Lecture IV **Economics 202A** **Fall 2007**

At the end of this morning's class I promised you that I would go over Sargent's model of the Lucas Critique.

That will be the topic of this class.

Before I begin the Sargent model, I must go over an important technical detail.

Sargent has terms in his equations, such as

$$p_t - p_{t-1}$$

where lower case p_t is the log of the price level at t and lower case p_{t-1} is the log of the price level at t-1.

In fact, such a term is almost equal to the percentage change in the price level. It is almost equal to the *rate of inflation.*

Let me show you why.

Let upper case P_t be the *Price Level.*

$$p_t - p_{t-1} = \ln P_t - \ln P_{t-1}$$
$$= \ln P_t / P_{t-1}.$$

Let me make an assertion.

My assertion is that

$\ln P_t / P_{t-1}$ is approximately equal to

or,

which is the percentage change in the price level.

How do I know that

Consider any number close to one.

I will show you that

$\ln x \cong x - 1$.

I know by Taylor series expansion that:

$\ln 1 = 0$, POINT AND ALSO

So $\ln x \cong (x - 1)$.

Using this approximation

If you do not follow what I have said here now I want you to be able to accept my interpretation of Sargent's formulas, and then you can later come back and verify that in fact this formula follows.

ERASE BB

I am now going to present to you Sargent's 3 equations. I will use his notation, which will make it easier for you to read it.

David Romer explains in detail how they arise out of microfoundations.

I am going to just write them down and explain why these correspond to standard macroeconomics from your intermediate course.

Equation (1) is an *aggregate supply equation.*

(1) $y_t = k_t + \gamma (p_t - {}_tp^*_{t-1}) + u_t$

where y_t is real income

k_t is potential GDP

p_t is the actual price level at time t

**${}_tp^*_{t-1}$ is the expected price level at time t,
with the expectations made at time t-1, and**

u_t is an uncorrelated random variable.

All of these variables are in *logarithms*, so I should have been more careful and said

y_t is *the log of* real income

k_t is *the log of* potential GDP

p_t is the *log of the* actual price level at time t, and

**${}_tp^*_{t-1}$ is *the log of the* expected price level at time t,
with the expectations made at time t-1.**

Equation (2) is an IS curve.

Again the variables are in *logs* with the exception of the *nominal* rate of interest r_t.

(2) $y_t = k_t + c (r_t - ({}_{t+1}p^*_t - p_t)) + d z_t + e_t$

where r_t is the nominal rate of interest,

**z_t is a vector of exogenous variables,
including *government spending and tax rates,* and**

e_t is a random error term.

Equation (3) is an LM curve

(3) $m_t = p_t + y_t + br_t + \eta_t$,

where m_t is the *log* of the money supply,
p_t, y_t, and r_t are as before
and η_t is a random error term.

While this notation is slightly hard to read, it turns out that this is *exactly* the standard Keynesian model with the standard Phillips Curve describing labor supply.

Let me now review the equations in reverse order.

Equation (3) is the demand for money. The level of this demand will be proportional to prices and real income. And it will depend negatively on the *nominal rate of interest.*

Equation (2) is an IS Curve.

k_t is potential GDP.

Demand depends on the real rate of interest and other variables.

r_t is the nominal rate of interest.

And using our previous reasoning you can check that

${}_{t+1}p_t^* - p_t$ is the expected rate of change of prices.

So $r_t - ({}_{t+1}p_t^* - p_t)$ is the *nominal rate* of interest *minus* the expected rate of price change. That then is the expected real rate of interest.

If we accept that $r_t - ({}_{t+1}p_t^* - p_t)$ is the expected real rate of interest, we can see that except for possible quibbles over functional form, this equation is an IS Curve for a closed economy.

The standard IS condition, to recall, is that
Sales = Production

in a closed economy.

This condition is therefore

$$C \quad + \quad I \quad + \quad G \quad = Y$$

C depends on real income and the real rate of interest and taxes.

