Lecture X **Economics 202A** **Fall 2007**

Today we will turn to something different.

We are now going to turn to the demand for money.
This is the next item on the reading list.

So you can see where this fits in, I just want to remind you of the structure of the course.
The structure of the course corresponds exactly to the structure of standard macroeconomics.

We first looked at general equilibrium concepts. This corresponds to looking at the intersection of the AD and AS curves, in intermediate macro.

After that we are exploring the components of the AD curve. The AD curve is the confluence of the IS and the LM curves.
We have just reviewed consumption and investment, which are the leading components of the IS curve.

We began the course with the New Classical view of the economy of Lucas, Sargent and Barro.

I do not know how Maury is going to pitch the rest of the course.

But my interpretation of David Romer's textbook is that the major theme of the book is that it begins with the classical model and growth theory.

And then in the second half ½ of the book, which is what we are covering in this course, we see time after time after time oversensitivity, relative to that model.

So far we have seen that there is oversensitivity of *income* relative to the money supply.

Next Tuesday we will see this oversensitivity in a different context.
We will see oversensitivity of the *exchange rate* relative to the money supply in the lecture next Tuesday.
In consumption, investment and finance you will again see, in each case, oversensitivity relative to the classical maximizing model.

This gives the textbook a general theme.

It is useful to consider standard maximizing models of the business cycle and its

components.
But that model never holds exactly.

The tell-tale that it is a bad fit is always *oversensitivity*, or *undersensitivity*, of the respective variable.

There is a remarkable unity here, all pointing in the direction of an economy that is much more Keynesian than a classical model.

The first part of this course discusses the general equilibrium of the system.

But now we turn to discuss each of the components of that model.

Maury and I divided these topics according to what we thought were our respective comparative advantages.

The demand for money, and S-s models, were thought to be my comparative advantage, and so we come to that topic now.

So let's now begin the section on the demand for money.

One crucial property of the demand for money is its *interest elasticity*.

It is commonly believed that if the interest elasticity is zero, the LM curve will be vertical and fiscal policy has no effect on equilibrium output or on employment.

Alternatively, in this view, if the interest elasticity is infinite, the LM curve is horizontal and monetary policy has no effect on equilibrium output or employment.

That is known as the liquidity trap.

In this section we are going to see leading models of the demand for money.

These models are interesting in their own right, but they can also be applied to other areas such as inventory demand, consumption demand, and investment demand.

Those other applications are of as much interest as our focus here on the demand for money.

I am going to begin with the inventory-theoretic demand for money by Baumol and Tobin, and then we will go over the Miller and Orr model of money demand.

I am going to put particular emphasis on Miller and Orr because that is the

prelude to examining a whole class of work on macroeconomic demands.

Before beginning discussion of money demand it is useful to review why money is so important.

I begin with the conventional wisdom regarding money, which may also seem a bit crazy.
Curiously, the reason that money is so important is that people do not have much use for it.
Not only is it not a terribly important commodity, but also the cost of holding money is a fairly small item in people's budgets.
But people *do need* it for transactions.

Thus it has a fairly low price elasticity of demand.
It is like salt.
You need salt.
But only so much of it.
So the demand for salt is quite inelastic.

Similarly, the demand for money is quite inelastic.
A small reduction in the availability of salt could change its price drastically.
The same is true of money.

Now let's consider what is the "price" of holding money.
The "price" of holding money is the opportunity cost of holding other assets.
The implicit rental cost of holding money is then the rate of interest.

What gives the Central Bank so much power is that they can buy and sell assets in such a way that the public *must* either get rid of money or acquire more money.
And people are reluctant either *to acquire* more money or *to get rid of* money.

Money is only a small fraction of their assets.
And the interest rate's being much higher or much lower has all but negligible effect on their gains or losses from money holding.

The net result is that in getting the public to hold a little more money, or a little less money, necessitates very large changes in interest rates.

But interest rates are, in fact, terribly important.

Interest rates determine the prices of *all assets*.
And there are *trillions and trillions* of other assets.

