Economics 202A Lecture Outline, November 27, 2007 (Version 1.2)

Maurice Obstfeld

Government Revenue from Money Creation

In general the central government has a monopoly right to issue money, and that privilege is a source of revenue. This lecture shows how to integrate money creation by the central government into national budgetary accounts. They relate the discussion to the notion of the revenue-maximizing rate of monetary growth. For a classic application, see the paper on "Unpleasant Monetarist Arithmetic" by Sargent and Wallace.

What is seigniorage?

If the private sector is willing to hold paper money that the government supplies, the government can buy real goods and services that the private sector produces with money that is (virtually) costless for the government to print.[1] The real resources that the government acquires in this way equal its *seigniorage* revenue. To define seigniorage we need not know how or why the private sector is willing to accept the government's fiat money; all that matters is that there is a demand for it.

In a discrete time mode, seigniorage in period t is given by

$$\frac{M_t - M_{t-1}}{P_t},$$

that is, it is the real resources the government acquires through increases in the nominal money balances the public is willing to hold. A useful way to rewrite this expression is as

$$\frac{M_t - M_{t-1}}{P_t} = \pi_t m_{t-1} + (m_t - m_{t-1}), \tag{1}$$

[1] Money that is not backed by a real commodity (such as gold) is called *fiat money*.

where $\pi_t \equiv (P_t - P_{t-1})/P_t$ and $m \equiv M/P$. This expression emphasizes two distinct sources of seigniorage. First is the inflation tax, the amount people must give to the government to hold their real money balances constant in that face of rising prices.[2] Second is the public's desire to alter its real money holdings, given the inflation rate. The same decomposition applies in continuous time. Seigniorage at time t is

$$\frac{\dot{M}(t)}{P(t)} = \pi(t)m(t) + \dot{m}(t),$$

as you can easily check. Observe that seigniorage *need not* equal inflation tax revenue, which is πm only.

The revenue-maximizing steady-state inflation rate

An important theoretical concept is the *revenue-maximizing steady-state inflation rate*: what is the highest rate at which we can squeeze golden eggs out of the proverbial goose? It turns out that the concept is slightly ambiguous, and for a reason that lies at the heart of discussions over time inconsistency in monetary policy.

One approach to the problem is to simply maximize $\dot{M}/P$, which does equal πm in a steady state (since $\dot{m} = 0$.) Thus, if r is the real interest rate and money demand is a declining function of $i = r + \pi$, we solve

$$\frac{\mathrm{d}}{\mathrm{d}\pi}\pi m(r + \pi) = 0,$$

which yields

$$m + \pi m'(i) = 0,$$

or

$$-\frac{\pi m'(i)}{m} = 1. \tag{2}$$

[2] You probably are used to defining inflation as $(P_t - P_{t-1})/P_{t-1}$. However, the inflation concept that concerns us here is the fraction of an agent's real balances that is "confiscated" through a rise in the price level, and that equals $(P_t - P_{t-1})/P_t = \pi_t$. Notice that as the rate of price level increase becomes arbitrarily big, $\pi_t \to 1$, meaning that an "infinite" rate of price increase reduces the value of real balances by 100 percent.

This formula instructs us to look for the point on the money demand curve where the inflation elasticity is 1. It is a standard monopoly pricing formula, which equates the marginal cost of producing money (zero) to the marginal revenue from creating it (zero, at the point where condition (2) holds).[3]

Conceptually this approach is a bit unsatisfactory because it apparently fails to answer the following dynamic question. Suppose we are in a steady state with inflation rate π. When will it be the case that we cannot raise present and future seigniorage revenue by raising inflation? The fundamental difference between this question and the one answered in the last paragraph is that now we must worry about how the *initial* inflation change, which occasions a price level jump and a jump in real money demand impacts seigniorage revenue. Suppose that the economy always jumps to its steady state in response to an unexpected change in inflation. Then, according to eq. (1)—which is appropriate because there will be discrete changes in P and in m at the moment of the change—the present discounted value of the change in seigniorage revenue resulting from a small change in inflation is

$$\underbrace{\tfrac{1}{P}\tfrac{\mathrm{d}P}{\mathrm{d}\pi}m + m'(i)}_{\text{initial stock adjustment}} \qquad + \qquad \underbrace{\textstyle\int_0^\infty \mathrm{e}^{-rt}\left[m + \pi m'(i)\right]\mathrm{d}t,}_{\text{ongoing inflation tax flow}} \tag{3}$$

where $\mathrm{d}P$ denotes the initial equilibrium price *level* change due to the change in the inflation rate.[4] Observe that since the *nominal* money supply M is

