

Now we are going to continue to review the usefulness of probabilistic systems such as we saw in the last class in Miller & Orr.

Let's re-begin where we ended last Thursday. I am going to go over the use of Miller & Orr in my own article, "Irving Fisher on His Head."

I want to show you that the Miller and Orr model helps us understand the effects of both fiscal and monetary policy.

I will begin by discussing some concepts in inventory models of the demand for money.

The Miller and Orr model also gives us a convenient way to categorize all *standard* models of the demand for money.

Those models are inventory models of the demand for money.

They can all be written in the form

$$L = L(P, S)$$

where Miller and Orr suggests three bits of terminology.

P = *autonomous payments* flows

S = *monitoring rules* whereby bank accounts are monitored to prevent them from having too high or too low a balance

An *induced payment* is a payment in application of the monitoring rule.

Let me illustrate these three bits of terminology in the context of standard models of money demand.

Example 1. In the *standard monetarist* story of Irving Fisher persons receive money in their bank account.

The flows are proportional to income:

$$T = k_T Y.$$

On average these receipts are kept for a period of λ .

Let's consider an example with some numbers.

λ , the period that receipts are retained is, say, two weeks, or 1/26 year.

$$\lambda = 1/26$$

$$k_T = 10.$$

The ratio of transactions to income is 10. I take this number from the actual ratio of check clearances to income.

In this case

$$M = 10/26 Y.$$

Now let's try to classify the terms in this example.

1. In this example the *autonomous* flows are $k_T Y$.
2. The *monitoring rule* is the average lag of two weeks between inpayment and outpayment.
3. *Induced payments* are the payments made on application of the monitoring rule.

Example II is the Baumol-Tobin model.

Autonomous payments are the periodic inflow pX .

The monitoring rule is:

when the bank account has reached the level $(1/n^*) pX$ it should be emptied, where n^* is the optimal number of transactions per period.

Example III is Miller and Orr.

autonomous payments flows are:

with probability p : gain \$1

with probability q : lose \$1

monitoring rule:

if the bank account hits an upper threshold h ,
buy interest-bearing securities in amount $h - z$.

if the bank account hits the lower threshold of 0,
sell securities in the amount z .

Most theories of the demand for money say:

$$L = L(P(Y), S(Y, r))$$

where P are autonomous payments flows and

S is the monitoring rule.

The following is the most important *folk theorem* of the demand for money.
This is what people's intuition tells them.

If the monitoring rule is *constant* and income changes, the demand for money will change proportionately, or

The implication of this is the belief that the short-run demand for money is proportional to income *unless* there is a rapid response of the monitoring rule to changes in interest rates or income.

Such a money demand function would make the LM curve vertical in the short-run, so shifts in the IS curve have no effect on equilibrium income.

Correspondingly, shifts in the money supply also have powerful effects on aggregate demand.

This corresponds to the simplest (and perhaps most common) monetarist view of the economy.

Now let's begin our application in the context of the Miller and Orr model.

I am going to show you that in a modified Miller and Orr model, if the thresholds and the targets are constant, a change in income will result in *no change* in money demand.

This is contrary to the intuition of the standard quantity theories of the demand for money.

Those theories suggest that if the *monitoring rules* are constant, the demand for

money will be proportional to income.

Here we can think of the levels of the targets and the thresholds as the monitoring rule that is determining money balances.

Let me now interpret this in terms of the Miller and Orr model.

Modify the Miller-Orr model that we have just seen to:

$$p + q + s = 1$$

p = probability of getting \$1

q = probability of losing \$1

s = probability of no transaction

Suppose that as Y increases p and q increase *proportionately*, while s decreases.

The justification for this assumption is that in continuous time the probability of gaining \$1 would be a differential dq and of losing \$1 would be a differential dp . Then $s = 1 - dp - dq$.

With h and z constant there is no change in money demand with such a proportionate change in p and q .

Here is the *Intuitive Explanation*.

Suppose everyone in this room formed an *economy* and *everyone* traded with one another.

In an equilibrium with *constant money supply* the expected addition to money holdings is 0.

The net addition to money holdings is the sum

$$\{\text{net autonomous payments}\} + \{\text{net induced payments}\}.$$

In equilibrium with constant money supply

$$E \{\text{autonomous payments}\} + E \{\text{induced payments}\} = 0.$$

Now suppose that there is an increase in income that changes all p 's and all q 's proportionately.

