Optimal Inefficient Production*

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Preliminary – Comments welcome

Abstract

This paper develops a model of optimal non-linear income and commodity taxation to analyze the desirability of aggregate production efficiency. In contrast to Diamond and Mirrlees (1971) individuals are assumed to operate individual-specific production technologies. It is demonstrated that the production efficiency theorem breaks down. Outputs of commodities should be taxed at higher (lower) rates if high- (low-)ability agents have a comparative advantage in producing them. In addition, outputs of commodities should be taxed relatively less when labor demand is more elastic. Aggregate production efficiency is obtained only when individual production functions are of the Gorman polar form. The breakdown of the Diamond-Mirrlees production efficiency theorem has potentially important policy implications.

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Key words: Diamond-Mirrlees production efficiency theorem, Atkinson-Stiglitz theorem, optimal non-linear income taxation, optimal commodity taxation

1 Introduction

Should the government distort production activities? That is the question raised by Diamond and Mirrlees (1971) in an article that is considered among the 20 most important papers of the *American Economic Review* during the last century. Diamond and Mirrlees demonstrated that it is optimal to operate an economy on the production-possibilities frontier even in second-best situations where the government employs distortionary taxation.[1,2] This finding is often referred to as the *production efficiency theorem*. It provides the theoretical foundation for many very important policy prescriptions, such as the desirability of equal taxation of production sectors, the optimality of not taxing intermediate goods, the optimality free trade, the undesirability of

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[1]Diamond and Mirrlees (1971) also note that two additional requirements are necessary: there need to be constant returns to scale in production or, if this is not the case, the government needs to have access to a perfect profit tax.

[2]Diamond and Mirrlees (1971) only considered linear taxes. Guesnerie and Seade (1982) demonstrated that the production efficiency theorem carries over in straightforward fashion to non-linear taxation.

source-based capital taxes like the corporate income tax, and it prescribes using market prices and discount rates in social cost-benefit analysis.[3]

This paper challenges the generality of the Diamond and Mirrlees (1971) production efficiency theorem. We develop a relatively standard optimal non-linear tax model with multiple commodities based on Diamond and Mirrlees (1971), Atkinson and Stiglitz (1976) and Mirrlees (1976). Individuals differ in their ability, which is private information. The model differs in one fundamental aspect from Diamond and Mirrlees (1971), Atkinson and Stiglitz (1976) and Mirrlees (1976): it is no longer assumed that every individual has access to the same (aggregate) production technology. Instead, it is assumed that every individual operates its own production technologies to produce different commodities. An individual's ability determines his/her productivity in transforming inputs in production into outputs for consumption. The main result of this paper is that when production technologies differ across individuals aggregate production efficiency is no longer socially desirable.

The fundamental reason that production is optimally inefficient is that the marginal rates of transformation provide valuable information on the hidden ability of individuals. If high-ability individuals have a comparative advantage in the production of a particular commodity, they will allocate more of their labor time to the production of that commodity. The incentive-compatibility constraints associated with income redistribution can thus be relaxed by distorting production choices. Production of commodities should be taxed at higher (lower) rates when when high-ability individuals have a stronger (weaker) comparative advantage in production of these commodities, i.e., when the marginal rates of transformation increase (decrease) with individual ability.

Moreover, we obtain a counterpart to Corlett and Hague (1953) and Atkinson and Stiglitz (1976) for the production side of the economy. Differential taxation of the production of commodities is shown to be optimal when the labor-demand elasticities in production of these commodities are different. Intuitively, by distorting allocation of labor over various production activities, the tax burden can be shifted to production activities with a more inelastic labor demand, so that the total distortions in the labor market can be alleviated.[4]

The Diamond and Mirrlees (1971) production efficiency theorem assumes that every individual has access to identical technological possibilities to transform his/her inputs into outputs. In that case, the incentive compatibility constraints do not depend on the production side of the economy and aggregate production efficiency prevails. Intuitively, no gain in redistribution can be obtained by distorting production if the marginal rates of transformation are the same for all agents, i.e., there is no comparative advantage in the labor market. Similarly, no reductions in labor-market distortions can be obtained by distorting production, since labor-demand elasticities are the same across sectors (i.e., infinite). Hence, production distortions should be avoided at all times. We derive the conditions under which aggregate production efficiency can be obtained. This is shown to depend on whether individual production technologies can be

[3] Acemoglu, Tsyvinski, and Golosov (2008) show that the production efficiency theorem may even be valid in dynamic political-economy settings where there is no benevolent planner.

[4] According to the Atkinson and Stiglitz (1976) theorem there should optimally be no commodity-tax differentiation when individuals have identical and weakly separable utility functions. Our paper reveals that the Atkinson-Stiglitz theorem should be interpreted as a *consumption efficiency theorem*, which is counterpart to the Diamond-Mirlees production efficiency theorem.

aggregated into an aggregate production technology. Gorman (1996) shows that aggregation is possible only when individual production functions are of the Gorman polar form.

Finally, Diamond and Mirrlees (1971) show that optimal tax expressions for both income and commodity taxation are the same in partial and general equilibrium and are independent from parameters from the production side. Saez (2004) refers to this finding as the 'Tax Formula' result. Our analysis demonstrates that this result is no longer applicable when individuals have different production technologies and production is optimally inefficient. Optimal tax formulae are then dependent on the parameters from the production side of the economy.

The policy implications of our analysis can potentially be far reaching. Our findings suggest that outputs from production activities in which high-ability agents have a comparative advantage should be taxed at higher rates compared to outputs from activities in which low-ability workers have a comparative advantage. Moreover, it is possible that minimum wages, industrial policies, trade restrictions and tariffs are socially desirable for that reason. Moreover, our model might explain why output of sectors with more (less) elastic labor demand should be taxed at relatively lower (higher) rates than other sectors. Moreover, taxation of intermediate goods can be generally socially desirable. Moreover, it is no longer guaranteed that using market prices in government production or social cost-benefit analysis is optimal. All in all, our analysis could provide a rationale as to why so many production inefficiencies are observed in the real world.

The rest of this paper is organized as follows. Section 2 discusses the relation to the existing literature. Section 3 develops the main argument Section 4 discusses the policy implications. Section 5 concludes.

2 Relation to the literature

Our analysis builds on several strands in the literature. First, numerous studies have explored the generality of the Diamond-Mirrlees production efficiency theorem. As already discussed in Diamond and Mirrlees (1971), the theorem is not applicable when not all transactions between firms and households can be taxed, including household production, see for example Stiglitz and Dasgupta (1971), Newberry (1986), and Kleven et al. (2000). Although these analyses are obviously important, we maintain the assumption from Diamond and Mirrlees (1971) that the government has access to a complete set of taxes on all outputs and commodities.

Second, Keen and Wildasin (2004) derive that the production efficiency does not generally apply in international settings, since international lump-sum transfers between countries are not available. These authors therefore cast doubts on the policy implications of the theorem for free trade, residence-based capital taxation and destination-based commodity taxation. We will assume a closed economy where such concerns do not arise.

Third, our findings are related to papers demonstrating that the production efficiency theorem is also not applicable when pure profits are not taxed at a 100 percent rate, see Diamond and Mirrlees (1971), Stiglitz and Dasgupta (1971) and Mirrlees (1972). In our model, the production efficiency theorem breaks down due to the presence of ability-specific rents, because individuals operate different production technologies. We derive that it is not the presence of rents as such that determines our results, but the fact that these rents are ability-specific.

Fourth, Naito (1999), Naito (2004) – analyzing commodity taxation – and Gaube (2005) – analyzing public-good provision – show that production efficiency breaks down when workers are imperfect substitutes in production. These authors use a Stiglitz (1982) 2-type model with two commodities being produced in two different sectors with constant-returns-to-scale production technologies. The Atkinson-Stiglitz theorem also no longer holds despite assuming weakly separable preferences. Their results are driven by general-equilibrium effects on prices and/or wages. Intuitively, changing factor prices affect the incentive-compatibility constraints, hence factor-price movements should be exploited for income redistribution. Production efficiency would be obtained in Naito (1999), Naito (2004) and Gaube (2005) when general-equilibrium effects on prices and wages are absent. Our findings differ from these studies, since we assume constant prices. Moreover, we demonstrate that even when production is distorted the Atkinson-Stiglitz theorem still applies, i.e., uniform commodity taxation is obtained when preferences are identical and weakly separable.

