# Thinking About the Long-run Economic Costs of AIDS

by

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## 1. Introduction

In his *Plagues and Peoples*, McNeill (1976) views history as the interplay between an array of parasites and their human hosts—a struggle in which communicable diseases and human responses to them have profound social, economic and cultural effects. Sweeping through Europe between 1347 and 1351, the Black Death, for example, carried off about one-third of the entire population, without sparing any particular social class or age group. It has been argued that this demographic catastrophe undermined the feudal system and freed these societies from the burden of overpopulation. McNeill and other historians[1] have argued that great plagues also left a cultural and psychological legacy of despair and pessimism. This latter thesis is vigorously challenged by Cohn (2002), who observes that the Plague was soon followed not only by the general optimism and individualism of the Renaissance, but also by the growing conviction, in medical and literate circles at least, that the Plague's causes lay not in God's wrath or the position of the stars, but rather in this natural world. In either case, the Black Death left a lasting impression upon the collective consciousness of European populations.

The first lesson to be drawn from this account is that any attempt to understand the effects of the AIDS epidemic must take a long-term perspective. The second is that any investigation of the long-run economic effects of AIDS must look at social factors. We take both to heart in this essay. Yet there are also some striking differences between the two diseases. In the first wave between 1347 and 1351, the Black Death struck down rich and poor, and young and old alike, usually in a matter days. AIDS is selective, and its individual course is both lengthy and, until the end stages, free of symptoms. Its victims are overwhelmingly young adults in their prime years, the great majority with

[1] See Cohn (2002: 223-4) for various references.

children to raise and care for. This is the fundamental difference in the economic and social effects of the two diseases. For if parents die while their children are still young, then all the means needed to raise the children so that they can become productive and capable citizens will be greatly reduced. The affected families' lifetime income will shrink, and hence also the means to finance the children's education. On a parent's death, the children will lose the love, knowledge and guidance which complement formal education. AIDS does much more, therefore, than destroy the existing abilities and capacities – or human capital – embodied in its victims; it also weakens the mechanism through which human capital is formed in the next generation and beyond. These ramifications will take decades to make themselves fully felt. Like the course of the disease in individuals, they are long-drawn out, insidious and, if the scale of the epidemic is sufficiently great, ultimately lethal to society. All the while, the growing burden on surviving adults can threaten institutions like the extended family, and the drumming reminders of an untimely death can seize society with a pessimism that hinders provision for the future. For these reasons, we shall argue that AIDS threatens the economic and social fabric on a scale that no other disease has done since the Black Death.

The plan of the paper is as follows. In section 2, we motivate our approach by pointing to the sheer scale of the wave of premature adult mortality that now threatens to engulf many countries in eastern and southern Africa. We then identify the main economic consequences of this wave. Section 3 gives a brief account of the standard model (the Solow model), which captures only one of these consequences. It acts as a foil to the detailed treatment of the overlapping-generations (OLG) model, which incorporates a wider array of effects, in section 4. Section 5 examines the policy problem, namely, to find the right combination of interventions to preserve economic growth in the face of the epidemic. We draw together our conclusions in section 6.

## 2. The Approach and its Motivation

The primary effects of premature adult mortality, which is defined here as pertaining to young adults and those in their prime years, whether it be due to AIDS or other causes, are:

(a) Firms and the government lose trained workers and must replace them. In particular, many teachers die prematurely of AIDS.
(b) Substantial expenditures, public and private alike, may be incurred in treating and caring for those who become sick. (This is certainly so in the case of AIDS.)
(c) Savings are also diverted out of net investment in physical and human capital into the treatment and replacement of workers who fall sick and die.
(d) Lifetime family income is greatly reduced.
(e) Children lose the love, care, guidance and knowledge of one or both parents.
(f) The tax base shrinks.
(g) Collaterization in credit markets becomes more difficult, and as a consequence credit markets function less well.
(h) Social cohesion and social capital decline.

Most of the earlier work on the macroeconomic effects of AIDS has focused on the first effect (a). These have been based on variants of the Solow (1956) model, in which the level of productivity in the long run depends on thrift and the rate of population growth. In this framework, a general increase in mortality, with unchanged fertility and thrift, will reduce the pressure of population on existing land and physical capital, and so increase productivity in both the short and the long run. When applied empirically to countries heavily afflicted by AIDS, the model yields predictable results, namely, that the epidemic tends to reduce the aggregate rate of growth—the estimates range from – 0.3 to – 1.5 percentage points per annum—but to *increase* the rate of growth of GDP per head.[3] The latter finding has driven some authors to tinker with other elements of the model, commonly in the form of diverting savings from the formation of physical capital into expenditures on health [effects (b) and (c)] and of lowering the productivity of infected

[3] See, for example, Cuddington (1993) and Over (1992). Multi-sector models of this genre are to be found in Kambou, Devarajan and Over (1992) and Arndt and Lewis (2000).

individuals, in an attempt to overturn it; but these exertions often bring about only modest 'corrections'. It is an odd twist in history that a model which is arguably well-suited to analyze the economic effects of the Black Death should be the first and main choice to investigate those of AIDS.

In this essay, it will be argued that a different framework, with a wholly different emphasis, is needed. The center stage is given over to the formation of human capital as the main wellspring of economic growth, in which the transmission of capacities and knowledge across generations within nuclear or extended family structures plays a vital role. Effects (d) through (f) weaken the mechanism through which human capital is accumulated by depriving the victims' children of parental upbringing and, very likely, much education. To the extent that education in general and needy children in particular are supported by public expenditures, and that treatment and survivors' pensions are publicly provided, the problem is exacerbated by lower tax revenues. Poorly functioning capital markets hinder economic growth, as do a lack of social cohesion and social capital understood in the broad sense, for both form part of the larger structure within which transactions are made. Effects (g) and (h) therefore intensify those of (d) through (f).

The magnitude of all the effects listed above is clearly heavily dependent on the level of premature adult mortality. The first step in any attempt to assess the economic effects of AIDS, therefore, is to establish the scale of such mortality before and after the outbreak of the epidemic. Mortality was high among all age-groups in sub-Saharan Africa in the 1950s and 1960s. It then began to fall, especially among infants and young children, so that by the middle of the 1980s, great improvements in life expectation at birth and substantial improvements at prime ages had been achieved. In most countries, however, premature adult mortality was still significant when the AIDS epidemic began to take its toll. Its impact on the profile of mortality is reduced by what some demographers call the 'substitution effect': some of the individuals who contract AIDS would have died prematurely of other causes. The higher the level of pre-existing mortality, the larger this effect will be. By taking South Africa and Zimbabwe, whose levels of premature adult mortality were comparatively low in the 1980s, but are now very high, we should therefore be able to get a good idea of how large the disease's net effect on such mortality can be, and hence on the gravity of the threat it poses to economic progress.

It is common practice among demographers who deal with AIDS to define premature adult mortality as the probability of dying before the age of 60, conditional on surviving to the age of 15 (Feeney, 1999), which is denoted by ${}_{45}q_{15}$. This is evidently unsuitable for the purposes of studying the effects of mortality on child-rearing: more natural choices are ${}_{20}q_{20}$ and ${}_{30}q_{20}$. The former better fits the cycle of child-rearing; the latter captures the substantial mortality due to AIDS among those in their forties, especially men, when adults are still very much in their productive years. Yet whatever the measure adopted, there has been a dramatic rise in premature adult mortality in both countries following the full-scale outbreak of the epidemic (see Table 1). The forecast levels for 2010 in Dorrington et al. (2001) are grim; those of the U.S. Bureau of the Census, as reported in *ibid.*, are grimmer still.

Table 1. Premature adult mortality by sex in South Africa and Zimbabwe

| Country | Year | ${}_{45}q_{15}$ $(M)$ | ${}_{45}q_{15}$ $(F)$ | ${}_{20}q_{20}$ $(M)$ | ${}_{20}q_{20}$ $(F)$ | ${}_{30}q_{20}$ $(M)$ | ${}_{30}q_{20}$ $(F)$ |
|---|---|---|---|---|---|---|---|
| S. Africa | 1990 | 0.265 | 0.265 | 0.106 | 0.040 | 0.182 | 0.098 |
| | 2000 | 0.419 | 0.419 | – | – | – | – |
| | 2010 | 0.790 | 0.790 | 0.359 | 0.541 | 0.616 | 0.707 |
| Zimbabwe | 1986 | 0.310 | 0.195 | n.a. | n.a. | 0.169 | 0.107 |
| | 1997 | 0.553 | 0.417 | n.a. | n.a. | 0.414 | 0.331 |

Sources.

South Africa: ${}_{45}q_{15}$, Dorrington et al. (2001), who report only the average over both sexes.
${}_{20}q_{20}$ and ${}_{30}q_{20}$, authors' own interpolations from the age-specific mortality profiles in *ibid.*

Zimbabwe: Feeney (1999, Table 1), who reports ${}_{35}q_{15}$ instead of ${}_{30}q_{20}$.

It should be emphasized that these measures refer to steady states, in the sense that the $q$ for each year are calculated on the basis of the continuation of that mortality profile. In this connection, it should be pointed out that the HIV/AIDS prevalence rate among adults in the age-group 15-49 in South Africa rose from about one per cent in 1990 to about 20 per cent in 2000. In the latter year, the rate in Zimbabwe had reached 25 per cent. Dorrington et al. (2001) forecast that the rate will peak in South Africa in about 2006. The above estimates of $q$ for 2010 can therefore be thought of as the values that will prevail if the disease establishes itself in the population at that level for good. The observed values for 1990 (1986 for Zimbabwe) correspond to the counter-factual case in which there is no epidemic at all.

To sum up, in these and other countries in southern and eastern Africa, AIDS has already had a dramatic effect on the level of premature adult mortality, and there is still worse to follow. With the potential scale of the problem firmly established, we now lay out two models within which the economic ramifications of this demographic upheaval can be analyzed.