FOOTNOTE: if we think of time as continuous then there are dp 's and dq 's, so there is no problem about $p + q$ being more than one. END FOOTNOTE

It's as if a *movie* of the *exchange* previously occurring had speeded up.

Everything increases proportionately — if there is no change in targets and thresholds.

Now if

$$E \{ \text{autonomous payments} \} + E \{ \text{induced payments} \} = 0$$

before the speed up, they will also equal zero after the speed up.

Thus with constant *targets* and *thresholds*, as might be expected in the short run, the *income* elasticity of money demand is zero.

Let me give you the “proof.”

The proof depends on two Facts.

Fact I.

Consider the distribution of money holdings:

$$f | p, q, s, h, z.$$

We can show that $f(x)$ is dependent only on p/q , h and z .

$f(x)$ is *independent* of s .

Proof of Fact I.

You find this by setting

$$f(x, t) = p f(x-1, t-1) + s f(x, t-1) + q f(x+1, t-1)$$

plus the 4 boundary conditions,
and *solving as we did in the last class*.

ERASE ALL BLACKBOARDS.

FACT II.

$$E(\text{autonomous payments}) + E(\text{induced payments}) = 0$$

both before and after the speed-up.

<FILL IN TABLE AS YOU GO THROUGH PROOF>

I am going to fill in a Table which I will use for the proof.

	AUTONOMOUS			INDUCED		
	Value	Probability	Expectation	Value	Probability	Expectation
Inpayments						
Outpayments						

Proof. With probability p there is an autonomous inpayment of \$1.
So expected autonomous payments are p dollars.

With probability q there is an autonomous outpayment of \$1.
So expected autonomous outpayments are q dollars.

Now let's turn to *induced payments*.

With probability $f(1)$ money holdings are 1.
In that case with probability q one dollar is lost.
Such a loss triggers an induced inpayment of z dollars.
This event occurs with probability $qf(1)$.

FILL IN TABLE

Similarly, with probability $f(h-1)$ money holdings are at $h-1$.
In that case with probability p one dollar is gained.
Such a gain triggers an induced outflow in amount $h-z$.
This event occurs with probability $pf(h-1)$.

Going back to the Table:

net autonomous inflows are

$$p - q.$$

Net expected induced flows are

$$q f(1)z - p f(h-1)(h-z).$$

So the net expected increase in money holdings will be:

$$p - q + z q f(1) - (h - z) p f(h-1).$$

Now let's not consider the general case with arbitrary p and q .
Instead let's consider the special case of Miller and Orr where p and q are both equal.

FN: This proof also works in the general case. END FN

In this special case Miller and Orr found:

$$f(1) = (1/z) (2/h)$$

$$f(h-1) = (1/(h-z)) (2/h)$$

Recall that calculation:

<GIVE DIAGRAM>

GIVE CALCULATION AGAIN

Last time we showed easily that the height of the large triangle is $2/h$.

I will not re-do the proof because it is easy, but also it is not important.

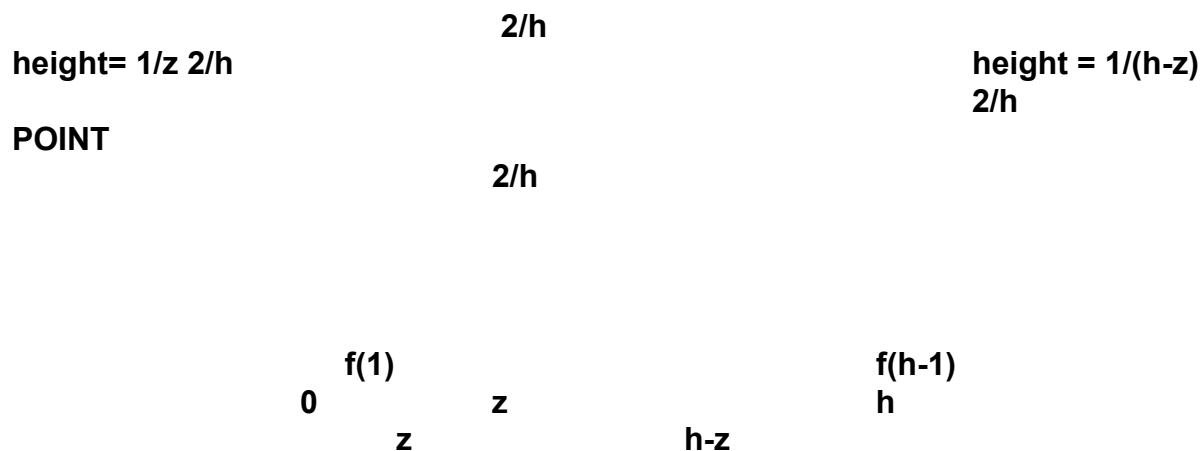
What is important is that we want to know the value of $f(1)$.

