University of California, Berkeley **Economics 230a**
Department of Economics **Fall 2003**

Problem Set #2

(due 11/25/03)

1. Consider an economy in which the government may provide two public goods, X and Y, each with unit cost 1, and household $h \in H$ has a utility function of the form $U^h(X,Y,M^h) = g^h(X,Y) + Z^h$, where Z^h is the household's consumption of private goods, equal to exogenous income M^h less the household's assessment for public goods. The functions $g^h(\cdot)$ may vary over h.

   A. Suppose that $g^h(X,Y) = \alpha^h \log(X+Y)$ and that household h is assigned a budget share β^h of the overall cost of each of the public goods. Show that preferences are single peaked with respect to the total level of public goods, $X+Y$.

   B. Now, suppose that $H = 3$, that $g^1(X,Y) = X \cdot Y$; $g^2(X,Y) = \max(X, Y)$; $g^3(X,Y) = 0$, and that $\beta^1 = \beta^2 = \beta^3 = 1/3$. The government has three spending options, $(X,Y) = (0,0)$, $(0,1)$, and $(1,1)$. Are preferences over public expenditures single-peaked?

2. Consider a one-period model of a small country that takes the world interest rate, r, as given. Other countries impose no capital income taxes. The home country has a representative individual with utility in private goods (C), public goods (G), and labor supply (L), $U(C, G, L) = C + log(G) - ½\, aL^2$. The individual has a fixed endowment of capital, K, which may be invested at home or abroad, and supplies labor to domestic production to maximize utility, taking tax rates, government spending, and factor prices as given. Private and public goods are produced with the same constant-returns-to-scale production function in domestic capital and labor. The government imposes a proportional tax, t, on wages, wL, and a residence-based capital income tax, τ.

   A. Solve for the individual's optimal labor supply, and substitute this into the expression for utility to obtain an indirect utility function for the individual, in terms of factor prices, tax rates, endowment income, and government spending.

   B. Show that the domestic wage rate, w, is independent of government policy or the level of individual labor supply.

   C. Solve for the optimal levels of t and τ. Show, in particular, that t should equal 0.

   D. Suppose, instead, that the country's only available capital income tax were on a source basis. How would this change your answers to parts A and B? Without deriving the optimal taxes in this regime, indicate whether it would still be optimal to set $t = 0$.

3. Consider an individual who wishes to invest initial wealth, W, to maximize the utility of terminal wealth one period hence. The investor's problem consists of two decisions:

   (1) how much of this wealth to place in bonds, which yield a certain return, $i > 0$, and how much to invest in stocks, which yield a stochastic return $r \in [0, R]$, $E(r) = \bar{r} > i$;

   (2) how to distribute these assets between a taxable account and a tax-sheltered account.

   Interest on bonds held in the taxable account (*TA*) is taxed at rate τ, while equity returns are taxed at rate $\alpha\tau$, where $0 < \alpha < 1$. Assets placed in the tax-sheltered account (*TSA*) are tax-exempt. An amount up to $V < W$ may be placed initially in the tax-sheltered account.

   A. Suppose that the individual is risk-neutral. Derive the optimal portfolio, in terms of the amounts of debt and equity held in each account. Now derive the optimal portfolio for an individual who is infinitely averse to risk.

   B. Show that, regardless of the individual's risk aversion, it will never be optimal to hold equity in the *TSA* and bonds in the *TA* at the same time. (Hint: by considering a portfolio shift, prove that such an initial allocation would permit the investor to achieve a higher after-tax return on debt for a given after-tax distribution of returns on equity.)

   C. In reality, some tax-sheltered accounts are not tax-free, but tax-deferred. Suppose that the tax treatment of assets in the *TSA* were changed, so that investors received a initial deduction at rate τ, with all returns (including the initial investment) taxed at rate τ when withdrawn. Assuming that the investor would still be permitted to place up to an amount V in the *TSA*, how would this change your answers to parts A and B?

4. Suppose that a risk-neutral investor faces a tax rate of c on long-term capital gains and a tax rate of $t > c$ on long-term capital losses. (That is, the taxpayer gets a refund for losses at rate t.) The investor has an asset purchased for P_0 a long time ago that is now worth P_1, and must decide whether to (1) sell the asset now and reinvest the proceeds for one more period, or (2) continue holding the asset for one more period. The period is long enough so that, under option (1), the new investment would qualify for long-term capital-gains treatment. The rate of return for the remaining period, r, is stochastic but will be the same under both options. r's mean is positive and its lower support is the ratio $(P_0 - P_1)/ P_1$.

   A. Derive a necessary and sufficient condition for the investor to realize the capital gain now, expressed in terms of some critical value of the ratio $R = P_1/P_0$, say R^*.

   B. How is the value of R^* derived in part A affected by a decrease in c, starting from the case in which t and c are initially equal?

   C. Given the impact on R^* derived in part B, what would happen to the level of capital gains realizations in the first period? to capital gains revenues in the first period? to revenues in present value?