**University of California, Berkeley** **Economics 230a**
**Department of Economics** **Fall 2003**

# Problem Set #1

(due 9/30/03)

1. Consider an economy in which relative producer prices are fixed and a representative household maximizes the following utility function:

$$U(c_1, c_2, l) = (c_1 - a_1)^{\beta_1}(c_2 - a_2)^{\beta_2} l^{1-\beta_1-\beta_2}$$

(where $c_1$ and $c_2$ are consumption goods and $l$ is leisure), subject to the budget constraint:

$$p_1 c_1 + p_2 c_2 = w(1-l)$$

A. Derive an explicit expression for the excess burden of taxes on $c_1$ and $c_2$ as a function of the original prices of the two goods, say $p_i^0$ ($i$ = 1,2), the distorted prices, $p_i^1$, and a fixed utility level.

B. Using this measure, show that the deadweight loss is positive for any tax or subsidy on good 2, and that it is strictly increasing (i.e., the marginal deadweight loss is positive) with respect to the size of the distortion, $\left|p_2^1 - p_2^0\right|$.

C. For a tax on good 2, how do the values of excess burden based on compensating and equivalent variations compare?

2. Consider a model of household production, in which the representative household maximizes the utility of market goods *X* and home goods *Z*, *U*(*X*,*Z*). The household has one source of income, labor, and derives no utility directly from leisure. It supplies some of its unit labor endowment to the market, and uses the rest in home production of *Z*. Labor supplied to the market goes into the production of *X* and an intermediate good, hired day care services, *M*. *X* and *M* are each produced in the market subject to constant returns to scale using labor, and the household produces *Z* subject to constant returns to scale using the labor it withholds from the market, *h*, and hired day care services, *M*.

A. Write down the household's utility function and budget constraint (without taxes) as a function of *h*, *X*, and *M*.

B. Suppose the government has imposed a proportional tax on labor income to raise revenue. It is suggested that efficiency might be enhanced by adding a subsidy to market day care services, *M*, in order to encourage individuals to work. Under what circumstances would this argument be correct? (Hint: consider the equivalent tax scheme involving taxes on $X$ and $M$.) If possible, interpret your answer in the context of this specific model.

3. There are several methods of imposing taxes on consumption. These methods can be evaluated and compared with the help of the national income identity, which states that

$$GDP = C + I + G + NX = W + R$$

where $C$ is consumption, $I$ is domestic investment, $NX$ is net exports, $W$ is income from wages and salaries, and $R$ is capital (i.e., all other) income.

   A. A *value-added tax* (*VAT*) is typically imposed on all domestic value added and all imports, with rebates then given on sales for investment, government use or export. Show that this ensures that the tax base equals consumption.

   B. Suppose that government purchases were not exempt from the *VAT*. What impact would this have? Be clear what your assumptions are about the government's budget constraint.

   C. The *flat tax* consists of two pieces: (1) a "cash-flow" tax on all returns to capital, with an immediate deduction for investment, and (2) a tax on wages and salaries at the household level. Show that the flat tax differs from the *VAT* in part B. in that it lacks "border adjustments," i.e., it does not tax imports and provides no rebate for exports. Discuss the possible impact of this difference, using the fact that the balance on foreign accounts requires that net foreign investment, $I^f$, equal net exports plus net foreign income, $Y^f$,

$$I^f = NX + Y^f$$

   (Hint: consider the incentive for foreign investment with and without the tax.)

4. In the Harberger two-sector model, capital bears 100% of the corporate (sector $X$) income tax if labor's share of *before-tax* income is unchanged as the term $T_{KX}$ changes, that is, if the following ratio stays fixed:

$$\frac{wL}{wL + rK + (T_{KX} - 1)rK_x}$$

   A. For $K$ and $L$ fixed and $T_{KX} = 1$, show that this implies that the following relationship holds for the relative changes in $w$ and $r$:

$$\hat{w} - \hat{r} = \lambda_{KX}\hat{T}_{KX}$$

   B. Using the expression for the corporate-tax response of $\hat{w} - \hat{r}$ derived in class, show that this expression is satisfied if sectors $X$ and $Y$ have the same initial factor proportions and the same elasticity of substitution in production.

C. Suppose instead that *X* and *Y* have the same initial factor proportions but that the production elasticity of substitution in sector *Y* is zero. What fraction of the tax does capital bear?