Fifth, Saez (2004) follows up on Naito (1999) and develops an optimal-tax model with occupational choice. Individuals can choose in which occupation/sector to work. Saez (2004) recovers both the Diamond-Mirrlees and Atkinson-Stiglitz theorems. In the long run, the relative supply to each occupation becomes infinitely elastic – even though labor types might be imperfect substitutes in production. This eliminates all differences in the production technology for individuals of different types. Since they only differ in their endowments of efficiency units of labor, all standard results are shown to be applicable again.

Sixth, Jacobs and Bovenberg (2011) refine the analysis Bovenberg and Jacobs (2005) to study optimal income taxation and optimal education subsidies in models with endogenous human capital formation and general specifications of the labor earnings functions. Human capital investment is the production of an intermediate good, which is then used in the final goods sector of the economy. Like the present paper, individuals are assumed to differ in their human capital production functions. Individuals with a higher ability are more productive in transforming human capital investment into labor earnings, that is high-ability individuals have a comparative advantage in skill formation. Jacobs and Bovenberg (2011) show that it is optimal to distort human capital formation, i.e distort the demand for intermediate goods, for redistributional or efficiency reasons. On the one hand, there should be a net tax (subsidy) on human capital investment when the ability bias in human capital formation is strong (weak). Taxing human capital then helps to redistribute income. On the other hand, there should be a higher subsidy (lower tax) on human capital formation if the complementarity of education with labor supply gets stronger. The reason is that labor supply increases by promoting human capital investment, so that the government can off-set some of the distortions of the income tax.

Seventh, Gomes, Lozachmeur, and Pavan (2014) analyze a 2-sector Roy model of occupational choice with linear production technologies. These authors show that besides labor supply sectoral choice is distorted in second best, which they interpret as a violation of the Diamond-Mirrlees production efficiency theorem. However, one may question whether an occupational distortion, i.e. a distortion on labor supply on the extensive margin, should be interpreted as a production inefficiency given that all marginal rates of transformation are constant and equal to unity by definition. Gomes, Lozachmeur, and Pavan (2014) also demonstrate that the Atkinson-Stiglitz theorem breaks down and non-uniform commodity taxation is generally desirable.

3 Model

3.1 Individuals

There is a unit mass of individuals that are heterogeneous with respect to their productive ability $n \in \mathcal{N} \equiv [\underline{n}, \overline{n}]$, where $0 < \underline{n} < \overline{n} \leq \infty$. Ability is continuously distributed according to $H(n)$, which is the cumulative distribution function of n. $h(n)$ is the corresponding density function. Ability is private information. In contrast to Diamond and Mirrlees (1971) there is no aggregate production technology in which all workers are assumed to be perfect substitutes. Hence, ability is no longer equal to the number of efficiency units of labor. Ability n now reflects an individual's productivity to transform inputs into outputs in production in a manner that will made precise below.

All individuals have a strictly concave, continuous and twice differentiable utility function $u(\cdot)$. Individuals derive utility from a numéraire commodity $c(n)$, indexed as commodity 0, I discrete consumption commodities $x_i(n)$, indexed $i \in \mathcal{I} \equiv \{0, \cdots, I\}$, and disutility from labor effort $\ell(n)$. Utility may depend on ability type n as in Mirrlees (1976) to allow for preference heterogeneity. Denote the vector of commodities consumed by a type-n individual $\mathbf{x}(n) \equiv (x_1(n), \cdots, x_I(n))$, then we can write utility as:

$$u(n) \equiv u(c(n), \mathbf{x}(n), \ell(n), n), \quad u_c, u_i, -u_\ell > 0, \quad u_c \leq 0, \quad u_{ii}, u_{\ell\ell} < 0, \quad \forall n, i. \tag{1}$$

where subscripts designate derivatives, and the subscript i stands for the derivative of utility with respect to commodity x_i for all i.

All individuals produce all commodities $c(n)$ and $\mathbf{x}(n)$, which are publicly traded on goods markets. The price of the numéraire c-good is normalized to unity without loss of generality, i.e. $p_0 = 1$. Prices of the x_i-goods are denoted by p_i. The vector of gross commodity prices is $\mathbf{p} \equiv (p_0, p_1, \cdots, p_I)$. For simplicity, we confine the analysis to a partial-equilibrium setting where commodity prices $\mathbf{p}$ are fixed. That is, the model can be viewed as a partial-equilibrium model or a small open economy where commodity prices $\mathbf{p}$ are determined in international goods markets.[5]

Each individual has access to a production technology to produce each commodity, including the numéraire good. This technology allows the individual to transform labor time $l_i(n)$ into output of each commodity i. Individual-specific production functions for commodity i are given by

$$y_i(n) \equiv f^i(n, l_i(n)), \quad f^i_n, f^i_l > 0, \quad f^i_{ll} \leq 0, \quad f^i_{nl} > f^i_{n'l} > 0, \quad \forall n, i. \tag{2}$$

[5] In Naito (1999) and Naito (2004) commodity prices are endogenously determined in general equilibrium on goods markets of a closed economy. We abstract from this as general-equilibrium effects as such also determine the desirability of aggregate production efficiency. Our results demonstrate that production inefficiency is optimal even in the absence of general-equilibrium effects, in contrast to Naito (1999) and Naito (2004). As the analysis of optimal second-best production distortions in general equilibrium would become analytically quite complex, this extension is left for future research.

Each production function $f^i(\cdot)$ exhibits positive and non-increasing marginal returns to labor effort. These production functions allow for the possibility that individuals of a different ability have different marginal returns on their labor effort in the production of different commodities. We allow for *comparative advantage*: individuals with a higher ability n can be relatively more productive in the production of x_j-commodities than x_i-commodities, i.e., $f^j_{nl}/f^j_l > f^i_{nl}/f^i_l$ (and vice versa). In addition, we assume *absolute advantage*: more able individuals, i.e. individuals with a higher n, are able to transform their labor input $l_i(n)$ in more output of all commodities, since $f^i_{nl} > 0$ for all n. Later we will demonstrate that absolute advantage is required to ensure implementability of the non-linear tax functions.[6] Outputs of commodities are designated by $\mathbf{y}(n) \equiv (y_0(n), y_1(n), \cdots, y_I(n))$. All outputs $y_i(n) \equiv f^i(n, l_i(n))$ of the household are traded at gross market prices p_i.

The vector of labor supplies is denoted $\mathbf{l}(n) \equiv (l_0(n), l_1(n), \cdots, l_I(n))$. The household time constraint is:

$$\ell(n) = \sum_{i=0}^{I} l_i(n), \quad \forall n. \tag{3}$$

The government can neither verify total individual labor effort $\ell(n)$ nor their productive ability n. However, the government is able to verify output $y_i(n)$ in each production activity i and it can tax it accordingly. This is obviously a strong assumption, but it corresponds to Diamond and Mirrlees (1971) who assume that all production and consumption activities are verifiable by the government. Hence, all transactions between firms and households can be taxed. Indeed, when some inter-sectoral transactions cannot be verified, e.g. due to an informal sector, aggregate production efficiency ceases to be optimal. By making these strong assumptions, we deliberately bias our findings towards the desirability of aggregate production efficiency.

The government levies non-linear taxes on the output volume of each commodity $T_i(p_i y_i(n))$, where the derivatives of the tax functions are assumed to be continuous, and denoted by $T_i'(p_i y_i(n)) \equiv \mathrm{d}T_i(p_i y_i(n))/\mathrm{d}(p_i y_i(n))$. In addition, the government is able to verify the consumption levels of all commodities. Hence, it can levy a set of non-linear ad valorem commodity taxes $t_i(p_i x_i(n))$ on all commodities, except for the numéraire commodity. Also, commodity tax functions are continuous and have derivatives $t_i'(p_i x_i(n)) \equiv \mathrm{d}t_i(p_i x_i(n))/\mathrm{d}(p_i x_i(n))$. It is not a priori clear that separate tax schedules on the outputs of each activity would support the optimal second-best allocation. However, under our assumptions the implementation through separate schedules on outputs and commodities works.[7] The individual budget constraint can thus be written as:

$$c(n) + \sum_{i=1}^{I} p_i x_i(n) + t_i(p_i x_i(n)) = \sum_{i=0}^{I} p_i y_i(n) - T_i(p_i y_i(n)), \quad \forall n. \tag{4}$$

[6]In principle it would be feasible to generalize the individual production functions to a production set $\mathbf{y}_n \equiv \mathbf{f}(n, \mathbf{l}_n)$, with $\mathbf{l}_n \equiv (l_n^1, l_n^2, \cdots, l_n^I)$. This would allow for different degrees of complementarity of labor effort in various individual production activities. This extension, however, gives rise to technical complexities so as to guarantee incentive compatibility, which would distract us from the main points this paper tries to make. Therefore, it is left for future research.

