Lecture XII   **Economics 202A**   **Fall 2007**

Exam: October 23. No class this Thursday.

Maury will continue with international next Tuesday.

It will be in the same mode as previous exams.

Today we will go over the theory of exchange rates.

Today I am going to go over the Mundell-Fleming model and the Dornbusch model.

Both of them serve as the basis for the determination of macroeconomic equilibrium with flexible exchange rates.

The Dornbusch model builds on Mundell-Fleming.

First, let me review the Mundell-Fleming model.
We are going to use IS-LM analysis.
To remind you the IS curve is the locus of income and the interest rate such that planned savings equals planned investment (or, equivalently so that sales equals production).
The LM curve is the locus of income and the interest rate such that the demand for money equals the supply of money.
Consider economies in which wages and prices are fixed, at least in the short-run which we will be considering.

r   **LM**

r*

IS

income

Suppose we have an IS and an LM curve.
Let us suppose, however, also that investors will not tolerate an interest rate in New York that is different from an international interest rate, which I will call r*.
And suppose further that there are flexible exchange rates.
We start in an initial equilibrium. <NOW PICTURE IT>
In that equilibrium the IS and the LM curve intersect at r*. (In what follows I will show why in fact that is the characteristic of an equilibrium).

Suppose the U.S. tries to shift the IS curve.
The IS curve shifts to IS′ at the old exchange rate.

r

LM

r*

$IS(e_o)$ $IS'(e_0)$

Y

Then the interest rate in the U.S. will rise relative to the international interest rate in London, and investors will buy *dollars* to take advantage of the higher U.S. interest rates.

Because investors are *buying* dollars the dollar will *appreciate.*
That appreciation will cause people to want to buy fewer U. S. goods.
As a result the demand for U.S. net exports will fall.

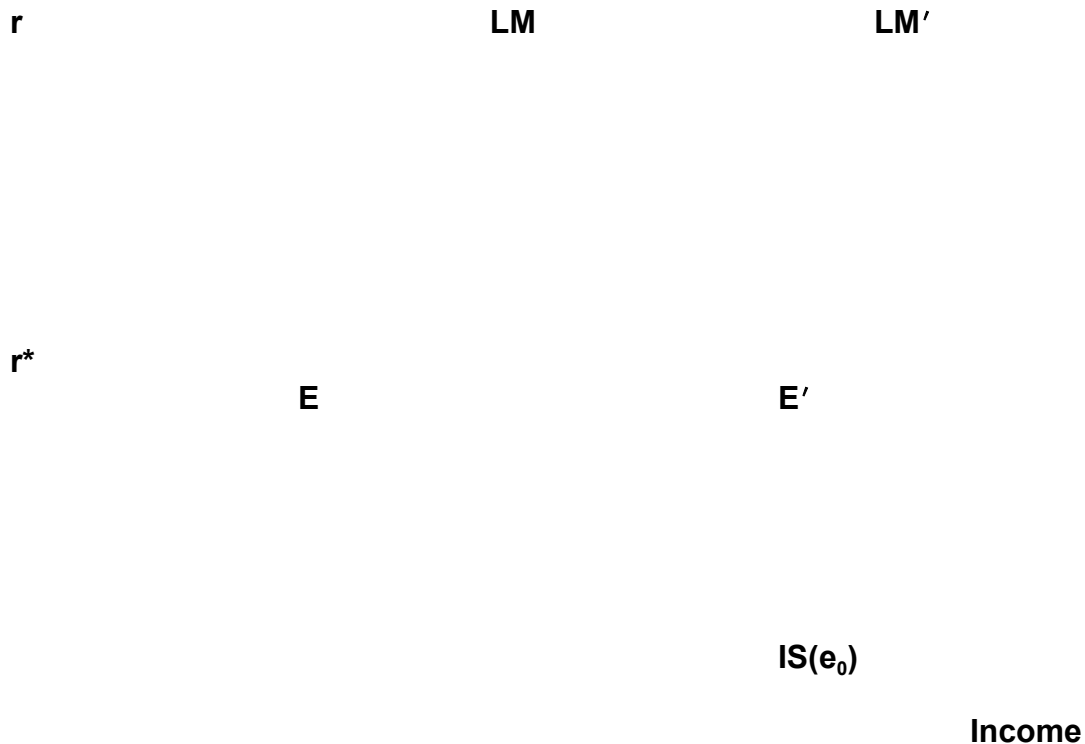**And the IS curve will be driven back as the dollar appreciates.**

The net result of the initial shift of the IS curve, however, will be an appreciated dollar and an _increased trade deficit._

So shifts in the IS curve in this model have _no effect_ on income.

