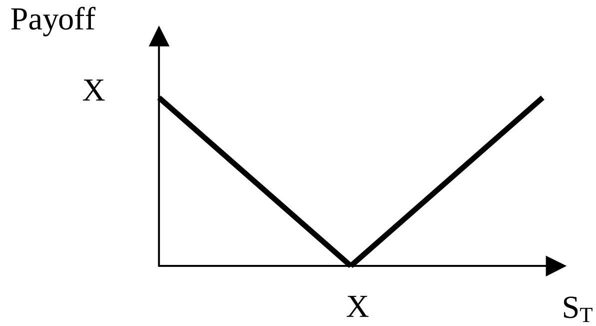
**Economics 136**
**Solutions to Problem Set #4**

1. Draw payoffs of following combinations of options. All the options have the same expiration date.

(a) Buying a call and a put with the same strike price.

Answer: Let X be the mutual strike price. Construct a table of payoffs:

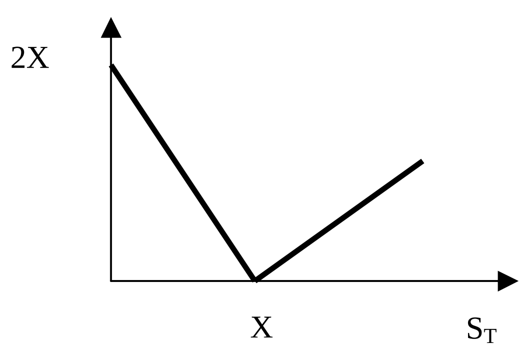
| State | Call Payoff | Put Payoff | Total Payoff |
|---|---|---|---|
| $S_T \leq X$ | 0 | $X - S_T$ | $X - S_T$ |
| $S_T > X$ | $S_T - X$ | 0 | $S_T - X$ |




(b) Buying a call and two puts with the same strike (this is called a *strip*).

Answer:

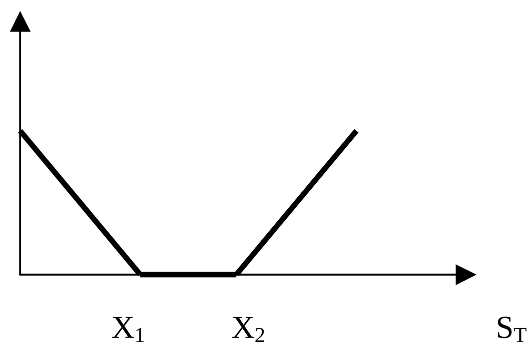
| State | Call Payoff | 2 x Put Payoff | Total Payoff |
|---|---|---|---|
| $S_T \leq X$ | 0 | $2(X - S_T)$ | $2(X - S_T)$ |
| $S_T > X$ | $S_T - X$ | 0 | $S_T - X$ |



(c) Buying a put with a strike price, $X_1$, and a call with a strike price, $X_2$, where $X_1 < X_2$ (called a *strangle*).

Answer:

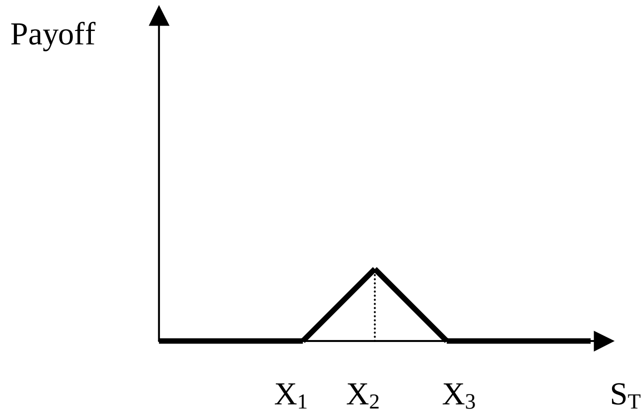
| State | Put($X_1$) | Call($X_2$) | Total |
|---|---|---|---|
| $S_T \leq X_1$ | $X_1 - S_T$ | 0 | $X_1 - S_T$ |
| $X_1 < S_T \leq X_2$ | 0 | 0 | 0 |
| $X_2 < S_T$ | 0 | $S_T - X_2$ | $S_T - X_2$ |




(d) Buying a call with a strike price, $X_1$, a call with a strike price, $X_3$, and selling two calls with a strike price, $X_2$, where $X_1 < X_2 < X_3$ and $X_3 - X_2 = X_2 - X_1$ (a *butterfly spread*).

