MIDDLEMEN MARGINS AND GLOBALIZATION

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Abstract

We provide a competitive theory of middlemen or entrepreneurs who develop brand-name reputations necessary to overcome product quality moral hazard problems, embedded in a Heckscher-Ohlin model of North-South trade. Agents with heterogeneous entrepreneurial abilities sort into different sectors and occupations, with endogenous intersectoral mobility. In some contexts competitive equilibrium is characterized by restricted inter-sectoral mobility; benefits of trade liberalization in the South accrue disproportionately to middlemen, North-South factor price differences grow, and offshoring reduces inequality in the South. In other contexts there is enough mobility; classical Stolper-Samuelson and factor price equalization results hold.

Keywords: middlemen, reputation, inequality, trade liberalization, North-South trade; JEL Classification Nos: D33, F12, F13, F21

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1 Introduction

Conventional trade theory focuses mainly on sources of production costs, ignoring the role of endogenous marketing costs and margins that accrue to trade intermediaries. Yet there is considerable evidence of the importance of intermediaries and associated markups that drive large wedges between consumer and producer prices.\(^5\) One needs to explain the role of trade intermediaries and analyze determinants of their markups, in order to understand their implications for distributive impacts of trade integration or offshoring.

In this paper we construct a theory of middlemen or entrepreneurs who develop brand-name reputations needed to overcome product quality assurance problems to final consumers, besides providing managerial inputs such as finance, technology, supervision or marketing. (In what follows we use the terms ‘middlemen’ and ‘entrepreneurs’ interchangeably). There is considerable evidence of the role of brand names and reputation in the context of trade, from consumer studies (Berges and Casellas (2007), Roth and Romeo (1992)); accounts of the role of trust and long-term relational contracting in international trade (Rauch (2001)), as well as econometric analyses of specific traded goods (Banerjee and Duflo (2000), Dalton and Goksel (2009), Machiavello (2010)).\(^6\) Our model of middlemen builds on the theory of Biglaiser-Friedman (1994), and extends it to an open economy general equilibrium setting. Reputational markups form part of overall product costs; the asso-

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\(^5\)Feenstra (1998) provides the following illustration of these markups: a Barbie doll sold to US customers for $10 returns 35 cents to Chinese labor, 65 cents covers the cost of materials, and $1 for transportation, profits and overhead in Hong Kong. Mattel, the US retailer earns at least $1 per doll. Morisset (1998) reports that the price of coffee declined 18% on world markets but increased 240% for consumers in the US between 1975–93. The average margin between US consumer price and world price for beef, coffee, oil, rice, sugar and wheat increased by 83% between 1975–94. Feenstra and Hanson (2004) calculate Hong Kong markups on re-exports of Chinese goods at 12% of its GDP, while manufacturing accounted for only 6%. The average markup rate accruing to Hong Kong intermediaries on re-exported Chinese goods was 24%. Ahn, Khandewal and Wei (2009) report that in the early 1980s, 300 trading firms accounted for 80% of all Japanese trade, with the ten largest accounting for 30%. In China they report that trading firms currently account for 22% of exports and 18% of imports.

\(^6\)Berges and Casellas provide evidence from Argentinian consumer surveys showing the greater importance of brand names compared with labels, seals and certification in consumer perceptions of food quality. Roth and Romeo and Chiang and Masson (1988) describe the role of reputations of countries-of-origin in consumer perceptions. Rauch provides a survey of evidence concerning the role of social and business networks in trade, and intermediaries that arise in the absence of such networks. Banerjee and Duflo, Dalton and Goksel, and Machiavello respectively test models of reputation formation and their role in exports of Indian software, of cars to the US, and Chilean wines to the UK respectively. Hudson and Jones (2003) discuss problems faced by developing countries in signalling their quality to export markets, owing to the kinds of goods they specialize in, and lower rates of ISO-9000 certification.
ciated rents would be sacrificed by entrepreneurs in the event of losing their reputation which provides them requisite incentives to maintain quality. The size of these rents are proportional to the size of the firm, which in turn is correlated with the entrepreneur’s productivity, or ability to ‘manage’ (i.e., finance working capital, supervise production workers or market the product). Agents in the economy differ in their latent entrepreneurial ability. Only those with ability above some (endogenously determined) threshold satisfy the incentive compatibility conditions for credible quality assurance, in any given sector of the economy. We focus on steady states of such a model, under the assumption of a given, unchanging ability of each agent in the economy. Agents with heterogeneous ability sort themselves into occupations and sectors. Low ability agents have no choice but to do manual or production work, and are thus prevented from selling directly to final consumers. High ability agents become middlemen or entrepreneurs that ‘manage’ production, pay production workers and sell to customers, earning rents or markups. To focus exclusively on the nonconvexities arising due to reputations, we assume that the underlying production technology satisfies constant returns. Moreover, all agents take prices as given; hence entrepreneurial markups represent competitively determined incentive rents, rather than market power.

The incentive constraints endogenously generate entry barriers, with associated restrictions on mobility of entrepreneurs across sectors. There are no technological restrictions on the mobility of agents: neither the ‘capital’ provided by entrepreneurs (working capital, human capital or marketing capital), nor the work performed by production workers is specific to any sector of the economy. We embed this in a two-by-two Heckscher-Ohlin trade model, and explore the resulting implications for classical results of this theory, such as patterns of comparative advantage, Stolper-Samuelson or factor price equalization results. Identifying the managerial services provided by entrepreneurs as ‘skilled’ labor, and production work as ‘unskilled’ labor, the main variables of interest are ‘skill premia’ or relative returns to entrepreneurs and workers in different sectors.

In one set of contexts, competitive equilibria turn out to be associated with restricted inter-sectoral mobility of entrepreneurs, with differences in skill premia across sectors. Conditions under which this happens involve differences in severity of the quality moral hazard problem across different goods, and relative TFP high enough in (or consumer tastes biased enough

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7Hence all variations in firm size across industries are generated only by underlying differences in moral hazard. Extending the model to make it more realistic would have to incorporate variations in scale economies.

8This interpretation would be valid if we extend the model to allow for skilled and unskilled workers employed by entrepreneurs, with relative productivity of skilled workers increasing in the entrepreneur’s ability.
in favor of) the high-moral-hazard sector. For instance, consider the case where the quality moral hazard problem is more severe in a less skill-intensive ‘low-tech’ good $L$ than the more skill-intensive ‘high-tech’ good $H$. This case is empirically plausible: quality is harder to control and inspect in farms or in labor-intensive manufactured good sectors, product warranties are rare, and such goods rarely feature in consumer reports. In such a case, skill premia are higher in the $L$-good sector under autarky as long as relative TFP in the $L$-good sector is not too low. Agents with the highest ability become entrepreneurs in the $L$-sector, those of intermediate ability become entrepreneurs in the $H$-sector, and those of the lowest ability become production workers (working in either sector). We illustrate our results for this context, though (as explained in (e) below) they also apply when the high-tech good is more prone to moral hazard under some additional assumptions on the technology.

