Economics 202A Final Exam Answers

Fall Semester 2007

1.(a) The Hamiltonian for this problem is

$$H = u(c) + \lambda\,(y + ra - c)\,.$$

The first-order conditions are

$$\begin{aligned}
\frac{\partial H}{\partial c} &= u'(c) - \lambda = 0, \\
\dot{\lambda} &= \lambda\delta - \frac{\partial H}{\partial a} = \lambda\,(\delta - r)\,. \\
0 &= \lim_{t\to\infty} \mathrm{e}^{-\delta t}\lambda(t)a(t).
\end{aligned}$$

(b) Since now $u'(c) = c^{-1/\sigma}$,we can write the last equation as

$$u''(c)\dot{c} = u'(c)\,(\delta - r)\,,$$

or as

$$\frac{\dot{c}}{c} = \frac{-u'(c)}{cu''(c)}\,(\delta - r) = \sigma\,(r - \delta)\,.$$

(c) We need to solve the equation

$$a(0) = \int_0^\infty \left[c(0)\mathrm{e}^{\sigma(r-\delta)t} - y\right]\mathrm{e}^{-rt}\mathrm{d}t$$

for the initial (optimal) consumption level, $c(0)$. The solution is

$$\begin{aligned}
c(0) &= \frac{a(0) + (y/r)}{\int_0^\infty \left[\mathrm{e}^{\sigma(r-\delta)t}\right]\mathrm{e}^{-rt}\mathrm{d}t} = \frac{a(0) + (y/r)}{\int_0^\infty \mathrm{e}^{\sigma(r-\delta)t-rt}\mathrm{d}t} \\
&= \frac{a(0) + (y/r)}{\left[\sigma(r-\delta) - r\right]^{-1}\left\{\mathrm{e}^{[\sigma(r-\delta)-r]t}\big|_0^\infty\right\}} \\
&= \left[\sigma\delta - (\sigma - 1)\,r\right]\left[a(0) + (y/r)\right].
\end{aligned}$$

The assumption that $(\sigma - 1)\,r - \sigma\delta = \sigma(r - \delta) - r < 0$ ensures that above, $\lim_{t\to\infty} \mathrm{e}^{[\sigma(r-\delta)-r]t} = 0$.

(d) Looking at the preceding consumption function, we see the three ways a rise in the interest rate r will affect saving:

1. The marginal propensity to consume out of total wealth is $\sigma\delta - (\sigma - 1)\, r$. When r rises, that coefficient falls with an effect proportional to σ. This is the substitution effect.

2. The substitution effect is counteracted by an effect proportional to unity that tends to make $\sigma\delta - (\sigma - 1)\, r$ rise when r rises. This is the income effect. The coefficient $\sigma - 1$ in the marginal propensity $\sigma\delta - (\sigma - 1)\, r$ captures the balance between the substitution and income effects.

3. In addition, y/r falls when r rises — there is a fall in lifetime wealth and so consumption falls. This is the wealth effect.

(e) (Extra credit) If $(\sigma - 1)\, r - \sigma\delta = \sigma(r - \delta) - r > 0$ then the preceding integrals do not converge. In economic terms, what does this mean? It means that no consumption path satisfying the first-order Euler condition $\frac{\dot{c}}{c} = \sigma\,(r - \delta)$ can be consistent with the individual intertemporal budget constraint. That, in turn, means that the individual's lifetime problem has no solution — there is no maximum. Why? No *feasible* plan can everywhere satisfy the first-order conditions for a maximum (i.e., the intertemporal Euler condition). This means that there is always room for a small intertemporal reallocation of consumption, say, between one period to the next, that still satisfies the intertemporal constraint but makes the consumer better off.

2. (a) See Lecture 16a.
   (b) See Lecture 19.

3.(a) The individual's Euler equation for tree holdings is

$$qu'(c_1) = \beta \mathrm{E}\left\{u'(c_2) y_2\right\}.$$

(b) If we substitute equilibrium consumption above, we get

$$qu'(y_1) = \beta \mathrm{E}\left\{u'(y_2) y_2\right\}.$$

Thus, the equilibrium tree price is

$$q = \frac{\beta \mathrm{E}\left\{u'(y_2) y_2\right\}}{u'(y_1)}.$$

(c) If y_1 goes up, people will want to spread the extra period 1 output over both periods of their lifetimes by saving (buying more tree). In the aggregate, though, they cannot save, because the output is perishable. So they will succeed only in bidding up the price of the tree. The price stops rising when for each individual, the amount of period 1 consumption forgone to purchase more tree (which is proportional to the asset price) exactly equals the expected future benefit from doing so.

(d) For the quadratic, $u'(c) - a - bc$. Applying the preceding formula to this special case, we get

$$q = \frac{\beta \mathrm{E}\left\{(a - by_2)\, y_2\right\}}{a - by_1} = \frac{\beta \mathrm{E}\left\{ay_2 - b\,(y_2)^2\right\}}{a - by_1}.$$

Since $\mathrm{E}\left\{(y_2)^2\right\} = \mu_y^2 + \sigma_y^2$, we may write this alternatively as

$$q = \frac{\beta a \mu_y - \beta b \mu_y^2 - \beta b \sigma_y^2}{a - by_1}. \tag{1}$$

Observe that when σ_y^2 rises, q falls. It seems intuitive that a rise in the uncertainty of dividends depresses the stock's price, but this actually is not a general result. Remember that the economic definition of risk, in this model, would relate to the covariance between y_2 and $u'(y_2)$, which is necessarily negative. (Why?) Does this covariance rise or fall when σ_y^2 rises? You might wish to think that through over winter break. (Hint: Think of Jensen's inequality.)

(e) Because $c_2 = c_2^* = \frac{1}{2}\,(y_2 + y_2^*)$, the equilibrium price of a tree for this case would be

$$q = \frac{\beta \mathrm{E}\left\{ay_2 - b\left(\frac{y_2 + y_2^*}{2}\right) y_2\right\}}{a - by_1}. \tag{2}$$

As the two countries are completely symmetric, we need only look at the home tree; the foreign tree looks the same. Notice that

$$\begin{aligned}
\mathrm{E}\left(\frac{y_2 + y_2^*}{2}\right) y_2 &= \frac{\mathrm{E}\,(y_2^2) + \mathrm{E} y_2 y_2^*}{2} = \frac{\mu_y^2 + \sigma_{y_2}^2 + (\mathrm{E} y_2)\,(\mathrm{E} y_2^*) + \mathrm{Cov}\,(y_2 y_2^*)}{2} \\
&= \frac{2\mu_y^2 + \sigma_y^2 + \rho \sigma_y^2}{2} = \mu_y^2 + \frac{1+\rho}{2}\sigma_y^2.
\end{aligned}$$

Thus, plugging into equation (2), we get

$$q = \frac{\beta a\mu_y - \beta b\mu_y^2 - \beta b\left(\frac{1+\rho}{2}\right)\sigma_y^2}{a - by_1}. \tag{3}$$

(f) The answer comes from comparing the old pricing equation (1) that holds before trade (i.e., in the closed economy) to equation (3), which holds after trade. You can see that if $\rho < 1$, allowing the two economies to trade with each other will lead to a general *rise* in tree prices — a global equities boom. Because people have now diversified their consumption, each country's marginal utility of consumption is less tightly linked to its own harvest, so trees are less risky for everyone in the world. If $\rho = 1$, however, so that the two trees are perfectly correlated, there is no such risk reduction — there is no diversification possible because the two assets have identical payoffs in every state of nature.