Answer:

| State | $C(X_1)$ | $-2C(X_2)$ | $C(X_3)$ | Total |
|---|---|---|---|---|
| $S_T \leq X_1$ | 0 | 0 | 0 | 0 |
| $X_1 < S_T \leq X_2$ | $S_T - X_1$ | 0 | 0 | $S_T - X_1$ |
| $X_2 < S_T \leq X_3$ | $S_T - X_1$ | $-2(S_T - X_2)$ | 0 | $2X_2 - X_1 - S_T$ |
| $X_3 < S_T$ | $S_T - X_1$ | $-2(S_T - X_2)$ | $S_T - X_3$ | $2X_2 - X_1 - X_3$ = 0 (why?) |




2. ACME Corp. stock currently trades at $25 a share. Next year it will either trade at $35 or $15. The stock will pay no dividends, and the risk-free one-year interest rate is 10%.

(a) What is the price of a European Call option with a strike price of $25? If the strike price were $0 instead, how would that change your answer. Explain.

**Answer**: Next year ACME's share price will either increase to $35 or decrease to $15. Thus, a call option written on that stock with a strike price of $25 will be worth either $10 or $0, respectively, when the option expires next year. Consider holding a portfolio that buys 1/2 of a share of ACME stock and writes one call on that stock with a strike price of $25. (If the concept of buying only half a share of stock bothers you, think of buying 1 share of a mutual fund that holds only ACME stock and issues 2 shares for every share of ACME it holds.) If the stock price rises then the stock in your portfolio is worth $35/2. However, the call option will be exercised by the holder, which will cost you $10. So the total value of your portfolio will be 35/2 - 10 = $7.50. On the other hand, if the stock falls in value your portfolio will also be worth $7.50 (the call is not exercised). Since this is a risk-free portfolio--it guarantees to payoff $7.50-- it must be worth the same as a risk-free bond promising to pay $7.50 in one year. Such a bond would sell for $7.5/1.1 = $6.82, which must also be the current value of your portfolio. But you know that the stock in your portfolio is currently worth $12.50 (= 25/2), so it must be that the call is worth 12.5 - 6.82 = 5.68. In this case a weight of 1/2, the hedge ratio, was chosen to make the hedge portfolio risk-free.

If the Strike price were $0 instead, then the call would have a payoff of $35 or $15 next period. But this is the same as the stock itself. If two assets have identical payoff profiles, they must also have the same price. So this call should sell for $25, the same as the stock.

(b) Redo part (a) for a put option.

**Answer**: To price the put options, just use the put-call parity condition (subscripts denote the strike price):

$$P_{(25)} = \frac{25}{1.1} + 5.68 - 25 = 3.41$$

$$P_{(0)} = \frac{0}{1.1} + 25 - 25 = 0$$

(c) Suppose the stock will be worth either \$40 or \$10 next year. What would be the price of a call option with a strike price of \$25. Explain intuitively.

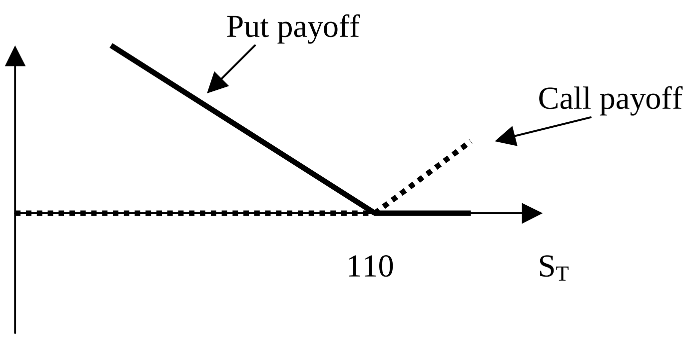
**Answer**: The call will be worth either \$15 or \$0 next period. Intuitively, this will make the call worth more than in part (a), since this call offers a greater potential payoff. A portfolio that buys *h* units of stock and shorts 1 call option will payoff 40*h* - 15 if the stock price rises and 10*h* if it falls. If *h* = 1/2, then these payoffs are the same. Regardless of what happens this portfolio will pay \$5, which makes it worth \$4.55 today. Thus this call option must be worth \$12.50 - \$4.55 = \$7.95. What has happened here is that we have increased the variance of the asset's return (without changing anything else), which will increase the value of an option written on that asset.