[7]Renes and Zoutman (2014) demonstrate that separate tax schedules indeed implement the optimal second-best allocation as long as there is a single-dimensional source of heterogeneity and there are no externalities.

A number of things are noteworthy in this specification of the household budget constraint. First, in view of the non-linearity of the tax schedules $T_i(\cdot)$ and $t_i(\cdot)$ abitrage would seems profitable. However, since we assumed that all individual outputs and consumptions are verifiable to the government, such abitrage is ruled out. Second, we explicitly allow for a different set of taxes on consumer and producer prices. Intuitively, the government would like to steer both the marginal rates of substitution in consumption and the marginal rates of transformation in production, since in our model each individual operates its own production technology. With a common, aggregate production technology, as in Diamond and Mirrlees (1971), the marginal rates of transformation do not vary by household type. We show later that this is the reason why aggregate production efficiency is optimal. Consequently, there would be no reason to have non-linear schedules on outputs resulting in production distortions.

Households maximize utility (1) subject to their time constraint (3), production technologies (2), and budget constraint (4). Setting up a Lagrangian for the individual's maximization problem, by substituting the time constraint in the utility function, the production technologies in the budget constraint, and defining $\lambda(n)$ as a multiplier for the individual budget constraint gives:

$$\begin{aligned}\mathcal{L}(n) \equiv\ & u\left(c(n), \mathbf{x}(n), \sum_{i=0}^{I} l_i(n), n\right) \qquad (5)\\ &+\lambda(n)\left[\sum_{i=0}^{I}\left[p_i f^i(n, l_i(n)) - T_i(p_i f^i(n, l_i(n)))\right] - c(n) - \sum_{i=1}^{I}\left[p_i x_i(n) - t_i(p_i x_i(n))\right]\right], \quad \forall n.\end{aligned}$$

Necessary first-order conditions for an optimum are denoted by:

$$\frac{\partial \mathcal{L}(n)}{\partial c(n)} = u_c(\cdot) - \lambda(n) = 0, \quad \forall n, \qquad (6)$$

$$\frac{\partial \mathcal{L}(n)}{\partial x_i(n)} = u_i(\cdot) - \lambda(n) p_i (1 + t_i'(\cdot)) = 0, \quad \forall n, i, \qquad (7)$$

$$\frac{\partial \mathcal{L}(n)}{\partial l_i(n)} = u_\ell(\cdot) + \lambda(n) p_i (1 - T_i'(\cdot)) f_l^i(\cdot) = 0, \quad \forall n, i. \qquad (8)$$

These first-order conditions are not sufficient to describe the individual optimum despite the concavity assumptions on the utility and production functions. This is due to the non-linearity of the tax schedules. Below we will derive the Spence-Mirrlees and monotonicity conditions, which ensure that the second-order sufficiency conditions for utility maximization are satisfied at the optimal second-best allocation.

These first-order conditions can be simplified to obtain the marginal rate of transformation $MRT_{ij}(n)$ of transforming x^i-goods into x^j-goods:

$$MRT_{ij}(n) \equiv \frac{(1 - T_j'(p_j f^j(n, l_j(n)))) f_l^j(n, l_j(n))}{(1 - T_i'(p_i f^i(n, l_i(n)))) f_l^i(n, l_i(n))} = \frac{p_i}{p_j}, \quad \forall n, i, j. \qquad (9)$$

Hence, if production of outputs i and j are taxed at the same rates (i.e., $T_i'(\cdot) = T_j'(\cdot)$), then (individual) production efficiency is obtained among these activities, since the marginal rate of transformation equals the relative output price.

Similarly, we can find the marginal rate of substitution $MRS_{ij}(n)$ between x^i-goods and x^j-goods:

$$MRS_{ij}(n) \equiv \frac{u_i(c(n), \mathbf{x}(n), \ell(n), n)}{u_j(c(n), \mathbf{x}(n), \ell(n), n)} = \frac{(1 + t_i'(p_i x_i(n)))p_i}{(1 + t_j'(p_j x_j(n)))p_j}, \quad \forall n, i, j. \tag{10}$$

Hence, if marginal commodity taxes are equal ($t_i'(\cdot) = t_j'(\cdot)$), then individual consumption choices for these commodities are efficient.

Finally, the first-order condition for labor supply is given by:

$$\frac{-u_\ell(c(n), \mathbf{x}(n), \ell(n), n)}{u_c(c(n), \mathbf{x}(n), \ell(n), n)} = (1 - T_0'(y_n^0)) f_l^0(n, l_n^0), \quad \forall n. \tag{11}$$

Hence, labor supply is undistorted if marginal tax rate on the output of the numéraire commodity is zero ($T_0'(\cdot) = 0$).

3.2 Social objectives and resource constraint

The government is assumed to maximize the utilitarian sum of utilities:

$$\int_{\mathcal{N}} u(c(n), \mathbf{x}(n), \ell(n), n) \mathrm{d}H(n). \tag{12}$$

We assume that marginal utility of income is always declining with income. Diminishing private marginal utility of income yields a social preference for redistribution.[8] We assume that the government is purely redistributive, as there is no revenue requirement. The aggregate resource constraint of the economy is given by:

$$\int_{\mathcal{N}} \left(\sum_{i=0}^{I} p_i f^i(n, l_i(n)) - c(n) - \sum_{i=1}^{I} p_i x_i(n) \right) \mathrm{d}H(n) = 0. \tag{13}$$

Satisfaction of these resource constraints and all the individual budget constraints implies that the government budget constraint will hold by Walras' law.

3.3 First-best allocation

In order to interpret the second-best results derived below, we will first characterize the first-best allocation in which the government maximizes social welfare (12) subject to the aggregate resource constraint (13) only. The Lagrangian for this optimization problem can be written as:

$$\mathcal{L} \equiv \int_{\mathcal{N}} u\left(c(n), \mathbf{x}(n), \sum_{i=0}^{I} l_i(n), n\right) + \eta \left(\sum_{i=0}^{I} p_i f^i(n, l_i(n)) - c(n) - \sum_{i=1}^{I} p_i x_i(n) \right) \mathrm{d}H(n), \tag{14}$$

where the multiplier η associated with the aggregate resource constraint (13) is the social marginal value of an additional unit of public resources. Necessary and sufficient conditions

[8] We could allow for a concave transformation of utilities, but this does not yield additional insights that are relevant to this paper.

for a first-best allocation are given by:

$$\frac{\partial \mathcal{L}}{\partial c(n)} = u_c(\cdot) - \eta = 0, \quad \forall n, \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial x_i(n)} = u_i(\cdot) - \eta p_i = 0, \quad \forall n, i, \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial l_i(n)} = u_\ell(\cdot) + \eta p_i f_l^i(\cdot) = 0, \quad \forall n, i. \tag{17}$$

Hence, production efficiency is obtained since all marginal rates of transformation in production are equalized for all individual production decisions of all x_i commodities:

$$MRT_{ij}(n) \equiv \frac{f_l^j(n, l_j(n))}{f_l^i(n, l_i(n))} = \frac{p_i}{p_j}, \quad \forall n, i, j. \tag{18}$$

Consumption efficiency is obtained since marginal rates of substitution between all commodities x_i and x_j are equal to marginal rates of transformation for these commodities:

$$MRS_{ij}(n) \equiv \frac{u_i(c(n), \mathbf{x}(n), \ell(n), n)}{u_j(c(n), \mathbf{x}(n), \ell(n), n)} = \frac{p_i}{p_j}, \quad \forall n, i, j. \tag{19}$$

Labor supply is not distorted:

$$\frac{-u_\ell(c(n), \mathbf{x}(n), \ell(n), n)}{u_c(c(n), \mathbf{x}(n), \ell(n), n)} = f_l^0(n, l_0(n)), \quad \forall n. \tag{20}$$

And, 'redistributional efficiency' is obtained as all marginal utilities of consumption are equalized across all individuals:

$$u_c(c(n), \mathbf{x}(n), \ell(n), n) = \eta, \quad \forall n. \tag{21}$$

3.4 Incentive compatibility

Given that ability n is private information, the first-best allocation cannot be obtained as it is not incentive compatible. Using a standard mechanism-design approach we will characterize the optimal second-best allocation using the revelation principle. First, we derive the incentive-compatible direct mechanism. Second, we will decentralize this mechanism as an outcome of a competitive market using the non-linear tax schedules on outputs and commodities.

