

**Problem Set #1**  
(due 10/14/14)

1. Consider an economy with three local communities of equal population size. Individuals are identical within each community  $i$ , with preferences over private goods  $c$  and local public goods  $g$  governed by the utility function  $\log(c - a_i) + \log(g)$  and endowment  $y_i$  of the private good. For simplicity, assume also that each community has one individual. Both private and public goods have a price of 1 and public goods must be purchased separately for each jurisdiction (i.e., there are no spillovers in public goods consumption across jurisdictions).
  - A. Suppose first that each community chooses its own level of spending on the public good. Solve for the level of public spending and the level of utility in each community as a function of the parameters  $a_i$  and  $y_i$ .
  - B. Now, suppose that all public spending is centrally financed by a proportional income tax at rate  $t$ , where  $t$  is the same across communities and each community's level of the public good equals one-third of revenue raised in the whole economy. Thus, once the level of  $t$  is determined, private and public goods consumption in each community is also determined. Assume that  $t$  is chosen by a simple majority vote.
    - i. Solve for each community's preferred level of  $t$  and show that preferences over  $t$  are single-peaked in each community, so that the level of  $t$  chosen by majority vote will be that of the median voter. What will determine who the median voter is?
    - ii. Under what condition will total public spending be greater under central provision than under local provision (case A.)?
    - iii. Is the equilibrium Pareto-optimal under central provision?
  - C. Now, suppose that before voting on the level of public goods, individuals vote on whether to use local provision or central provision. Also suppose that  $\bar{y} < y_1$ , where  $\bar{y} = (y_1 + y_2 + y_3)/3$  is average community income. Show that community 1 will vote for local provision. (*Hint*: First compare the outcomes in the case for which community 1 is the median voter under central provision.)
2. Consider an economy with fixed producer prices and a representative household that maximizes utility, which is a function of one consumption good and two types of leisure, perhaps the leisure of two spouses,

$$U(c, l_1, l_2) = c^{\alpha_0} l_1^{\alpha_1} l_2^{\alpha_2}$$

(where  $\alpha_0 + \alpha_1 + \alpha_2 = 1$ ), subject to the budget constraint:

$$pc = w_1(1 - l_1) + w_2(1 - l_2)$$

- A. In order to set the problem up in terms of transactions between households and firms, rewrite the household's utility function and budget constraint in terms of consumption and labor supply,  $L_1 = (1-l_1)$  and  $L_2 = (1-l_2)$ .
- B. Derive expressions for the compensated demand for  $c$  and compensated supplies of  $L_1$  and  $L_2$ .
- C. Suppose that the government wishes to raise a fixed amount of revenue from the household using separate proportional taxes on  $L_1$  and  $L_2$ . Based on the standard three-good analysis, use your answers to part B to derive a condition in terms of exogenous variables – wage rates and utility function parameters – for uniform taxation to be optimal.
3. In the Harberger two-sector model, with overall supplies of labor and capital fixed and earning rates of return  $w$  and  $r$ , respectively, labor bears a fraction  $\psi$  of an incremental tax burden  $\Delta$  if the ratio

$$R = \frac{wL + \psi\Delta}{wL + rK + \Delta}$$

is unchanged as  $\Delta$  increases from its initial value of 0; that is,  $dR/d\Delta = 0$ .

- A. Show that labor's share of the burden,  $\psi$ , equals its share of initial income,  $wL/(wL+rK)$ , if there is no change in the ratio  $w/r$  (i.e.,  $\hat{w} - \hat{r} = 0$ ) as the tax is introduced.
- B. Suppose that the tax introduced is on capital income in sector  $X$ , so that  $\Delta = T_{KX}rK_X$ . Derive a condition for  $\hat{w} - \hat{r} = 0$ , using the expression for  $\hat{w} - \hat{r}$  derived in class for this tax experiment.
- C. Now suppose that  $\sigma_D = \sigma_X$  and that sector  $X$  uses both capital and labor in production. Show that the condition you derived in part B cannot be satisfied, and hence that  $\hat{w} - \hat{r} > 0$ : capital's relative share of the tax must be higher than labor's. (*Hint*: you will need to use the fact that  $a_X = \lambda_{LX}\theta_{KX} + \lambda_{KX}\theta_{LX}$ ).