University of California, Berkeley **Economics 230a**
Department of Economics **Fall 2009**

Problem Set #2

(due 11/24/09)

1. Consider an economy in which there are two commodities, musical composition (X) and home construction (Y). Each is produced competitively using a single factor, labor, that is in fixed supply. Producers of X face the production function $X = (\alpha e^{-\beta Y})L_x$, and producers of Y face the production function $Y = \gamma L_y$, where $L = L_x + L_y$ is the total labor supply. That is, each producer perceives constant returns to scale, but the productivity of labor in sector X is reduced via a negative production externality by the aggregate level of production of Y.

   A. Derive the economy's production possibilities frontier as an expression for X in terms of Y, and show that the production set is not convex.

   B. Letting labor be numeraire, derive the cost functions for producers of each good, $c(X;Y)$ and $c(Y)$, assuming that producers of X take Y as given and that producers of Y ignore their impact on sector X. Solve for the competitive prices at given values of X and Y.

   C. Now, solve for the social cost function for X and Y, $c(X,Y)$, and the marginal costs of X and Y at given production levels. What Pigouvian taxes on producers of X and Y would cause competitive prices to equal marginal social costs, given X and Y?

   D. Suppose that consumers are identical, with preferences that satisfy the utility function $U(X,Y) = X+Y$. How much revenue does the Pigouvian tax raise at the social optimum?

2. Suppose that an economy has two goods, education and housing, and that every family has preferences over the two goods defined by the common utility function, $U(E,H) = E^{\alpha} H^{1-\alpha}$. Households differ only with respect to income level, with household i's exogenous income equal to y^i. Housing is produced subject to constant unit cost $p_H = 1$, and may be purchased in any quantity. Education may be produced using one of two technologies: by the private sector as a regular private good with unit cost $p_E = 1$ per family, and by the public sector as a pure public good with unit cost q. The level of public education, $\overline{E}$, is financed by a proportional tax at rate τ on income, and no individual household may use public and private education at the same time.

   A. For fixed values of the tax rate, τ, and the level of public education, $\overline{E}$, show that there exists a critical level of income, y^*, above which households choose private school, and below which households choose public school. Show that y^* is increasing in $\overline{E}$, given τ.

   B. Start with your solution for y^* as a function of $\overline{E}$ and τ from part A. Letting Y equal aggregate income in the economy, substitute for τ using the government's budget constraint that relates τ to $\overline{E}$ and Y. Calculate the full effect of $\overline{E}$ on y^*. Show that this effect is larger than the partial effect you solved for in part A, and explain why.

C. Show that, among individuals who choose public education, there single level of public education, say E^*, that is most preferred by all. If the existing level of public education is initially slightly below E^*, under what condition would a majority vote to increase public education spending to E^*? (Hint: relate y^* to the income of the median voter.)

3. Consider an individual who wishes to invest initial wealth, W, to maximize the utility of terminal wealth one period hence. The investor's problem consists of two decisions:

   (1) how much of this wealth to place in bonds, which yield a certain return, $i > 0$, and how much to invest in stocks, which yield a stochastic return $r \in [0, R]$, $E(r) = \bar{r} > i$;

   (2) how to distribute these assets between a taxable account and a tax-sheltered account.

   Interest on bonds held in the taxable account (TA) is taxed at rate τ, while equity returns are taxed at rate $\lambda\tau$, where $0 < \lambda < 1$. Assets placed in the tax-sheltered account (TSA) are tax-exempt. An amount up to $V < W$ may be placed initially in the tax-sheltered account.

   A. Derive the optimal portfolio, in terms of the amounts of debt and equity held in each account, for an individual who is risk neutral and for one who is infinitely averse to risk.

   B. Show that, regardless of the individual's risk aversion, it will never be optimal to hold equity in the TSA and bonds in the TA at the same time. (Hint: by considering a portfolio shift, prove that such an initial allocation would permit the investor to achieve a higher after-tax return on debt for a given after-tax distribution of returns on equity.)