A rise in U.S. government spending will only appreciate the dollar and increase the deficit in the balance of payments.

Now let's see what happens with shifts in the LM curve.
We will see that expansionary monetary policy will increase output.

r   **LM**   **LM′**

r*

                    E   **E′**

                                                        IS(e$_0$)

                                                                    Income

First we shall picture the initial equilibrium.
Then the LM curve shifts outward to LM′.

The interest rate in the US is now low. <point>
People sell dollars.
That _depreciates_ the dollar.
And that depreciation makes imports from the U.S. attractive to the rest of the

world, which shifts out the IS curve.

The interest rates will be below r* until the exchange rate has depreciated the dollar to the point where the new IS curve crosses the new LM′ curve at E′.

This is the Mundell-Fleming model.

It explains the behavior of trade deficits, exchange rates and income to shifts in the IS and LM curves.

For example, historically, it offers an excellent explanation why the Reagan Tax cuts in the U.S., provoked large fiscal deficits, were also accompanied by large trade deficits and appreciation of the U.S. dollar.

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Let's now shift gears.
Let me go over a slightly more dynamic version of the model.
This is the Dornbusch model.

Let me give you a warning.
The natural thing to do while I am presenting this lecture is to spend your time figuring out whether or not I have made a sign error in the math.
But do not do that.
If you do you are just going to get yourselves hopelessly confused, and you will miss the logic.
Even if I do make a sign error, and I do not think that I will, there is no chance that Dornbusch made one, and it is important that you get the intuition behind his model.

With this prefatory note let's proceed.

It has been observed that there is a great deal of variation in exchange rates.
In a perfect world of complete monetary neutrality there would be a relation between countries' money supplies and their exchange rates.

Let me give you an example.
In a world of perfect monetary neutrality, if Britain has a *constant* money supply and the U.S. has a 5 % increase in its money supply, the *dollar* should depreciate relative to the *pound* by 5 %.

It turns out that application of the principle of *monetary neutrality* understates the amount of exchange rate variation.

If you have not thought about this for more than 13 seconds, this may be surprising.
It may be surprising, because insofar as the relative money supplies of two countries determine the long-run exchange rates, if exchange rates are slow to adjust, as they might be in a world with wage and price stickiness, then we might (wrongly) expect there to be *less* fluctuation in exchange rates than warranted by the variation in long-run equilibrium.

If you think about it, however, for more than 13 seconds, you should not find this *such* a surprising result. In competitive markets, prices very frequently overshoot their long-run equilibrium in response to a shock.

I will give a homely example.

For example, if there is a sudden and persistent increase in the *supply of onions,* the initial decrease in the price of onions will be larger than the long-run price change, because the *short-run demand* for onions is *less price elastic* than *the long-run demand* for onions*.*

Closer to home, in terms of the Dornbusch model, suppose the demand for money is of the form:

$$(M/p)^D = y^\phi \, r^{-\lambda}.$$

To make matters easy, assume income *y* is fixed both in the *Short-Run,* and in the *Long-Run.*
Also assume that the price level, p, is fixed in the *Short-Run,* but *not* in the *Long-run.*

In the *Short-run* the interest rate equilibrates the demand and the supply for money.
In contrast, in the *long-run,* the price level changes, so this economy is *money neutral.*
In the long run *y* and *r* are unchanged by a change in M.

Then, if the money supply rises by 10 %, the *Short-run* interest rate falls by 10/λ %.
And the interest rate *overshoots* its long-run value, which was unchanged.

This is the basic principle that underlies the Dornbusch model.
The Dornbusch model relates what is happening to the domestic interest rate to what is happening to the exchange rate.
The interest rate overshoots *its* long-run equilibrium.

We shall see that this, in turn, causes the exchange rate also to overshoot its long-run equilibrium.
The model describes the interaction between the domestic interest rate and the exchange rate and why that occurs.
Let me proceed to review it and to explain its basic mechanism.

ERASE BB
USE RHBB

Let r be *the domestic interest rate* (in dollars) and
let   r* be the *world interest rate.*

In equilibrium

$$r + x = r*$$

where x is the *expected rate of appreciation of the dollar.*

Let's be specific:

r* let's say is the interest rate in London (quoted in £'s)
r is the interest rate in New York (quoted in $'s)

Let's suppose

r* = 10 %
r = 9 %

Then the dollar must be expected to appreciate against the £ by 1 % per year for people to be indifferent between holding funds in New York or holding funds in London.

