Economics 202A
Fall 2002

Midterm Exam

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Answer each part of all four questions.

Each question counts 20 points.
There are 80 points on the whole exam.

1. Shapiro-Stiglitz.

In this problem you are supposed to price an asset using the simple dynamic programming technique that you learned while studying the efficiency-wage model of Shapiro and Stiglitz.

Suppose the asset pays \$10 when the economy is in state *H* and \$0 when it is in state *L*. The transition probability from state *H* to state *L* is ½. The transition probability from state *L* to state *H* is ¼. The interest rate is 25 %.

How much is a risk-neutral investor willing to pay for the asset when the economy is in state *H*.

2. Near-rationality.

Demand for the product of an individual firm is

$$m - p + \bar{p}.$$

p is the price of the product of the individual firm. $\bar{p}$ is the aggregate price level.
Cost is zero.
Firms' own prices have negligible effect on the aggregate price level.

1. In an initial equilibrium all producers charge their individually optimal price.
What is the long-run price that all firms are charging with an initial money supply m_0?

2. The money supply then changes to $m_0\,(1 + \epsilon)$.
All firms keep their initial (sticky) price.
How much does an individual firm lose by its failure to change its price?

3. Is the firm's behavior described in (2) near rational? If so, why? If not, why not?

Continue to the next page.

3. Exchange Rates.

Consider a Dornbusch model of the economy. The short-run demand for money is of the form:

$$m - p = \phi y - \lambda r$$

where m is the log of the money supply
p is the log of the price level
y is the log of real income
and r is the nominal interest rate.

Log income is fixed at $\bar{y}$.

The international interest rate is fixed at r*.

Prices change in the domestic economy as a function of the difference between aggregate demand and supply:

$$dp/dt = \pi\,[\mu + \delta\,(e - p) + (\gamma - 1)\,y - \sigma\,r\,].$$

Initially at time zero the log money supply is in long-run steady state at m_0. It changes discontinuously and unexpectedly at this time to m_1.

1. What is the value of the log price p_0^* at time 0 before the change in the money supply?

2. What is the long-run value of the log price p_1^* after the change in the log money supply to m_1?

3. What is the initial value e_0 of the log of the exchange rate at time 0 immediately after the change in the money supply.

Express e_0 in terms of the rate of adjustment of the exchange rate toward its long run equilibrium (θ) and the long-run exchange rate $\bar{e}_1$.

4. By how much did the log exchange rate overshoot? Express your answer in terms of the parameters given and the change in the money supply.

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4. Consumption.

A consumer maximizes her expected 3-period utility, which is

$$U = u(c_1) + u(c_2) + u(c_3)$$

where

$$u(c_1) = c_1 - \tfrac{1}{2} a\, c_1^2 \qquad 0 < a < 1$$

$$u(c_2) = c_2 - \tfrac{1}{2} b\, c_2^2 \qquad 0 < b < 1$$

and

$$u(c_3) = c_3 - \tfrac{1}{2} d\, c_3^2 \qquad 0 < d < 1.$$

In period 1 she receives income $Y + \epsilon_1$; in period 2, $Y + \epsilon_2$; in period 3, $Y + \epsilon_3$.

Each shock is *i.i.d.* with mean 0 and standard deviation σ.

The interest rate is zero.

She is unaware of the shock in income *before* she plans her consumption for each respective period.

1. What is her first period consumption, c_1?

2. What is her second period consumption, c_2?

Express c_1 and c_2 in terms of income Y and the shocks ϵ_1, ϵ_2, and ϵ_3, as they apply.

Midterm Exam's Answer Key
University of California at Berkeley
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1. Shapiro-Stiglitz

   The investor solves:
   $$\begin{cases} p_H = 10 + \frac{1/2}{1.25} p_L + \frac{1/2}{1.25} p_H \\ p_L = 0 + \frac{1/4}{1.25} p_H + \frac{3/4}{1.25} p_L \end{cases}$$
   The solution is: $p_H = 25.0$, $p_L = 12.5$. Thus, a risk neutral investor is willing to pay no more than \$25.

   Note: if one assumes that fundamental asset equation is of the form interest rate time asset value equals flow benefits plus expected (not discounted) capital gains, then p_H is \$20.

2. Near-rationality

   (a) $p_0^* = \arg\max_p p(m_0 - p - \overline{p}) \Rightarrow p_0^* = \overline{p}_0^* = m_0$ since in equilibrium $p - \overline{p}$.

   (b) $p_1^* = \arg\max_p p(m_1 - p - \overline{p}_0^*) = \arg\max_p p(m_0 + \varepsilon - p - m_0) \Rightarrow p_1^* = m_0(1 + .5\varepsilon)$
   Thus, by not reoptimizing the firm looses $p_1^* q_1(p_1^*) - p_0^* q_1(p_0^*) = .25\varepsilon^2 m_0^2$

   (c) Yes, it is near-rational because the loss is of an order of magnitude smaller than the shock.

3. Exchange Rates

   (a) Using the LM equation: $p_0^* = m_0 - \phi\overline{y} + \lambda r^*$.

   (b) Thus, $p_1^* = m_1 - \phi\overline{y} + \lambda r^* = p_0^* + \Delta m_1$.

   (c) Recall that in equilibrium: $e_t - \overline{e} = -(\lambda\theta)^{-1}(p_t - \overline{p})$.
   Thus, $e_0 = e_1 - (\lambda\theta)^{-1}(p_0 - p_1) = e_1 + (\lambda\theta)^{-1}\Delta m_1$.

   (d) Thus, the exchange has to overshoot by: $e_0 - e_1 = (\lambda\theta)^{-1}\Delta m_1$.

4. Consumption

   The Euler condition for the problem is: $u'(c_t) = E_t u'(c_{t+1})$, or since utility is quadratic and so marginal utility is linear: $u'(c_t) = u'(E_t c_{t+1})$.

   (a) Thus, $1 - ac_1 = 1 - bE_1 c_2 = 1 - dE_1 c_3$, or $E_1 c_2 = \frac{a}{b} c_1$ and $E_1 c_3 = \frac{a}{d} c_1$. When she plans consumption in period 1, her expected income is $3Y$, she sets c_1 so that $c_1 + \frac{a}{b}c_1 + \frac{a}{d}c_1 = 3Y \Rightarrow c_1^* = 3\frac{bd}{bd+ab+ad}Y$.

   (b) When she plans consumption in period 2, her expected income is $3Y + \varepsilon_1 - c_1$, she sets c_2 so that $c_2 + \frac{b}{d}c_1^* = 3Y + \varepsilon_1 - c_1^* \Rightarrow c_2^* = 3\frac{ad}{bd+ab+ad}Y + \frac{d}{d+b}\varepsilon_1$.