I depends upon real income and the real rate of interest.

FILL IN:

$$C\ (Y,\ \text{re r},\ T) + I\ (Y,\ \text{re r}) \quad + \quad G \quad = Y$$

We can linearize and find as an approximation:

$$Y = k + c\ \text{re r} + d\ z$$

where z includes taxes and government spending.

So equation (2) is just new compact notation for an IS curve.

Now let's return to equation (1).

Equation (1) turns out to be just a standard Phillips Curve in a new notation.

k_t is GDP potential.

y_t - k_t is then the deviation between current income and GDP potential, and since it is in log form it is really the percentage deviation from GDP potential.

According to Okun's law the unemployment rate is a multiple of the percentage deviation from GDP potential.

So we could rewrite

y_t - k_t as $b(U^* - U_t)$ where $U^* - U_t$ is the deviation of the unemployment rate from the natural rate.

So at this point we can see that

$$b(U^* - U_t) = \gamma\ (p_t - {}_tp^*_{t-1}) + u_t,$$

merely by substituting for y_t - k_t its equivalent in unemployment rates rather than in percentage deviation from GDP potential.

Let me now take one minor algebraic step and I will show you that this equation is equivalent to the Standard Accelerationist Phillips Curve.

Let me make the merely algebraic operation of *adding and subtracting* γp_{t-1} to get

$$b(U^* - U_t) = \gamma (p_t - p_{t-1}) - \gamma({}_t p^*_{t-1} - p_{t-1}) + u_t.$$

Now $p_t - p_{t-1}$ is the difference in log prices and is therefore approximately the rate of inflation, which I will denote π_t.

And, ${}_t p^*_{t-1} - p_{t-1}$ is approximately the expected rate of inflation, which I will denote π^e_t.

We then get

$$b(U^* - U_t) = \gamma(\pi_t - \pi^e_t) + u_t.$$

We can solve for π_t and obtain:

This is the standard accelerationist Phillips Curve that you must have been taught in your intermediate macroeconomics class.

Sargent gives a rather different interpretation of this equation. I will go over that later.

So far I have shown you that all three equations here correspond to what you were taught in the equivalent of Economics 100B, or Economics 101B.

To that I am going to add the assumption of *rational expectations*.

The assumption of rational expectations is the following:

$${}_t p^*_{t-1} = E(p_t | \theta_{t-1}).$$

<Add this as (4) below other three equations.>

That is: the expectations people make at t-1 about the price level at t is:

the expected value of the price level which will actually occur given the information available at t-1. The symbol θ_{t-1} here represents the information available at t-1.

Two implications follow from Sargent's model + Rational expectations.

(1) FIRST, a systematic monetary policy will have no *expected effect* on equilibrium income in this model.

(2) SECOND, deviations in income from GDP potential will not be serially correlated.

And, *yet more strongly,* these deviations will have no correlation with any previously observable economic variable.

I now want to put this exact model on hold and show you these propositions in a simpler model, so you can understand why these propositions are going to hold.

They follow easily once one uses our time series notation in the right way.

Let me remind you of the two key equations in the model that we are going to use.

The first is the accelerationist Phillips Curve.
Sargent writes that equation in the form:

(1) $$y_t = k_t + \gamma\,(p_t - {}_tp^*_{t-1}) + u_t$$

The second equation is rational expectations regarding the price level.

That equation is:

(4) $${}_tp^*_{t-1} = E(p_t|\theta_{t-1}).$$

I will review the model and its implications presently.

What you are going to see is that with rational expectations ${}_tp^*_{t-1}$ is going to mimic p_t so closely that the only difference between the two will be a random term uncorrelated with any past variable. It will be easy to see this using the time series analysis that we have just reviewed.

Thus what we are going to find invariably is that rational expectations makes the

Phillips Curve equation into:

$$y_t - k_t = \gamma (p_t - {}_tp^*_{t-1}) + u_t$$

$$= \gamma \epsilon_t + u_t,$$

where ϵ_t is a random variable uncorrelated with any prior variable.