Now we are going to try on for size the assumption that the exchange rate behaves according to *partial adjustment*. By *partial adjustment* I mean that it will adjust by a fraction of the difference between its current value and its long-run equilibrium value.

Later we shall, in fact, choose the single parameter for partial adjustment that will coincide with *rational expectations.*
But that is way ahead of our story.

Assume for now that there is such an adjustment in the exchange rate.

If we define the exchange rate *e* as how many *dollars you pay for a pound*,

with partial adjustment you will find that the rate of *appreciation* of the dollar *x* will be:

$$x = \theta(e - \bar{e})$$

where $\bar{e}$ is the log of the long-run exchange rate,
       e is the log of the current exchange rate, and,
       θ is the rate of adjustment of the current exchange rate, e, to its long-run value, $\bar{e}$.

And, as I said, e is how many dollars you pay for a pound.

FN: e is how many dollars you pay for a pound, so the rate of *depreciation* of the dollar is $\dot{e}$. Partial adjustment says that $\dot{e} = \theta(\bar{e} - e)$. Thus the rate of *appreciation* of the dollar, x, will be $\theta(e - \bar{e})$. END FN

This is a so-called "partial adjustment" formula because the change in e depends on the fraction of the gap between long-run value and the current value.

FOOTNOTE on terminology:

We saw last time such an adjustment model for both money demand and for investment demand. Log money made up a difference between its long run value m* and its value in the previous period, m_{t-1}, at the rate γ. For Caballero, Engel and Haltiwanger, the null hypothesis was stock adjustment-investment:

$$I_t = K_t - K_{t-1} = \alpha(K_t^* - K_{t-1}), \text{ where}$$

the change (in the capital stock) is a fraction α of the difference between the desired stock and the current stock).

END FOOTNOTE

I should reiterate that we do *not know* the rate θ at which e will converge to $\bar{e}$.

       POINT TO θ

But we will solve our system and will use the technique of matching coefficients to choose the unique value of θ that yields rational expectations.

For now, however, it is extremely useful just to see how the system behaves with

the partial adjustment formula.

<center>**************</center>

We have talked about the foreign exchange market.
Now let's talk about the domestic demand for money.

I am now going to derive an incomprehensible equation, which I shall then interpret.
This interpretation should tell you what is going on in this model.

In *log form* the demand for money is:

LHBB

$$m - p = \phi y - \lambda r$$

where m = log of nominal money supply
**   p = log of price level**
**   r = nominal interest rate**
**   y = log of real income.**

Remember that r + x = r*. <write on the side>

So I can re-write this equation as:

$$p - m = - \phi y + \lambda r^* - \lambda x$$

So

$$p - m = - \phi y + \lambda r^* - \lambda \theta(e - \bar{e})$$

In the long run <SIDE BB>

$$e = \bar{e}$$

and $$p = \bar{p}, \quad \text{<note: superscript bars are hard to read.>}$$

and so

$$\bar{p} - m = - \phi y + \lambda r^*.$$

By subtraction we get:

$$e - \bar{e} = - (1/\lambda\theta)(p - \bar{p}).$$

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Let me now pause to interpret this equation because I want you to understand it.

I think that I can show you that this equation has an *interpretation.*
And once you understand this interpretation you will understand the Dornbusch model.

Let me give you a homely example.
Suppose that markets wake up one *fateful* morning and discover that Mr. Bernanke has increased the money supply by 10 %.
In the short run the price level is constant, and income is constant.
They are constant because they take some time to adjust.

So the domestic interest rate must adjust the demand for money to the supply.

Let's say that the domestic interest rate had been 20 % in the old equilibrium, and the interest elasticity of the demand for money is ½.

So the domestic interest rate must adjust by 10 percent of 20 percent divided by ½ to induce domestic holders to hold Mr. Bernanke's extra dollars of money supply.
So the domestic interest rate must fall by 4 percentage points to 16 percent.

The exchange markets see the US domestic interest rate at 16 percent, and the London rate is 20 percent.

So people begin *selling dollars* and *buying pounds.*

This will drive down the value of the dollar.
Remember, however, that no dollar assets will ever be held in this model until it is foreseen that the dollar is going to *appreciate* against the pound.

Since the domestic rate is 16 % and the foreign rate is 20 percent, the dollar must be *appreciating* at the rate of 4 percent per year.

In the long-run the 10 percent change in the U.S. money supply will cause U.S. prices to rise by 10 percent, and thus the dollar should *depreciate* relative to the pound by 10 percent in the long-run.