An allocation is said to be incentive compatible, when an individual of type n weakly prefers the bundle $\{c(n), \mathbf{x}(n), \ell(n)\}$ of net consumption and total labor effort that the government intends for type n over the bundle $\{c(n'), \mathbf{x}(n'), \ell(n')\}$ intended for another type n'. Hence $u(n) = \max_{n'} u(c(n'), \mathbf{x}(n'), \ell(n'), n)$, $\forall n, n' \neq n \in \mathcal{N}$. As in Mirrlees (1971), we will apply the first-order approach to derive the second-best optimal allocation. The first-order incentive compatibility constraint is derived in Lemma 1.

Lemma 1 *The first-order incentive-compatibility constraint is given by:*

$$\frac{\mathrm{d}u(n)}{\mathrm{d}n} = -u_\ell(c(n),\mathbf{x}(n),\ell(n),n)\varphi(n,\mathbf{l}_n)+u_n(c(n),\mathbf{x}(n),\ell(n),n), \tag{22}$$
$$\varphi(n,\mathbf{l}(n)) \equiv \sum_{i=1}^{I}\frac{f_n^i(n,l_i(n))}{f_l^i(n,l_i(n))}, \quad \forall n.$$

Proof. Totally differentiating the utility function (1) gives – omitting the n-indices and function arguments:

$$\frac{\mathrm{d}u}{u_c} = \mathrm{d}c + \sum_{i=1}^{I}\frac{u_i}{u_c}\mathrm{d}x_i + \frac{u_\ell}{u_c}\mathrm{d}\ell + \frac{u_n}{u_c}\mathrm{d}n, \quad \forall n. \tag{23}$$

Totally differentiating the time constraint (3) gives:

$$\mathrm{d}\ell = \sum_{i=0}^{I}\mathrm{d}l_i, \quad \forall n\in\mathcal{N}. \tag{24}$$

Totally differentiating the individual budget constraint (4) yields:

$$\mathrm{d}c + \sum_{i=1}^{I}p_i(1+t_i')\mathrm{d}x_i = \sum_{i=0}^{I}(1-T_i')p_i f_n^i\mathrm{d}n + \sum_{i=0}^{I}(1-T_i')p_i f_l^i\mathrm{d}l_i, \quad \forall n. \tag{25}$$

Next, use the first-order conditions for the individual problem (11), (10), and (9) to substitute out the prices in the last equation. Derive $\frac{u_i}{u_c}=(1+t_i')p_i$ and $\frac{-u_\ell}{u_c}=(1-T_0')f_l^0$ so that we find:

$$\mathrm{d}c + \sum_{i=1}^{I}\frac{u_i}{u_c}\mathrm{d}x_i = \sum_{i=0}^{I}\frac{(1-T_i')p_i f_n^i}{(1-T_0')f_l^0}\frac{-u_\ell}{u_c}\mathrm{d}n + \sum_{i=0}^{I}\frac{(1-T_i')p_i f_l^i}{(1-T_0')f_l^0}\frac{-u_\ell}{u_c}\mathrm{d}l_i, \quad \forall n. \tag{26}$$

Then, use $\frac{(1-T_i')p_i}{(1-T_0')}=\frac{f_l^0}{f_l^i}$ and $\frac{(1-T_i')f_l^i p_i}{(1-T_0')f_l^0}=1$ and $\mathrm{d}\ell=\sum_{i=0}^{I}\mathrm{d}l_i$ to find

$$\mathrm{d}c + \sum_{i=1}^{I}\frac{u_i}{u_c}\mathrm{d}x_i + \frac{u_\ell}{u_c}\mathrm{d}\ell + \frac{u_\ell}{u_c}\sum_{i=0}^{I}\frac{f_n^i}{f_l^i}\mathrm{d}n = 0, \quad \forall n. \tag{27}$$

Substitute this result into (23) to establish Lemma 1. ■

In what follows, we will assume that the first-order approach is valid to characterize the optimal allocation. Second-order sufficiency conditions for utility maximization are respected under the constraint that is provided in Lemma 2.

Lemma 2 *Invert the production functions $y_i(n)\equiv f^i(n,l_i(n))$ to express labor inputs $l_i(n)$ as functions of outputs $y_i(n)$ and ability n: $l_i(n)\equiv\phi^i(n,y_i(n))$, where derivatives are $\phi_n^i = -f_n^i/f_l^i<0$, $\phi_y^i=1/f_l^i>0$ and $\phi_{ny}^i=-f_{nl}^i/\left(f_l^i\right)^2<0$. Then, use the time constraint (3) and write the utility function (1) in terms of observables $c(n)$, $\mathbf{x}(n)$, $\mathbf{y}(n)$, and ability n as*

$$u(n)\equiv u\left(c(n),\mathbf{x}(n),\sum_{i=0}^{I}l_i(n),n\right) = u\left(c(n),\mathbf{x}(n),\sum_{i=0}^{I}\phi^i(n,y_i(n)),n\right)\equiv U(c(n),\mathbf{x}(n),\mathbf{y}(n),n). \tag{28}$$

Now, let $V(c(n), \mathbf{X}(n), n) \equiv U(c(n), \mathbf{x}(n), \mathbf{y}(n), n)$, where $\mathbf{X}(n) \equiv (\mathbf{x}(n), \mathbf{y}(n))$. The following constraint on the Spence-Mirrlees and monotonicity conditions must hold at the optimal allocation:

$$\frac{\mathrm{d}(V_{\mathbf{X}}/V_c)}{\mathrm{d}n} \cdot \frac{\mathrm{d}\mathbf{X}_n^{\mathrm{T}}}{\mathrm{d}n} \geq 0. \tag{29}$$

Proof. See Mirrlees (1976, 334–335). ■

This constraint generalizes the Spence-Mirrlees ('single crossing') and monotonicity conditions to our multi-commodity setting with individual production functions. Intuitively, constraint (29) ensures that consumption and output are increasing with ability, and the high-ability types have a preference to produce more output in all sectors (i.e., supply more labor in all activities) when confronted with the same bundle $\{c(n), \mathbf{x}(n), \mathbf{y}(n)\}$ of consumption goods and outputs. Since we assumed that high-ability individuals have an absolute advantage in all production activities, i.e., $f_{nl}^i > 0$, the partial Spence-Mirrlees conditions for output in each production activity can be signed, provided that the single-crossing in labor supply holds, i.e., $\frac{\partial(-u_\ell/u_c)}{\partial n} < 0$, like in the standard Mirrlees (1971)-model:

$$\frac{\partial(-V_{y_i}/V_c)}{\partial n} = \frac{\partial(-u_\ell/u_c)}{\partial n}\frac{1}{f_l^i} - \left(\frac{-u_\ell}{u_c}\right)\frac{f_{nl}^i}{\left(f_l^i\right)^2} < 0. \tag{30}$$

If, in addition, we assume that also the Spence-Mirrlees condition for the consumption vector $\mathbf{x}$ holds, i.e., $\frac{\partial(V_{\mathbf{x}}/V_c)}{\partial n} > 0$, monotonicity in consumption vector $\mathbf{x}(n)$ and in the output vector $\mathbf{y}(n)$ would be sufficient for the first-order approach to be applicable. In the remainder it is assumed that constraint (29) is always respected.

In what follows we will write consumption of the numéraire good $c(n)$ as a function of the allocation, that is $c(n) \equiv c(\mathbf{x}(n), \ell(n), u(n), n)$, which is obtained from inverting the utility function $u(n) = u(c(n), \mathbf{x}(n), \ell(n), n)$. Derivatives of the consumption function $c(\cdot)$ are found using the implicit function theorem:

$$\frac{\partial c}{\partial \ell} = \frac{-u_\ell}{u_c} = (1 - T_0')f_l^0, \quad \frac{\partial c}{\partial x_i} = -\frac{u_i}{u_c} = -(1 + t_i')\frac{(1 - T_0')f_l^0}{(1 - T_i')f_l^i}, \quad \frac{\partial c}{\partial u} = \frac{1}{u_c}, \quad \forall n. \tag{31}$$

The Hamiltonian for maximizing social welfare can then be formulated as:

$$\begin{aligned}
\mathcal{H} \;\equiv\;& u(n)h(n) + \eta\left(\sum_{i=0}^{I} p_i f^i(n, l_i(n)) - c\left(\mathbf{x}(n), \sum_{i=0}^{I} l_i(n), u(n), n\right) - \sum_{i=1}^{I} p_i x_i(n)\right) h(n) \\
+\;& \theta(n) u_\ell\left(c\left(\mathbf{x}(n), \sum_{i=0}^{I} l_i(n), u(n), n\right), \mathbf{x}(n), \sum_{i=0}^{I} l_i(n), u(n), n\right) \varphi(n, \mathbf{l}(n)) \\
-\;& \theta(n) u_n\left(c\left(\mathbf{x}(n), \sum_{i=0}^{I} l_i(n), u(n), n\right), \mathbf{x}(n), \sum_{i=0}^{I} l_i(n), u(n), n\right), \quad \forall n.
\end{aligned} \tag{32}$$

