Problem Set #2

(due 11/16/04)

1. In the Harberger two-sector model, with labor and capital fixed, labor bears a fraction ψ of an incremental tax burden Δ if the ratio

$$\frac{wL + \psi\Delta}{wL + rK + \Delta}$$

is unchanged as Δ increases from its initial value of 0.

A. Show that labor's share of the burden, ψ, equals it share of initial income, $wL/(wL+rK)$, if there is no change in the ratio w/r (i.e., $\hat{w} - \hat{r} = 0$) as the tax is introduced.

B. Suppose that the tax introduced is on capital income in sector X, so that $\Delta = (T_{KX} - 1)rK_X$. Derive a condition for $\hat{w} - \hat{r} = 0$, using the expression for $\hat{w} - \hat{r}$ derived in class.

C. Now suppose that $\sigma_D = \sigma_X$ and that sector X uses both capital and labor in production. Show that the condition you derived in part B cannot be satisfied, and that $\hat{w} - \hat{r} > 0$.

2. Consider an economy with overlapping generations, each with a single agent who lives for two periods. The world interest rate is fixed at r. Government debt is issued at the beginning of the period, and taxes, transfer payments and government purchases occur at the end of the period. Initially, the government has no national debt outstanding, and operates a social security system that transfers 1 unit of the numeraire commodity from the older individual to the younger individual in each period.

A. Write down the government's year-t intertemporal budget constraint (GIBC) in terms of national debt, government purchases and government net taxes (taxes less transfers), and show that the government's policy satisfies the GIBC.

B. Now write down the GIBC in its alternative formulation, in terms of the initial level of debt, government purchases, and the generational accounts for all existing and future generations. Solve for the generational account for each generation, and show that this version of the GIBC is also satisfied under current government policy.

C. Suppose that, at the end of the current period, t (i.e., at the beginning of period t+1) the government eliminates the social security system by issuing bonds to pay for the current elderly agent's benefit. Assuming that government services the debt using equal taxes on each future elderly generation, solve for the tax needed to satisfy the GIBC. Solve for all generational accounts after this policy change, and show that they are the same as for the original social security system in part B.

D. Now, compare the generational accounts as of date <u>t+1</u> for the initial social security system and the post-reform system. Do the accounts differ? Is the GIBC still satisfied?

3. Consider an economy in which there are two commodities, deliveries by bicycle messengers (X) and taxi rides (Y). Each is produced competitively using a single factor, capital, that is in fixed supply. Producers of X face the production function $X = (\alpha e^{-\beta Y})K_x$, and producers of Y face the production function $Y = \gamma K_y$, where $K = K_x + K_y$ is the total capital stock. That is, each producer perceives constant returns to scale, but the productivity of capital in sector X is reduced via a negative production externality by the aggregate level of production of Y.

   A. Derive the economy's production possibilities frontier as an expression for X in terms of Y, and show that the production set is not convex.

   B. Letting capital be numeraire, derive the cost functions for producers of each good, $c(X;Y)$ and $c(Y)$, assuming that producers of X take Y as given and that producers of Y ignore their impact on sector X. Solve for the competitive prices at given values of X and Y.

   C. Now, solve for the social cost function for X and Y, $c(X,Y)$, and the marginal costs of X and Y at given production levels. What Pigouvian taxes on producers of X and Y would cause competitive prices to equal marginal social costs, given X and Y?

   D. Suppose that consumers are identical, with preferences that satisfy the utility function $U(X,Y) = X+Y$. How much revenue does the Pigouvian tax raise at the social optimum?

4. Consider an economy in which every family has preferences over two goods, education and housing, defined by the common utility function, $U(E,H) = E^{\alpha}H^{1-\alpha}$. Households differ only with respect to income level, with household i's exogenous income equal to y^i. Housing is produced subject to constant unit costs $p_H = 1$, and may be purchased in any quantity. Education may be produced using one of two technologies: by the private sector as a regular private good with unit cost p_E per family, and by the public sector as a pure public good with unit cost q_E. The level of public education, $\overline{E}$, is financed by a proportional tax at rate τ on income, and no individual household may use public and private education at the same time.

   A. For fixed values of the tax rate, τ, and the level of public education, $\overline{E}$, show that there exists a critical level of income, y^*, above which households choose private school, and below which households choose public school. Show that y^* is increasing in $\overline{E}$, given τ.

   B. Start with your solution for y^* as a function of $\overline{E}$ and τ from part A. Substitute for τ using the government's budget constraint that relates it to $\overline{E}$. Calculate the full effect of $\overline{E}$ on y^*. Show that this effect is larger than the partial effect you solved for in part A, and explain why.

   C. Suppose that the government proposes a small increase in $\overline{E}$, and that voters understand what the impact of this increase on τ would be. As measured by income y, indicate which voters would favor this increase. Identify the possible equilibrium values of $\overline{E}$, at which voters would defeat any proposal for a small change in public education spending.