

Problem Set #1

(due 10/3/05)

1. Consider an economy in which relative producer prices are fixed and a representative household maximizes the following utility function in an aggregate consumption good (C) and two types of leisure, perhaps the leisure of two spouses, l_1 and l_2 ,

$$U(C, l_1, l_2) = C^{\beta_0} l_1^{\beta_1} l_2^{\beta_2}$$

(where $\beta_0 + \beta_1 + \beta_2 = 1$) subject to the budget constraint (with each type of labor having unit endowment):

$$c + w_1 l_1 + w_2 l_2 = w_1 + w_2$$

- A. Rewrite the household's problem in terms of consumption and labor supplies, $L_i = 1 - l_i$.
- B. Solve for the expenditure function in terms of C , L_1 and L_2 .
- C. Derive an explicit expression for the excess burden of taxes on L_1 and L_2 as a function of original wages, say w_1^0, w_2^0 , distorted wages, w_1^1, w_2^1 , and a fixed utility level.
- D. Using the measure you derived in Part C, show that the deadweight loss is zero when the two taxes are zero, and that, starting from zero taxes, there is no first-order deadweight loss (i.e., the derivative with respect to any tax change is also zero).
- E. Starting from arbitrary values for the taxes on the two types of labor, solve for the change in deadweight loss with respect to an increase in the tax on type-1 labor. Interpret the condition that determines whether this change in deadweight loss is positive.
- F. Assume that taxes on the two types of labor must be used to raise a fixed amount of revenue, R , from the representative agent. Derive a condition in terms of the exogenous parameters of the utility function and the budget constraint for the taxes on the two types of labor income to be equal.
2. In the Harberger two-sector model, capital bears 100% of an excise tax on sector- X output if the ratio of labor income to gross expenditures (including the excise tax) is unchanged.
- A. Show that this requires that sector X be more capital intensive than sector Y .
- B. Derive an expression for the change in the ratio of labor income to gross output.
- C. Using the expression you derived in part B, show that, in the limit as sector X accounts for all output in the economy, it is not possible for capital to bear the entire excise tax.

3. In class, we showed that a consumption tax is equivalent to a tax on labor income plus a tax on existing assets. That derivation assumed that assets took the form of homogeneous capital. This question reconsiders the question for a wider class of assets.
- A. Write down the budget constraint for a household that lives for two periods, works in the first period, has initial assets in the first period, and consumes in both periods. Assume that the household initially faces a uniform tax on capital income and labor income at rate t .
 - B. There is no “price level” in the budget constraint in part A, because there are no nominal magnitudes in the model – only relative prices matter. But suppose now that the household holds two types of assets, real capital and government bonds, each yielding the same rate of interest. Rewrite the budget constraint from part A for this case, letting A_1 be the real quantity of capital and A_2 be the nominal stock of government bonds. Assume that the price level is the same in periods 1 and 2, i.e., that there is no inflation.
 - C. Now, suppose that, before period 1, the government replaces the income tax with a sales tax on consumption in both periods, and that the real before-tax wage and the real before-tax interest rate remain the same. Assume also that the price level net of the sales tax does not change in either period. Rewrite the budget constraint from part B for this tax system, and show that the consumption tax is equivalent to a tax on labor income and on all existing wealth.
 - D. Now, change the assumption about the price level in part C. Suppose that, when the sales tax is imposed, the Fed successfully adjusts monetary policy to keep the consumer price index (which includes the sales tax) constant in both periods. How does your answer to part C change?
 - E. Now, go back to the price level assumption from part C, but assume instead that the government bonds are initially not taxed (as is the case in the United States for bonds issued by state and local governments), and so in capital market equilibrium yield a before-tax rate of return under the income tax of $r(1-t)$, where r is the before-tax return to capital. Suppose also that these bonds are consols. How does your answer to part C change?