## 3. The Solow Model

What follows is a brief account of the Solow (1956) model, which is used to capture the effect of premature adult mortality on labor supply, and the attendant consequences for economic growth. We set out the model's salient features to show why, when applied to the question at hand, it tends to yield the results it does.[4] An aggregate good is produced by means of capital and labor under conditions of constant returns to scale (CRS). The total output produced in period $t$ is

(1) $Y(t) = F(K(t), L(t), t)$,

where $K(t)$ and $L(t)$ denote the corresponding total inputs of capital[5] and labor, respectively, and the production function $F(.)$ has all the usual 'nice' properties. The appearance of time, $t$, as an argument of the production function in its own right allows for the possibility that there is technical progress, in the sense that any given bundle of inputs yields ever higher levels of output simply through the passage of time. By virtue of CRS, eq. (1) may be re-written in the intensive form:

(2) $y(t) \equiv Y(t)/L(t) = F(K(t)/L(t), 1, t) \equiv f(k(t), t)$.

This states that output per unit of labor employed in production is a function of the amount of capital with which each unit of labor is equipped and the state of technology,

[4] For a full exposition, see, for example, Barro and Sala-I-Martin (1995).

[5] In order to avoid awkward index number problems, capital should be thought of as being made of the same stuff as aggregate output.

as represented by $t$. For the present, let there be no technical progress, in which case, we may suppress the argument $t$ and write (2) as

(2′) $y(t) \equiv Y(t)/L(t) = F(K(t)/L(t), 1) \equiv f(k(t))$.

A special case must be mentioned at this point, for it is closely connected with one feature of the OLG-framework that both underpins and structures our entire approach. Suppose aggregate output is produced by means of capital alone, where the term 'capital' is now understood to be a suitably defined aggregate of all produced means of production, including human capital. Then CRS implies that (1) specializes to

(1′) $Y(t) = AK(t)$,

where the scalar $A$ reflects the state of technology and the economy's endowment of fixed factors. Models that can be written in this form are called, mnemonically enough, AK-models. It is evident that since aggregate output is proportional to the inputs of aggregate 'capital' $K(t)$, output per head will grow so long as $K(t)$ grows faster than the population.

We turn to behavior. Production is undertaken by firms under conditions of perfect competition. Since there are CRS, all output accrues to households in the form of the wages and rentals paid for the use of labor and capital employed in production. In each period, the stock of capital is supplied completely inelastically, and if there is enough substitutability between capital and labor, all of it will find employment in production. At the end of each period, the current stock is augmented by current investment and diminished by depreciation. Let a fixed fraction $s$ of output be saved and invested, and let depreciation take the form of 'radioactive decay' at the fixed rate $\delta$.

In order to analyze the behavior of this system, both $L(t)$ and its instantaneous rate of growth, $n(t)$, must also be determined. Suppose, initially, that the entire population of working age is homogeneous and that all supply their labor completely inelastically. Given flexible factor prices and enough substitutability between capital and labor, there will be full employment, so that $L(t)$ is then none other than the size of the labor force, and $k(t)$ and $y(t)$ become, respectively, the capital stock per individual of working age and the amount of output he or she produces.

The actual course of $L(t)$ depends on demography. The simplest case is a steady state, in which there is a stable age structure and $L(t)$ grows at the constant rate $n$. By definition, a steady state with respect to $k(t)$ and $y(t)$ holds when the level of savings per worker is just sufficient to cover both current depreciation and the capital needed to equip the increase in the workforce in the same way as those currently employed. The determination of the value of $k$ in the steady state, $k^*$, is depicted graphically in Figure 1, wherein the line through the origin with slope $(n + \delta)$ intersects the curve $sf(k)$ at $k^*$. It is clear that from Figure 1 that $k^*$ is increasing in $s$ and decreasing in $n$. A full derivation is set out for interested readers in Appendix A.

[Figure 1 about here]

Now suppose that there is an outbreak of a deadly disease that afflicts those of working age. This will reduce $n(t)$ sooner or later. If the effect is permanent and $s$ remains unchanged, then once the demographic structure has settled down into its new steady-state, both $k^*$ and $y(k^*)$ will be higher than before the outbreak. Here, it should be emphasized that this demographic adjustment will normally take a rather long time, and that the new value of $k^*$ will be attained only asymptotically. In order to see what happens in the shorter run, observe that a fall in $n(t)$ with $s$ unchanged will result in an immediate increase in the savings available to equip all workers with additional capital. This (temporary) acceleration in the accumulation of capital per worker is bound up with the associated rise in $k^*$. It is also perfectly consistent with a slower rate of growth of, or even a contraction in, total output as $n(t)$ falls. As we have seen in section 2, these results exert a heavy influence on the findings obtained from applying the model empirically.

How sensitive are they to the assumption that labor is homogeneous? At any point in time, workers may differ in their levels of skill or, more broadly, productivity. If they are perfect substitutes for one another, they can be aggregated at once, using relative wage rates as weights. The variable $L(t)$ now denotes the total input of labor measured in efficiency units, with some category of workers serving as the reference group, and the analysis still goes through exactly as above. It follows that whether a disease afflicts one sort of worker more severely than another has no effect on $k^*$ defined as capital per efficiency unit of labor. If, however, skilled and unskilled workers are not perfect substitutes, then a disease that afflicts one more heavily than the other will require a specific response on the supply side, since the numbers of both kinds must grow at the same rate in a steady state. In any event, one must not lose sight of the fact that the outbreak of a disease that afflicts those of working age will affect the average level of output per *natural* unit of labor (i.e., average output per worker), whatever be the degree of substitutability among different types of worker.

An epidemic of such a disease is almost sure to have an effect upon thrift and investment. First, to the extent that it carries off those of working age rather than children or the old, it will raise the dependency ratio. Second, bouts of sickness will result in both a loss of earnings and the burden of expenditures on treatment. Third, the loss of lifetime income among those who succumb to it reduces their families' capacity to invest in the education of their children; and if the disease gets permanently established, higher premature adult mortality in the future will reduce the attractiveness of such investment to all families. Thus, the course of $L(t)$ – measured in efficiency units – following the outbreak depends in a complicated way on the ensuing pattern of morbidity and mortality.

The conclusions from this discussion are clear. Treating the growth of labor supply and thrift as exogenously given in the form of the parameters $n$ and $s$ is perhaps defensible when the demographic regime is fairly stable, but it is potentially dangerous when the outbreak of an epidemic causes that regime to shift sharply. For the response of $n(t)$ and $s(t)$ will almost surely enrich the dynamical behavior of the whole system, including the set of possible steady states.[6] That the adjustment process is likely to be a long drawn-out affair also calls for more attention to be paid to the transition paths to steady states than they commonly receive in analyses based on the Solow model. The behavior of $n(t)$ and $s(t)$ outside a steady state, moreover, involves considerations such as family structure and parents' decisions concerning the education of their children that do not fit naturally into that model. A unified analysis of output, labor supply, accumulation and mortality is, therefore, arguably essential, so that we need an alternative framework, in which these elements do find a natural, common home.

## 4. An Overlapping Generations Framework

[6] In existing applications of the Solow model to the effects of AIDS, the changes in $n(t)$ and $s(t)$ are treated as once-and-for-all events that occur at some point in time. The interactions of output, population, thrift and productivity are not examined.

The structure that follows draws upon, and in some ways extends, that in Bell, Devarajan and Gersbach (2003), whereby the emphasis here is on an accessible discussion of the factors listed in section 2 rather an exhaustive technical analysis of the system. For simplicity, we confine the exposition to the case of two overlapping generations, so that the two periods of life are called childhood and adulthood. With AIDS very much in mind, premature adult mortality is assumed to occur about a decade into adulthood, this being the median time from infection to death in the absence of treatment with anti-retroviral drugs.

### *4.1 The basic model*

The place to start is with the formation of families and of human capital. For the present, let the family structure be nuclear. At the beginning of each period $t$, every young adult chooses a partner with the same level of human capital, which is denoted by $\lambda_t$, that is to say, there is assortative mating. Each cohort begins with equal numbers of young men and women, and all find a partner. All couples have their children soon afterwards. A decade or so later, when the children have just started school, some parents sicken and die, leaving their children as half- or full orphans. At this stage, therefore, the family will find itself in one of the following four states:

1. both parents survive into old age ($s_t$ = 1),
2. the father has died ($s_t$ = 2),
3. the mother has died ($s_t$ = 3),
4. both parents have died ($s_t$ = 4).

Let $\Lambda_t(s_t)$ denote the surviving adults' total human capital when the family is in state $s_t$. Then,

(3) $\Lambda_t(1) = 2\lambda_t,\ \Lambda_t(2) = \Lambda_t(3) = \lambda_t,\ \Lambda_t(4) = 0.$

Let the probability that a family formed at the beginning of period $t$ winds up in state $s_t$ be denoted by $\pi(s_t)$. The relationship between $\pi(s_t)$ and the mortality statistic $q$ will be taken up in section 5.

Apart from innate ability, the two main factors that influence the level of human capital a young adult has attained are the quality of child-rearing and formal education. The former involves not only care and a loving upbringing, but also the transfer of knowledge. As a rule, it is surely both increasing in the parents' human capital and complementary with formal education. Let the latter be represented by the fraction of childhood, $e_t \in [0, 1]$, spent in school, where this phase of childhood can be thought of as spanning the decade six to fifteen years. Then the process whereby these factors yield human capital in adulthood for a child born in period $t$ can be represented by

(4) $\lambda_{t+1} = z(s_t)g(e_t)\Lambda_t(s_t) + 1$, $s_t = 1,2,3,4$.

The term $z(s_t)$ may be thought of as a transmission factor, in the sense that its magnitude expresses the strength with which the parents' human capital creates in their children a potential capacity to attain human capital. If, as is plausible, fathers and mothers are not perfect substitutes for one another, then $z(1) > \max\ [z(2), z(3)]$. It is plausible, too, that $z$ depends on the number of children within the family, but we defer discussion of this point for the moment. The function $g(e_t)$ represents the educational technology, where $g$ is increasing in $e_t$. That $g(e_t)$ and $\Lambda_t(s_t)$ enter into (8) multiplicatively expresses the complementarity between the quality of child-rearing and formal education: the stronger the transmission factor and the greater the parents' human capital, the more productive is any given level of the child's schooling. Now suppose further that some formal education is needed if a child is to realize at least some of the potential $z(s_t)\cdot\Lambda_t(s_t)$ created by child-rearing: formally, $g(0) = 0$. It then follows that any child deprived of all formal schooling will attain $\lambda = 1$ as an adult, whatever be its parents' level of human capital, whereby the value $\lambda = 1$ is simply a convenient normalization.