(d) Redo part (c) with a risk-free rate of 15%. Explain your result intuitively.

**Answer**: With a risk-free rate of 15%, the certain payoff of \$5 in part (c) is only worth \$5/1.15 = \$4.35. Thus the call is now worth \$12.50 - \$4.35 = \$8.15. A higher interest rate makes holding the call more attractive relative to holding the stock itself, since by holding the call investors can still benefit from increases in the stock's price without paying much up front--money that now has a higher opportunity cost in forgone interest income. This drives up the call's price.

3. (a), (b) and (c)

| State | Probabilities | Price of Daily Granite | Call Option Payoff | Put Option Payoff |
|---|---|---|---|---|
| 1 | 1/5 | \$120 | \$10 | \$0 |
| 2 | 1/5 | \$110 | \$0 | \$0 |
| 3 | 1/5 | \$100 | \$0 | \$10 |
| 4 | 1/5 | \$90 | \$0 | \$20 |
| 5 | 1/5 | \$80 | \$0 | \$30 |



(d) Put-Call parity tells us the if we buy a stock, buy a put, and sell a call, we will have a risk-free portfolio:

| State | Stock | Long Put | Short Call | Total Payoff |
|---|---|---|---|---|
| 1 | $120 | $ 0 | - $10 | $110 |
| 2 | $110 | $ 0 | $ 0 | $110 |
| 3 | $100 | $10 | $ 0 | $110 |
| 4 | $ 90 | $20 | $ 0 | $110 |
| 5 | $ 80 | $30 | $ 0 | $110 |

4. The current price of one share Widgets U.S.A. stock is $50. It's expected that Widgets will have a return of 10% and a standard deviation of 60% over the next year. The risk-free rate is 5%.

(a) Use the Black-Scholes formula (consult section 21.4 of BKM) to calculate the price of a call option on Widgets stock with an exercise price of $60 expiring in one year.

**Answer**: The Black-Scholes formula is $C_0 = S_0 N(d_1) - Xe^{-rT} N(d_2)$. Thus to calculate the call price we need to fill-in the values of all the parameters on the right hand side of this equation. We know right off the bat that $S_0$ = 50 , X = 60, T = 1, r = .05, and $\sigma$ = .60. (Note: This assumes that the rate given is the continuously compounded risk-free rate, and that the standard deviation is for the stock's continuously compounded rate of return. These are what are required for the Black-Scholes formula. If we were given discrete variables instead, then we would have to transform them into continuous variables. For example, if the discrete risk-free rate, $r_f$, is .05, then we find the continuously compounded rate, r, by solving: $1 + r_f = e^r \Rightarrow r = \ln(1 + r_f) = \ln(1.05) = .049$. As you can see this should not make a big difference.)

Now we can calculate the d's:

$$d_1 = \frac{\ln(50/60) + \left(.05 + \frac{(.6)^2}{2}\right)}{.6} = \frac{-.1823 + .23}{.6} \approx .08$$

$$d_2 = .08 - .6 = -.52$$

Consulting the standard normal table, we find that

$$N(d_1) = .5319$$
$$N(d_2) = .3015$$

Thus the call option's price is
$C_0 = 50(.5319) - 60e^{-.05}(.3015) = 9.39$

(b) Redo part (a) for a similar option that expires in two years. Explain your result intuitively.

Answer: Perform the same calculation as above except now let T=2. You should get

$$
\begin{aligned}
d_1 &\approx .33 \\
d_2 &\approx -.52 \\
N(d_1) &\approx .6255 \\
N(d_2) &\approx .3015 \\
C_0 &\approx 14.91
\end{aligned}
$$

Intuitively, increasing the time to expiration makes the call more valuable because there is a greater chance that the call can be exercised profitably.

(c) Redo part (a) for a put option with the same terms.

Just use the put-call parity condition:

$$P = \frac{60}{e^{.05}} + 9.39 - 50 = 16.46$$