(a) A preliminary issue concerns the validity of the Rybczynski theorem and resulting North-South patterns of comparative advantage. If the elasticity of substitution between production work and managerial inputs is small enough, Northern countries with a larger endowment of ‘skill’ may end up with a comparative advantage in the less skill-intensive $L$-good. This is an instance of a Leontief paradox, echoing a result of Wynne (2005), though the reasoning is somewhat different. Nevertheless, in what follows, we consider the situation where the South has a comparative advantage in the $L$-good.

9This is as long as product prices are fixed. More generally, when product prices are endogenously determined, this is true under the additional assumption that the elasticity of demand for the relative outputs of two goods with respective to their relative price is less than one.

10Scandals over safety of Chinese exports of farmgoods and toys have erupted in recent years, highlighting quality concerns for less skill-intensive goods exported from developing countries. Using data spanning a large number of countries, Hudson and Jones (2003) show ISO-9000 certification rates are highest in electrical and optical equipment, basic metal and fabricated metal products, machinery and equipment sectors. Conversely, agriculture and farm products, textiles, wood and pharmaceuticals have the lowest accreditation rates. Accreditation take up rates are also lowest in less developed countries. In similar vein, Dobrescu (2009, Table 1) shows that that 14% of Slovenian manufacturing exporting firms in textiles/tobacco, wearing apparel, leather/shoes, wood between 1995-2005 had ISO certification, compared with 36% in more capital intensive sectors (chemicals, rubber, machinery, communication equipment, instruments, motor vehicles).

11Wynne’s argument rests on greater financial development between North and South countries, wherein the North may acquire comparative advantage in labor-intensive goods because these goods are also more subject to a financial market friction. In our model, the underlying distortion does not vary across countries. Instead, it results from differences in skill premia across sectors: if the $L$-sector premium is large relative to the $H$-sector premium to start with, a small increase in skill endowment ends up raising the relative premium in the $L$-sector, as the $L$-sector premium falls by a lower proportion than in the $H$-sector. This raises relative production of the $L$-good.
(b) If the skill premium is higher in the $L$-sector to start with, a rise in the price of the $L$-good results in a rise in the skill premium in this sector, and a fall in the $H$-sector. The return earned by production workers will fall, relative to earnings of $L$-sector entrepreneurs — reversing the Stolper-Samuelson result. The intuitive reason for this is that the relative output of the $L$-good must rise in this case, for which it is essential for entrepreneurs to enter the $L$-sector. These newly entering entrepreneurs must be less able than incumbents, so the margins in the $L$-sector must rise relative to wages to allow them to function credibly in the $L$-sector.\textsuperscript{12}

In this situation a rise in the price of $L$-good causes inequality to rise in the $L$-sector, and fall in the $H$-sector; the implication for the economy-wide average skill premium is thus ambiguous. If the $L$-sector is large enough to start with, the economy-wide skill premium will rise. Moreover, if the South has a comparative advantage in the $L$-good, differences in some factor prices (e.g., $L$-sector skill premium and profit rates) across countries are magnified with trade integration. Production workers earn lower real wages in the South under free trade. Trade integration can thus raise the average skill premium in both North and South, if the $L$ and $H$-sectors are the dominant sectors respectively in those countries, with corresponding rises in intra-firm inequality within those sectors. Owing to induced impacts on entrepreneurial rents, it is also possible for trade integration to lower welfare in one of the two countries.

(c) Northern entrepreneurs have incentives to offshore production to the South even under free trade, owing to North-South differences in profit and wage rates. The distributive effects of offshoring are the opposite of trade integration: differences in skill premia and wages across countries fall, as does inequality within the Southern $L$-sector. The key difference between trade integration and offshoring concerns the pool of potential entrepreneurs. With trade integration, the most skilled entrepreneurs are already in the lucrative $L$-sector; expansion of the sector must therefore involve entry of less skilled entrepreneurs from within the South, which necessitates a rise in the skill premium. In contrast offshoring is associated with entry of high skilled entrepreneurs from the North, motivated by South-North differences in profit rates. Such entry can coexist with (and in fact induces) a decline

\textsuperscript{12}The more precise reasoning is based on the clearing of the market for production workers. A higher price of the $L$-good initially induces some entrepreneurs to switch from the $H$-sector into the $L$-sector — besides inducing $L$-sector entrepreneurs to employ more production workers – both of which causes the demand for production workers and hence their wage to rise. This depresses the skill premium in both sectors. The drop in the $H$-sector premium causes the labor market to slacken, owing to an exit of entrepreneurs from the $H$-sector and a drop in employment of workers by each remaining $H$-sector firm. So the $L$-sector premium must end up higher, to ensure that the labor market clears. The key feature here is that a rise in the $L$-sector skill premium tightens the labor market.
in the Southern $L$-sector skill premium.

(d) Trade integration raises North-South offshoring incentives in the $L$-sector, and lowers them in the $H$-sector, essentially because trade integration magnifies profit differences in the $L$-sector and shrinks them in the $H$-sector.\(^{13}\)

(e) In the case where the $H$-good is more subject to moral hazard, anti-Stolper-Samuelson results similar to (b) above obtain, provided the skill-premium in the $H$-sector is larger to start with, under the additional assumption that the elasticity of substitution between skilled and unskilled labor is large enough. This assumption ensures that changes in within-firm employment outweigh effects of cross-sector employment differences resulting from some entrepreneurs switching sectors.

(f) On the other hand, if both goods are subject to the same degree of moral hazard, or if initial differences in skill premia are absent despite differences in moral hazard (e.g., if relative TFP is sufficiently low in the high moral hazard sector), competitive equilibria involve enough inter-sectoral mobility of entrepreneurs within each country to ensure that classical results of the mobile factor version of the Heckscher-Ohlin theory obtain.

Existing empirical evidence (e.g., as surveyed by Goldberg and Pavcnik (2007), Harrison, McLaren and McMillan (2010), Winters, McCulloch and McKay (2004) or Wood (1997)) concerning the distributive impact of trade integration on developing countries shows the results are ambiguous, with increases in skill premia in some contexts and decreases in others. Our results provide a potential explanation for these findings, in terms of conflicting effects on export and import-competing sectors, and different results for cases corresponding to different degrees of inter-sectoral mobility of entrepreneurs, which could be tested in future research.\(^{14}\) The evidence also shows limited

\(^{13}\)Antras and Caballero (2009) consider a model where one good is subject to financial frictions, and the South is more subject to financial frictions, acquiring a comparative advantage in the good lacking any frictions. They show that factor prices are not equalized with free trade, and trade integration raises incentives for North capital owners to lend to entrepreneurs in the financially unconstrained sector in the South. Their analysis thus predicts complementarity of trade integration with capital flows, whereas our results pertain to offshoring wherein the results end up being sector specific.