To understand this proposition I am going to start with a simple comprehensible example.

Rational expectations is particularly useful in describing expectations of ARMA processes.

Suppose p_t is an AR(1) process.

$$p_t = \alpha p_{t-1} + \epsilon_t .$$

What would ${}_tp^*_{t-1}$ be?

With rational expectations:

$${}_tp^*_{t-1} = E(p_t|\theta_{t-1})$$

$$= E [(\alpha p_{t-1} + \epsilon_t)|\theta_{t-1})]$$

$$= E [\alpha p_{t-1}|\theta_{t-1}] + E [\epsilon_t|\theta_{t-1})]$$

$$= \alpha p_{t-1} + 0$$

αp_{t-1} part since p_{t-1} is known at t; POINT; 0 part since ϵ_t is uncorrelated with prior information: POINT

Sargent later claims that

$$E[(p_t - {}_tp^*_{t-1})|\theta_{t-1}] = 0.$$

Here in our simple example we can see that

$$p_t = \alpha p_{t-1} + \epsilon_t$$

$${}_tp^*_{t-1} = \alpha p_{t-1}$$

So $\quad p_t - {}_tp^*_{t-1} = \alpha p_{t-1} + \epsilon_t - \alpha p_{t-1} = \epsilon_t.$

So $E[(p_t - {}_tp^*_{t-1})|\theta_{t-1}] = E[\epsilon_t|\theta_{t-1}] = 0.$

I wanted to illustrate this because I am going to follow Sargent and also David Romer in a very abstract proof that follows and I want you to associate what's *very* abstract with something more concrete, like an ARMA process.

Let me now repeat the implications of Sargent's model.

(1) A systematic monetary policy will have *no expected effect* on equilibrium income.

(2) Deviations in income from GDP potential will not be *serially* correlated with any past observed variables.

You will understand that proof if you think about what is happening in the preceding example.

I will give his proof, which is much too abstract. Then I will discuss it at some length to make what is happening intuitive.

Using equation (1) POINT

$$E(y_t\,|\theta_{t-1}) = E(k_t\,|\theta_{t-1}) + \gamma\, E[(p_t - {}_tp^*_{t-1})|\theta_{t-1}] + E(u_t\,|\theta_{t-1})\,.$$

We can rewrite this as

$$E[(y_t - k_t)|\theta_{t-1}] = \gamma\, E[p_t|\theta_{t-1}] - \gamma\, E[{}_tp^*_{t-1})|\theta_{t-1}] + E(u_t\,|\theta_{t-1})\,.$$

Now let's consider the last two terms. <POINT>

<SIDE BOARD>

$E(u_t\,|\theta_{t-1}) = 0$ because u_t is an innovation uncorrelated with *past* events.

It is defined as this period's random supply shock.

Now let's examine

$$E[{}_tp^*_{t-1})|\theta_{t-1}].$$

By the definition of rational expectations it is:

$$E[\,E(p_t|\theta_{t-1})|\theta_{t-1}]\,.$$

If you think a long time about it you will see that

$E(p_t|\theta_{t-1})$ is a *constant*.

It is a number independent of events that will actually occur at time t.

SO:

$E[\,E(p_t|\theta_{t-1})|\theta_{t-1}] = E[\,p_t|\theta_{t-1}]$.

And as a result, <GO BACK TO MAIN BOARD>

$E[(y_t - k_t)|\theta_{t-1}] = \gamma\, E[\,p_t|\theta_{t-1}] - \gamma\, E[\,p_t|\theta_{t-1}] = 0.$

This result says that there is nothing in the information set at time t-1 that is useful in predicting $y_t - k_t$, which is the potential deviation of income from GDP potential.

From this result there are two consequences.

First,

Last period's *deviation* of income from GDP potential is part of last period's information set.

Since nothing in last period's information set is helpful in predicting this period's income deviation, this period's income deviation and last period's income deviation must be uncorrelated.