And, the necessary first-order and transversality conditions to characterize an optimal allocation are denoted by – omitting the indices n and the function arguments:

$$\frac{\partial \mathcal{H}}{\partial x_i} = -\left(\eta p_i + \eta \frac{\partial c}{\partial x_i}\right) h(n) + \theta\varphi\left(u_{\ell i} + u_{\ell c}\frac{\partial c}{\partial x_i}\right) - \theta\left(u_{ni} + u_{nc}\frac{\partial c}{\partial x_i}\right) = 0, \quad \forall n, i, \tag{33}$$

$$\frac{\partial \mathcal{H}}{\partial l_i} = \left(\eta p_i f_l^i - \eta \frac{\partial c}{\partial \ell}\right) h(n) + \theta\varphi\left(u_{\ell\ell} + u_{\ell c}\frac{\partial c}{\partial \ell}\right) + \theta u_\ell \varphi_{l_i} - \theta\left(u_{n\ell} + u_{nc}\frac{\partial c}{\partial \ell}\right) = 0, \forall n, i, \tag{34}$$

$$\frac{\partial \mathcal{H}}{\partial u} = \left(1 - \eta\frac{\partial c}{\partial u}\right) h(n) + \theta\varphi u_{\ell c}\frac{\partial c}{\partial u} - \theta u_{nc}\frac{\partial c}{\partial u} = \frac{\mathrm{d}\theta}{\mathrm{d}n}, \quad \forall n \neq \underline{n}, \overline{n}, \tag{35}$$

$$\lim_{n \to \underline{n}} \theta_n = \lim_{n \to \overline{n}} \theta_n = 0. \tag{36}$$

3.5 Optimal consumption inefficiency

It will be useful for our discussion to first derive the optimal consumption taxes, since the expressions for the optimal production taxes will yield a mirror image. The next Proposition demonstrates under which conditions it is optimal to distort consumption patters. Basically, this proposition generalizes Atkinson and Stiglitz (1976) and Mirrlees (1976) to a setting with production distortions.

Proposition 1 *The optimal non-linear marginal commodity tax rates on the demand for x_i-commodities are given by:*

$$\frac{t_i'(p_i x_i(n))}{1 + t_i'(p_i x_i(n))} = \frac{u_c(\cdot)\theta/\eta}{nh(n)}\left(\frac{\partial \ln(u_i(\cdot)/u_c(\cdot))}{\partial \ln n} - \frac{\varphi(n, \mathbf{l}(n))n}{\ell(n)}\frac{\partial \ln(u_i(\cdot)/u_c(\cdot))}{\partial \ln \ell(n)}\right), \quad \forall n, i. \tag{37}$$

Proof. Substitute the derivatives of consumption in (31) in (33), rewrite using the first-order conditions (10) and derive that $\frac{u_{lx}}{u_x} - \frac{u_{lc}}{u_c} = \frac{1}{l_n}\frac{\partial \ln(u_x/u_c)}{\partial \ln l_n}$ – omitting the indices and functional arguments:

$$\frac{p_i - \frac{u_i}{u_c}}{\frac{u_i}{u_c}} = \frac{u_c\theta/\eta}{nh(n)}\frac{\varphi n}{\ell}\left(\frac{\ell u_{\ell i}}{u_i} - \frac{\ell u_{\ell c}}{u_c}\right) - \frac{u_c\theta/\eta}{nh(n)}\left(\frac{n u_{ni}}{u_i} - \frac{n u_{nc}}{u_c}\right)$$

Note that $\frac{\ell u_{\ell i}}{u_i} - \frac{\ell u_{\ell c}}{u_c} = \frac{\partial \ln(u_i/u_c)}{\partial \ln \ell}$ and $\frac{n u_{ni}}{u_i} - \frac{n u_{nc}}{u_c} = \frac{\partial \ln(u_i/u_c)}{\partial \ln n}$. Finally, the tax implementation uses the household's first-order condition from (10), which can be used to eliminate $\frac{u_i}{u_c}$ to establish the proposition. ■

The left-hand side of equation (37) gives the non-linear marginal tax wedge on commodity x_i. The right-hand side of equation (37) gives the marginal benefits of taxing commodity x_i. The marginal benefits of (differential) commodity taxation are twofold. First, commodity taxation helps to complement the income tax to achieve the distributional goals of the government, as captured by the first term in brackets on the right-hand side. In particular, if high-ability individuals have a stronger (weaker) taste for commodity x_i than low-ability individuals do, then the marginal rate of substitution u_i/u_c between both commodities increases (declines) with n, so that $\frac{\partial \ln(u_i/u_c)}{\partial \ln n} > 0$ (< 0). Consequently, commodity x_i needs to be taxed (subsidized). Of course, this distributional benefit also comes at a cost of distorting commodity demands. In the optimum, the government equates marginal redistributional benefits of commodity taxes and marginal distortions in commodity demands. This motive for differential commodity taxes is known since Mirrlees (1976), and has later been explored further by Saez (2002).

Second, differential commodity taxation is employed to alleviate the distortions of income taxation on labor supply, as the second term in brackets on the right-hand side demonstrates. Differential commodity taxation is optimal when the marginal rate of substitution between commodity x_i and the numéraire commodity c varies with labor supply: $\frac{\partial \ln(u_i/u_c)}{\partial \ln \ell} \neq 0$. That is, when good x_i is more (less) complementary to labor than good c is, it will be optimal to subsidize (tax) x_i. Intuitively, by introducing a distortion in commodity demands, the government is able to alleviate some of the distortions in labor supply created by the non-linear income tax. See also Atkinson and Stiglitz (1976) and Jacobs and Boadway (2014).

The famous Atkinson-Stiglitz theorem is recovered when if the utility function is identical for all n, so that $\frac{\partial \ln(u_i/u_c)}{\partial \ln n} = 0$, and preferences are weakly separable between commodities and labor, so that $\frac{\partial \ln(u_x/u_c)}{\partial \ln \ell} = 0$. In that case, $t'(p_i x_i(n)) = 0$, since commodity taxes have no redistributional benefit over and above income taxes and commodity taxes are impotent to boost downward distorted labor supply. Hence, all commodities should be uniformly taxed (at zero rates). One may therefore interpret the Atkinson-Stiglitz theorem as the 'consumption efficiency theorem' of public finance.

3.6 Optimal production inefficiency

The next proposition states the main result of this paper.

Proposition 2 *Aggregate production efficiency is not socially optimal. The optimal non-linear tax wedge on output of commodity x_i relative to output of the numéraire commodity c is given by:*

$$\frac{T_i'(y_i(n)) - T_0'(y_0(n))}{(1 - T_i'(y_i(n)))(1 - T_0'(y_0(n)))} = \frac{u_c(\cdot)\theta(n)/\eta}{h(n)}(\varphi_{l_i}(n, \mathbf{l}_n) - \varphi_{l_0}(n, \mathbf{l}_n)), \quad \forall n, i, \tag{38}$$
$$\varphi_{l_i}(n, \mathbf{l}_n) - \varphi_{l_0}(n, \mathbf{l}_n) = \frac{1}{n}\left(\frac{\partial \ln(f_l^i(\cdot)/f_l^0(\cdot))}{\partial \ln n} - \frac{n f_n^i(\cdot)}{l_i f_l(\cdot)}\frac{\partial \ln f_l^i(\cdot)}{\partial \ln l_i} + \frac{n f_n^0(\cdot)}{l_0 f_l^0(\cdot)}\frac{\partial \ln f_l^0(\cdot)}{\partial \ln l_0}\right).$$

Proof. Take (34) for commodity i and commodity $i = 0$ – omitting the indices and functional arguments:

$$\left(\eta p_i f_l^i - \eta \frac{\partial c}{\partial \ell}\right) h(n) + \theta\varphi\left(u_{\ell\ell} + u_{\ell c}\frac{\partial c}{\partial \ell}\right) + \theta u_\ell \varphi_{l_i} - \theta\left(u_{n\ell} + u_{nc}\frac{\partial c}{\partial \ell}\right) = 0, \tag{39}$$

$$\left(\eta p_0 f_l^0 - \eta \frac{\partial c}{\partial \ell}\right) h(n) + \theta\varphi\left(u_{\ell\ell} + u_{\ell c}\frac{\partial c}{\partial \ell}\right) + \theta u_\ell \varphi_{l_0} - \theta\left(u_{n\ell} + u_{nc}\frac{\partial c}{\partial \ell}\right) = 0. \tag{40}$$