[3] I assume the second-order condition that the function $\pi m(i)$ is concave where the preceding condition holds:

$$2m'(i) + \pi m''(i) < 0.$$

[4] It may be helpful to note that the path of the price level P may not be differentiable. Strictly speaking, I am thinking of $\pi(t)$ here as the expected rate of inflation *going forward* — in technical terms, the right-hand derivative of the price level, or

$$\lim_{h\to 0}\frac{P_{t+h} - P_t}{hP_{t+h}}.$$

The effect we are considering is the impact of this change in the expected future inflation rate on the price level and real money demand today.

not changed at time 0 when π rises,

$$m'(i) = \frac{\mathrm{d}m}{\mathrm{d}\pi} = -\frac{M}{P^2}\frac{\mathrm{d}P}{\mathrm{d}\pi} = -\frac{1}{P}\frac{\mathrm{d}P}{\mathrm{d}\pi}m.$$

Thus, the equation for total discounted seigniorage revenue reduces to

$$\int_0^\infty \mathrm{e}^{-rt}\left[m + \pi m'(i)\right]\mathrm{d}t.$$

(Intuition for the cancellation of initial effects: The initial nominal money supply does not jump, so the government cannot gain any seigniorage at the initial instant.) Plainly, maximizing this with respect to π simply leads to the same answer we found before, eq. (2).

This solution is still somewhat problematic, however, because it entails an unexpected expropriation of private sector real wealth equal to

$$\frac{1}{P}\frac{\mathrm{d}P}{\mathrm{d}\pi}m.$$

This surprise inflation tax receipt offsets the decline in seigniorage revenue caused by the initial fall in real money demand when π is raised.

We could well imagine, however, that the government has promised to avoid surprise changes in the value of real balances; perhaps, like Brazil in 1998, it is allowing prices to rise gradually over time but has pledged not to engage in a "maxi-devaluation" that reduces the value of the currency by a discrete amount. In the presence of such an "honest government" constraint,[5] a small rise in inflation would raise government seigniorage revenue by only

$$m'(i) + \int_0^\infty \mathrm{e}^{-rt}\left[m + \pi m'(i)\right]\mathrm{d}t,$$

and not by the amount in eq. (3). The reason: to ensure that $\mathrm{d}P = 0$ when inflation rises (say), the government must reduce the nominal money supply sharply; it might finance this loss in seigniorage by selling bonds, for example, but it cannot finance it by a surprise inflation tax on the private

[5] The terminology comes from Leonardo Auernheimer, "The Honest Government's Guide to the Revenue from the Creation of Money," *Journal of Political Economy* 82 (May/June 1974): 598-606.

sector, as before. Setting the last expression equal to 0, we find that the optimal constrained inflation rate satisfies

$$m'(i) + \frac{m + \pi m'(i)}{r} = 0,$$

or

$$-\frac{im'(i)}{m} = 1. \tag{4}$$

This solution, which sets the *interest* elasticity of money demand to 1, results in a lower inflation rate than solution (2) because the government is now concerned for the seigniorage it will lose in the initial jump to the new steady state.