Second,

**The systematic part of monetary policy is part of last period's information set. And nothing in last period's information set is *helpful* in predicting this period's income.
So the *expected effect of systematic* monetary policy on this period's income is 0.**

ERASE BLACKBOARD

The preceding proof, which is Sargent's proof, is too abstract.

At the same time it gets what is fundamentally happening.

So let me try to motivate it by presenting some examples.

I want you to see what that proof actually means.

Let's go back to our earlier example, which I shall repeat so that you can now see it in the context of Sargent's proof.

Suppose p_t followed an AR(1) process.

$$p_t = \alpha p_{t-1} + \epsilon_t \,.$$

With rational expectations

$$\begin{aligned} {}_tp^*_{t-1} &= E\,[(\alpha p_{t-1} + \epsilon_t)|\theta_{t-1})] \\ &= E\,[\alpha p_{t-1}|\theta_{t-1}] + E\,[\,\epsilon_t|\theta_{t-1})] \\ &= \alpha p_{t-1} + 0 \\ &= \alpha p_{t-1}. \end{aligned}$$

Thus in Sargent's model

$$\begin{aligned} y_t - k_t &= \gamma\,(\,p_t - {}_tp^*_{t-1}) + u_t \\ &= \gamma\,(\,\alpha p_{t-1} + \epsilon_t - \alpha p_{t-1}) + u_t \\ &= \gamma\,\epsilon_t + u_t. \end{aligned}$$

ERASE BLACKBOARD

Thus

$$\begin{aligned} E[(y_t - k_t)|\theta_{t-1}] &= E[(\gamma\,\epsilon_t + u_t)|\theta_{t-1}] \\ &= 0. \end{aligned}$$

Now let's pause:

Was the preceding example in any way special because we said that p_t followed the simplest AR(1) process?

In general, with time series models we can write:

$$p_t = \beta x_{t-1} + \epsilon_t$$

where β is a vector and

x_{t-1} is a vector of information available at t-1,
and ϵ_t is an innovation uncorrelated with any term in θ_{t-1}.

Then, with rational expectations

$$ {}_tp^*_{t-1} = E[(\beta x_{t-1} + \epsilon_t)|\theta_{t-1})] = \beta x_{t-1} $$

$$ p_t = \beta x_{t-1} + \epsilon_t. $$

So $E[(p_t - {}_tp^*_{t-1})|\theta_{t-1}] = E[\epsilon_t|\theta_{t-1}] = 0.$

What is the message?

Because of rational expectations, the *expectations of p_t* exactly mimics the systematic part of
p_t. So the net result is that the gap between p_t and ${}_tp^*_{t-1}$ is only the *unsystematic* part.

With Sargent's supply equation, which is only dependent on the gap between actual and expected inflation, we then see that the gap between income and GDP potential,
$y_t - k_t$, is only dependent on the error.

In this case

$$ y_t - k_t = \gamma\, \epsilon_t + u_t. $$

ERASE BLACKBOARD

Let me emphasize these results by giving you another example.

This example has a *monetary rule.*

Suppose that the AS equation is Sargent's:

(1) $y_t - k_t = \gamma (p_t - {}_tp^*_{t-1}) + u_t$

and we have the simplest LM curve:

(2) $m_t = p_t$

Let's also suppose that there is a monetary rule:

(3) $m_t = \alpha x_{t-1} + \epsilon_t,$

where α is a vector conforming to x_{t-1} and x_{t-1} is a vector of variables known at t-1.

The *only part* of the monetary rule that will have an effect on current income will be the random error, ϵ_t.

How do we know?

$$p_t \quad = \quad m_t \quad = \quad \alpha x_{t-1} + \epsilon_t$$

$${}_tp^*_{t-1} \quad = \quad E\,[p_t|\theta_{t-1})] = \quad \alpha x_{t-1}$$

So $\quad y_t - k_t$

$$= \gamma\,(\,p_t - {}_tp^*_{t-1}) + u_t$$

$$= \gamma(\,\alpha x_{t-1} + \epsilon_t - \alpha x_{t-1}) + u_t$$

$$= \gamma\,\epsilon_t + u_t.$$

What are the policy implications of this model?