Suppose the instant reaction in exchange markets was a reduction of *less than* 10 percent. Could that be an equilibrium?
        The answer is *no.*

Why not?

Because the domestic interest rate is 16 %.
And with a depreciation of less than 10 percent people will expect further *depreciation* of the dollar.

Let's try another depreciation and see if it could be an equilibrium.

Suppose the immediate depreciation is the long-run depreciation of 10 percent. Could *that* be an equilibrium?

No. In that case the expected rate of appreciation of the dollar is

$$\theta(e - \bar{e}),$$

which is 0.

But domestic rates are 16 percent, while foreign rates are 20 percent, so there must be expected appreciation of the dollar.

Will there be an equilibrium anywhere?
Suppose the depreciation on this *fateful* morning *overshoots* the long-run depreciation.
Suppose the *dollar* becomes *very low* relative to the pound.
Then an *appreciation* is expected.
        The greater the depreciation on this fateful morning the faster the subsequent *appreciation.* This follows from the partial adjustment formula.

So there will be a level of the exchange rate sufficiently low, at which the rate of appreciation is just 4 %. That will be just enough to make investors indifferent between holding their portfolio in London and holding their portfolio in New York.

This is what underlies the equation:

$$e - \bar{e} = - (1/\lambda\theta) (p - \bar{p}).$$

1. $p - \bar{p}$ is the 10 percent on that morning.

2. the domestic interest rate went down by: $(p - \bar{p})/\lambda$.

3. Now how much must be the difference between e and ē to make the dollar appreciate at the rate: $(p - \bar{p})/\lambda$?

$$e - \bar{e} \text{ must be equal to } - (1/\lambda\theta) (p - \bar{p}).$$

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So this gives us our key equation for the exchange rate:

$$e - \bar{e} = -(1/\lambda\theta)(p - \bar{p}).$$

And this equation *always holds*, including outside of equilibrium.
It holds because it is the *instant* arbitrage condition between London and New York.

I will summarize:
Insofar as p falls short of $\bar{p}$, money demand departs from long-run money demand by that percent.
To re-equate money demand to supply the domestic interest rate must depart from its long- run equilibrium by

$$(p - \bar{p})/\lambda.$$

This means that the rate of appreciation *x* must be:

$$x = -(p - \bar{p})/\lambda.$$

And if x has the partial adjustment form, then:

$$\theta(e - \bar{e}) = -(1/\lambda)(p - \bar{p}).$$

FN: x is the negative of $\dot{e}$, where $\dot{e}$ is the rate of change of the log of the exchange rate or the percentage change in the exchange rate. END FN

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I have now explained Dornbusch's key equation. Let us now proceed.
If the price level had moved immediately to its long-run level $\bar{p}$, so would the exchange rate have moved to its long-run level $\bar{e}$.

The real problem in this economy is that prices are a *state variable.*
They do not move quickly.
That is reflected in the Phillips Curve.
That is why the interest rate had to adjust in the *short run.*
And in turn that is the reason that the exchange rate overshoots.

If we want to complete our model we must have an equation for the rate of change of prices, for $\dot{p}$.

Let us assume that the rate of price change depends on the gap between supply and demand.
That, of course, is a Phillips Curve.

$$\dot{p} = \pi \ln D/Y = \pi[\mu + \delta(e - p) + (\gamma - 1) y - \sigma r]$$

        where e - p is the real exchange rate
             y is real income and
             r is the nominal rate of interest.

FOOTNOTE: real demand depends of course on real income, the real exchange rate, and the *real* interest rate, which is r - $\dot{p}$, but then we can reach this equation after eliminating $\dot{p}$ from the RHS. So this equation is a reduced form.
END FOOTNOTE

We are now ready for Dornbusch's key diagram.

Let's put p on one axis.
Let's put e on the other.

p

e

PUT IN DIAGRAM

In the long run p̄ is proportional to ē:

      i.e.: a 10 % change in the money supply results in a 10 percent change in the long-run value of p and the long-run value of e.

So the long-run equilibria are along a 45° line.
We can adopt units so that it goes through the origin, which makes the picture look nicer.
DRAW IT IN

Suppose that we were initially at the long-run equilibrium: $(\bar{e}_0, \bar{p}_0)$. SHOW IT.

p

$(\bar{e}_1, \bar{p}_1)$

$(\bar{e}_0, \bar{p}_0)$

45°

e

Then Mr. Bernanke moved in the night.
The long-run equilibrium changes to: $(\bar{e}_1, \bar{p}_1)$,
with $\bar{e}_1$ proportional to $\bar{p}_1$.   SHOW IT.