So Central Bank operations can have very *large* effects on the economy.
With a change in the money supply the *interest rates* very much *overshoot* simply

**because the Short-Run interest elasticity is so very small.
This is, of course, a point that has always been appreciated at Chicago.
We will see next Tuesday that it is the key to the Dornbusch article on exchange rate volatility.**

**With these prefatory notes, let us now turn to the demand for money.
First, we shall begin with a bit of history.
The standard view of the money demand function is:**

$$M^D = pL(X, r - r_m)$$

**X = real income
r = nominal rate of interest
r_m = rate of interest on money holdings
p = price level.**

**It's easy to write down this function.
But theorists have also derived such demands from fundamental assumptions.**

**The standard way to view money demand comes from viewing money as an "inventory."
People need money for their transactions, just as shopkeepers need goods in inventory.**

**I will now give you a simple version of the Baumol-Tobin model.
It will also serve as a useful prelude to the Miller-Orr model, which is the next item in the lecture.**

**Randomly, money is depleted, or added to.
If it gets to zero people add it to their holdings.
If it gets too high people subtract from their holdings by the purchase of nonmonetary assets.
There is a fixed cost to such buying and selling transactions.
So people hold more than zero money.
They hold it as an *inventory*.**

This is analogous to shopkeepers' holding significant inventories because there are fixed costs to ordering a batch of goods.

The simplest model is one in which there is a constant real income flow X, a fixed cost of transactions and an interest rate.

**Suppose also that the fixed cost of transactions is *a* in real terms.
In this case the nominal cost of a transaction will be $a \cdot p$.**

In this case to derive the demand for money it is necessary to solve for the optimal frequency of transactions, n*.
Let's do a quick derivation of the demand for money.

n* is determined by the best trade-off between the *costs* of holding more money against the *benefits* of holding more money.
The costs are the interest foregone for holding that money.
The benefits are the reduced number of transactions that are necessary, each at a cost *a ·p.*

Suppose the person in question has a net income inflow of pX dollars.
X is proportional to her real income.
Then if she makes n transactions per period, the length of time between her transactions will be 1/n.

And her money holdings will have the following look to it.

SHOW TRIANGLE WITH BASE 1/n.

We can then calculate her average bank balance.

The person begins the period with 0 money.
Money then flows in at the rate pX.
At the end of the period of length 1/n her bank balance will be 1/n pX.

SHOW THAT ON TRIANGLE.

Her average holding of money will be (1/n)(pX/2).

So her opportunity costs of holding money, in terms of interest foregone, will be

$$r\,(1/n)\,(pX/2).$$

Her costs of transactions per period will be

$$npa.$$

Her total costs of holding money, including both her opportunity cost and also the costs of transactions, are then:

$$r\,(1/n)\,(pX/2) + npa.$$

The individual in question then chooses n at an optimal value n* to minimize her costs of holding money.

So she sets

$$- r (pX/2n^2) + pa = 0.$$

As a result $n^* = (rX/2a)^{½}$

This gives is money demand of the form

$$M^D = ½ (pX/n^*) = ½ pX/([(rX/2a)^{½}]) = p (aX/2r)^{½}.$$

So, solving for n*, that gives you an average money demand of the following form:

$$M^D = p (aX/2r)^{½}.$$

This is the famous Baumol-Tobin formula, which shows an interest elasticity of money demand of - ½ and an income elasticity of demand of ½.

FOOTNOTE:

Here is the proof that this is the elasticity of demand. I will prove it since it is so very important that it is automatic for you to recognize elasticities directly from demand formulae.

$$\{X/M\} \ \{\partial M/\partial X\} = \{X/[(p\, a^{½} X^{½})/(2^{½}\, r^{½})]\} \{½ (p\, a^{½} X^{-½})/(2^{½}\, r^{½})\} = ½.$$

Alternative proof.

$$\ln M = \ln p + ½ \ln a + ½ \ln X - ½ \ln r.$$

$$d \ln M/ d \ln X = ½$$

$$(dM/M)/(dX/X) = ½.$$

END FOOTNOTE

The special interest in this formula comes from a fairly high *interest elasticity* of money demand of ½.

ERASE BLACKBOARD

Let me now go over the Miller and Orr inventory model of money demand. The reason for doing that is because this type of demand system is useful not just in money demand but also for analyzing changes in prices, investment in plant and equipment, inventories and consumer durables.

With this prefatory note let me introduce the Miller and Orr model of money demand.

That model makes 7 assumptions.

1. There are just 2 assets:
money* and *bonds.

2. The return on money is 0.
The return on bonds is r.

3. The fixed cost of transactions between *money* and *bonds*, and also between *bonds* and *money* is *a.*

4. The *minimum* balance a firm may hold is 0.

5. In any given period there is a probability

p of gaining \$1
q of losing \$1

and p + q = 1.

6. The firm lets cash holdings wander until they hit an *upper* threshold *h.* The firm lets cash holdings wander until they hit a *lower* threshold 0.