There are two results.

***First*, monetary policy cannot affect the *expected value* of the GDP gap.**

$$E(y_t - k_t) \quad = E(\gamma\,\epsilon_t + u_t) = E(\gamma\,\epsilon_t) + E(u_t) = 0.$$

Let's explore some of the further implications of this proposition.

Monetary policy can be stabilizing *only* if there is a negative correlation between ϵ_t and u_t –between this period's shock to the money supply ϵ_t and this period's shock to aggregate supply, u_t.

Let's measure the stability of income by

$$\sigma^2(y_t - k_t) = \sigma^2(\gamma\,\epsilon_t + u_t).$$

The general formula for that is:

$$\gamma^2\,\sigma_\epsilon^{\,2} + 2\,\gamma\,\text{cov}\,(\epsilon_t, u_t) + \sigma_u^{\,2}.$$

Now let's consider two cases.

In case I, there is no way that ϵ_t and u_t have a correlation because that would depend upon
current information.

In that case cov (ϵ_t, u_t) = 0.

So $\sigma^2(y_t - k_t) = \gamma^2 \sigma_\epsilon^{\ 2} + \sigma_u^{\ 2}$.

In this case the monetary policy that is most stabilizing is a monetary policy with *no random term*. By that I mean a monetary policy with ϵ identically zero.

If, however, we could devise a monetary policy with automatic stabilizers, which was negatively correlated with current supply shocks, *u,* then we could stabilize output.

Indeed a monetary policy where

$$\epsilon_t = -(1/\gamma)u_t$$

would *exactly* stabilize output.

You can see that because in that case:

$$y_t - k_t = \gamma\,\epsilon_t + u_t = \gamma(-(1/\gamma))u_t + u_t = 0.$$

Or equivalently we could have used the covariance formula to calculate that
$\sigma^2(y_t - k_t) = 0$.

POINT TO IT.

So let's do a bit of a summary here.

It is useful to reflect on the reasons for the neutrality of any monetary rule in this model.

There are two ingredients for Sargent's result:

First there are rational expectations.

Because of rational expectations $_tp^*_{t-1}$ mimics p_t except for an error term.

***Secondly,* and *probably much more importantly* aggregate supply depends *critically* on the difference between actual and expected inflation, plus a random error term.**

But, in reality, aggregate supply *may* depend on the *price level* because there is some form of money illusion such as *sticky money wages.*

If you believe that money wages may be sticky in some form or other, or in some way behave irrationally, then you have probably rejected Sargent's model and his conclusion of the neutrality of the monetary rule.

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Let me now give you a brief description of the difference between David Romer's coverage and my coverage of Lucas and Sargent.

David derives the aggregate Supply of Sargent from a microeconomic model of production and labor supply, and he also explains why the nonstochastic part of the system is money neutral.

This is in contrast to my approach, which was to show you that the AS curve of Sargent corresponds to the expectations augmented Phillips Curve.

David's derivation of Sargent's supply curve *exactly* corresponds to Sargent's *description of it,* which is contained in a paragraph just following the equation for it.

I think that it is helpful to understand that paragraph if you are to understand what David is doing in his write-up of Chapter 6.

So I am going to leave a footnote in the lecture that I think will help you read Romer. Without this footnote I think his whole exposition is very hard to read.

FOOTNOTE:

Quote from page 435, top paragraph in Sargent as his explanation for his Phillips Curve:

"Equation (1) is an aggregate supply schedule relating the deviation of output from normal productive capacity directly to the gap between the current price level and the public's prior expectaion of it."

Why does that occur?

"Unexpected rises in the price level thus boost aggregate supply, because producers mistakenly interpret surprise increases in the aggregate price level as increases in the relative prices of the labor or goods that they are supplying.