Since the distribution is exactly tent shaped it is a fraction $1/z$ the height of the large triangle, or $1/z 2/h$.

We get this because the small triangle between 0 and 1 is similar to the large triangle between 0 and z .

Similarly $f(h-1)$ will be a fraction $1/(h-z)$ of $2/h$.

We get this because the large triangle between $h-z$ and h is similar to the small triangle between $h-1$ and h .



So if $p = q$ we find

Expected value of autonomous payments + Expected value of induced payments =

$$p - q + qz [2/hz] - p (h-z) [2/(h-z)h]$$

$$= p - q - (p - q)(2/h)$$

$$= 0 \text{ as long as } p = q, \text{ and as long as } h \text{ and } z \text{ have not changed.}$$

If we multiply p and q by a factor λ after the speed up due to the increase in income the flow of money into the bank account will be

$$\lambda(p - q) - \lambda(p - q)(2/h).$$

Since the sum was 0 before, it will continue to be zero after the speed-up.

Note: this property holds for general choice of p and q , not just the special case where p and q are exactly equal. I just gave you the proof in the special case.

Restating this result:

with this type of *monitoring rule*, until h and z change, Money Demand does not respond to changes in income.

This has important implications for the effectiveness of both fiscal policy and monetary policy in changing income in the short run.

Let's first turn to fiscal policy.

We can also see from this how fiscal policy can be effective in the short run, even if there is a *low* short-run elasticity of money demand.

Let me explain further.

Namely, the interest elasticity of money demand may be *small* in the short-run because persons are sluggish in adjusting their monitoring rules in response to changes in their rate of interest r . That is the leading reason why it is commonly thought that there is a low short-run interest elasticity of money demand.

But similar sluggishness in response to changes in *income* will also make the *income* elasticity of money demand comparably small.

Aggregate demand is determined by the point where the IS curve crosses the LM curve.

The effectiveness of fiscal policy, which shifts the IS curve, depends on the slope of the LM curve. If the LM curve is vertical then shifts in the IS curve will have no effect on the equilibrium level of income.

The standard textbook logic regarding money demand is that if the demand for money is interest inelastic, then fiscal policy is ineffective. And monetarists (such as Irving Fisher) tend to believe that the demand for money is interest inelastic.

But the slope of the LM curve is

I can now re-phrase the proposition.

The short-run LM curve is not *almost* vertical because the short-run interest elasticity of the demand for money is very small.

Why not?

Because the short-run *income* elasticity is also small.

Furthermore, the same reason why the short-run elasticity of interest rates is small, also explains why the short-run elasticity with respect to income is small.

What will make them *both* small is that the monitoring rule, the targets and the

thresholds of holders of money, are slow to adjust.

We will later see that the standard estimates of the demand for money have this precise property.

The short run and the long run demands for money are comparably small —they are comparably small relative to their long-run values.

Fiscal policy will have an effect here because the folk theorem that the short-run income elasticity of the demand for money is *one* is not true with target-threshold monitoring.

On the contrary, the folk theorem is exactly false.

We can also use the Miller and Orr model to give an explanation why open market operations are so effective in changing interest rates.

The Federal Reserve thinks that by its open market operations it controls interest rates. Here we see why a very small amount of open market operations will have a large effect on interest rates.

Suppose the Fed is driving down interest rates by buying government bonds. The money supply is increasing.

Who is going to be the holder of that additional money?

Let's draw our tent-shaped distribution.

0 z h

The interest rate must fall sufficiently to get these people (POINT) in the *tail* of the distribution not to cash their money for bonds.

<NOTE: it is the upper tail of the distribution that would raise h and they would

not buy the bonds that they would have otherwise purchased if there had been no fall in the interest rate.>

However, there are not very many people in the tail of this distribution. So, in the short-run it may take a significant reduction in the rate of interest to get them to hold small additional amounts of money.

Although it is not now fashionable, I think that IS-LM theory is remarkably open for research.

There is a paper by Greg Mankiw and Larry Summers which indicates that very small changes in assumptions yield very large changes in multipliers. This suggests that we have not understood how the system works. These crazy multipliers come from the nature of money demand, and this type of analysis we have done here is the place to begin.

We are now going to look at two papers on pricing behavior—one by Caplin and Leahy and one by Caplin and Spulber.