\(^{14}\)This is based on the presumption that entrepreneurial rents in our model are interpreted as returns to skilled or non-production workers, which is valid once we extend the model to allow for heterogeneity of worker skills, with relative productivity of skilled workers increasing in the entrepreneur's productivity. The only paper we are aware of examining differences in effects of trade liberalization on wages in exporting and import-competing firms is Amiti and Davis (2008). Using Indonesian manufacturing census data for 1991-2000, they find average wages were lower in import-competing firms and higher in exporting firms following a cut in tariffs. This is consistent with our model, provided average wages include (at least some fraction of) earnings of entrepreneurs. Amiti and Davis do not examine effects on wage inequality.
factor mobility across sectors in response to trade liberalization, consistent both with our theory as well as models with exogenous factor-specificity (e.g., Anderson (2010)). In contrast to standard specific sector models, the extent of specificity in our theory is endogenously determined.\footnote{Other differences are that in Anderson’s theory, inequality rises between specific capital in export and import-competing sectors, with no implications for inequality between capital owners and workers. Our focus is on the latter form of inequality. Moreover, with a fixed pattern of factor specificity, the former kind of inequality must rise as a result of trade integration. In our approach the nature of sector specificity is endogenously determined, generating different kinds of inequality effects of trade integration depending on the nature of associated specificity.} Moreover, explanation of some key results are qualitatively different from models with exogenously imposed specificity: e.g., a rise in the $L$-sector premium in (b) above following a rise in the product price of $L$ is associated with entry of less efficient entrepreneurs into the $L$-sector, rather than the absence of entry. This is consistent with evidence provided by Fafchamps and Hill (2008) concerning the rising gap between wholesale and farmgate coffee prices in Uganda following increases in export prices during 2002-03. They show this could not be explained by accompanying changes in transport or storage costs. Instead, the rising demand for coffee exports and accompanying rise in middleman markups induced entry of a less efficient set of middlemen. Similar findings are reported by McMillan, Rodrik and Horn (2002) in the context of rising trader margins for cashews in Mozambique during the 1990s: a falling ratio of farmgate to export prices was accompanied by an increase in the number of traders, especially informal, unlicensed traders buying in smaller quantities directly from farmers’ homes.

The surveys of empirical evidence cited above also highlight the importance of context within which trade liberalization takes place, such as policy environment, infrastructure access or local institutions. Our theory provides one possible source of context-specificity pertaining to inequality and other determinants of entry barriers into entrepreneurship in specific sectors. In the presence of imperfect capital markets, agents’ wealth represents an important determinant of access to capital, and thus of entrepreneurial ‘ability’ in our model, as stressed by occupational choice models in development (e.g. Banerjee and Newman (1993)). Economies with highly polarized wealth distributions that lack a middle class are ones where there is limited scope for entry into lucrative sectors of trade intermediation. Our theory predicts that the output response of economy to trade liberalization will then be sluggish, with high increase in inequality (see Proposition ??, part (i)). This could potentially explain differences between effects of trade liberalization in Latin America and sub-Saharan Africa during the 1980s and 90s compared with East Asia during earlier decades that have been highlighted by Winters, McCulloch and McKay (2004) or Wood (1997).
The next section describes relation to existing literature in more detail. Section 3 introduces the model of the closed economy. Section 4 considers the case where the \( L \)-good is more prone to moral hazard. We first describe the equilibrium of the supply-side, where product prices are taken as given. This is followed by the economy-wide equilibrium, and the main comparative static properties. We then extends it to a two country context and studies effects of trade liberalization and offshoring. Section 5 discusses the extension to the case where the \( H \)-good is more prone to moral hazard, while Section 6 concludes.

\section{Related Literature}

The only other work we are aware of concerning intermediation and its implications for trade theory is that of Antras and Costinot (2010a,b). In their theory, it is exogenously assumed that producers cannot sell to consumers directly, and must search for traders who can. The allocation of bargaining power between producers and traders is exogenous, whereas in our context it is determined endogenously by occupational and sectoral choices of agents. They abstract from heterogeneity within producers or traders. The results of our respective approaches also differ markedly. The benefits of trade integration are evenly divided between producers and traders in one version of their model, and accrue entirely to producers in another. Offshoring by Northern traders may render Southern producers worse off, if the bargaining power of the former is large enough. This is in contrast to our model where offshoring always makes Southern production workers better off. A common prediction, on the other hand, is that Southern traders will be worse off as a result of offshoring.

Similar to many recent trade models and consistent with empirical evidence (e.g., see the survey by Harrison, McLaren and McMillan (2010)), our model predicts Southern entrepreneurs locating in the export sector are of higher ability than other entrepreneurs, and earn correspondingly more. An obvious extension of our model wherein reputations depend not just on the product characteristics but also the markets in which they are sold — specifically, where international reputations are harder to build than domestic ones — would imply that the productivity thresholds for exporting would be higher than for domestic production in all countries, not just in the South. In such a context, trade integration would generally raise inequality between exporting and non-exporting firms. In this respect our approach has some similarity to that of Helpman, Itzhoki and Redding (2010), in which trade liberalization may raise inequality by inducing high productivity firms to search more intensively for high ability workers. In similar vein, Costinot and Vogel
(2010) model matching between heterogeneous productive tasks and workers of heterogeneous abilities. However their analysis generates generalized Stolper-Samuelson predictions when the source of trade is differences in factor endowments across countries. Matsuyama (2007) provides an alternative approach wherein the activity of exporting — involving transport, finance, marketing and communication — is itself more skill-intensive than production. A rise in export activities owing to improved technology of transport or communication can then end up increasing the demand for skilled workers, and eventually the skill premium. Verhoogen (2008) provides a theory and supporting empirical evidence that cars marketed in the US are of higher quality than those in Mexico (owing to non-homothetic preferences), whence increased exports of Mexican-produced cars to the US following an exchange rate devaluation of the Mexican peso generated higher demand and relative wages of skilled Mexican workers. Zhu and Trefler (2005) provide evidence with a cross-country panel wherein the rise in skill premia across middle income and developing countries was positively correlated with a shift in export shares towards more skill-intensive goods. All these approaches stress the correlation between firm productivity and export activities, which generates rising inequality as an outcome of trade integration, a feature shared by our approach, though it operates through a different (reputational) mechanism.\textsuperscript{16}