In the short-run for reasons already explained:

$$(\bar{e}_1 - e) = -(1/\lambda\theta)(\bar{p}_1 - p).$$

The line $(\bar{e}_1 - e) = -(1/\lambda\theta)(\bar{p}_1 - p)$.

goes through $(\bar{e}_1, \bar{p}_1)$ with slope $-(1/\lambda\theta)$.

DRAW IN LINE

At the initial time, p_0 is fixed so e shoots up to e_{SR}, and then falls on the path indicated towards $(\bar{e}_1, \bar{p}_1)$.

DRAW IN ARROWS

So the initial response to the money supply is that the exchange rate overshoots its long-run equilibrium.

Now the $\dot{p}$ equation determines how fast we are moving up the S-R equilibrium line toward the new equilibrium.
So, as you can see, the exchange rate is changing.

POINT TO LINE WITH ARROWS

We have one last task.
Can we find the value of θ that is consistent with the $\dot{p}$ equation.
It should not be too hard to find.

First remember

$$\dot{e} = \theta(\bar{e}_1 - e).$$

Now define v as the rate at which prices are approaching $\bar{p}_1$.

It turns out that $v = \theta$.

FOOTNOTE:
Proof:

Since $(\bar{e}_1 - e) = -(1/\lambda\theta)(\bar{p}_1 - p)$,

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differentiating

$$\dot{e} = -(1/\lambda\theta)\,\dot{p} = -(1/\lambda\theta)\,v(p_1 - p) = v\,(\bar{e}_1 - e)$$

But $\dot{e}$ **is also equal to** $\theta\,(\bar{e}_1 - e)$.

So $\theta\,(\bar{e}_1 - e) = v\,(\bar{e}_1 - e)$,
and $\theta = v$. **END FOOTNOTE**

So our very last task is to see if we can find out what v actually might be.

That way we find the unknown value of θ **that we postulated at the very beginning.**

We will use all of our equations to find out what v must be.
We will use the method of matching coefficients to find the consistent value of θ.

To recall,

$$\dot{p} \qquad = \pi[\mu + \delta(e - p) + (\gamma - 1)\,y - \sigma r]$$

$$= \pi[\mu + \delta(e - p) + (\gamma - 1)\,y - \sigma r^* + \sigma x].$$

We are going to substitute for e and for x:

$$e = \bar{e}_1 + (1/\lambda\theta)\,(p_1 - p),$$

$$x = \theta\,(e - \bar{e}_1) = \theta\,(1/\lambda\theta)\,(p_1 - p).$$

**You substitute in and collect terms, and you will find
that** $\dot{p}$ **is of the linear form:**

$$\dot{p} = A\,(p_1 - p) + B,$$

where A and B have the following values:

$$A = \pi[\,(\delta/\lambda\theta) + \delta + \sigma\,\theta\,(1/\lambda\theta)]$$

$$B = \pi[\mu + \delta\,\bar{e}_1 - \delta\,p_1 + (\gamma - 1)\,y - \sigma r^*].$$

Now I am going to use a small trick.

In the long run

$$\dot{p} = 0, \text{ and also}$$

$\overline{p}_1$ = p, so that

$$B = 0.$$

Thus

$$\dot{p} = \pi[\,(\delta/\lambda\theta) + \delta + \sigma\,\theta\,(1/\lambda\theta)](\,\overline{p}_1 - p).$$

In other words,

$$v = \pi[\,(\delta/\lambda\theta) + \delta + \sigma\,\theta\,(1/\lambda\theta)] = \theta.$$

Solution of this equation gives the consistent value of θ, given δ, λ, σ and π:

This is the exact trick of matching coefficients.

The exact solution choosing the positive root is:

$$\theta = \pi/2\,(\delta + (\sigma/\lambda)) + (\tfrac{1}{2})\,[\pi^2\,(\delta + (\sigma/\lambda))^2 + 4\pi\delta/\lambda]^{1/2}.$$

We have used the exact trick of matching coefficients.

We have by definition:

$$\dot{p} = \theta\,(\,\overline{p}_1 - p).$$

    We derived from the model:

$$\dot{p} = k(\theta)\,(\,\overline{p}_1 - p).$$

This yields an equation for θ:

$$\theta = k(\theta).$$

So we have completely solved the problem!!!

This is the basic model of exchange rates.
There may be all kinds of variations that reflect reality better, but this is the fundamental model.

It depends a great deal on the traditional concept of the demand for money, which we visited last week.

And it also depends on perfect international arbitrage of interest rate variations.