This mistake occurs because suppliers receive information about the price of their own goods faster than they receive information about the aggregate price level."

In other words the suppliers look out and they see the price they are receiving. They then try to make an inference regarding the extent to which the price they are receiving is relatively high or low is due to their own price is being relatively high or low, or that all prices are relatively high. Insofar as they think that their own price is relatively high then they will supply more. Insofar as they think that their own price is relatively low they will supply less.

David derives how the aggregate supply depends then on the gap between the expected relative price and the actual price.

END FOOTNOTE

The textbook chapter also reviews one of the first empirical observations consistent with rational expectations.
Lucas and Rapping looked at international data.

They found that countries with more rapid rates of *inflation* have *steeper* Phillips Curves.
This is what rational expectations would predict.

As an aside let me mention that they do not necessarily prove what they said they proved.
Rational expectations says that higher inflation countries will have steeper Phillips Curves.
But there are many conceivable models that would also have this property but would not also have exact rational expectations.

In contrast to the textbook, I have emphasized the *Time Series* basis for the Lucas-Sargent model.

Let me now give you some standard criticisms of RE models that are *un*interesting because they are too knee-jerk.

Criticism 1.* The Sargent model has ineffective monetary policy *only because the labor market clears.

When you read the Sargent article you will see that he assumes that all markets clear <that is implicit in the above footnote>. The reason that he gives for output expanding if inflation exceeds expected inflation is that suppliers are fooled into supplying more goods than they really want to.
If inflation exceeds expected inflation, producers think that the price level is, say,

only 95 whereas in fact it is 100.
When they are offered 100 for their goods or services, they think that they are getting quite a good deal, and they supply more.

This is Sargent's own explanation for his equation (1) and David Romer's explanation for it too. But in fact equation (1), as we saw, could be the result of many different explanations.

For example, it could describe economies such as those with efficiency wages or union bargaining, in which labor markets do not clear at all.

In fact, in any model with optimizing behavior over *only real variables* changes in the money supply will have no effect on equilibrium output if the changes were fully anticipated.

Systematic monetary policy will have no effect on equilibrium in *real models,* whether or not there is labor market clearance.

In fact there seems to be only one way to construct a model with a systematic effective monetary policy.

Somewhere in that model there must be *systematic* money illusion.

As an example, consider the Keynesian model with a *fixed money wage.*

In that model changes in the money supply cause changes in output because there is one variable, the wage, that is fixed *in money terms.*

That is the form of *money illusion.*

Now let's consider *Unfair Criticism 2.*

If you took Sargent's *exact* model–equations (1), (2) and (3), it has a very simple prediction. The prediction is that unemployment will be *serially* uncorrelated.

Such a prediction will be *untrue.* It will be *untrue* in spades. Unemployment has a high degree of serial correlation.

Using the test of his model that serial correlation should be absent is unfair to Sargent.

It is unfair because any reasonable person would expect *supply shocks* to be serially correlated. Those supply shocks are represented in his model by the error

term *u_t*.

As you will see later, potential serial correlation of the *u_t's* makes his model very difficult to test.

The idea that serially correlated *u_t*'s could account for business cycles is the beginning of *real business cycle theory.*

The extreme view of real business cycle theory is that

(1) the Sargent model, including the voluntary notion of unemployment, holds.

(2) business cycles– that is the serial correlation of the unemployment rate, can be explained by serially correlated supply shocks. In terms of the model, that is serial correlation of *u_t* in equation (1).

There is, however, one major empirical problem with Real Business Cycle theory. It predicts that when there are negative shocks workers will be unhappy with their work conditions, and so they will *quit* their jobs.

But that is the exact opposite of what happens.
In cyclical downturns there are more layoffs, but *quits* are extremely sensitive to the unemployment rate.
There are far fewer quits.
That is the exact opposite of what is being predicted by the supply-side explanations of the business cycle.