Differences between effects of trade integration and offshoring have been stressed by a number of recent papers, for reasons quite different from those in this paper. Feenstra and Hanson (1996) pioneered the literature on offshoring and inequality, showing how inequality could rise in both North and South as a consequence of offshoring low-skill tasks in the North to the South where these are relatively high-skilled. Such a mechanism relies on heterogeneity of production worker skills, something our model abstracts from. In a model with a continuum of worker skills, Grossman and Rossi-Hansberg (2008) elaborate how offshoring can benefit domestic workers via employer cost-savings through better matching, that are passed on to workers in a competitive labor market. Antras, Garicano and Rossi-Hansberg (2006) and Kremer and Maskin (2003) study related models in which agents of heterogeneous abilities sort into hierarchical teams. Inequality rises in the South in these models owing to the matching of high ability agents in the South with worker teams from the North. Karabay and McLaren (2010) examine effects on risk and long term employer-employee contracting that coexist with spot markets. Our theory abstracts from risk considerations, or the possibil-

\textsuperscript{16}One qualitative distinction between these theories and ours is that quality, productivity or profits vary continuously in the former, whereas there is a discontinuous rise in profits of entrepreneurs at the thresholds for entry into each sector in our theory. This has implications for welfare effects of changes in trade costs or policies.
ity of some production tasks within any given sector being offshored while others are not. Instead, we emphasize how offshoring and trade integration may have opposite effects on inequality between entrepreneurs and workers, owing to differences in the associated entry patterns and pools of potential entrepreneurs that can enter any given sector.

Wynne (2005) and Antras and Caballero (2009, 2010) present trade models with financial frictions which affect production of one good more than another, with North countries less subject to financial frictions than South countries. Our model is based instead on frictions arising from quality moral hazard which affect different goods in different ways, where the nature of the moral hazard problem is assumed to be the same between North and South. These give rise to some features which are similar, though there are many differences in the detailed way in which these appear. As noted in the Introduction, shared features include the possibility of a Leontief paradox, complementarity between trade and capital flows, and the role of wealth distributions.

3 Closed Economy Model

3.1 Endowment and Technology

We normalize the size of the population to unity. Each agent is characterized by a level of entrepreneurial ability or skill \( a \), a nonnegative real variable. We provide alternative interpretations of ‘skill’ below. A fraction \( 1 - \mu \) of agents have no skill at all: \( a = 0 \): we refer to them as unskilled. The remaining fraction \( \mu \) are skilled; the distribution of skill is given by a cdf \( G(\tilde{a}) \) on \((0, \infty)\). We shall frequently use the notation \( d(a) \equiv \int_{a}^{\infty} \tilde{a}dG(\tilde{a}) \). The cdf \( G \) will be assumed to have a density \( g \) which is positive-valued. Then \( d \) is a strictly decreasing and differentiable function.

There are two goods \( L \) and \( H \), and two occupations: labor or production work, and entrepreneurship. Each entrepreneur can operate at most one firm, which produces any one of the two goods upon combining labor with ‘managerial services’ provided by the entrepreneur. Labor is hired on a competitive market. There is no market for hiring managerial services, owing to moral hazard problems not explicitly modeled here.\(^{17}\) Hence man-

\(^{17}\)In practice, of course, firms may employ more than one manager in order to grow, but problems of managerial moral hazard and coordination across managers eventually limit the size of firms (as emphasized by a large literature on ‘organizational diseconomies of scale’, e.g., Williamson (1967), Calvo and Wellisz (1978), Keren and Levhari (1983), Qian (1994) or van Zandt and Radner (2001)). In order to explore the industry or general equilibrium implications of limits to the size of firms created by such problems, we adopt the simplifying assumption that a firm is managed by a single entrepreneur, similar to
agement services must be self-supplied by the entrepreneur. Examples of these services are: provision of essential raw materials, supervising and coordinating production undertaken by hired workers, or marketing the product. These correspond to alternative interpretations of entrepreneurial skill: if credit markets are imperfect, skill can be interpreted as the entrepreneurs wealth which determines her ability to finance purchase of raw materials. Or skill may refer to the entrepreneurs ability to supervise workers, as in Lucas (1978).

Any given agent makes the following decisions: (a) whether to become an entrepreneur or worker; (b) if she decides to become an entrepreneur, she selects which good to produce; (c) how many workers to employ; and (d) one of two quality levels for the good to be produced. The production function for good $i$ is $X_i = A_iF_i(n_i, a)$ for the high quality version of the good, and $A_iF_i(n_i, z_ia)$ for the low quality version, where $z_i > 1$ is a technology parameter representing the severity of the quality moral hazard problem, $A_i$ is a TFP parameter, $a$ denotes the skill of the entrepreneur, and $n_i$ the units of labor hired. $F_i$ is a CRS, smooth and strictly concave production function. Producing lower quality enables an entrepreneur to produce a larger quantity of the good with the same number of workers, as it increases the number of effective units of skill.

In what follows we shall refer to production work as unskilled labor, and entrepreneurship as skilled labor. Note that neither unskilled or skilled labor is specific to either sector. Given the assumption that a firm can have only one manager, there will be no market for entrepreneurs: every entrepreneur works for herself, managing her own firm. Entrepreneurial rents will correspond to imputed prices of ‘skill’ which will be equalized across all agents, which clear the market for skill. In other words, optimal employment of production workers and an entrepreneur’s own skill will be the profit-maximizing factor combinations that would have been chosen by an ‘as if’ firm owner who pays for both unskilled and skilled inputs at the (imputed) factor prices, and ends up with zero profit. Returns earned by entrepreneurs in any given sector will be linear in their own skill. This allows a simple strategy.


In some cases we specialize to the case of a Leontief technology, a limiting case of such a technology.

In the imperfect capital market interpretation, producing low quality goods requires fewer raw materials per unit of output, so corresponds to a higher number of effective units of entrepreneurial skill. The same happens in the production supervision interpretation, since producing a lower quality version requires less intensive supervision by the entrepreneur. An alternative formulation of the moral hazard problem would be one where lower quality goods are produced at lower cost (i.e., with fewer workers) rather than in higher quantity corresponding to a given number of workers. This is closely related to our formulation and the two versions coincide with a Leontief technology.
measure of the returns to skill within any sector.

Consider the cost-minimizing factor combinations in each sector, when skill is imputed a cost $\gamma$ relative to unskilled labor: $(\theta^H_n(\gamma), \theta^H_a(\gamma))$ and $(\theta^L_n(\gamma), \theta^L_a(\gamma))$ are defined as

$$(\theta^H_n(\gamma), \theta^H_a(\gamma)) \equiv \arg \min \{\theta^H_n + \gamma \theta^H_a \mid F_H(\theta^H_n, \theta^H_a) = 1\}$$

and

$$(\theta^L_n(\gamma), \theta^L_a(\gamma)) \equiv \arg \min \{\theta^L_n + \gamma \theta^L_a \mid F_L(\theta^L_n, \theta^L_a) = 1\}$$

The following assumption states that good $L$ is more labor intensive than good $H$: for any common ratio of factor costs, production of $L$ uses a higher ratio of unskilled labor to skilled labor in the cost-minimizing factor choice. One can think of $L$ as corresponding to agricultural products or low-end manufactured goods, while $H$ corresponds to high-tech manufactured goods or services.