As it stands, the difference equation (4) is a purely 'technical' relationship, in the sense that it yields the resulting formation of human capital for any given level of education, but says nothing about how that level is chosen. This difference equation is also a stochastic one, in that the child of a union formed in period $t$ with human capital $2\lambda_t$ can attain one of four, arguably different, levels of human capital as an adult in period $t + 1$, depending on the incidence of premature mortality among its parents in period $t$. Given the state $s_t$, and hence $\Lambda_t(s_t)$, the choice of $e_t$ determines the outcome for the child, in the form of the level of $\lambda_{t+1}$.

How, then, is $e_t$ chosen? One possibility is that school attendance is rigorously enforced by the authorities, and that full orphans are taken into first-rate care. Remedial measures might also be needed to offset the disadvantages suffered by half orphans. Such a policy would ensure the continued formation of human capital in society at large, while holding inequality within reasonable bounds. Yet the chances of actually implementing it in most poor countries are remote, to say nothing of the financial demands it would make on the Treasury. In view of these difficulties, it seems much more compelling to treat $e_t$ as the parents' decision, which they make in the light of the resources available to them and the expected returns to education.

We start with output and income. As in the Solow model, there is an aggregate consumption good, which is taken to be the numéraire, and there are CRS in its production, but the only input is labor – measured in efficiency units. In this setting, it is natural to define an adult's endowment of labor so measured as $\lambda_t$, which he or she supplies completely inelastically. Children will be less productive workers than their parents, and given the reasoning underlying (4), it seems plausible to assume that a child could supply at most $\gamma\,(< 1)$ efficiency units of labor to production. Then a family with $N_t$ children has the following level of full income, measured in units of the aggregate consumption good, in state $s_t$:

(5) $$\Omega_t(\Lambda_t, N_t, s_t) \equiv \alpha[\Lambda_t(s_t) + N_t\gamma], \quad s_t = 1,2,3,4$$

where the positive scalar $\alpha$ denotes the productivity of human capital, measured in units of the numéraire, and the expression in brackets is the total amount of labor, measured in efficiency units, that the household can supply. Recalling (1′), it is seen at once that we are employing an AK-model, in which the sole means of production is human capital, which is itself produced through a process involving childrearing and formal education.

The allocation of full income among competing uses lies in the parents' hands – so long as at least one of them survives into old age. We rule out bequests at death, so that full income is spent on the consumption good and the children's education. For simplicity, let the adults behave as equal partners, and let each child receive the fraction $\beta \in (0, 1)$ of a surviving adult's consumption. Since children of school-going age can also work, let this be the alternative to attending school, and let all siblings be treated in the same way. For the present, let the only costs of schooling be the opportunity cost of the children's time. Then, in the absence of taxes or subsidies, the household's budget line may be written as

(6) $$[(3 - s_t) + \beta N_t]c_t(s_t) + \alpha\gamma N_t e_t(s_t) = \Omega_t(s_t), \qquad s_t = 1,2$$

where $c_t(s_t)$ denotes the level of each adult's consumption and the assumption of assortative mating implies that states 2 and 3 are identical in this regard. Observe that, given $\lambda_t$ and $N_t$, single-parent families have less full income and face a higher relative price of education than do two-parent families. Full orphans are left to fend for themselves: they do not attend school and consume whatever income they earn as child laborers.

To complete this account of the family's decision problem, we must specify its preferences. Let mothers and fathers have identical preferences over their consumption of the aggregate good and their children's welfare as adults, the level of which they can influence by choosing the level of schooling $e_t(s_t)$.[7] It is clear from (6) that they will maximize their own consumption by using the children as full-time workers, so that one can say that their altruism towards their children is operative only when they choose $e_t(s_t) > 0$. When both parents survive, let there be no 'joint' aspect of the bundle $(c_t(1), e_t(1))$: each adult enjoys $c_t(1)$ as a private good, whereas the children's resulting level of human capital as adults, $\lambda_{t+1}$, as given by (4), is a public good within the marriage. Since all their children will attain that value of $\lambda_{t+1}$, the only uncertainty that arises concerns the number of children who will die prematurely as adults in period $t + 1$, each such death being regarded as a 'wasted' investment, as it were. To be exact, we assume that parents in period $t$ form expectations about the premature mortality that will afflict their children as adults in period $t + 1$ and take the average number of survivors in weighting the payoff to schooling in the form of $\lambda_{t+1}$.

A formal statement of the household's decision problem is set out in Appendix B. Let $(c_t^0(s_t), e_t^0(s_t))$ denote the household's optimum bundle of current consumption and schooling. It can be shown that, under weak assumptions, $e_t^0(s_t)$ is increasing in $\lambda_t$ whenever $0 < e_t^0(s_t) < 1$. For any given value of $\lambda_t$: (i) children in two-parent families receive at least as much schooling as those in single-parent families, and strictly more if

[7] This implicitly assumes that adults are concerned directly only with their own consumption and the size of their children's budget sets, whereby the grandchildren's opportunities appear in the latter, and so on. Thus, all generations are effectively connected. For further discussion of this formulation in the context of this OLG-structure, see Bell and Gersbach (2003).

the latter choose some, but not full, schooling; (ii) children in two-parent families attain, as adults, at least as much human capital as those in one-parent families, and strictly more if fathers and mothers are not perfect substitutes in child-rearing; and (iii) an increase in expected premature mortality in period $t + 1$ will reduce schooling in period $t$ if $0 < e_t^0(s_t) < 1$, and may do so if $e_t^0(s_t) = 1$ ($s_t = 1,2,3$). All these results accord with elementary intuition. To complete matters, we introduce a further, plausible assumption, namely, that uneducated couples ($\Lambda_t = 2$) are so poor that, in the absence of compulsory education, they choose not to educate their children (i.e., $e_t^0(s_t) = 0$), if even neither dies prematurely.

*4.2 Dynamics*

The next step is to investigate the system's dynamic behavior. By replacing the non-specific $e_t$ in (4) with $e_t^0(s_t)$, we obtain the system's equation of motion, as governed by the rational, forward-looking behavior of individual households in the technical and mortality environment in which they find themselves:

(7) $$\lambda_{t+1} = z(s_t)g\left(e_t^0(\Lambda_t(s_t), s_t, \kappa_{t+1})\right)\Lambda_t(s_t) + 1, \qquad s_t = 1,2,3,4$$

where $\kappa_{t+1}$ is a measure of the survival chances of the children after they have reached adulthood in period $t + 1$, as assessed by the parents in period $t$. Like (4), eq. (7) is a stochastic difference equation, a full treatment of which would go well beyond the scope of this paper. In particular, there are sixteen possible cases: a child in any of the four family states in period $t$ may wind up, as an adult, in any of the four states in period $t + 1$. What follows, therefore, is an intuitive sketch of the main idea.

Full orphans ($s_t = 4$) can be dealt with at once. In the absence of support, they do not attend school, and each will marry another uneducated individual. In the absence of support or compulsion, the offspring of these unions will also go uneducated, and so on. Observe that any premature adult mortality will produce a new crop of orphaned children in each period, and that these lineages will fall into poverty and illiteracy, even if they were not in that condition before. Hence, as time progresses, a steadily increasing proportion of the whole population finds itself in poverty. Caring for orphans is not, of course, a new problem for mankind, and societies have devised various ways of dealing with this problem. Whether these arrangements can withstand the burden of an epidemic like AIDS, however, remains to be seen. We shall return to this question below.

At the other extreme, consider children who have the good fortune, not only to see both parents survive into old age, but also to experience the same outcome themselves in adulthood (this is the case where $s_t = s_{t+1} = 1$). If the level of premature adult mortality is not too large, this is the typical case, the essentials of which are captured in the so-called phase diagram depicted in Figure 2. Let $\Lambda^d(1)$ (> 2) be the parents' endowment of human capital such that for all larger values, their children will receive some schooling, but otherwise none. Observe that $\Lambda^d(1)$ is determined by both the parents' altruism and family income. Similarly, let $\Lambda^a(1) (= 2\lambda^a)$ be the smallest value of the parents' human capital such that their children will receive complete schooling. As the value of $\Lambda_t(1)$ rises from the value 2 to $\Lambda^d(1)$, the children will remain wholly uneducated. As it rises further, from $\Lambda^d(1)$ to $\Lambda^a(1)$, increasing affluence will cause $e_t^0(1)$ will rise from zero to unity; so that, from (7), $\lambda_{t+1}$ will increase from unity to $[z(1)g(1)\Lambda^a(1) + 1]$. Suppose that $[z(1)g(1)\Lambda^a(1) + 1] > \Lambda^a(1)$, that is, every child of couples with $\Lambda^a(1)$ attains a level higher than $\Lambda^a(1)/2$, the human capital of each such parent. Now consider the graph of $\Lambda_{t+1}(1) (= 2\lambda_{t+1})$ against $\Lambda_t(1)$. For all $\Lambda_t(1) \in [2, \Lambda^d(1)]$, the children will not attend school, with the outcome $\Lambda_{t+1}(1) = 2$. For all $\Lambda_t(1) \in [\Lambda^d(1), \Lambda^a(1)]$, $\Lambda_{t+1}(1)$ is increasing in $\Lambda_t(1)$, and its graph cuts the 45-degree line through the origin at least once, by virtue of the assumption that $[z(1)g(1)\Lambda^a(1) + 1] > \Lambda^a(1)$. Suppose it does so just once, at $\Lambda_t(1) = \Lambda^*(1)$.[8] Then the system possesses two stationary equilibria: $\Lambda_t(1) = 2$ and $\Lambda_t(1) = \Lambda^*(1)$. It is clear from Figure 2 that: (i) the former is stable, (ii) the latter unstable, and (iii) the system exhibits a poverty trap. It is shown in Appendix B.2 that when the transmission factor and educational technology combine to ensure that

[8] A sufficient condition for this to hold is that the graph be convex over the said interval. For a full discussion, see Bell and Gersbach (2003).

the condition $2z(1)g(1) > 1$ holds, long-term growth is, in principle at least, possible. In what follows, we assume this condition indeed holds, and Figure 2 is drawn accordingly.