At either juncture the firm pays a current transactions cost of *a* dollars to return money balances to the optimal level z.

GIVE PICTURE

time

7. The firm chooses the threshold *h* and the target *z* to minimize the expected cost of money holdings *inclusive* of the interest foregone *and* the transactions costs.

In other words the firm minimizes

$$E(c) = a\, p_T + r\, E(M),$$

where c **is the expected cost of money holdings per period**
p_T **is the probability of a transaction in a given period**
$E(M)$ **is the expected value of cash holdings.**

$E(M)$ will be dependent on h and on z.

Let me explain the formula:

The cost of holding money is the sum of transactions costs plus foregone interest.
Transactions costs per period are the product of the expected number of transactions per period, p_T and the cost per transaction, which is a. POINT

Interest foregone per period because of money holding is

$rE(M)$. <POINT>

where r is the interest rate on interest-bearing assets and E(M) is expected money holdings.

Given these assumptions I am going to solve for the *demand for money.*

I will present 10 FACTS. These facts will lead us to the demand for money.

Fact I.

Given p, q

$$p_T = p_T(z, h)$$
$$E(M) = E(z, h)$$

So

$$E (c) = a\, p_T(z, h) + r\, E(z, h).$$

Therefore all we have to do is calculate

$p_T(z, h)$
and $E(z, h)$.

Then we choose the optimal
$z(r,a)$,
and $h(r,a)$
that minimizes the cost of holding money.

And then we get

$$E(M) = E(z^*(r,a), h^*(r,a)).$$

This is our demand for money.

ERASE BB

Fact II.

Let $f(x,t)$ be the probability of holding x dollars at t.

Then $f(x,t) = p\, f(x-1, t-1) + q\, f(x+1, t-1)$

$x \neq z$,
$1 \leq x \leq h-1$.

Let me explain:

the probability of holding x dollars at t is the probability of holding x-1 dollars at t-1 and gaining \$1

plus

the probability of holding x+1 dollars and losing a dollar.

This holds for all x that is not on the boundary, at 0 and h;
it does not hold for the target z either.

There are four ways to get to the target z.

$$f(z,t) = p\, f(z-1, t-1) + q\, f(z+1, t-1) + p\, f(h-1, t-1) + q\, f(1, t-1).$$

One could have z-1 and gain \$1, which occurs with probability p.
One could have z+1 and lose \$1, which occurs with probability q.

One could also have h-1 and gain \$1, hitting the upper threshold h.
In that case bonds are purchased to reduce money holdings and to reach the target z.

And finally one could have only \$1, and lose \$1, and hit the lower threshold of 0. In this case bonds are sold to reach the money target of z.

In addition, on the boundaries

$$f(0, t) = 0 \text{ and}$$
$$f(h,t) = 0.$$

And finally because these are probabilities they must all sum to one, so that:

$$\sum_{x=1,\ldots,h-1} f(x,t) = 1.$$

Fact III.

If p, q, h and z are *constant*

f approaches a steady state with the property

$$f(x,t) = f(x, t-1) = f(x).$$

So we can rewrite the preceding equations in terms of the steady-state probabilities:

(1) $f(x) = p\, f(x-1) + q\, f(x+1)$

$$x \neq z,$$
$$1 \leq x \leq h-1$$

and the four boundary conditions:

(2) $f(z) = p\, f(z-1) + q\, f(z+1) + p\, f(h-1) + q\, f(1).$

(3) $f(0) = 0$

(4) $f(h) = 0$

(5) $\sum_{x=1,\ldots,h-1} f(x) = 1.$

Fact IV.

(1) is a second-order linear difference equation in two parts:

from 1 to z-1,

and from z+1 to h-1.

(2), (3), (4) and (5) are boundary conditions.

The system is easily *soluble.*

If we change to the more standard notation for difference equations:

$f(x) = y_t$	**with t substituting for x**
$f(x-1) = y_{t-1}$	**with t-1 substituting for x-1**
$f(x+1) = y_{t+1}$	**with t+1 substituting for x+1,**

then

$$y_t = p\, y_{t-1} + q\, y_{t+1}.$$

There is, as you may remember, a standard way to solve this equation.

I will remind you how to solve that second order difference equation in a footnote to the lecture.