Another way to look at the problem is to ask what discounted government revenue would be, given initial private real balances m_0 and the initial price level, at different levels of i.[6] There are two components. First, the money the government must sell to peg the price level at the moment it sets i, equal to $m(i) - m_0$, equals the discounted revenue

$$\int_0^\infty \mathrm{e}^{-rt} r \left[m(i) - m_0\right] \mathrm{d}t,$$

which represents the real interest savings from being able to issue an initial supply of interest-free debt. Second, there is the inflation-tax component

$$\int_0^\infty \mathrm{e}^{-rt} \pi m(i) \mathrm{d}t,$$

which is levied on the totality of real balances. The sum of these two components is

$$\int_0^\infty \mathrm{e}^{-rt} i m(i) \mathrm{d}t - m_0. \tag{5}$$

This formulation is useful in thinking about the government and private sector budget constraints.

[6]In the special case $m_0 = 0$, we have the problem: at what level should the nominal interest be set to maximize the seigniorage from introducing a *new* currency?

Aggregate private-sector budget constraint

Let A be aggregate private-sector nominal assets

$$A = M + B,$$

where B denotes nominal bond holdings by the private sector The (flow) finance constraint of the private sector is

$$\dot{A} = Py + iB - P\tau - Pc$$

where y is real output, τ real taxes, and c real consumption. This equation can be expressed in real terms as

$$\begin{aligned}
\dot{a} &= \frac{\dot{A}}{P} - \pi a = y + ib - \tau - c - \pi a \\
&= y + i(m+b) - \tau - c - \pi a - im \\
&= y + ra - \tau - c - im,
\end{aligned}$$

where $a = A/P$ and $b = B/P$. Above, think of r as a given (and, for simplicity, constant) world real interest rate.

Integrate this last expression forward from $t = 0$ and apply the terminal condition, $\lim_{t\to\infty} \mathrm{e}^{-rt} a(t) = 0$, to obtain the (stock) intertemporal budget constraint

$$m(0) + b(0) = \int_0^\infty \mathrm{e}^{-rt} \left[c(t) + i(t)m(t) - y(t) + \tau(t)\right] \mathrm{d}t. \tag{6}$$

The constraint's interpretation is straightforward. The present discounted value of private expenditure (on consumption and the services of real money balances) can exceed that of after-tax labor earnings by the value of initial financial assets $m(0) + b(0)$, but by no more.

Public-sector budget constraint

Let D stand for the nominal value of the government's interest-bearing debt. The government issues interest and non-interest bearing debt (the latter being money) to cover its deficit:

$$\dot{D} + \dot{M} = Pg + iD - P\tau,$$

where g is real government consumption. We may alternatively express this relation in real terms as

$$\begin{aligned}\dot{d}+\dot{m} &= g+id-\tau-\pi d-\pi m \\ &= g+rd-\tau-\pi m \\ &= g+r(d+m)-\tau-im.\end{aligned}$$

Integrating forward from $t=0$ yields

$$m(0)+d(0)=\int_0^\infty \mathrm{e}^{-rt}\left[i(t)m(t)+\tau(t)-g(t)\right]\mathrm{d}t, \tag{7}$$

provided $\lim_{t\to\infty}\mathrm{e}^{-rt}\left[m(t)+d(t)\right]=0$.

Compared to a nonmonetary economy, the novel element in (7) is that government resources are augmented by a net revenue from issuing money equal to

$$\int_0^\infty \mathrm{e}^{-rt}i(t)m(t)\mathrm{d}t-m(0).$$

We can now understand how this expression—which necessarily equals the net cost of real money balances to the private sector in (6)—arises. It equals

$$\int_0^\infty \mathrm{e}^{-rt}r\left[m(t)-m(0)\right]\mathrm{d}t+\int_0^\infty \mathrm{e}^{-rt}\pi(t)m(t)\mathrm{d}t,$$

the sum of (i) real interest savings due to the ability to issue non-interest bearing debt and (ii) inflation tax proceeds.

The economy's real *net foreign assets* are given by

$$f=b-d.$$

Observe that by subtracting (7) from (6) we get the economy-wide resource constraint from $t=0$ onward,

$$f(0)=\int_0^\infty \mathrm{e}^{-rt}\left[c(t)+g(t)-y(t)\right]\mathrm{d}t.$$

This simple "real" constraint follows because domestic money is not held by foreigners, and domestic residents hold no money issued by foreign governments.