[Figure 2 about here]

Children in single-parent families ($s_t$ = 2,3) face a less favorable situation. All of $\Lambda^d(s_t)$, $\Lambda^a(s_t)$ and $\Lambda^*(s_t)$ will be larger than their counterparts $\Lambda^d(1)$, $\Lambda^a(1)$ and $\Lambda^*(1)$, respectively: in other words, the size of the trapdoor into poverty will be correspondingly larger. The long-term rate of growth will also be lower if fathers and mothers are not perfect substitutes in child-rearing, for if they are not, then $2z(1) > \max[z(2), z(3)]$. Indeed, unbounded growth may not be possible at all for one or both of these sub-groups, even when it is so for two-parent groups.

Summing up, premature adult mortality in a nuclear family setting is a powerful force making for inequality in the future, as well as in the present.

*4.3 The social response to premature adult mortality*

The death of parents while their children are still young poses such grave problems in strictly nuclear families that societies have been driven to find solutions to them. In Africa, the widespread practice of fostering and adoption within the circle of kinship is such a response. In effect, this is a collective – or pooling – arrangement to deal with the individual risks of premature adult mortality as they affect the rearing of children, and hence the future well-being of the group or society. Since any such pooling arrangement also needs rules, suppose all children must be treated alike. We now analyze such an arrangement, albeit in a starkly simplified form.

Let there be complete pooling, in the sense that all the surviving adults take on responsibility for all children. This family structure, or 'state', will be denoted by $s_t = 0$. The rule that all children be treated the same then ensures that there will be no inequality within each generation. A proportion $\kappa_t$ of all adults survive into old age in period $t$. For simplicity, let premature mortality afflict men and women equally, so that each surviving 'couple' will raise, not $N_t$ children, but

(8) $$N_t(0) = N_t/\kappa_t\ .$$

The couple's budget constraint is

(9) $$[2 + \beta N_t / \kappa_t]c_t(0) + \alpha\gamma(N_t / \kappa_t)e_t(0) = \alpha[2\lambda_t + (N_t / \kappa_t)\gamma] \equiv \Omega_t(0)\ ,$$

a comparison of which with (6) reveals that, relative to an otherwise identical two-parent nuclear family, the burden of pooling implies, first, a lower relative price of current consumption, and second, a lower level of full income, measured in units of an adult's consumption – provided $\beta > \gamma$. Both effects work in the direction of reducing schooling, relative to the two-parent nuclear family.

Turning to preferences, let the 'couple' go beyond the requirements of the social rule of equal treatment, and regard all the children in their care, natural and adopted alike, with the same degree of altruism. Altruism in this degree therefore furthers investment in education by increasing the weight attached to the payoff in the form $\lambda_{t+1}$, since $N_t(0) > N_t/\kappa_t$. Whether it, or even equal treatment, prevails in practice will be taken up in section 5.

Given the burden of rearing $N_t(0)$ as opposed to $N_t$ children, it is natural to ask whether the transmission factor will not be correspondingly weakened. To allow for this possibility, we write the latter as $z(0, \kappa_t)$, where it is plausible that $z(0, \kappa_t)$ is increasing in $\kappa_t$. Note also that in the absence of premature adult mortality, pooling will be superfluous and all children will be raised by their natural parents, so that $z(0, 1) = z(1)$. The fundamental difference equation becomes

(10) $$\lambda_{t+1}(0) = z(0,\kappa_t)g\left(e_t^0(\Lambda_t(0),0,\kappa_t,\kappa_{t+1})\right)\Lambda_t(0) + 1,$$

where it should be noted that the current level of premature adult mortality influences both investment in schooling and the transmission factor.

The dynamics are comparatively simple, in the sense that there is only one family state in all periods – so long as the institution of pooling can bear the weight of mortality among young adults. For any given $\kappa_t$, we effectively have a two-parent family with $N_t(0)$ children and transmission factor $z(0, \kappa_t)$, and Figure 2 may be used once more. Whether the trapdoor into poverty is larger than its counterpart in the corresponding nuclear family setting depends on whether the effects of pooling on the family's budget line and the transmission factor outweigh those of altruism. In any event, the long-term rate of growth will be lower, since $z(0, \kappa_t) < z(1)$ whenever $\kappa_t < 1$. When all the single-parent households and full orphans are brought into the reckoning, however, the average growth rate of a society of nuclear families may well be smaller than the growth rate under pooling, namely, $[2z(0, \kappa_t)g(1) - 1]$.[9]

*4.4 The outbreak of an epidemic*

What happens when the outbreak of a hitherto unknown disease brings about a dramatic rise in premature adult mortality – albeit with a lag of a decade or so? At first, very little, for those infected show no symptoms. As time wears on, however, they begin to sicken and die, and the survivors begin to revise their assessments of the chances that their children will die prematurely on reaching adulthood. The first wave of deaths leaves behind orphans on a scale not seen in earlier generations.

A careful distinction between nuclear and collective family structures is needed. In the nature of AIDS, infection of one partner in a marriage is rather likely to be followed by infection of the other, so that the proportion of full orphans in the child population will rise dramatically too. Yet the effects of the adverse shift in the proportions of nuclear families falling into the four states will make themselves fully felt only in the next generation and beyond, in the form of lower levels of human capital averaged over the population as a whole. Under pooling, the fall in $\kappa_t$ has both an immediate, adverse effect on the common budget set by increasing $N_t(0)$ and a damaging long-term effect on the accumulation of human capital.

If the disease persists – or rather, if the adults expect it to do so – then there will be a further effect in the present, certainly adverse and perhaps devastating. If there are nuclear families, a fall in the expected level of $\kappa_{t+1}$ will cause $\Lambda^d(s_t)$, $\Lambda^a(s_t)$ and $\Lambda^*(s_t)$ to increase for two-parent and single-parent families alike, for it will reduce the expected returns to education. In other words, it will make the trapdoor larger, so that groups that were enjoying sustained growth before the outbreak could slide into poverty. Again, this effect will make itself felt only with a long lag, but if the (expected) increase in mortality is large enough, the whole system could switch regimes from one generation to the next. Under pooling, the sharing of resources and responsibilities will stave off

[9] It should be remarked that with a representative family unit of this kind, the OLG model is equivalent to a Solow model in which output is proportional to inputs of efficiency units of labor. The difference is that whereas the OLG model provides the structure which yields the evolution of human capital, the latter must be supplied exogenously to the Solow equivalent, which is then, in effect, the fifth wheel of the coach.

such a collapse if neither $\kappa_t$ nor the expected level of $\kappa_{t+1}$ falls too strongly, even though a permanent fall in $\kappa$ will tend to reduce the long-run rate of growth by weakening the transmission factor. Otherwise, the entire group will slide into poverty together, a disaster that may well undermine the institution's rules – or even the institution itself.

*4.5 Empirical evidence*

While a mere glance at Table 1 reveals that the situation is already grave and threatens to become catastrophic, what we actually need in order to apply the OLG-model are the probabilities of the four family states, that is, the $\pi(s_t)$, which also yield the survival statistic $\kappa$ needed in the pooling case. In order to derive the $\pi(s_t)$ from the mortality measures $q$, an assumption has to be made about the occurrence of mortality within marital unions. In the absence of AIDS, one could perhaps make a case for treating the premature deaths of spouses as independent events, at least as a working approximation for the population as a whole. Given the nature of that disease, however, it is tempting to assume that the infection of one partner outside the relationship would soon be followed by the infection of the other within it. Viewed in a time frame of twenty or thirty years, single-parent households would become rather rare. In fact, the probability of transmission within a union appears to be of the order of 10 per cent per annum under the conditions now prevailing in East Africa (Marseille, Hoffman and Kahn, 2002). Cumulated over the median course of the disease from infection to death, namely, about a decade, this implies that the probability that both partners will become infected, conditional on one of them getting infected outside the relationship, is about 0.65 – high, but still far removed from infection being perfectly correlated within a union. Since it would take anywhere from one to two decades for both to die, the chances that all their children will lose both parents before reaching the end of adolescence are correspondingly reduced. This has a strong bearing on deriving the state probabilities using ${}_{20}q_{20}$, which is the natural measure in connection with analyzing the effects of premature adult mortality on the distribution of family types. To err on the side of caution where the numbers of full orphans are concerned, let us therefore assume that infection is indeed an independent event within a union. This yields the state probabilities corresponding to any choice of $q$ as follows:

$$\pi(1) = (1 - q(M)) \cdot (1 - q(F)),$$

$\pi(2) = q(M) \cdot (1 - q(F))$,

$\pi(3) = (1 - q(M)) \cdot q(F))$,

$\pi(4) = q(M) \cdot q(F)$,

the values of which are set out in Table 2. Recalling the assumption that each cohort begins adulthood with equal numbers of males and females, the proportion of adults surviving beyond the age defined by $q$ is given by

(13) $\kappa = [2\pi(1) + \pi(2) + \pi(3)]/2 = [1 + \pi(1) - \pi(4)]/2$,

the values of which are also reported therein.

The devastating effects of the epidemic on families are appalling to contemplate. In its absence, about 85 per cent of South African children in nuclear families would have grown up in the happy circumstances of having both parents to care for them, and less than one per cent would have been completely orphaned. In the mature phase of the epidemic, as described by the steady state corresponding to 2010, these proportions will lie close together, at 29 and 19 per cent, respectively, and just over one half of all children will be raised by a single parent. The fraction of adults surviving beyond the age of forty would have been 93 per cent; instead, only about 55 per cent will actually do so, which speaks eloquently of the burden that is already beginning to fall on survivors under pooling arrangements. In Zimbabwe, where the epidemic had broken out earlier, the observed shift over the period 1986 to 1997 is scarcely less dramatic, and the epidemic still had not peaked.

One should not, of course, make too much of a few percentage points here or there; but the broad qualitative nature of the results is surely robust to any reasonable amendments to the underlying mortality profiles. Viewed in the light of how human capital is formed, these drastic shifts in the family state probabilities and adult survival rates allow only one conclusion, namely, that there is a very real threat of an economic collapse if the epidemic continues unabated on its present course. That raises the question of how much it will cost to contain, and then reverse it.