I am going to give you the intuition behind their results in terms of the Miller and Orr model.

In fact these papers are a bit fancier mathematically than what I am going to give you in class.

But once you know Miller and Orr you will understand the intuition behind these papers.

Let me first go over the paper by Caplin and Leahy.

Let's start off with a model of pricing.

In this model there is a long-run optimal price p^* .

p^* is proportional to the money supply.

Let's denote the *log* of the optimal price as p^* and the log of the money supply as m .

In Sargent's notation

$$p^* = m.$$

Now let's say that firms suffer a loss insofar as p^* is not proportional to m . They suffer a loss insofar as they have not set the optimal price.

This loss is

$$k(p - p^*)^2.$$

Let's also suppose that there is a fixed cost, denoted a , of changing the price p .

Now suppose that the log of the money supply follows a random walk.

So in the absence of any adjustment to the price level,

with probability $\frac{1}{2}$ the gap $p - p^*$ rises by 1
 with probability $\frac{1}{2}$ the gap $p - p^*$ falls by 1.

Then the optimal policy is to have an upper threshold:

$p^* + U$ at which p is set equal to its target value p^* .

And there is a symmetric lower threshold:

$p^* - U$,

at which point also p is set equal to the target value p^* .

Indeed, we have already analyzed this situation.
 We know that is almost the Miller and Orr model.

We know from Miller and Orr that the distribution of p around p^* is tent-shaped.
 Since the problem is symmetric the distribution must also be symmetric.

So this is the distribution.

DIAGRAM

$$H = 1/U$$

$p^* - U$ p^* $p^* + U$
 Now let's ask the following question.

Suppose there is a change in p^* by +1.

This could occur because of a change in m by +1.

Suppose also firms' pricing was in their steady state equilibrium for the many firms in the economy.

So there is a tent-shaped distribution of p that centers on p^* .

Then the *expected* change in p from the change in m is the probability of being at $(p^* - U + 1) \times$ the size of the adjustment that firms will make.

That adjustment is of size U .

So the expected immediate effect on actual prices from the one unit upward change in p^* will be:

$$f(p^* - U + 1) \times U.$$

What is $f(p^* - U + 1)$?

We must first calculate H .

We know the area of the triangle POINT TO IT is the base $\times$ the height $\times \frac{1}{2}$.

The base is $2U$. The height is H .

So the area of the triangle is

$$\frac{1}{2} \times 2U \times H = UH$$

But since this is a distribution its area is 1.

So

$$UH = 1.$$

$$H = 1/U.$$

We also know $f(p^* - U + 1) = H/U$.

I will remind you how we know this:

This occurs because of the similarity of the smaller triangle between $p^* - U$ and $p^* - U + 1$, and the larger triangle between $p^* - U$ and p^* .

$$\text{So } f(p^* - U + 1) = 1/U^2.$$

So the expected short-run response in price to the change in m by one unit will be

$$1/U^2 \times U = 1/U < 1 \text{ for } U > 1.$$

U is greater than or equal to one, since 1 is the minimal number of steps that could be considered in this problem.

So in this model we find that money supply changes are not neutral. A change in the money supply will result in a less than proportionate change in the price level.

You find that when you add money that follows a random walk and the transactions cost to changing money prices, a , that the system will then act with a *lagged* response to changes in the money supply.

This is the basic idea behind Caplin and Leahy.

In contrast Caplin and Spulber consider a case where the money supply only increases.

In this case prices are uniformly distributed between the lower and the upper threshold.

The tent-shape of prices disappears, and the change in expected prices will be proportional to the money supply.

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I now want to give you some further reason for looking at this type of probabilistic demand.

It turns out that it is a generalization of a very standard demand system.

Ideally we would like to estimate a demand system that had a low short-run elasticity of demand, and a high long-run elasticity of demand.

That exactly corresponds to a stock-adjustment demand system.

We can use stock adjustment demand systems in a very simple way to estimate short-run and long-run elasticities of demand.

I will illustrate from the demand for money.

I will give you the simplest such system.

In this system current money demand is a geometrically weighted average of the long run demand for money and money demand in the previous period.

The long-run demand for money m_t^* .

The demand for money in the previous period as m_{t-1} .

This means that:

$$m_t = (m_t^*)^\gamma (m_{t-1})^{1-\gamma}$$

So m_t is the geometric mean of long-run real balances, which are m_t^* and last period's real balances, which are m_{t-1} .