**Assumption 1** For any $\gamma > 0$,

$$\frac{\theta^L_n(\gamma)}{\theta^L_a(\gamma)} > \frac{\theta^H_n(\gamma)}{\theta^H_a(\gamma)}$$

### 3.2 Entrepreneur’s Profit Maximization and Equilibrium Price-Cost Relations

Consider an entrepreneur in sector $L$, facing a product price of $p_L$ (with the $H$ good acting as numeraire, so $p_H \equiv 1$). Suppose the wage rate for unskilled labor is $w$. If this entrepreneur were to decide to produce the high quality version of product $L$, she would solve the following problem:

$$\max_{n_L} p_L A_L F_L(n_L, a) - w n_L. \tag{1}$$

The optimal employment of unskilled labor $n^*_L$ is a function of $p_L/w$, characterized by the first-order condition

$$(p_L/w) A_L \partial F_L(n^*_L, a)/\partial n_L = 1. \tag{2}$$

Let $\Pi^*_L(p_L, w; a)$ denote the resulting level of profit earned by the entrepreneur. Define

$$\gamma_L \equiv \frac{\Pi^*_L}{wa} \tag{3}$$

the skill premium in the $L$-sector.
Using standard analysis of the profit-maximization conditions, we obtain the following price-cost relations in each sector (using $\gamma_i$ as the imputed price of skill in sector $i$):

\[
\frac{p_L}{w} \leq \frac{\theta^L_n(\gamma_L) + \gamma_L \theta^L_a(\gamma_L)}{A_L} \quad \text{with } \frac{1}{w} = \text{if } X_L > 0. \quad (4)
\]

\[
\frac{1}{w} \leq \frac{\theta^H_n(\gamma_H) + \gamma_H \theta^H_a(\gamma_H)}{A_H} \quad \text{with } \frac{1}{w} = \text{if } X_H > 0. \quad (5)
\]

The left-hand-side of the preceding conditions are the product wage in the two sectors respectively, which are decreasing functions of their respective skill premia. Hence the skill premium is the key measure of inequality between entrepreneurs and workers in any given sector.

Equation (4, 5) yields the following equation for ratio of prices of the two goods to their respective unit costs:

\[
p_L = \frac{A_H}{A_L} \frac{\theta^L_n(\gamma_L) + \gamma_L \theta^L_a(\gamma_L)}{\theta^H_n(\gamma_H) + \gamma_H \theta^H_a(\gamma_H)} \quad (6)
\]

in the case where both goods are produced in positive quantities, with a corresponding inequality in the case of complete specialization. Note that the right-hand-side is increasing in the skill premium in sector $L$, and decreasing in the skill premium in sector $H$. Hence (4) expresses a relation between the skill premia in the two sectors, and the price $p_L$ of product $L$ relative to $H$. This can be expressed as follows:

\[
\gamma_L = \lambda(\gamma_H; p_L, A_H), \quad (7)
\]

For any given product price $p_L$ and set of TFP parameters, it expresses a monotone increasing relation between the skill premia in the two sectors. And for any given $\gamma_H$ and TFP parameters, it expresses a monotone increasing relation between $p_L$ and $\gamma_L$.

Various properties of this relationship between skill premia in the two sectors will be used subsequently. For now we note one property which plays an important role in establishing uniqueness of equilibrium.

**Lemma 1** $\frac{d\gamma_L}{d\gamma_H} \equiv \lambda_1(\gamma_H; p_L, \frac{A_H}{A_L}) > 1$ whenever $\gamma_L \geq \gamma_H$.

Assumption ?? plays an important role here. Since sector $L$ is less skill-intensive, an equal increase in the skill premium in the two sectors will cause unit cost in the $L$-sector to increase by less than in the $H$-sector. Hence the premium must rise by more in the $L$-sector to ensure that the ratio of unit costs remains the same.
3.3 Quality Moral Hazard Problem

Customers do not observe the quality of the product at the point of sale. We assume they value only the high quality version of the product, and obtain no utility from the low quality version. Entrepreneurs will be tempted to produce the low quality version which enables them to produce and sell more to unsuspecting customers. The short-run benefits of such opportunism can be held in check by possible loss of the seller’s reputation. With probability $\eta_i$, an entrepreneur selling a low-quality item in sector $i$ will be publicly exposed (say by a product inspection agency or by investigating journalists). In this event the entrepreneur’s brand-name reputation is destroyed, and the agent in question is forever barred from entrepreneurship. In equilibrium, customers will purchase only from entrepreneurs for whom the threatened loss of reputation is sufficient to deter short-term opportunism. Hence in order for an entrepreneur with skill $a$ to be able to operate in sector $i$, the following incentive constraint must be satisfied:

$$\gamma_i a \geq \gamma_i w z_i a + \delta \eta_i \frac{w}{1 - \delta} + (1 - \eta_i) \gamma_i a \frac{w}{1 - \delta}$$

where $\delta \in (0, 1)$ denotes a common discount factor for all agents. The left-hand-side of (8) is the present value of producing and selling the high quality version of good $i$ for ever. The first term on the right-hand-side, $\gamma_i w z_i a$ represents the short-term profit that can be attained by the entrepreneur upon deviating to low quality. With probability $\eta_i$, this deviation results in the entrepreneur losing her reputation for ever, in which case the agent is forced to work as an unskilled agent thereafter. With the remaining probability the entrepreneur’s reputation remains intact.

The incentive constraint can be equivalently expressed as

$$a \geq m_i / \gamma_i$$

where

$$m_i \equiv \frac{\delta \eta_i}{\delta \eta_i + (1 - \delta)(1 - z_i)} > 1$$

is a parameter representing the severity of the moral hazard problem in sector $i$.

Equation (9) represents a reputational economy of scale, which also translates into a sector-specific entry barrier in terms of entrepreneurial skill required. Intuitively, higher skilled entrepreneurs produce and earn profits at a higher scale (conditional on being able to operate as an entrepreneur), while the consequences of losing one’s reputation are independent of the level

\[\text{Recall that a deviation to low quality is equivalent to an increase in the entrepreneur’s effective skill from } a \text{ to } z_i a.\]
of skill. The stake involved in losing reputation is thus proportional to the entrepreneurs skill, which has to be large enough for the agent to be a credible seller of a high-quality good.\footnote{We assume customers can infer quality from observing the size of the corresponding firm and existing prices, by checking whether the incentive constraint is satisfied.}

The sector-specific entry barriers represent elements of a specific factor model. However unlike most specific-factor models, these barriers are endogenously determined rather than exogenously imposed: e.g., the skill threshold for entry into a particular sector is decreasing in the skill premium in that sector. The reason is simple: a higher skill premium means entrepreneurs have more to lose from losing their reputations.