Table 2. Family state probabilities and survival rates.

| Country | Year | $\pi(1)$ | $\pi(2)$ | $\pi(3)$ | $\pi(4)$ | $\kappa$ |
|---|---|---|---|---|---|---|
| S. Africa | 1990 | 0.855 | 0.101 | 0.039 | 0.005 | 0.925 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | 2010 | 0.294 | 0.168 | 0.347 | 0.194 | 0.550 |
| Zimbabwe | 1986 | 0.742 | 0.151 | 0.089 | 0.018 | 0.862 |
| | 1997 | 0.392 | 0.277 | 0.194 | 0.137 | 0.628 |

Basis: for South Africa, ${}_{20}q_{20}$; for Zimbabwe, ${}_{35}q_{15}$.

The implications of these estimates for long-run economic growth can be discerned by calibrating the rest of the model in sections 4.3 and 4.4 to data from South Africa, the full details of which are set out in Bell, Devarajan and Gersbach (2003). We incorporate the data series on education, output and mortality in South Africa from 1960 to 1990, and simulate the evolution of the economy with and without the AIDS epidemic from 1990 onwards. The results are quite striking. In the absence of the AIDS epidemic, the South African economy would enjoy modest economic growth with universal and complete primary education (of ten years) in about two generations (by 2050). Per-capita income would quadruple in three generations.

With the AIDS epidemic, and with no interventions, this salutary path will be interrupted. Instead of steadily increasing, primary education levels will decline progressively to the point where, in two generations, the society is full of uneducated adults. Economic performance declines accordingly, and per-capita income, instead of quadrupling by 2080, is only one-half its value in 1990.

Table 3. Two growth paths for the South African Economy

| | *No AIDS* | | | *AIDS* | | |
|---|---|---|---|---|---|---|
| Year | $\lambda$ | $e$ | $y$ | $\lambda$ | $e$ | $y$ |
| 1960 | 2.62 | 0.5 | 19,500 | 2.62 | 0.5 | 19,500 |
| 1990 | 3.14 | 0.64 | 22,340 | 3.14 | 0.2 | 26,370 |
| 2020 | 4.32 | 0.97 | 29,590 | 2.01 | 0.0 | 17,770 |
| 2050 | 7.86 | 1.0 | 53,720 | 1.00 | 0.0 | 12,900 |
| 2080 | 13.85 | 1.0 | 94,720 | 1.00 | 0.0 | 12,900 |

Source: Bell, Devarajan and Gersbach (2003).

## 5. Policy

The real possibility that the economy possesses two strikingly different equilibria – a poverty trap and steady growth – means that there is a job for the government to do, namely, to bring about the latter state by ensuring the reproduction of human capital and its accumulation. Yet although the need for intervention is clear enough, the rationale for it must be sharpened if the government is to intervene in the right way. The first reason is paternalistic; for the failure to attain, or maintain, growth may stem from parents' weak altruism. This altruism has two aspects. First, there is the value parents place on their children's level of human capital in adulthood, which competes with their own current consumption. Second, although parents are not necessarily myopic, in that they recognize that their children will, in turn, care about their own children and so on (see footnote 7), they may not *directly* value any generation's welfare beyond their children's (as is the case here – see Appendix B). The fewer the number of future generations whose well-being is directly valued by today's parents, the weaker are the incentives to invest in education today. The second is that AIDS, as a communicable disease, involves an externality whose full ramifications, it has been argued above, can be enormously damaging. The third has to do with information: the course of the disease is long and insidious, and in many communities, knowledge of how it is transmitted and how to prevent it is often sketchy and sometimes woefully wanting and distorted. Fourthly, there is – or should be – a social aversion to inequality, a condition which premature adult mortality does much to promote.

In formulating policies, it will be useful to begin by drawing a distinction between preventive and remedial measures, with a firm emphasis on their economic and social consequences. A second, more conventional, distinction is between socio-economic sectors: education, health, and support of the needy. We take them up in that order. Once formulated, the spending programs must be financed, and raising the additional revenues will itself have an effect on the accumulation of human capital through the channels identified above. The framework in section 4 enables us to treat interventions in a way that reveals their full ramifications in a long-run setting.

Premature adult mortality in the present and expectations concerning its level in the future emerge in section 4 as key factors influencing long-term economic performance. The outbreak of a disease like AIDS calls for containment on both scores.

The first *economic* preventive measure is, almost tautologically, the vigorous pursuit of public health measures to stem the spread of the epidemic – ideally, in the very early stages. It bears repeating that AIDS is very much a preventable disease. The second such measure, however, involves medical treatment of the infected, in order to maintain their productivity and prolong their lives, and so enable them to provide more of the vital things their children need to become productive members of society. Although there is no medical remedy in the sense of a cure, treating opportunistic infections and providing anti-retroviral therapies still come into the reckoning, at least potentially. The second measure is unquestionably much more expensive than the first, but both directly further the formation of human capital and equality in the next generation.

Premature adult mortality cannot be banished, however, so that remedial action in the form of supporting the survivors, adults and children alike, is also essential. In a nuclear family setting, this will involve payments to single-parent and foster families, and establishing orphanages for those children who would otherwise lack a home. In extended family settings, the aim must be to support the institution itself, while seeing to it that all children enjoy equal treatment. Observe that all these measures are also preventive, to the extent that they further investment in the children's education, and so help them to become productive adults.

It is here that social and educational policy overlap. The case for the government to intervene rests heavily on market failures that result in socially sub-optimal levels of schooling, especially when the family structure is nuclear. Received theory tells us that distortions of this kind should be attacked as closely as possible to their source, which suggests that the right form of intervention is a subsidy payable to the family contingent on each child's attendance at school. In poor societies, this intervention is generally desirable even in the absence of premature adult mortality (Bell and Gersbach, 2003; Siemers, 2002); but in the face of the AIDS epidemic, such a sharp instrument is surely of great importance. It is quite possible, of course, that the government is unable to implement such a policy, in which case, it might have to fall back on unconditional transfers to all needy households – though this is an administratively troublesome policy too.

What, then, is the right balance between preventive and remedial measures? This is a complicated question, to which there is no ready, general answer. The framework of section 4 will, however, enable us to formulate the problem in such a way as to gain qualitative insights. If the relevant quantitative information were available, the framework would even yield answers for a particular setting. There remains the need to finance any bundle of policies selected from the above menu, which has its own influence on the matter. Foreign aid, in the form of outright grants, will relieve the burden, but no society can expect help on a scale that will make increases in domestic taxation unnecessary. For clarity of exposition, therefore, we go to the other extreme of self-reliance. We treat pooling first, for this is relatively straightforward, and then outline the additional considerations that arise when there are nuclear families.

*5.1 Optimal policy under pooling*

There is neither scope nor need for redistribution among families under pooling, so the government can impose poll taxes to finance its expenditures. Let each 'couple' pay the amount $\tau_t$ in period *t*. For the moment, let us rule out school-attendance subsidies, so that $\tau_t$ will be used solely for 'economic' preventive measures, that is, effectively to increase the survival statistic $\kappa_t$. To emphasize the connections, we write $\kappa_t = \kappa_t(\tau_t)$ and hence $z(0) = z(0, \kappa_t(\tau_t))$. These are two of the factors yielding benefits, whereby diminishing returns will set in at some point. The drawback is the reduction in full income in the amount $\tau_t$. The budget line (9) becomes

(11)

$$[2+\beta N_t/\kappa_t(\tau_t)]c_t(0)+\alpha\gamma(N_t/\kappa_t(\tau_t))e_t(0)=\alpha[2\lambda_t+(N_t/\kappa_t(\tau_t))\gamma]-\tau_t\equiv\Omega_t(0,\tau_t)\,.$$

Thus, parents must form expectations about not only the general mortality environment in the next period, but also the level of the government's expenditure (equals tax) on mitigating it. Hopes of a less dangerous future will induce more schooling, pessimism will reduce it. A failure to act swiftly and publicly can therefore do enormous damage through this channel alone.

Armed with this pair of instruments, the government's task is to choose a sequence of taxes over some time horizon starting in period 0, $\{\tau_t\}_{t=0}^{T}$, with the aim of maximizing social welfare. This is a difficult problem, for not only are all the periods connected through the formation of parents' (rational) expectations about the course of future policy, but there may be problems of credibility if the government is unable to commit itself to a certain future course of action. This is not the place to go into the details. A sketchy account is given in Appendix B.4, and the interested reader is referred to Bell, Devarajan and Gersbach (2003).

To complete the formulation of the government's decision problem, we must define social welfare. Since there is no inequality within each generation, but there is the ever-present threat of a collapse, it is intuitively appealing to put a heavy emphasis on accumulating human capital rapidly. It is argued in Bell, Devarajan and Gersbach (2003) that the following claim is valid:

*The pooling case: If, in each and every period, the government chooses the level of the tax in that period so as to maximize the level of human capital attained by a child on reaching adulthood in the next, the resulting plan will be 'good', in the sense of not departing very far from the optimum* .

That is to say, it is enough for the government to look just one generation ahead in order to attain a 'good' result, given the expenditure instruments at its disposal. Appendix B.4 sets out the government's problem in a formal way.

We now introduce school-attendance subsidies, which, if they are available, will yield a further improvement. Let the amount paid per child in period $t$ be $\sigma_t$ for each unit of time he or she spends in school. That the government must allocate its revenues between combating mortality and subsidizing schooling directly calls for a little additional notation. Denote total spending on the former by $\eta_t$, where this expenditure should be thought of as producing a public good within the family, so that $\kappa_t(\eta_t)$ replaces $\kappa_t(\tau_t)$. The family's budget line then reads:

$$(12)\quad \begin{aligned}&[2+\beta N_t/\kappa_t(\eta_t)]c_t(0)+(\alpha\gamma-\sigma_t)(N_t/\kappa_t(\eta_t))e_t(0)\\ &\quad =\alpha[2\lambda_t+(N_t/\kappa_t(\eta_t))\gamma]-\tau_t\equiv\Omega_t(0,\eta_t,\tau_t)\end{aligned}$$

a comparison of which (11) reveals that the subsidy works by reducing the opportunity cost of the child's time, and so encourages investment in schooling directly. The government's budget constraint will always bind at the optimum, so that the ministries of health and education will be competing for funds.