Suppose that the long run demand for money depends upon income and interest. In that case (forgetting the constant term):

$$m_t^* = y_t^\alpha r_t^{-\beta} \quad \text{where}$$

$m_t^* = \text{L-R demand}$

$\alpha = \text{L-R income elasticity of demand}$

$\beta = \text{L-R interest elasticity of demand.}$

Then we can write:

$$m_t = y_t^{\alpha\gamma} r_t^{-\alpha\beta} m_{t-1}^{1-\gamma}$$

and

$$(*) \quad \ln m_t = \alpha\gamma \ln y_t - \beta\gamma \ln r_t + (1 - \gamma) \ln m_{t-1}$$

where

$\gamma = \text{rate of adjustment}$

$\alpha\gamma = \text{short run income elasticity of demand}$

$\beta\gamma = \text{short-run interest elasticity of demand}$

$\alpha = \text{long-run income elasticity of demand}$

$\beta = \text{long-run interest elasticity of demand.}$

Now (*) is very useful.

It is useful because a linear regression of (*) gives you an estimate of $\alpha\gamma$, $\beta\gamma$, and γ .

A linear regression immediately gives estimates of $\alpha\gamma$ and $\beta\gamma$, which are the short-run income and interest elasticities.

But then this estimate also gives you estimates of the long-run elasticities of demand.

How?

You also get an estimate for γ , so you can back out the values of α and β , which are the long-run elasticities.

For example, estimates by Stephen Goldfeld gave a long run income elasticity of the demand for money of .68 and of the interest elasticity is .23.

The short-run (one-quarter) income elasticity is .19 and the short run (one quarter) interest elasticity is .06.

This is a standard estimation of a money demand function.

Of course there are simultaneity problems, but they are very hard to solve, and this may be doing as well as you will ever be able to do.

But also I want you to note that it is a *particular* representation of Friedman's use of the permanent income hypothesis.

How?

You can solve the difference equation.

Ignoring the constant term you find that:

$$\ln m_t = \alpha \gamma \sum_{i=0}^{\infty} (1 - \gamma)^i \ln y_{t-i} - \beta \gamma \sum_{i=0}^{\infty} (1 - \gamma)^i \ln r_{t-i}.$$

So $\ln m_t = \alpha \ln y_p - \beta \ln r_p$

$$\text{where } y_p = \gamma \sum_{i=0}^{\infty} (1 - \gamma)^i \ln y_{t-i}$$

$$\text{and } r_p = \gamma \sum_{i=0}^{\infty} (1 - \gamma)^i \ln r_{t-i}.$$

But y_p corresponds exactly to what Friedman considered to be permanent income.

He considered it to be a geometric weighted average of past income with declining weights.

In this sense then estimating the stock adjustment equation gives a way of *estimating* Friedman's concept of y_p and r_p , and also the demand for money at the

In terms of what I wrote previously: $K_t^* - K_{t-1}$ is mandated investment.

A more general approach than the stock-adjustment approach that I just put on the board POINT would say that for individual firms the rate of adjustment would vary with the percentage gap between K_t^* and K_{t-1} .

Thus we might posit that if x is the percentage gap between K_t^* and K_{t-1} , then investment will make up a fraction $A(x)$ of that gap.

So

In other words the investment rate of a firm will be

$$I/K = A(x) x.$$

And if we posit that there is a distribution $f(x)$ over x for different firms that is independent of the capital stock K , we will find that for the economy as a whole the investment capital ratio will be:

$$I = \int A(x) x f(x) dx.$$

For convenience of notation I am now going to define I as I/K .
Let me illustrate such an $A(x)$ adjustment function.

Suppose that we have the following S-s, or target-threshold, adjustment policy.

If the desired capital stock is 20 % above the actual there is immediate adjustment. If the desired capital stock is 30 % below the actual there is immediate adjustment.
Otherwise no action is taken.

This is the picture of $A(x)$.

0

-0.3 0 +0.2

Now with a rich micro data set on firm level investment we could in fact plot $A(x)$.

How?

We know the capital stock of each firm i ,

K_{it} at time t .

We know the sales.

We know the industry.

We can estimate the long-run capital stock as

where A_j and θ_j are industry-specific constants.

Y is the firm's value added;

c is the cost of capital.

How do you know A_j , θ_j ?

A_j and θ_j are the unique values of these coefficients so that

is stationary. They use a neat trick from cointegration theory, which in fact gives rather tight estimates of K^* . I explain that in a footnote.

FN on COINTEGRATION

I want to give you a somewhat important footnote.

