University of California, Berkeley **Economics 230a**
Department of Economics **Fall 2005**

Problem Set #2

(due 11/15/05)

1. Consider an economy in which relative producer prices are fixed and there are H identical households, each with the following utility function in household consumption of goods 1 and 2, c_1 and c_2, household leisure, l, and aggregate consumption of good 1, $C_1 = Hc_1$:

$$u(c_1, c_2, l, C_1) = c_1^{\alpha_1} c_2^{\alpha_2} l^{1-\alpha_1-\alpha_2} C_1^{-\beta}$$

Each household maximizes this utility function subject to the budget constraint:

$$p_1 c_1 + p_2 c_2 + wl = y$$

where p_i is the consumer price of good i, w is the wage rate, and y equals the value of the household's labor endowment, $w\overline{L}$, plus any lump-sum transfers (or minus any lump-sum taxes) from the government. In this optimization process, the household ignores the effect of its own consumption of good 1 on C_1, i.e., it treats C_1 as fixed when choosing c_1. Let labor be the numeraire commodity (w =1) and let producer prices for goods 1 and 2 be q_1 and q_2.

A. Solve for the household's indirect utility function, conditional on the value of C_1, $V(p_1, p_2, y; C_1)$. Using the household's demand function for c_1 to express C_1 in terms of income and prices, rewrite this indirect utility function solely in terms of prices and income, $\widetilde{V}(p_1, p_2, y)$. Letting the social welfare function be the sum of the utilities of the H identical households, use your expression for $\widetilde{V}(\cdot)$ to obtain a solution for social welfare in terms of prices and aggregate income $Y = Hy$, i.e., $W(p_1, p_2, Y)$.

B. Suppose that the government must raise an exogenous level of revenue R for public expenditures (which, for simplicity, we assume do not enter directly into the utility function). It may do so by imposing lump-sum taxes as well as taxes on consumption goods 1 and 2, with any revenue in excess of R being divided equally among households and rebated in the form of lump-sum transfers. Let θ_i be the proportional tax on good i, i.e., $\theta_i = (p_i - q_i)/p_i$ or $p_i = q_i/(1-\theta_i)$. Solve for θ_1 and θ_2 in terms of the problem's exogenous parameters, showing that the optimal tax on good 2 is zero and that the optimal tax on good 1 is $\theta_1^p = \beta/\alpha_1$.

C. Now, suppose that the government must still raise revenue R but cannot use lump-sum taxes or make lump-sum transfers. Solve for the optimal taxes in this case, θ_1^* and θ_2^*, and show that the optimal ratio of consumer prices should be the same as in case B. That is, show that $\dfrac{q_1/(1-\theta_1^*)}{q_2/(1-\theta_2^*)} = \dfrac{q_1/(1-\theta_1^p)}{q_2}$, where $\theta_1^p = \beta/\alpha_1$ is the Pigouvian tax solution obtained in part B. Relate this result to the "double-dividend" hypothesis.

2. Consider an economy with three local communities of equal population size. Individuals are identical within each community i, with preferences over private goods c and local public goods g governed by the utility function $log(c - a_i) + log(g_i)$ and endowment y_i of the private good. The marginal rate of transformation between public and private goods is 1, and public goods must be purchased separately for each jurisdiction (i.e., there are no spillovers in public goods consumption across jurisdictions); for simplicity, assume also that each community has one individual.

   A. Suppose first that each community chooses its own level of spending on the public good. Solve for the level of public spending and the level of utility in each community as a function of the parameters a_i and y_i.

   B. Now, suppose that all public spending is centrally financed by a proportional income tax at rate t, where t is the same across communities and each community's level of the public good equals one-third of revenue raised in the whole economy. Thus, once the level of t is determined, private and public goods consumption in each community is also determined. Assume that t is chosen by a simple majority vote.

      i. Show that preferences over t are single-peaked in each community, so that the level of t chosen will be that of the median voter.

      ii. Under what condition will total public spending be greater under central provision than under local provision (case A.)?

      iii. Is the equilibrium Pareto-optimal under central provision?

   C. Now, suppose that before voting on the level of public goods, individuals vote on whether to use local provision or central provision. Also suppose that $y_1, y_2 < \bar{y} < y_3$, where $\bar{y} = (y_1 + y_2 + y_3)/3$ is average community income. Show that community 3 will vote for local provision. (*Hint*: First compare the outcomes for a community in which $y_i = \bar{y}$.) What can you say about the preferences of communities 1 and 2? Is it possible that both would vote for central provision? Is it possible that both would vote for local provision?

3. Suppose that there are two countries that use source-based taxes to finance public goods. Each country has one variable factor of production, capital, and a representative agent endowed with such capital, in amounts equal to A in the home country and A^* in the foreign country. Capital flows freely between the two countries, so that the domestic capital stocks in the home and foreign countries, K and K^* (with $K + K^* = A + A^*$) adjust to the point where the after-tax marginal products of capital in the two countries, $r(1-\tau)$ and $r^*(1-\tau^*)$, are equal. In addition to taxes on capital, each country also expropriates all domestic profits, so that government revenue in the home country is $[F(K) - rK] + \tau rK$, with a similar expression holding in the foreign country. All revenue in each country is spent on that country's public goods. Each agent's after-tax income (which, because the after-tax rates of return are equal

in the two countries, can be written, respectively, as $r(1-\tau)A$ and $r(1-\tau^*)A^*$) is spent on private consumption. The production function is identical in the two countries, with $F(K) = log(K)$ and $F(K^*) = log(K^*)$. The utility function in the home country is $log(C) + \beta G$, where C is domestic consumption and G is public expenditures. The utility function in the foreign country is $log(C^*) + \beta^* G^*$, with $\beta, \beta^* \geq 1$.

A. Assuming that the choice of the tax rates τ and τ^* are determined in a Nash equilibrium, derive the first-order condition for the home country's choice of τ in terms of A, A^*, β, β^*, τ and τ^*.

B. Using the first-order condition you derived in part A (without solving explicitly for τ), show that the two taxes are strategic complements, i.e., that $d\tau/d\tau^* \geq 0$. (Assume that $\tau^* \geq 0$.)

C. Suppose that $\beta^* = \beta$. Solve for the optimal home tax rate, τ.