

### Partial-Equilibrium vs. General-Equilibrium Incidence Expressions

According to the Harberger model, the relative change in the consumer price of taxed good X is:

$$(1) \quad \hat{q}_x - \hat{p}_y = \frac{\frac{\bar{\sigma}}{\lambda^* \theta^*}}{\frac{\bar{\sigma}}{\lambda^* \theta^*} + \sigma_D} \hat{t}_x$$

Note that the numerator term comes from the expression for the production possibilities frontier,

$$(2) \quad \hat{X} - \hat{Y} = \frac{\bar{\sigma}}{\lambda^* \theta^*} (\hat{p}_x - \hat{p}_y)$$

Under profit maximization,  $p_x dX + p_y dY = 0 \Rightarrow \hat{Y} = -\frac{p_x X}{p_y Y} \hat{X}$ , so (2) implies:

$$(2') \quad \hat{X} \left( 1 + \frac{p_x X}{p_y Y} \right) = \frac{\bar{\sigma}}{\lambda^* \theta^*} (\hat{p}_x - \hat{p}_y)$$

Letting good Y be numeraire ( $\hat{p}_y = 0$ ) this may be rewritten:

$$(3) \quad \frac{\bar{\sigma}}{\lambda^* \theta^*} = \frac{\hat{X}}{\hat{p}_x} \left( 1 + \frac{p_x X}{p_y Y} \right) = \eta_X^S \left( 1 + \frac{p_x X}{p_y Y} \right)$$

where  $\eta_X^S$  is the elasticity of supply of good X with respect to its producer price.

Now consider consumer demand, which is determined by the elasticity of substitution,  $\sigma_D$ :

$$(4) \quad \hat{X} - \hat{Y} = -\sigma_D (\hat{q}_x - \hat{p}_y)$$

Under utility maximization,  $q_x dX + p_y dY = 0 \Rightarrow \hat{Y} = -\frac{q_x X}{p_y Y} \hat{X}$ , so (4) implies:

$$(4') \quad \hat{X} \left( 1 + \frac{q_x X}{p_y Y} \right) = -\sigma_D (\hat{q}_x - \hat{p}_y)$$

Again letting good Y be numeraire ( $\hat{p}_y = 0$ ) this may be rewritten:

$$(5) \quad \sigma_D = -\frac{\hat{X}}{\hat{q}_x} \left( 1 + \frac{q_x X}{p_y Y} \right) = \eta_X^D \left( 1 + \frac{q_x X}{p_y Y} \right)$$

where  $\eta_X^D$  is the elasticity of demand of good X with respect to its consumer price.

Substituting (3) and (5) into the incidence expression (1), and noting that  $q_X = p_X$  in the initial equilibrium, we have:

$$(6) \quad \hat{q}_x - \hat{p}_y = \frac{\eta_X^S}{\eta_X^D + \eta_X^S} \hat{t}_x$$

which is the partial-equilibrium expression for the impact on the taxed good's consumer price.