It turns out that there is a nice generalization of stock adjustment under special conditions. Often economic time series are not stationary. For example prices and nominal income seem to have graphs like

p, y

time

They do not seem to want to revert to some level.

Let me give you an interesting example.

Suppose

$$m_t - m_{t-1} = \gamma (m_t^* - m_{t-1}) + \epsilon_t$$

$$\text{where } m_t^* = a y_t \\ \text{and } \epsilon_t = \text{i.i.d. } N(0, \sigma_\epsilon^2)$$

and where y_t follows a random walk.

Then y_t is not stationary.

And m_t is not stationary either.

But it turns out that in this case

$$m_t - a y_t$$

will be stationary.

So while m_t is not stationary and y_t is not stationary

a linear combination of the two

$$m_t - a y_t$$

will be stationary.

If that is the case, that some linear combination of variables is stationary, even if the variables themselves are not stationary, the two variables are said to be cointegrated.

The linear combination of the variables that makes them stationary is the so-called cointegrating vector.

In fact, the stock adjustment equation is a specific example of an *error correction mechanism*.

The fraction

$\gamma(m_t - \alpha y_t)$,
which is the change, $m_t - m_{t-1}$, can be viewed as an *error correction*.

The error correction component gives the long-term relation between m and y , or the relation between m^* and y .

The long-term relation between m and y is that the error correction term

$m_t - \alpha y_t$ is zero,
or

$$m^* = \alpha y_t$$

We now have an econometric method for identifying the long-term relation between m and y .
 α will be the unique value of α that makes

$$m_t - \alpha y_t \text{ stationary.}$$

It turns out that it is fairly easy to estimate α , because the wrong value of α makes m and y deviate by a nonstationary amount, and that is likely to be large.

So the *cointegrating vector* will also give the long-run relation in an error correction model here such as we have between m_t and y_t .

This error correction approach was begun by Hendry, Davidson, Mizon and Srba, and the terminology and significance of cointegration in error correction processes were seen by Engel and Granger.

END COINTEGRATION

With this data we can then compute the value of x for each firm in each period.

It is

For each firm we know the level of investment, since that is observable.
So we know the adjustment:

So we can get the joint distribution of $A(x)$ and x .
With that we can obtain a picture of the adjustment function. We can verify whether it really follows something like an S-s policy.

Now with all this data we can play a very neat trick. We can estimate the adjustment function.

Remember

$$(*) \quad I_{j,t} = \sum_x A_j(x) \times f(x,t),$$

where industry j has an adjustment function $A_j(x)$.

Now we want to look at the shape of the adjustment function $A_j(x)$.

Suppose $A_j(x)$ is a polynomial. Suppose it is of the form:

$$A_j(x) = a_{0,j} + a_1 x + a_2 x^2 + a_3 x^3,$$

where j is the industry subscript.

The different industry adjustments vary by a constant term.

Feeding into (*) we find

$$I_{j,t} = a_{0,j} M_{j,t}^1 + a_1 M_{j,t}^2 + a_2 M_{j,t}^3 + a_3 M_{j,t}^4,$$

where $M_{j,t}^1$ is the first moment of x in industry j at t
 $M_{j,t}^2$ is the second moment of x in industry j at t
 $M_{j,t}^3$ is the third moment of x in industry j at t and
 $M_{j,t}^4$ is the fourth moment of x in industry j at t .

C, E & H know these moments because they can compute the distribution of x at t in each industry j .

From this equation and the data they have, we can estimate $a_{0,j}$ for each industry j , and also each of the three coefficients, a_1 , a_2 , and a_3 .

This gives the shape of the adjustment functions.

C, E, & H can plot the $A(x)$ function for the average industry once they know these three parameters, a_1 , a_2 , and a_3 .

The adjustment function is strikingly non-constant—indicating that firms invest proportionately more when there are large deviations of K^* from K than when there are small deviations.



This is the C, E, & H picture of $A(x)$.

The higher moments of $A(x)$ play a major role in the fluctuations of investment.

This gives added reason why changes in actual investment will not be one-for-one with changes in the L-R demand for capital stock in this type of system.

So that finishes our discussion of Miller and Orr. I want you to see C, E & H as a generalization of Miller and Orr with an application to investment demand.

It is also a significant advance over stock adjustment theory. Such stock adjustments are standard fare for macro functions such as consumer durables, inventories, money demand and also investment. What we have here is a way of simultaneously estimating long-run and short-run demands, which, of course, is at the heart of many, many economic questions.