Note also that $m_i > 1$ implies that entrepreneurs with skills above the required threshold for sector $i$ will strictly prefer to be entrepreneurs in sector $i$ rather than work as an unskilled agent. The per period profit from the former option is $\gamma_i wa \geq wm_i > w$ if (??) is satisfied.

The results will turn out to depend critically on the relative seriousness of the moral hazard problem across the two goods. For good $i$ this is represented by $m_i$, which is a function of exogenous parameters. We shall contrast two cases:

(A) $m_L > m_H$: the $L$-good is more prone to moral hazard.

(B) $m_L < m_H$: the $H$-good is more subject to moral hazard.

Quality moral hazard problems could be larger in the less skill-intensive good, owing to problems in quality control or regulation of these goods, and the relative lack of product warranties for farm or light manufactured goods compared with high-tech durable goods. The $H$-good is more durable; it is produced in a more automated and regulated production process which is easier to inspect, and thus allows less scope for skimping on labor or other essential raw material requirements. An offsetting factor would be the greater technological complexity of these goods, combining a larger number of components in the production process. This may lead to high costs of ensuring high quality, as emphasized in the O-ring theory of Kremer (1993). Owing to the absence of any concrete empirical evidence on which of these two cases is more plausible, the purpose of our analysis is to examine how the results differ between the two cases. We focus first on case A. A subsequent section explains how the results extend to Case B.
4 Case A: Where the $L$-good is More Prone to Moral Hazard

We break up the analysis of competitive equilibrium into two steps. First we take product prices as given, and derive the resulting equilibrium of factor markets: occupational choices and the market for production workers, which comprise the supply-side of the economy. At the second step we shall bring in consumer demands and thereby determine product prices.

Definition Given $p_L$ the price of good $L$ relative to $H$, a factor market equilibrium of the economy is a wage rate $w$ and skill-premia $\gamma_L, \gamma_H$ such that: (i) every agent takes these prices as given and selects between different occupations (unskilled worker, $L$-sector entrepreneur, $H$-sector entrepreneur) to maximize earnings subject to incentive constraints represented by $\ldots$; (ii) entrepreneurs within each sector select employment of production workers to maximize their profits; and (iii) the market for production workers clears.

The analysis of factor market equilibria proceeds as follows. Skill-premia in the two sectors define the entry thresholds into each sector, which determine the occupations that any given agent can feasibly choose from while respecting the incentive constraints. The maximum profit that the agent can earn in any sector is the product of her own entrepreneurial skill, the wage rate and the skill premium in that sector. Agents select between occupational options to maximize their earnings. The allocation of agents across occupations combined with output prices and the wage rate determines demand for labor from entrepreneurs in each sector. The aggregate demand for labor must equal the supply of labor, i.e., the mass of agents that do not meet the entry thresholds for entrepreneurship in either sector. The equilibrium wage rate must in turn be consistent with the skill premia in the two sectors, satisfying the price-cost equations $\ldots$ characterizing profit-maximization by entrepreneurs within each sector.

We shall represent the factor market equilibrium by the intersection of two conditions involving the skill-premia in the two sectors: one which corresponds to clearing of the factor markets, the other to the profit maximization condition $\ldots$. We start with the former next.

First we take the skill premia in different sectors as given, and describe occupational choices of agents in the economy. There are four different situations to consider, depending on the relation between skill premia and moral hazard parameters in different sectors.

Case A1: $\gamma_L \geq \gamma_H \frac{m_L}{m_H}$

Since we are in Case A and $\frac{m_L}{m_H} > 1$, it follows that in this situation $\gamma_L > \gamma_H$ also holds. This implies that entrepreneurship in sector $L$ is more profitable
than in sector $H$. The entry threshold for this sector is also lower, as $m_L < \frac{m_H}{\gamma_H}$. Hence all those with skill above $\frac{m_L}{\gamma_L}$ will enter the $L$-sector, and those below will become production workers. Clearing of the labor market requires

$$\mu\left[\frac{\theta^L_n(\gamma_L)}{\theta^L_a(\gamma_L)}\right]d\left(\frac{m_L}{\gamma_L}\right) = \mu G\left(\frac{m_L}{\gamma_L}\right) + (1 - \mu).$$  \hspace{1cm} (10)$$

The production levels will be $X_H = 0, X_L = \mu A_L d\left(\frac{m_L}{\gamma_L}\right)/\theta^L_a(\gamma_L)$: the economy specializes in production of good $L$.

**Case A2**: $\gamma_H < \gamma_L < \gamma_H \frac{m_L}{m_H}$

Here $\gamma_L > \gamma_H$ implies that the $L$-sector is more profitable. On the other hand, the entry threshold is higher in the $L$-sector: $a_L = m_L/\gamma_L > m_H/\gamma_H = a_H$. So agents with $a \geq a_L$ will choose to become $L$-sector entrepreneurs, while agents with $a \in [a_H, a_L)$ are unable to enter the $L$-sector and so have to be content with becoming $H$-sector entrepreneurs. And agents with $a < a_H$ become workers.

The labor market clears if

$$\mu\left[\frac{\theta^H_n(\gamma_H)}{\theta^H_a(\gamma_H)}\right]d\left(\frac{m_H}{\gamma_H}\right) + \mu\left[\frac{\theta^H_n(\gamma_H)}{\theta^H_a(\gamma_H)}\right]d\left(\frac{m_L}{\gamma_L}\right) = \mu G\left(\frac{m_H}{\gamma_H}\right) + (1 - \mu).$$  \hspace{1cm} (11)$$

Condition (??) provides a relation between skill premia in the two sectors that must hold for the labor market to clear. This relationship is downward-sloping, because an increase in the skill premium in either sector tightens the labor market condition. To see this, note that a rise in $\gamma_H$ lowers the entry threshold into the $H$-sector, and raises the demand for production workers for any given $H$-sector entrepreneur. And on the other hand a rise in the $L$-sector skill premium causes the skill threshold for the $L$-sector to fall, motivating some entrepreneurs to switch from the $H$ to the $L$-sector. By Assumption ?, and by hypothesis $\gamma_L > \gamma_H$, $L$-sector entrepreneurs demand more production workers than the $H$-sector entrepreneurs. So the switch of entrepreneurs between the two sectors tightens the labor market. This is accentuated by the rise in demand for workers by incumbent $L$-sector entrepreneurs.