The trade-off is subtle and complex. For any level of taxation, additional spending on reducing mortality will increase not only current full income, but also expectations about future mortality. Both work to increase schooling, and so offset the reduction therein brought about by the correspondingly smaller subsidy on school-attendance. Finding the right balance between them involves solving problem (B.9) in Appendix B.4; but it is intuitively clear that when a society is threatened by very high mortality, the best policy will always involve a fairly substantial effort to combat it.

*5.2 Optimal policies with nuclear families*

In societies with strictly nuclear family structures, premature adult mortality brings about inequality within each generation – unless there is countervailing action by communities or government. In this respect, therefore, the government's task is more complicated than it is under pooling. Indeed, avoiding such inequality may not be possible, depending on the instruments available and the pressures exerted by the need to ensure long-term growth. Yet most of the elements that make up the policy program are clear. Suppose, for simplicity, that the authorities are able to observe each family's status. The tax base is normally provided by two-parent households, for only under conditions of some affluence will single-parent households have any taxable capacity when their children attend school full-time. The society's needy individuals, therefore, are the members of single-parent households and full orphans, who must be cared for in special institutions. The budget lines for households with adults can be written down as before, taking into account whatever taxes and subsidies are payable. Suitable standards must be drawn up for orphanages, whose staffing must suffice to provide decent care for their charges, and whose staff must be paid their opportunity cost in the production of the aggregate consumption good. At the very least, the children should receive the package of consumption and education enjoyed by their counterparts in single-parent households. The government's budget constraint is written out accordingly.

The absence of pooling as a form of insurance and an instrument to ensure equality within each generation requires a reformulation of the policy program. It is argued in Bell, Devarajan and Gersbach (2003) that one way of arriving at a 'good' program is as follows:

*The nuclear family case: In each and every period, choose a tax and expenditure plant so as to maximize the society's expected taxable capacity in the next period.*

Observe the switch from the future attainment of the representative child under pooling to future *aggregate* taxable capacity. The intuition here is that given the need to undertake redistribution within a generation sooner or later, it is this capacity that ultimately determines whether the whole society can eventually escape from want and illiteracy. In dire circumstances, however, it may happen that it is not optimal to grant support to all those in need in some periods. The full problem is written out formally in Bell, Devarajan and Gersbach (2003), and will not be repeated here. Suffice it to say that the tension between providing direct support to families, whether conditional on school-attendance or otherwise, and combating mortality necessarily arises once more, with the further twist that avoiding premature mortality in period $t$ has an immediate effect both on the tax base and on the numbers and types of the needy through its influence on the distribution of family types in period $t$.

*5.3 Empirical estimates of the costs of policies*

In section 5.1, we introduced public spending on combating premature adult mortality from all sources as an instrument to improve both economic performance and well-being. What especially concerns us in this paper, of course, is spending on combating AIDS, and here we know much less than we would like about its effects on the level of mortality.

We proceed in six steps, beginning with two useful reference cases. The first is the counterfactual, in which the human AIDS virus never came into existence, a disease environment which will be denoted by $D = 0$. This, we have argued, can be taken as corresponding to the mortality profile that prevails in the very early stages of the epidemic, as exemplified by Zimbabwe in 1986 and South Africa in 1990. The second step involves the second reference case, which arises when the epidemic simply runs its course, unhindered by public action of any kind. Specifying this alternative poses various problems; for even here, the course of the epidemic depends on individual behavior, which may, in its own turn, respond to the experience of the epidemic, and on whether the virus adapts to its human hosts by becoming less virulent over the longer run. Nor is there a single historical example of this particular epidemic simply running its full course in some society or other to offer us any guidance. In neither South Africa nor Zimbabwe had the epidemic reached maturity by the end of the 1990s, grim though the situation had already become in both countries. Describing this second case is therefore a task more for epidemiologists and virologists than for social scientists. For South Africa, Dorrington et al.'s (2001) forecast for 2010 reflects such considerations, and it seems to us to be a good working approximation of what is needed, with the reservation that palliative care of the sick and the treatment of opportunistic infections are already absorbing resources on a substantial scale.

The third step is to connect these two reference cases through the plausible assertion that very heavy spending on combating the disease would restore the *status quo ante* profile of mortality. If this much be granted, then the probability of premature mortality, viewed as a function of the level of spending on combating the disease, namely, $q(\eta; D = 1)$, will be anchored at both ends of the spending range. Formally,

$$q(\infty; D = 1) = q(D = 0), \tag{14}$$

where the distinction between males and females has been suppressed. In order to supply the shape in between, it is natural, for economists at least, to appeal to diminishing returns; so that $q(\eta; D = 1)$ would be downward-sloping and convex, steep when $\eta$ is small and flat when it is large. It is also tempting to associate these 'end points' of the spending range with preventive measures and anti-retroviral treatments, respectively, and these yield steps four and five, in that order.

One intervention that commends itself in connection with the control of all sexually transmitted diseases (STDs) is to target prostitutes and their clients, whereby the use of condoms is also strongly promoted. Marseille, Hofmann and Kahn (2002) give the corresponding cost of averting a single case of AIDS in Kenya, for example, as 8 to 12 US dollars. This is cheap indeed, but in the nature of the disease and people, it must be inferred that this is an expenditure that will recur annually. They also present evidence that other preventive measures, such as ensuring a safe blood supply and treating mothers at birth with nevirapine, are less cost-effective by a factor of ten or more. Choosing a bundle of diverse preventive measures, they estimate the resulting cost per DALY so saved at $12.50. This figure can be translated into the OLG framework as follows. Taking productive adulthood as spanning the ages 20 to 40, a reduction in ${}_{20}q_{20}$ of 0.01

will yield 0.2 (expected) DALYs for each adult. Hence, spending an additional \$12.50 on such a bundle when $\eta$ is very small will yield a net reduction in ${}_{20}q_{20}(\eta; D = 1)$ of

$$(0.01) \cdot (1/0.2) \cdot (1 - {}_{20}q_{20}(D = 0)),$$

where the presence of the counterfactual term ${}_{20}q_{20}(D = 0)$, which differs between the sexes, allows for 'substitution' among diseases. The slope of the function ${}_{20}q_{20}(\eta; D = 1)$ when $\eta$ is very small is therefore

$$q'(0; D = 1) = -(1 - {}_{20}q_{20}(D = 0)) / (20 \cdot 12.50) = -(1 - {}_{20}q_{20}(D = 0)) / 250. \tag{15}$$

Recall that the expenditure on the bundle is treated as a public good within the family, for both functions are written as a function of $\eta$. Observe that in the absence of diminishing returns to preventive measures, it would be possible to attain ${}_{20}q_{20}(D = 0)$ by spending

$$\xi \equiv [{}_{20}q_{20}(0; D = 1) - {}_{20}q_{20}(D = 0)] \cdot 250 \tag{16}$$

per family.

The fifth step involves going to the other end of the range, where the overwhelming bulk of expenditures goes on treating those with the disease. Such treatment would cover not only opportunistic infections, especially in the later stages of the disease, but also anti-retroviral therapies. These measures keep infected individuals healthier and can extend their lives for a few years, thereby raising lifetime family income and improving parental care. It seems perfectly defensible, therefore, to interpret these gains as equivalent to a reduction in $q$ within the OLG-framework. Marseille et al. (2002) put the cost of saving a DALY by these means at \$395, on the very conservative assumption that the drugs take the form of low-cost generics and that the costs of the technical and human infrastructure needed to support an effective HAART regimen[10] of this kind can be wholly neglected.

The sixth step is to establish how quickly diminishing returns to total expenditures set in, which puts us in very speculative territory. Bell, Devarajan and Gersbach (2003) simply assert that HAART will not become cost-effective until spending on preventive measures and the treatment of opportunistic infections is at least triple the amount $\xi$. This implies that the ratio of the derivatives of $q(.)$ at very low and very high levels of expenditure, respectively, is

[10] Highly active anti-retroviral therapy.

(17) $$\frac{q'(0;D=1)}{q'(3\xi;D=1)}=\frac{395}{12.5}.$$

In order to complete the argument, we need to choose a specific functional form for $q(\cdot)$. Since it must be very flexible to satisfy all of the conditions (14) – (17), the four-parameter logistic is a natural choice. The details of the derivation are set out in Bell, Devarajan and Gersbach (2003).

At this point in the proceedings, the reader can be forgiven for hankering after some idea of how much a comprehensive HAART program might cost. The elements of an estimate for Burkino Faso, a poor west African country in which the prevalence rate is about 8 per cent, are set out in World Bank (2003). If generic anti-retroviral drugs can be purchased from Indian firms, the annual cost of treating each individual would be about \$810; under the next best, negotiated alternative, they would more than double, to \$1730. At the prevailing prevalence rate, the lower of the two estimates translates into an aggregate outlay that is about 80 per cent of the Health Ministry's current budget and about 1.8 per cent of GDP. In Kenya, where the prevalence rate is about 15 per cent and the level of GDP per head is similar, the aggregate outlay would be roughly twice as large. These are sobering numbers, but broadly in line with those emerging from the optimum programs derived in Bell, Devarajan and Gersbach (2003), albeit for South Africa.

*5.4 Other economic losses*

An individual who dies prematurely in adulthood will, all else being equal, produce less over the lifecycle than one who does not. This loss of output, as well as all its consequences, are fully taken into account in the OLG-framework set out in section 4, and there is no real need to estimate it independently in its own right.

Much is also sometimes made of the loss of trained workers in particular. Again, to the extent that productivity in adulthood depends only on the quality of childrearing and formal education, this loss of human capital appears in full in the above framework. To the extent that workers acquire specific skills through training on the job, however, premature mortality among them will indeed entail losses not allowed for in that framework, so that the results derived from it will be on the optimistic side. Assuming that firms are rational in their investments in workers through such training, the costs of training replacements places a lower bound on such losses.

