University of California, Berkeley **Economics 230a**
Department of Economics **Fall 2021**

Problem Set #1

(due 10/19/21)

1. Consider an economy in which relative producer prices are fixed and a representative household, with a unit endowment of labor, maximizes the following utility function:

$$U(c_1, c_2, l) = (c_1 - a_1)^{\beta_1}(c_2 - a_2)^{\beta_2} l^{1-\beta_1-\beta_2}$$

(where c_1 and c_2 are consumption goods and l is leisure), subject to the budget constraint:

$$p_1\, c_1 + p_2\, c_2 + w(l\text{-}1) = 0$$

A. Derive an explicit solution (i.e., in terms of prices and preference terms a_i and β_i) for the excess burden of taxes on c_1, c_2, and l as a function of the original, undistorted prices of the three goods (p_1^0, p_2^0, and w^0), the distorted prices (p_1^1, p_2^1, and w^1), and a fixed utility level.

B. Show that excess burden equals zero if $p_i^1 = (1+\theta)p_i^0$, i = 1, 2, and $w^1 = (1+\theta)w^0$ for some constant θ.

C. Using the measure derived in part A, show that the marginal excess burden for an increase in a tax *or* subsidy on good 2 is positive. (*Hint*: relate the change in excess burden to the sign of $(p_2^1 - p_2^0)$.)

2. Suppose that a risk-neutral investor seeking to maximize terminal wealth faces a tax rate of c on capital gains, while facing a tax rate of $t \leq c$ (i.e., getting a refund at rate t) on capital losses. The investor has an asset originally purchased for P_0 that is now worth $P_1 > P_0$, and must decide whether to (1) sell the asset now, pay a tax on $(P_1 - P_0)$ at rate c, and reinvest the remaining proceeds for one more period; or (2) continue holding the asset for one more period before selling. In either case, the rate of return over the next period is r, which is stochastic with pdf $f(r)$. Also, $r \in [r_{min}, r_{max}]$, $r_{min} < 0$, and $E(r) = \bar{r} > 0$. Under choice (1), subsequent gains will be taxed at rate c and subsequent losses will be taxed at t. Under choice (2), total gains, $(P_1(1+r) - P_0)$ will be taxed at rate c, for we assume that $P_1(1+r_{min}) > P_0$, i.e., that the investor will have net gains when selling.

A. Derive an expression for the critical value, say R^*, of the ratio $R = P_1/P_0$, that determines whether the investor will realize gains now (i.e., the investor realizes gains now if and only if $R < R^*$.)

B. Using the expression you derived for R^*, show that $dR^*/dc < 0$, starting from the case in which t and c are initially equal.

C. Also starting from the case in which $t = c$, show that $dR^*/dt > 0$. Explain your result.

3. In class, we observed that a consumption tax is equivalent to a tax on labor income plus a tax on existing assets. This question reconsiders the issue in the case of nominal and real assets.

   A. Write down the budget constraint, expressing consumption in terms of real labor income and real assets, for a household that lives for two periods, supplies labor L in the first period for wage rate w, has initial assets with fixed nominal value B in the first period, and consumes goods in both periods, c_1 and c_2, with price level p in both periods applicable to all quantities (i.e., there is no inflation). Assume the household faces a tax at rate t on capital income and labor income and that saving yields a before-tax rate of return r.

   B. Suppose now that the household initially holds two types of assets, real assets (say, capital) A that have a nominal value determined by the producer price level, and nominal assets (say bonds) B, that have a fixed nominal value, with each yielding a rate of return r. Rewrite the budget constraint for this case, assuming again that the price level is p in both periods and noting that the nominal value of real assets equals pA.

   C. Suppose that, at the beginning of period 1, the government replaces the income tax with a sales tax at rate τ on consumption in both periods, and that the real wage (in terms of the producer price of consumption goods, i.e., the price level *net* of sales tax) and the before-tax return, r, are unaffected by the tax. Also assume that the producer price level remains equal to p in both periods. Rewrite the budget constraint from part B for this tax system, showing that the consumption tax is equivalent to a tax on labor income plus all initial wealth.

   D. Now, change the assumption about the price level in part C. Suppose that, when the sales tax is imposed, the Fed uses monetary policy to keep the <u>consumer</u> price level (which now includes the sales tax), rather than the <u>producer</u> price level, at its original value. How does your answer to part C change, assuming again that the real wage (relative to the producer price level) and interest rate are unaffected by the tax reform?