In this case, production levels are:

$$X_H = \mu A_H [d(a_H) - d(a_L)]/\theta^H_a(\gamma_H)$$

$$X_L = \mu A_L d(a_L)/\theta^L_a(\gamma_L)$$

This case will be of particular interest in the subsequent analysis since skill premia are not equalized across the two sectors. Entrepreneurs in the
Case A3: $\gamma_H = \gamma_L$

$\gamma_L = \gamma_H = \gamma$, say, implies that entrepreneurs are indifferent between the two sectors. The $L$-sector is harder to enter, as $a_L = m_L/\gamma > m_H/\gamma = a_H$. Hence agents with $a \in [a_H, a_L)$ have no option but to enter sector $H$, while those with $a \geq a_L$ can enter either of the two sectors. The equilibrium in this case will involve a fraction of those with $a \geq a_L$ who will go to sector $L$, the remaining going to sector $H$. This fraction must be such as to permit the factor market to clear. We show now that this in turn translates into an upper and lower bound for the common skill premium $\gamma$.

Denoting the production levels by $X_L, X_H$ respectively, the factor market clearing conditions are

$$[\theta_n^L(\gamma) / A_L]X_L + [\theta_n^H(\gamma) / A_H]X_H = \mu G(a_H) + (1 - \mu)$$

$$[\theta_a^L(\gamma) / A_L]X_L + [\theta_a^H(\gamma) / A_H]X_H = \mu d(a_H)$$

These equations are equivalent to

$$X_L = A_L \frac{\theta_a^H(\gamma) [\mu G(a_H) + (1 - \mu)] - \theta_n^H(\gamma) \mu d(a_H)}{\theta_n^L(\gamma) \theta_a^H(\gamma) - \theta_n^H(\gamma) \theta_a^L(\gamma)}$$

$$X_H = A_H \frac{-\theta_a^L(\gamma) [\mu G(a_H) + (1 - \mu)] + \theta_n^L(\gamma) \mu d(a_H)}{\theta_n^L(\gamma) \theta_a^H(\gamma) - \theta_n^H(\gamma) \theta_a^L(\gamma)}.$$

However since only agents with $a \geq a_L$ have the option to become $L$-sector entrepreneurs,

$$X_L \leq \mu A_L d(a_L) / \theta_a^L(\gamma)$$

which implies

$$\mu [\frac{\theta_n^L(\gamma)}{\theta_n^L(\gamma)}] d(m_L / \gamma) + \mu [\frac{\theta_n^H(\gamma)}{\theta_a^H(\gamma)}] [d(m_H / \gamma) - d(m_L / a_L)] \geq \mu G\left(\frac{m_H}{\gamma}\right) + (1 - \mu). \quad (12)$$

On the other hand, $X_L \geq 0$ implies

$$\mu [\frac{\theta_n^H(\gamma)}{\theta_a^H(\gamma)}] d(m_H / \gamma) \leq \mu G\left(\frac{m_H}{\gamma}\right) + (1 - \mu). \quad (13)$$
Inequalities (??, ??) provide lower and upper bounds on the common premium rate $\gamma$. Note that (??) is the inequality version of the factor market clearing condition (??) in Case A2. Hence the lower bound in Case A3 exactly equals the limiting premia in Case A2 as $\gamma_L$ and $\gamma_H$ approach each other (see Figure ?? below).

This situation involves equal skill premia across the two sectors, thus corresponding to a non-specific factor setting. The relationship between the skill premia is upward-sloping (in contrast to Case A2): it coincides with part of the 45 degree line of equality in Figure ??.

**Case A4: $\gamma_H > \gamma_L$**

$\gamma_H > \gamma_L$ implies that sector $H$ is more profitable. Also the entry threshold in sector $H$ is lower. In this case no entrepreneur enters sector $L$. Those with skill $a \geq a_H$ enter sector $H$, the rest become workers. The labor market condition is

$$\mu\left[\frac{\theta^H_n(\gamma_H)}{\theta^H_a(\gamma_H)}\right]d(a_H) = \mu G(a_H) + (1 - \mu).$$

(14)

The production levels are

$$X_L = 0$$

$$X_H = \mu A_H d(a_H)/\theta^H_a(\gamma_H).$$

Hence the economy specializes in production of the $H$-good here.

Figure ?? shows the relationship between skill premia in the two sectors consistent with clearing of the factor market. The entry thresholds depicted $\gamma^1_H$, $\gamma^2_H$, and $\gamma^3_H$ are defined by the solutions to the following equations.

$$\mu\left[\frac{\theta^L_n(m_L/\gamma^1_H)}{\theta^L_a(m_L/\gamma^1_H)}\right]d(m_H/\gamma^1_H) = \mu G(m_H/\gamma^1_H) + (1 - \mu)$$

$$\mu\left[\frac{\theta^L_n(\gamma^2_H)}{\theta^L_a(\gamma^2_H)}\right]d(m_L/\gamma^2_H) + \mu\left[\frac{\theta^H_n(\gamma^2_H)}{\theta^H_a(\gamma^2_H)}\right][d(m_H/\gamma^2_H) - d(m_L/\gamma^2_H)] = \mu G(m_H/\gamma^2_H) + (1 - \mu).$$

$$\mu\left[\frac{\theta^H_n(\gamma^3_H)}{\theta^H_a(\gamma^3_H)}\right]d(m_H/\gamma^3_H) = \mu G(m_H/\gamma^3_H) + (1 - \mu)$$

For future reference, we shall denote this relationship by the equation

$$\gamma_L = \psi(\gamma_H).$$

(15)

Note that this function depends on parameters $\mu, m_L, m_H$ but is independent of $p_L$ or TFP parameters $A_L, A_H$. Note also that this function is well defined for $\gamma_H < \gamma^3_H$, and is not a monotone relationship: it is decreasing below $\gamma^2_H$.
but increasing thereafter. The downward-sloping part corresponding to Case A2 is the ‘non-classical’ region where skill premia are not equalized across sectors. The upward-sloping part corresponding to Case A3 coincides with the line of equality, so this is the ‘classical’ region where skill premia are equalized. Note that the greater the relative severity $\frac{m_L}{m_H}$ of the moral hazard problem in the $L$-sector, the greater the range occupied by the non-classical region.

4.1 Factor Market Equilibrium

We are now in a position to characterize the factor market equilibrium, by putting together the condition that the labor market clears (which incorporates reputation effects, occupational and sectoral choices by entrepreneurs), with the relation between prices and costs representing profit maximization by active entrepreneurs in each sector.