Some idea of the order of magnitude of these losses can be gained from estimates prepared by Schneider and Kelly (2003) for financial services companies in South Africa. In the absence of AIDS, the combined costs to a hypothetical company of the following items: main risks costs, defined pension benefits, replacement and retraining, sick leave, economic costs of absences, maternity benefits and ancillary insured benefits, accounted for 24.8 per cent of the basic payroll. (The contribution of replacement and retraining is a modest 1.1 per cent.) In the current phase of the epidemic, this total is greater by 2.8, 2.9 and 2.3 percentage points in Gauteng, Kwa Zulu Natal and Western Cape Provinces, respectively. Of these increases, only 0.1, 0.2 and 0.1 percentage points, respectively, arise from additional replacement and retraining costs (*ibid.*: 9-10). For a hypothetical manufacturing company in Gauteng Province, the contribution of replacement and retraining costs is both larger in the absence of AIDS, namely, 1.4 per cent of the basic payroll, and more sensitive to the epidemic, which induces an increase of 0.9 percentage points (*ibid.*: 75). None of these estimates is strikingly large, but they are costs all the same.

What we have called the transmission factor $z(s_t)$, $s_t = 0,1,2,3,4$, and the educational technology $g(.)$ play a vital role in determining the dynamical behavior of the system, especially where its long-run rate of growth, $2zg(1) - 1$, is concerned. Our direct empirical knowledge of these elements, so formulated, is limited; but there are ways of getting what is needed to apply the OLG-framework. If a plausible functional form $g(.)$ is simply chosen, then it is possible to use the available series for GDP, the labor force and the average number of completed years of education of the population over 25 years of age in conjunction with eq. (4) in order to extract estimates of $z$, the productivity parameter $\alpha$ and the level of human capital per head, $\lambda_0$, in the base year of the time interval to which the model is calibrated. One such procedure is set out in Bell, Devarajan and Gersbach (2003), using South Africa as an example. By their very nature, these processes are long-term ones, so that the initial calibration must be based on series that stretch back several decades into the past, before AIDS had begun to take hold. How, then, are the effects of the epidemic on $z$ and $g(.)$ to be estimated?

Beginning with the transmission factor, for strictly nuclear families, the value of $z(s_t)$ is given, and changes in mortality work their effects by changing the state probabilities $\pi_t(s_t)$. If, however, surviving couples take in orphans – pooling arrangements provide an extreme 'ideal' – then the sheer burden of caring for more children will, at some point, surely reduce the quality of child-rearing they can provide, though the magnitude of the effect remains a matter for speculation. What with the related financial stress, it is only to be expected that these parents might favor their natural children over their adopted or foster ones in matters of nutrition, education, health and that vital intangible, loving care and attention. Case, Paxson and Ableidinger (2002) find that in a group of African countries, the schooling of orphans depends heavily on how closely related they are to the head of the adopting household. In another recent study of twenty-eight countries, twenty-two of them African, Ainsworth and Filmer (2002) arrive at a more cautious conclusion.[11] While enrollment rates in the majority of the countries studied are lower among orphans than children with two living parents, the differences therein are frequently modest in comparison with those between children from rich and poor households, so that targeting on the basis of orphan status is not always obviously the right option. As Ainsworth and Filmer emphasize, moreover, the ultimate aim is not enrollment, important though that is, but rather learning; yet we know little about how orphans perform compared to children with two living parents. As the numbers of orphans swell with the wave of adult mortality that is now beginning to sweep through Sub-Saharan Africa, winning such knowledge has become a pressing need.

Turning to the educational technology, or more broadly, the supply side of education, much has been made of the very high mortality among teachers, of its grave consequences if replacements are not trained or found, and of the costs of replacing those who sicken and die.[12] None of these considerations appears in sections 4 and 5, but they are readily introduced into the OLG-framework. The essentials are fully captured by looking at pooling arrangements, which have evident expositional advantages. Since a teacher's time in the classroom is spread over the children who are present, it bears a

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[11] In the great majority, the source of the data is a Demographic and Health Survey conducted in the 1990s. These do not cover children who do not live in households, and are therefore likely to miss those full orphans who have been left to fend for themselves.

[12] For a brief account with some references, see Hamoudi and Birdsall (2002).

relation to the average value of $e_t$ in generation $t$. For simplicity, let it be a fixed fraction $r$ thereof for each child. Teachers in period $t$, like all other adults, are endowed with human capital $\lambda_t$, and are correspondingly paid $\alpha\lambda_t$. Eq. (9), the extended family's budget constraint when normalized to a single 'couple', becomes

$$(18)\qquad \begin{aligned}&[2+\beta N_t/\kappa_t(\tau_t)]c_t(0)+\alpha(\gamma+r\lambda_t)(N_t/\kappa_t(\tau_t))e_t(0)\\ &\quad=\alpha[2\lambda_t+(N_t/\kappa_t(\tau_t))\gamma]-\tau_t\equiv\Omega_t(0,\tau_t)\end{aligned}$$

from which it is seen that the need for teachers to bring about learning expresses itself as the component $r\alpha\lambda_t(N_t/\kappa_t(\tau_t))$ of the total 'price' of education. This component is increasing in the current levels of both productivity and premature mortality among adults. Thus, even if mortality among teachers is no different from that in other sections of the population, it still discourages investment in education. If, further, teachers require special training, then the costs discussed above compound the problem by imposing an additional burden on the Treasury, and hence on families through taxation.

It has been argued above that parents effectively choose the level of schooling, so that $e_t$ is influenced by the whole range of factors discussed above, including premature adult mortality. Disentangling them empirically constitutes a very tall order indeed, but one can still attempt to establish whether there is an association between such mortality and schooling, and thereby provide indirect support for the approach chosen in section 4. This has been undertaken by Hamoudi and Birdsall (2002), using Demographic and Health Surveys conducted in twenty-three Sub-Saharan African countries. Employing two specifications, they settle on the estimate that a reduction in life expectancy at birth of ten years is associated with a fall of 0.6 years in the average schooling attained by that cohort (*ibid.*: 23). In view of the fact that life expectancy at birth in most countries in Southern and East Africa fell by ten years or more from 1985 to 2000 (Dorrington and Schneider, 2001), and that the average years of schooling among the populations aged 25-49 lay in the modest range of three to six years, this is a disturbing finding. In the light of the OLG-framework, it would be useful to know whether the general magnitude of this effect also holds good for the more pertinent indicators ${}_{20}q_{20}$ and ${}_{30}q_{20}$, but no such results appear to be available.

## 6. Concluding Discussion

Like the Black Death, AIDS has the potential to transform the societies in which its victims live. But unlike the Plague, AIDS can have this effect largely by undermining the transfer of human capital from one generation to the next—arguably the core mechanism by which societies grow and flourish. The reason is that, in contrast with other epidemics, AIDS is a fatal disease of young adults. Not only does it cause unspeakable human suffering, but AIDS also makes it difficult for these young men and women to provide for the education of their children, not to mention the love and care their offspring need to complement their formal schooling. The result is possibly a whole generation of under-educated (and hence under-productive) youth who, in adulthood, will find it difficult to provide for *their* children's education, and so on. In this way, an otherwise growing economy could, when hit with the AIDS epidemic, spiral downwards into a low-level subsistence economy in three or four generations.

This progressive collapse of the economy is particularly insidious because it will not be felt immediately. Thus, estimates of the economic impact of AIDS that look only at the short-term to medium-term effects of reductions in labor supply are dangerously misleading. They risk lulling policymakers, especially those concerned with short-term economic fluctuations, into a sense of complacency. As we showed in this paper, it is possible to avert the downward spiral, but only with an aggressive set of policies aimed at shoring up the faltering mechanisms of human capital transmission between generations—policies that prevent AIDS, prolong the lives of its victims, and support the education of its victims' children. These policies are expensive but, when viewed against the specter of a collapse of the economy, and possibly of society itself, they seem like a bargain.

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[13] In the great majority, the source of the data is a Demographic and Health Survey conducted in the 1990s. These do not cover children who do not live in households, and are therefore likely to miss those full orphans who have been left to fend for themselves.

[14] For a brief account with some references, see Hamoudi and Birdsall (2002).

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## Appendix A. The Steady State in the Solow Model

Given the assumptions about thrift and depreciation, the instantaneous rate of change of the capital stock is given by

(A.1) $\dot{K}(t) = sY(t) - \delta K(t)$.

Recalling that $k(t) \equiv K(t)/L(t)$, we have

$$\dot{k}(t) = \dot{K}(t)/L(t) - n(t)k(t),$$

where $n(t) \equiv \dot{L}(t)/L(t)$, namely, the proportional rate of change in $L(t)$. Dividing both sides of (A.1) by $L(t)$ and substituting, we obtain the following fundamental differential equation in $k(t)$ and $n(t)$:

(A.2) $\dot{k}(t) = sf(k(t)) - (n(t) + \delta)k(t)$.

In order to analyze this equation, both $L(t)$ and $n(t)$ must be determined.

If $L(t)$ grows at the constant rate $n$, then eq. (4) specializes to the familiar form:

(A.3) $\dot{k}(t) = sf(k(t)) - (n + \delta)k(t)$.

By definition, a steady state with respect to $k(t)$ and $y(t)$ holds when $\dot{k}(t) = 0$, which implies that any $k$ satisfying

(A.4) $sf(k) - (n + \delta)k = 0$

supports such a state. If both inputs are necessary in production (equivalently, $f(0) = 0$), then $k = 0$ is one such value – albeit not a very interesting one. Since the above assumptions on the technology imply that $f(k)$ is a strictly concave function, there will also be a single positive value of $k$, denoted by $k^*$, that satisfies (A.4). The analysis of steady states corresponds to the thought experiment in which there is a chain of autarkic island economies, all endowed with the same technology ($f(.)$ and $\delta$), but exhibiting different levels of thrift ($s$) and population growth.

## Appendix B. The OLG Framework: Some Technical Notes

*B.1 The household's decision problem*

Let preferences be separable in $c_t$ and $\lambda_{t+1}$, with representation in terms of the (expected) utility function

(B.1) $EU_t(s_t) = (3 - s_t)[u(c_t(s_t)) + E_t A_{t+1} v(\lambda_{t+1})]$, $s_t = 1,2$

where the random variable $A_{t+1} \in \{0, 1, \ldots, N_t\}$ is the number of the $N_t$ children born in period $t$ who survive into old age in period $t + 1$, and $E_t$ is the expectations operator. To put it in somewhat different words, parents in period $t$ form expectations about the premature mortality that will afflict their children as adults in period $t + 1$, and take the average number of survivors in weighting the payoff $v(\lambda_{t+1})$.