FOOTNOTE

Rewrite it as

(*) $\quad q\, y_{t+2} - y_{t+1} + p\, y_t = 0.$

Conjecture that there is a solution of the form

$$y_t = A\, r_1^t + B\, r_2^t$$

and put the values of y_t in the equation and we find the LHS of (*) is:

$$Ar_1^t(q\, r_1^2 - r_1 + p) + B\, r_2^t(q\, r_2^2 - r_2 + p) = 0.$$

If $q\, r_1^2 - r_1 + p = 0$,
and if $q\, r_2^2 - r_2 + p = 0$,

then the LHS of (*) is 0 for all t.

So r_1 and r_2 should be chosen as the roots of the associated polynomial.

END FOOTNOTE.

ERASE BB.

This takes us to:

FACT V.

Miller and Orr choose the special case, where

$p = q = ½$.

This yields the special equation:

$½\, y_{t+2} - y_{t+1} + ½\, y_t = 0$.

The associated polynomial of this equation is then:

$x^2 - 2x + 1 = 0$.

The roots are $x = +1$, $x = +1$.

If you go back to the solutions to difference equations you will see that the solution to this special case with double roots is:

$Ar_1^t + Btr_1^t$.

In this special case with $r_1 = 1$, this has the special form

$A + B\,t$,

since r_1 is *exactly one.*

Thus the distribution of money holdings is linear on either side of z.

Since f(0) = 0, it must rise from 0 to z.

PICTURE f IN A DIAGRAM.

f(z)

0 z h

And similarly, since f(h) = 0 it must fall from z to h.

It is very important that this distribution has a tent shape.
I will return to those implications of being tent-shaped in the next class.

This takes us to:

FACT VI

The height of the distribution is 2/h.

How do I know?
The area under the whole distribution is 1.

1= Area of Triangle.

MAKE DIAGRAM OF TRIANGLE

	H	
0	**z**	**h**

Base of triangle = h
Height of triangle = H

This is a distribution so the total area of the triangle will be 1. So:

$$1 = hH/2.$$

So $H = 2/h$.

Fact VIII. We can now calculate the E of money holdings as

$$(z + h)/3.$$

Actually this is a slightly obscure fact.
The mean of a *triangular distribution* is the *mode* plus the base divided by 3.

In fact it is fairly easy to calculate this. I have left the calculation in a footnote.

FOOTNOTE:

END FOOTNOTE.

Remember that we want the formula

$$ap_T(z,h) + r\,E(z, h).$$

We now know that this is:

$$ap_T(z,h) + r\,(h + z)/3.$$

The next step then is to calculate

$$ap_T(z,h)\ .$$

Fact VII.

$$p_T(z,h) = 1/[z(h-z)].$$

Proof.

$$p_T = q\,f(1) + p\,f(h-1)$$

$$= ½\,(1/z)\,(2/h) + ½\,(1/(h-z))\,(2/h)$$

How do we know?

GIVE TRIANGLE DIAGRAM

2/h

0 1 z h-1 h

The small triangle (between 0 and 1) is similar to the larger triangle between 0 and z,
so

$f(1)/1 = (2/h)/z$. The heights are in the same ratio as the bases.

So
$f(1) = 2/hz$.

Similarly, the large triangle between z and h is similar to the small triangle between h-1 and h. So the ratio of the heights have the same ratio as the bases, and

$f(h-1)/1 = (2/h)/ (h-z)$.

Doing algebra, feeding in the values of f(1) and f(h-1) into the formula for p_T we find that

$p_T = 1/z(h-z)$.

FOOTNOTE.

$1/zh + 1/[(h-z)h] = [(h-z) + z]/h(h-z)z = 1/z(h-z)$.

FACT IX.

Hence, to minimize the cost of money holding one minimizes

$E(c) = a\ [1/z(h-z)] + r [(h + z)/3]$.

We choose h, z so that

$\partial E(c)/\partial h = 0$

$\partial E(c)/\partial z = 0$.

You go through this algebra and you discover

$z = h/3$

$h = 3^{4/3} (a/r)^{1/3}$.

FACT X.

$M^D = E (h,z) = (h+z)/3 =(4/9)\ 3^{4/3} (a/r)^{1/3}$.

This gives an interest elasticity of $-\frac{1}{3}$.

We are now going to use what we learned here to discuss

> **money demand**
> **pricing**
> **consumption and investment demand.**

We have already seen one important property of the Miller and Orr money demand, which is the tent-shaped distribution for money demand.

Show diagram

0 z h

Why is this tent-shape important?

It is important because the only people in the short-run who will respond to a shock are those who are near the thresholds, near 0 or near h.

But there are relatively few of these.

Next time we will see three applications of these models.