Subtract both equations to find:

$$\eta(p_i f_l^i - p_0 f_l^0) h(n) + \theta u_\ell(\varphi_{l_i} - \varphi_{l_0}) = 0. \tag{41}$$

Simplify (41) using $p_0 = 1$ and the first-order condition (11) to obtain

$$\frac{f_l^0/f_l^i - p_i}{f_l^0/f_l^i} = \frac{u_c\theta/\eta}{h(n)}(1 - T_0')(\varphi_{l_0} - \varphi_{l_i}), \quad \forall n. \tag{42}$$

Next, take the derivatives of φ with respect to l_i and l_0 to find:

$$\varphi_{l_i} - \varphi_{l_0} = \frac{f^i_{nl} f^i_l - f^i_{ll} f^i_n}{(f^i_l)^2} \frac{f^0_{nl} f^0_l - f^0_{ll} f^0_n}{(f^0_l)^2} = \frac{f^i_{nl}}{f^i_l} - \frac{f^0_{nl}}{f^0_l} - \frac{f^i_{ll} l_i}{f^i_l} \frac{f^i_n}{f^i_l l_i} + \frac{f^0_{ll} l_0}{f^0_l} \frac{f^0_n}{f^0_l l_0}. \tag{43}$$

Derive that $\frac{f^i_{nl}}{f^i_l} - \frac{f^0_{nl}}{f^0_l} = \frac{1}{n} \frac{\partial \ln(f^1_l / f^0_l)}{\partial \ln n}$ and $\frac{f^i_{ll} l_i}{f^i_l} \frac{f^i_n}{f^i_l l_i} - \frac{f^0_{ll} l_n}{f^0_l} \frac{f^0_n}{f^0_l l_0} = \frac{f^i_n}{f^i_l l_i} \frac{\partial \ln f^i_l}{\partial \ln l_i} - \frac{f^0_n}{f^0_l l_0} \frac{\partial \ln f^0_l}{\partial \ln l_0}$, so that

$$\varphi_{l_i} - \varphi_{l_0} = \frac{1}{n} \left(\frac{\partial \ln(f^i_l / f^0_l)}{\partial \ln n} - \frac{n f^i_n}{f^i_l l_i} \frac{\partial \ln f^i_l}{\partial \ln l_i} + \frac{n f^0_n}{f^0_l l_0} \frac{\partial \ln f^0_l}{\partial \ln l_0} \right). \tag{44}$$

Substitution of (44) in equation (42) and using the implementation with taxes from the household first-order condition (9) establishes the proposition. ■

Recall from the section on first-best policies above that aggregate production efficiency is obtained when $f^0_l / f^i_l = p_i$. In that case, the marginal rates of transformation between x_i- and c-goods f^0_l / f^i_l are equalized across all commodities for all individuals in the economy. From Proposition 2 follows that commodity outputs are generally taxed at differential rates, hence production inefficiency is generally desirable. To see why, we have $f^0_l / f^i_l - p_i > 0$ (< 0) if the output of x_i goods is taxed at a higher (lower) rate than output of the numéraire commodity c, i.e., $T'_i > T'_0$ ($T'_i < T'_0$), see first-order condition (9). Consequently, aggregate production decisions are not efficient (i.e., $f^0_l / f^i_l \neq p_i$). Production inefficiencies are optimal for two distinct reasons, which are the mirror image for the reasons why consumption inefficiencies are optimal.

First, if better skilled workers have a comparative advantage in producing the x_i-commodity over producing the numéraire c-commodity, then we have $f^i_{nl} / f^i_l > f^0_{nl} / f^0_l$ (and vice versa). Consequently, the marginal rate of transformation increases in ability as higher ability workers are able to produce *relatively* more output of x_i-commodities than the c-commodities with an additional unit of labor effort: $\frac{\partial \ln(f^i_l / f^0_l)}{\partial \ln n} > 0$. Thus, when the marginal rate of transformation between x_i- and c-goods f^i_l / f^0_l increases (decreases) with ability n, output of x_i-commodities should be taxed at higher (lower) rates than output of c-commodities. Intuitively, when an individual of a higher ability is relatively more (less) productive in producing x_i-goods, this individual will allocate more (less) of his working effort to producing these goods. Consequently, the output from this activity reveals information on the hidden ability of this individual, which should optimally be exploited for income redistribution.

Second, the term $\left(-\partial \ln f^i_l / \partial \ln l_i\right)^{-1} > 0$ can be interpreted as an *implicit* labor-demand elasticity in the production of commodity x_i. If the implicit labor-demand elasticity in production of x_i commodities $\left(-\partial \ln f^i_l / \partial \ln l_i\right)^{-1}$ is higher relative to the implicit labor-demand elasticity in production of the numéraire commodity $(-\partial \ln f^0_l / \partial \ln l_0)^{-1}$, the output of commodity x_i should optimally be taxed less than the output of the numéraire commodity c. Intuitively, the government wishes to introduce production distortions to alleviate the distortions on total labor supply. By distorting the composition of production activities, the government induces individuals to allocate their labor time towards production sectors where taxing output gives fewer labor-market distortions. Doing so reduces distortions in total labor supply $\ell(n)$ at the cost of distorting the composition of labor supply $l_i(n)$ over the different production activities.

The term $nf_n^i/(l_i f_l)$ is relative weight for the implicit labor-demand elasticity. It captures the importance of comparative advantage in the output of commodity x_i – in the numerator – and the contribution of labor effort to output – in the denominator. If ability rents become more (less) important compared to the contribution of labor effort in total output of commodity x_i, then skimming off ability rents becomes more important than alleviating labor-market distortions, and the optimal tax on this commodity should be relatively higher (lower).

We thus uncovered a production counterpart of the results by Mirrlees (1976) and Saez (2002). These authors showed that if the marginal rates of *substitution* in consumption vary with ability, then differential commodity taxation is optimal to redistribute income, see also the previous section. Intuitively, the government wishes to tax commodities that the high-ability types like to consume. We, in contrast, demonstrated that when marginal rates of *transformation* vary with ability, the government should set higher taxes on the production of commodities in which the high ability types have a comparative advantage. The government thus wishes to tax outputs of commodities that high-ability individuals like to produce most.

The latter result is most clearly demonstrated in the special case where the production functions are given by $f^i(n, l_i(n)) \equiv l_i(n)\phi_i(n)$, with $\phi_i' > 0$, $\phi_i'' < 0$. Then, we still have comparative advantage, since $\frac{\partial \ln(f_l^i/f_l^0)}{\partial \ln n} = \frac{\partial \ln(\phi_i(n)/\phi_0(n))}{\partial \ln n} \neq 0$. But the implicit labor-demand elasticities are infinite, cf. $\frac{\partial \ln f_l^i}{\partial \ln l_i} = \frac{\partial \ln f_l^0}{\partial \ln l_i} = 0$. Consequently, production distortions are introduced only to exploit comparative advantages, but not to alleviate labor-market distortions.

We also derived a production counterpart of the Corlett and Hague (1953), Atkinson and Stiglitz (1976) and Jacobs and Boadway (2014) results for non-uniform commodity taxation. By taxing commodities that are relatively more complementary to leisure at higher rates, distortions from income taxation on labor supply are reduced. Commodities are not equally complementary to leisure when the marginal rates of *substitution* between commodities vary with labor effort. We demonstrate that a similar intuition applies to the production side of the economy since non-uniform output taxation is optimal. To see why, consider the special case where $f^i(n, l_i(n)) \equiv n\phi(l_i(n))$, with $\phi' > 0$, $\phi'' < 0$. Here, comparative advantage is completely absent, since $\frac{\partial \ln(f_l^i/f_l^0)}{\partial \ln n} = 0$. However, production distortions are still desirable, since implicit labor-demand elasticities and the shares of n and l_i in f are not equal across production activities. Hence, production is distorted only to alleviate labor-market distortions created by taxation.

How do our findings relate to Diamond and Mirrlees (1971)? Aggregate production efficiency optimal in their analysis because all individuals have access to identical production technologies so that there are no ability-specific rents in production. We can recover the production efficiency theorem if individuals would differ only in their endowments of efficiency units of labor, and all production activities would entail identical production technologies as the next Corollary shows.