We are now in a position to write out the household's problem formally. As a preliminary, we substitute for $\lambda_{t+1}$ in (B.1) using (4), which yields

(B.2) $EU_t(s_t) = (3 - s_t)[u(c_t(s_t)) + E_t A_{t+1} v(z(s_t) f(e_t)\lambda_t + 1)]$, $s_t = 1,2$

A family in state $s_t$ (=1,2,3) chooses the bundle ($c_t(s_t)$, $e_t(s_t)$) so as to

(B.3) maximize $EU_t$ subject to (10), $c_t(s_t) \geq 0$ and $e_t(s_t) \in [0, 1]$.

A full analysis of problem (B.3) is to be had in Bell, Devarajan and Gersbach (2003).

*B.2 Dynamics*

In order to establish whether unbounded growth is possible, consider the interval $[\Lambda^a(1), \infty)$, in which $e_t^0(1) = 1$. The growth rate of human capital in period $t$ in those families such that $s_t = s_{t+1} = 1$ is

$$\frac{\Lambda_{t+1}(1)}{\Lambda_t(1)} - 1 = [2z(1)g(1) - 1] + \frac{1}{\Lambda_t(1)}.$$

Hence, for all families in the group such that $s_t = s_{t+1} = 1$ for all $t$ and $\Lambda_t(1) > \Lambda^*(1)$ at some point in time, human capital per head will indeed grow without bound if and only if $2z(1)g(1) \geq 1$, the asymptotic growth rate being $2z(1)g(1) - 1$.

*B.3 Preferences under pooling*

Given the assumptions about altruism in the text, the 'couple's preferences are represented as

(B.4) $EU_t(0) = 2[u(c_t(0)) + E_t A_{t+1}(0)v(\lambda_{t+1})]$,

where the random variable $A_{t+1}(0) \in \{0, 1, \ldots, N_t(0)\}$ is the number of the $N_t(0)$ children born in period $t$ who survive into old age in period $t + 1$. The term $E_t A_{t+1}$ depends on the parents' expectations in period $t$ about premature adult mortality among their children in period $t + 1$. In the presence of the poll tax $\tau_t$, the variable $N_t(0)$ must be rewritten accordingly: the random variable $A_{t+1}(0) \in \{0, 1, \ldots, N_t(0, \kappa_t(\tau_t)\}$ and

(B.5) $E_t A_{t+1} = [E_t \kappa_{t+1}(\tau_{t+1})] \cdot N_t / \kappa_t(\tau_t)$.

Thus, they must form expectations about not only the mortality environment in the next period, but also the level of the government's expenditure (equals tax) on mitigating it. If these expectations are stationary, that is, that the future will be like the present, then (B.5) will specialize to the simple form

(B.6) $E_t A_{t+1} = N_t$.

*B.4 Optimum policy*

Where expectations are concerned, it is argued in Bell, Devarajan and Gersbach (2003) that a good approximation to a full optimum can be achieved as follows. Suppose there are stationary expectations, so that (B.6) holds and the forward connection among periods is cut. Then, starting from period $t$, the sequence $\{\tau_t\}_{t=0}^{T}$ can be constructed as a series of taxes, each element of which is derived independently of all future values as the optimum for the particular period in question. There is, however, an important connection with the immediately preceding period: the condition $\tau_{t-1} \leq \tau_t$ must not be violated, for otherwise the extended family will have chosen the level of education in period $t - 1$ on the basis of falsely optimistic expectations about mortality in period 1. Observe that if the sequence settles down into a stationary one, it will also involve a rational expectations equilibrium.

Formally stated, the government's problem is as follows: starting in period 0,

(B.7) $\max_{\tau_t}[2z(0, \kappa(\tau_t))g(e_t^0(\Omega_t(0, \tau_t), 0, \kappa(\tau_t)))\lambda_t + 1]$, s.t. $\tau_t \geq 0$, $\tau_t \geq \tau_{t-1}\ \forall t$

Observe that the potential problem of credibility is implicitly assumed away: when the family forms its (stationary) expectations in period $t$, the government has found some way to commit itself to $\tau_{t+1} = \tau_t$ .

Following the introduction of school-attendance subsidies as an additional instrument, the government's own budget constraint reads

(B.8) $$\eta_t + \sigma_t (N_t / \kappa_t(\eta_t)) e_t^0 (\Omega_t(0, \eta_t, \tau_t), 0, \sigma_t, \kappa_t(\eta_t)) \leq \tau_t .$$

Problem (B.7) becomes

(B.9) $$\max_{\eta_t, \sigma_t, \tau_t} \left[ 2z(0, \kappa(\eta_t)) g(e_t^0 (\Omega_t(0, \eta_t, \tau_t), 0, \sigma_t, \kappa(\eta_t))) \lambda_t + 1 \right],$$

s.t. $\tau_t \geq 0$, $\tau_t \geq \tau_{t-1}$ and (B.8) $\forall t$.

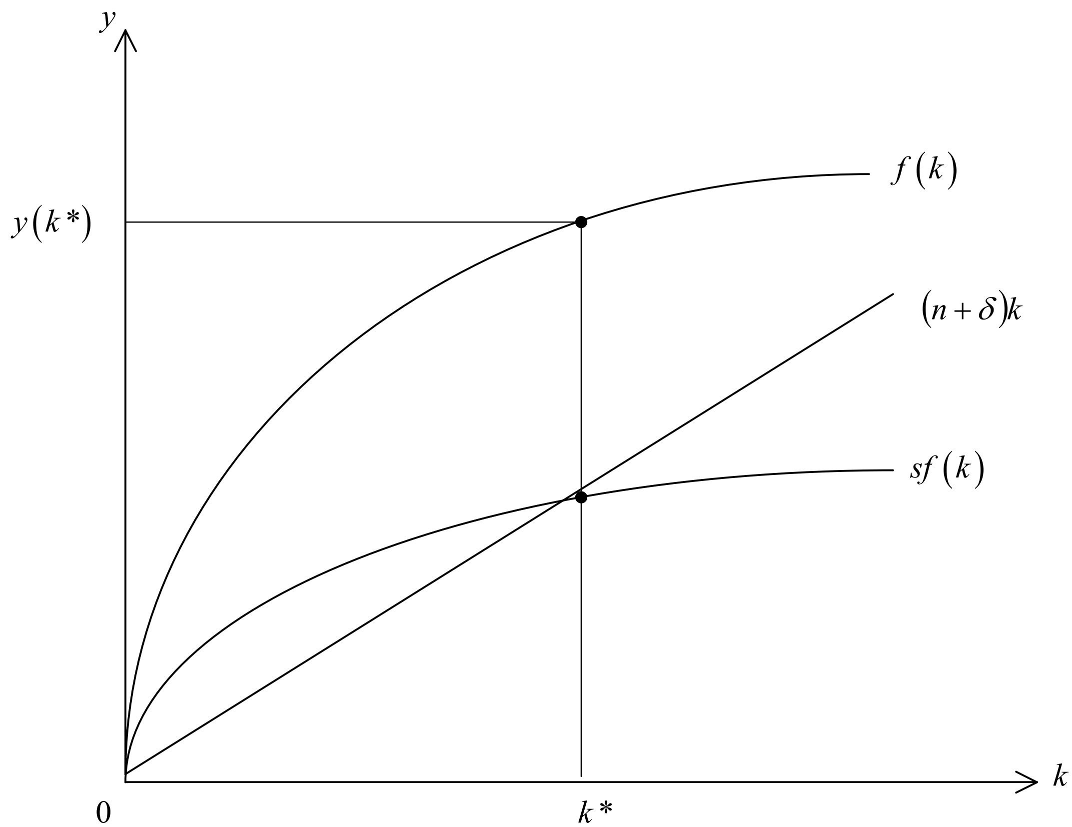


Figure 1.   The steady state in the Solow-Swan Model

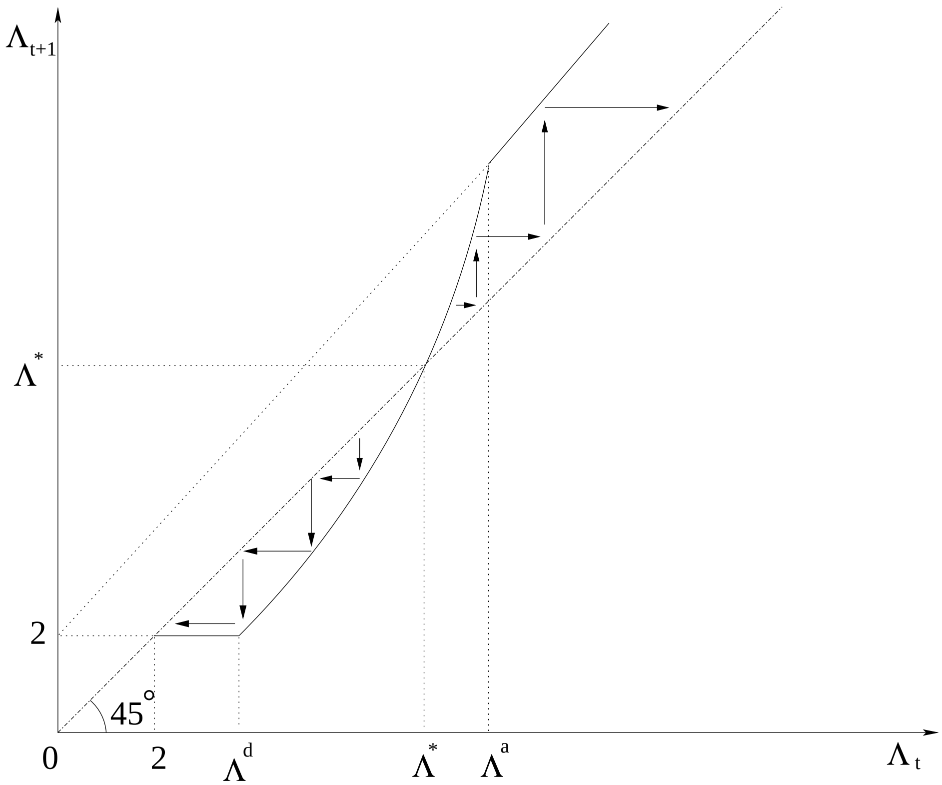


Figure 2. The phase diagram in the absence of premature adult mortality