Corollary 1 *If the production functions are given by $f^i(n, l_i(n)) \equiv nl_i(n)$, then aggregate production efficiency is obtained:*

$$\frac{f_l^0(n, l_0(n))}{f_l^i(n, l_i(n))} = 1 = p_i, \quad \forall n, i. \tag{45}$$

In this particular case, there is, first, no comparative advantage, so that $\frac{f^i_{nl}}{f^i_l} = \frac{f^0_{nl}}{f^0_l}$ and $\frac{\partial \ln(f^i_l/f^0_l)}{\partial \ln n} = 0$. And, second, implicit labor-demand elasticities are equal in all production activities (and infinitely elastic), since $\frac{f^i_{ll} l_i}{f^i_l} = \frac{f^0_{ll} l_0}{f^0_l} = \frac{\partial \ln f^i_l}{\partial \ln l_i} = \frac{\partial \ln f^0_l}{\partial \ln l_0} = 0$. In this case, there are neither distributional nor efficiency reasons to distort production activities, and we find aggregate production efficiency as the social optimum in second best.

We can reveal a second intuition behind the desirability of aggregate production inefficiency in Diamond and Mirrlees (1971). Note that if $f^i(n, l_i(n)) \equiv n l_i(n)$, we can write for $\varphi(n, \mathbf{l}_n)$ in the incentive compatibility constraint (22):

$$\varphi(n, \mathbf{l}_n) \equiv \sum_{i=1}^{I} \frac{f^i_n(n, l_i(n))}{f^i_l(n, l_i(n))} = \frac{\ell(n)}{n}. \tag{46}$$

In this case the incentive-compatibility constraint (22) is the same as in Mirrlees (1971). Thus, when production technologies of all individuals are equal, then the allocation of inputs in production of various commodities $\mathbf{l}(n)$ does not affect the incentive-compatibility constraints. Intuitively, individuals of a higher ability type cannot mimic individuals of a lower ability by reallocating their labor to production of different commodities, as their labor productivity per hour worked is the same in the production of all commodities. Naturally, production decisions should be overall efficient then.

Corollary 2 generalizes the special case of Diamond-Mirrlees to the case where each production activity features a Cobb-Douglas production function.

Corollary 2 *If production functions are Cobb-Douglas, i.e.,* $f^i(n, l_i(n)) \equiv n l_i(n)^\alpha$, $0 < \alpha < 1$, *then aggregate production efficiency is obtained:*

$$\frac{f^0_l(n, l_0(n))}{f^i_l(n, l_i(n))} = \left(\frac{l_0(n)}{l_i(n)}\right)^{\alpha-1} = p_i, \quad \forall n. \tag{47}$$

In this case we also have that $\varphi_{l_i} - \varphi_{l_0} = 0$. Intuitively, high-ability individuals have no longer a comparative advantage in the production of any commodity, since $\frac{\partial \ln(f^i_l/f^0_l)}{\partial \ln n} = 0$. Moreover, the implicit labor-demand elasticities are equalized across all activities, but not necessarily infinite: $\frac{\partial \ln f^i_l}{\partial \ln l_i} = \alpha - 1$. And, finally, the weights $\frac{n f^i_n}{l_i f^i_l} = \frac{1}{\alpha}$ are constant. Thus, there would be neither redistributive reasons – exploiting comparative advantage – nor efficiency reasons – alleviating labor-supply distortions – to employ differential output taxes. This special case is interesting because it demonstrates that it is not decreasing returns to scale in production as such – giving rise to rents – that generates the violation of production efficiency. Indeed, production distortions are desirable only to the extent that they help to redistribute *ability-specific* rents.

This last example gives a condition under which it is possible to aggregate individual-specific production technologies into an aggregate technology. This is not the only example where individual production functions can be aggregated. Gorman (1996) shows that as long as production functions belong to the Gorman polar form, such aggregation is generally feasible. Our Cobb-Douglas example belongs to this class of production functions.

Finally, the analysis demonstrates that optimal output taxes generally depend on parameters from the production side of the model. Hence, the property in Diamond and Mirrlees (1971) that optimal tax formulae only depend on the parameters from the consumption side of the economy – dubbed the 'Tax Formula result' in Saez (2004) – is no longer applicable.

3.7 Optimal income taxation

Finally, we derive the optimal non-linear income tax on the output of the numéraire good, which we will refer to as the non-linear income tax.

Proposition 3 *The optimal non-linear income tax schedule on output in the c-sector is given by:*

$$\frac{T_0'(y_0(n))}{1-T_0'(y_0(n))} = \frac{u_c(\cdot)\theta(n)/\eta}{nh(n)}\left(n\varphi_{l_0}(n,\mathbf{l}(n)) + \frac{n\varphi(n,\mathbf{l}(n))}{\ell(n)}\frac{\partial\ln(u_\ell/u_c)}{\partial\ln\ell(n)} - \frac{\partial\ln(u_\ell/u_c)}{\partial\ln n}\right) \quad (48)$$

$$\frac{\theta(n)}{\eta} = \int_n^{\overline{n}}\left(\frac{1}{u_c(\cdot)}-\frac{1}{\eta}\right)\exp\left[-\int_n^m z(s)\frac{\mathrm{d}s}{s}\right]h(m)\mathrm{d}m > 0, \quad (49)$$

$$z(s) \equiv \frac{\partial\ln u_c(\cdot)}{\partial\ln s} - \frac{s\varphi(n,\mathbf{l}(n))}{\ell(s)}\frac{\partial\ln u_c(\cdot)}{\partial\ln\ell(s)}, \quad \forall n\neq\underline{n},\overline{n}. \quad (50)$$

Proof. We can rewrite first-order condition (11) for l_0 using the derivatives of c (31) and the first-order condition (11) to find – omitting the indices and functional arguments:

$$\frac{T_0'}{1-T_0'} = \frac{u_c\theta/\eta}{nh(n)}\left[n\varphi_{l_i} + \frac{n\varphi}{\ell}\left(\frac{\ell u_{\ell\ell}}{u_\ell}-\frac{\ell u_{\ell c}}{u_c}\right) - \left(\frac{nu_{n\ell}}{u_\ell}-\frac{nu_{nc}}{u_c}\right)\right], \quad \forall n. \quad (51)$$

Next, note that $\frac{\ell u_{\ell\ell}}{u_\ell}-\frac{\ell u_{\ell c}}{u_c} = \frac{\partial\ln(u_\ell/u_c)}{\partial\ln\ell}$ and $\frac{nu_{n\ell}}{u_\ell}-\frac{nu_{nc}}{u_c} = \frac{\partial\ln(u_\ell/u_c)}{\partial\ln n}$ to establish the first part of the Proposition. Further, use the derivatives in equation (31) in the first-order condition for utility (35). Note that $\varphi\frac{u_{lc}}{u_c} = \frac{\varphi}{\ell}\frac{\partial\ln u_c}{\partial\ln\ell}$ and $\frac{u_{nc}}{u_c} = \frac{1}{n}\frac{\partial\ln u_c}{\partial\ln n}$, hence we find:

$$\frac{\mathrm{d}\theta}{\mathrm{d}n} + \theta\left(\frac{1}{n}\frac{\partial\ln u_c}{\partial\ln n} - \frac{\varphi}{\ell}\frac{\partial\ln u_c}{\partial\ln\ell}\right) = \left(1-\frac{\eta}{u_c}\right)h(n), \quad \forall n\neq\overline{n},\underline{n}. \quad (52)$$

This is a linear differential equation in θ of the form $\frac{\mathrm{d}\theta(n)}{\mathrm{d}n} + a(n)\theta(n) = b(n)$, with $a(n) \equiv \left(\frac{1}{n}\frac{\partial\ln u_c(\cdot)}{\partial\ln n} - \frac{\varphi(n,\mathbf{l}(n))}{\ell(n)}\frac{\partial\ln u_c(\cdot)}{\partial\ln\ell(n)}\right)$ and $b(n) \equiv (1-\frac{\eta}{u_c(\cdot)})h(n)$. This differential equation can be integrated, using a transversality condition from (36) to find: $\theta(n) = -\int_n^{\overline{n}}\exp\left[\int_n^m a(s)\mathrm{d}s\right]b(m)dm$. Substituting for $a(n)$ and $b(n)$ yields the second part of the proposition. ■

The major difference of our expression for the optimal non-linear tax with the one derived by Mirrlees (1971) is the elasticity term $n\varphi_{l_0} + \frac{n\varphi}{\ell}\frac{\partial\ln(u_\ell/u_c)}{\partial\ln\ell} - \frac{\partial\ln(u_\ell/u_c)}{\partial\ln n}$. This term consists of three elements. First, it encompasses the distortions of the income tax on the allocation of labor over the sectors as captured by $n\varphi_{l_0}$. When the tax on the output of the numéraire commodity c is increased, the individual will allocate less labor time to the production of the numéraire, and more to the production of other commodities. This effect is new compared to Mirrlees (1971), since that analysis only considers one production sector.

Second, $\frac{n\varphi}{\ell}\frac{\partial\ln(u_\ell/u_c)}{\partial\ln\ell}$ captures the standard Frisch elasticity of labor supply. The higher are marginal income taxes the larger are distortions in the supply of total labor time ℓ. This term is the main ingredient of the elasticity of the tax base in Mirrlees (1971). In our setting there is a correction $\frac{n\varphi}{\ell}$ reflecting the fact that outputs and earnings are determined by individual production functions. Finally, $\frac{\partial \ln(u_\ell/u_c)}{\partial \ln n}$ captures how the willingess to supply labor varies with the skill level – conditional on labor income – since we allowed for preference heterogeneity. When the marginal cost of work effort in terms of consumption increases (decreases) with ability optimal marginal tax rates at higher income levels should be lower (higher) – ceteris paribus. All three elements determine the effective elasticity of the total tax base. Note again that the elasticity term generally depends on parameters on the production side of the economy.

The social marginal value of income redistribution $\theta(n)/\eta$ at skill level n is the same as in Mirrlees (1971), except for the presence of $\frac{\partial \ln u_c(\cdot)}{\partial \ln n}$ in the bracketed term inside the integral. This term, again, originates from the fact that we allowed for preference heterogeneity. If the utility function would be the same for all individuals, it would disappear. Saez (2001) and Jacobs and Boadway (2014) show that the term in brackets is associated with income effects in labor effort.

The expression for the optimal non-linear income tax is otherwise very similar to the expression found in Mirrlees (1971). To see this, suppose that the assumptions of the Diamond and Mirrlees (1971) production efficiency theorem would hold and there would be no preference heterogeneity. In particular, if individual production technologies are given by $f^i(n, l_i(n)) \equiv nl_i(n)$ and the utility function is $u(c, \mathbf{x}, \ell, n) = u(c, \mathbf{x}, \ell)$, then we can derive that $\varphi_{l_0} = 1/n$, $\varphi = \ell/n$, and $\frac{\partial \ln(u_\ell/u_c)}{\partial \ln n} = 0$. Then, the optimal non-linear income tax would be given by:

$$\frac{T_0'(y_0(n))}{1 - T_0'(y_0(n))} = \frac{u_c(\cdot)\theta(n)/\eta}{nh(n)} \left(1 + \frac{\partial \ln(u_\ell(\cdot)/u_c(\cdot))}{\partial \ln \ell(\cdot)}\right) \tag{53}$$

Which is exactly the same expression as in Mirrlees (1971). Note that under these assumptions, there would be overall production efficiency, hence the tax schedule is the same for all outputs x_i: $T_i'(y_i(n)) = T_0'(y_0(n))$. The interested reader may wish to consult Diamond (1998) and Saez (2001) for more interpretations and intuitions of the non-linear tax schedule.

4 Policy implications

When not everyone has the same technological possibilities to transform inputs into outputs, the practical desirability of free trade, no taxation of intermediate goods, the use of market prices in social-cost benefit analysis or public sector production, might all be called into question. Our analysis has therefore a number of potentially important policy-relevant implications, which we will shortly discuss.

4.1 Capital income taxation

Our model could be given an intertemporal interpretation, with commodity c denoting consumption today and commodities x_i consumption levels at future dates. In such a context, the marginal rates of transformation f_l^j/f_l^0 can be viewed as the technological opportunities to transform current consumption into future consumption. These production functions may differ by individuals for various reasons. First, high-ability individuals might generate larger returns on their savings than low-ability individuals do. For example, when they earn returns on assets in their own, closely-held firms, which are also determined by individual productive abilities, see for example Gerritsen et al. (2015). Second, more able individuals might earn larger returns on their assets due scale effects in portfolio management (Piketty (2014)). Third, some individuals might be barred from entering capital markets. If high-ability individuals are less liquidity constrained than low-ability individuals are, then they are typically more efficient to transform current consumption into future consumption. Fourth, when insurance markets are missing, different individuals have different possibilities to transform current consumption into *expected* future consumption. If risk risk aversion falls with productive ability, the consumption-possibilities frontier in expected consumption will be less concave. For all these reasons, the marginal rates of transformation in consumption differ by individual's abilities. Consequently, taxes on saving can be socially desirable for redistributional reasons, as Gerritsen et al. (2015) formally demonstrate.

4.2 Intermediate goods taxation

Jacobs and Bovenberg (2011) develop a completely worked out example where taxes or subsidies on intermediate goods taxation are socially desirable. In their model, educational investment is an intermediate good used in human capital production. Human capital is employed in final goods production. Education should be taxed if high-ability individuals have a comparative advantage in human capital formation. Education should be subsidized if this alleviates labor-market distortions. Whether human capital should be taxed or subsidized on a net basis remains ambiguous as it is the result of these two off-setting effects.

This example demonstrates that taxes or subsidies on the use of intermediate goods can be optimal more generally. Intermediate goods should be taxed (subsidized) if high-ability (low-ability) individuals have a comparative advantage in using these intermediate goods in final-goods production. Moreover, intermediate goods should be taxed less (more) if they are stronger (weaker) complements with labor supply.

4.3 Differential sector taxation

Our results demonstrate that outputs of different outputs should be taxed at different rates. This might have implications for the taxation of different sectors or occupations. Although our model does not explicitly distinguish production sectors and occupations, it provides some important intuitions. In particular, it seems quite plausible that the playing field between sectors/occupations should not be level. Outputs produced in sectors/occupations in which high-ability individuals have a comparative advantage – for example IT – should be taxed at higher rates than the outputs from sectors/occupations in which low-ability individuals have a comparative advantage – for example restaurants.

Similarly, our findings might also explain why optimal tax rates should be lower in sectors/occupations in which labor demand is relatively more elastic. Examples include the construction sector, bars and restaurants or personal services. Due to the presence of close substitutes (household production, black/grey labor market) labor demand might be relatively more elastic in these sectors compared to other sectors/occupations.

4.4 Free trade

Our analysis demonstrated that it is desirable to tax the output in which high-ability agents have a comparative advantage, or to subsidize the outputs those sectors in which low-ability individuals have a comparative advantage. This implies that in order to redistribute income in the most efficient way, trade tariffs, production subsidies, and so on, could be socially desirable to raise the net incomes of low-skilled workers. Similarly, deviations from residence-based factor taxes or destination-based consumption taxes might be optimal.

4.5 Public production and social cost-benefit analysis

The findings of this paper imply that the desirability of public production activities may not be properly evaluated when costs or benefits are measured using market prices. Indeed, the government may oversupply (undersupply) public goods if low-skilled (high-skilled) workers are have a comparative advantage in producing them. Similarly, the government may produce more public goods than dictated by conventional social cost-benefit analysis, if these public goods help to lower labor-market distortions in the private sector, for example, through child-care facilities. Finally, there appears to be no obvious candidate for the correct social discount rate if individuals have different intertemporal marginal rates of transformation, see also our the discussion above.

5 Conclusions

The Diamond and Mirrlees (1971) production efficiency theorem is derived under the assumption that all individuals have access to the same technological possibilities to transform their inputs into outputs. These technological possibilities are described by the aggregate production function. Although an aggregate production technology is a useful device to describe many simple market transactions of homogeneous commodities or publicly traded assets, it does not seem to be very plausible that all individuals have access to the same technological opportunities to transform their labor, assets and other resources into outputs, which can either be sold on the market, consumed, or saved. The main message of this paper is that, in contrast to Diamond and Mirrlees (1971), when individuals face different technological possibilities, the production efficiency theorem generally breaks down.

We have shown that aggregate production efficiency is not desirable for either equity or efficiency reasons. Production of goods should be taxed at higher (lower) rates if individuals with higher (lower) earnings abilities have a comparative advantage in the production of these goods. Outputs of certain goods should also be taxed at higher rates if labor demand in the production of these goods is less elastic. Only when production technologies are of the Gorman polar form, aggregation of individual production functions is possible, and the production efficiency theorem can be recovered.

For future research it is important to empirically examine to what extent individuals indeed operate different production technologies. We believe that this is not an easy task. Individual's factor incomes (labor and capital incomes) are the result of potentially many forces such as individual productive abilities, occupational and human capital decisions, access to markets (for labor, capital and insurance), general-equilibrium effects on factor prices, and so on. Nevertheless, if one is willing to accept that not all individuals have access to the same technological opportunities, then many of the very strong policy prescriptions that follow from Diamond and Mirrlees (1971) – free trade, no intermediate goods taxation, no sectoral differentiation in taxation, use of market prices in public production and social cost-benefit analysis – need not be applicable. Many economic policies that appear to be distortionary at first sight could turn out to be socially desirable after all.

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