The former is represented by the relation between skill premia that clears the factor markets, shown in the previous section. The latter is represented by the upward-sloping relation ($\gamma_L = \gamma_H$) between premia in the two sectors for any given product price $p_L$. Geometrically it is represented by the intersection of the corresponding relations between the two skill premia. This is shown in Figure ?? for different values of $p_L$. 

\[
\gamma_L = \gamma_H = \frac{m_L}{m_H} \gamma_H 
\]
Lemma 2 Consider Case A, where the $L$-good is more prone to moral hazard: $m_L > m_H$.

(a) For any given $p_L > 0$, a factor market equilibrium exists and is unique.

(b) There exist thresholds $p_L^1 > p_L^2 > p_L^3$ such that below $p_L^3$ the economy specializes in producing good $H$ while above $p_L^1$ it specializes in good $L$. Between $p_L^1$ and $p_L^3$ both goods are produced. Skill premia are equalized in the two sectors (i.e., Case A3 arises) when $p_L$ is between $p_L^2$ and $p_L^3$, or conversely when relative TFP $\frac{\lambda_L}{\lambda_H}$ of the $L$-good is low relative to $p_L$. The skill premium is strictly higher in the $L$-sector, i.e., case A2 arises, when $p_L$ is between $p_L^1$ and $p_L^2$, or conversely when relative TFP $\frac{\lambda_L}{\lambda_H}$ of the $L$-good is high relative to $p_L$.

We sketch the proof. The price thresholds which mark the transition between Situations 1, 2, 3 and 4 are calculated as follows:

$$p_L^1 = (A_L/A_H) \frac{\theta_n(\gamma_L)}{\theta_n(\gamma_H)} + \frac{m_L}{m_H} \theta_a(\gamma_L)$$

$$p_L^2 = (A_L/A_H) \frac{\theta_n(\gamma_H^2)}{\theta_n(\gamma_H)} + \frac{\gamma_H^2}{\theta_a(\gamma_H)}$$

$$p_L^3 = (A_L/A_H) \frac{\theta_n(\gamma_H^3)}{\theta_n(\gamma_H)} + \frac{\gamma_H^3}{\theta_a(\gamma_H)}$$

Now consider the following price ranges A1, A2, A3, A4 (illustrated in Figure ??).

**Case A1:** $p_L \geq p_L^1$

In this case, there is an equilibrium with $\gamma_H \leq \gamma_H^1$, with complete specialization in product $L$, and production levels $X_L = \mu d(\frac{m_L}{\gamma_L})/\theta_a(\gamma_L)$, $X_H = 0$. Since the price-cost relation (??) is upward-sloping, it is evident there cannot be any other equilibrium. In the interior of this range, equilibrium outputs are locally independent of $p_L$.

**Case A2:** $p_L^1 > p_L \geq p_L^2$

If the price-cost relation does not intersect the horizontal segment of the skill-premium frontier associated with labor market clearing, the skill premium in the $L$-sector is defined by the point where this horizontal segment intersects the vertical axis. Here the price-cost relation takes the form of an inequality for the $H$-sector and equality for the $L$-sector.

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22If the price-cost relation does not intersect the horizontal segment of the skill-premium frontier associated with labor market clearing, the skill premium in the $L$-sector is defined by the point where this horizontal segment intersects the vertical axis. Here the price-cost relation takes the form of an inequality for the $H$-sector and equality for the $L$-sector.
Figure 2: Factor Market Equilibrium: Case A
Here there is an equilibrium corresponding to the downward sloping stretch in the relation between $\gamma_H, \gamma_L$ expressing labor market clearing. This follows from the fact that at $p_1^L$ there is an equilibrium corresponding to $\gamma_1^H$, and at $p_2^L$ there is an equilibrium corresponding to $\gamma_2^H$. Moreover in this case there cannot be any other equilibrium owing to Lemma 23.

In the interior of this range of prices, increasing $p_L$ results in an increase in $X_L$ and $\gamma_L$, and a decrease in $X_H$ and $\gamma_H$.

**Case A3**: $p_L^2 > p_L \geq p_L^3$

Now there will be an equilibrium in which skill premia are equalized across the two sectors. The same argument as in Case A2 ensures the equilibrium is unique. The equilibrium skill premium in this case $\gamma_L = \gamma_H = \gamma^*$ is determined by the condition

$$p_L = \frac{A_H}{A_L} \frac{\theta^L_n(\gamma^*)}{\theta^H_n(\gamma^*)} + \frac{\gamma^* \theta^L(a)}{\theta^H(a)(\gamma^*)}.$$  \hfill (16)

It is evident that an increase in $p_L$ will increase $X_L$, reduce $X_H$ and the common skill premium $\gamma^*$. The latter results as the shift in production towards the $L$-sector raises the demand for labor, inducing a rise in the wage rate.

**Case A4**: $p_L < p_L^3$

In this case, there is a unique equilibrium with perfect specialization in sector $H$. The production level is $X_L = 0$ and

$$X_H = \mu A_H d(a_H)/\theta^H_n(\gamma_H).$$

An increase in $p_L$ in this region will raise $\gamma_L$, while leaving $X_H, \gamma_H$ unchanged.

---

23If there were another equilibrium, it would have to lie in the range $\gamma_H > \gamma^2_H$. But this would require the slope of the skill premium relationship expressing (??) to have a slope smaller than one somewhere above the 45 degree line, which is ruled out by Lemma 23.

24Note in particular that Lemma ?? ensures that the slope of the relation between skill premia expressing (??) strictly exceeds unity even on the 45 degree line. Hence a tangency of this relation with the 45 degree line is ruled out.

25Again, if the price-cost relation does not intersect the vertical segment of the skill-premium-frontier represent labor market clearing, the equilibrium is located at a skill premium of $\gamma_3^H$ in the $H$-sector and 0 in the $L$-sector. Here prices equal cost in the $H$-sector while they fall below cost in the $L$-sector.
4.2 Effect of Changes in Skill Endowment on Factor Market Equilibrium: Validity of the Rybczynski result

Now consider the effect of an increase in $\mu$, the proportion of agents in the economy with skills. As shown in Figure ??, the frontier between skill premia corresponding to the factor market-clearing condition (??) shifts inwards, owing to the resulting tightening of the labor market. Excepting the case that $\gamma_H = \gamma_L$ is maintained before and after the change in $\mu$, both skill premia fall. What is the effect on the relative production levels $X_L/X_H$? This determines the pattern of comparative advantage in an open economy setting, an issue addressed by the Rybczynski Theorem in classical trade theory.

Since the $H$-good is more skill-intensive, one would intuitively expect an increase in endowment of skilled labor in the economy to raise the production of $H$ relative to the $L$-good, an implication of the classical Rybczynski theorem. This indeed is true in the ‘classical’ region A3 with equal skill premia in the two sectors. From (??) which is independent of $\mu$, it is evident that a rise in $\mu$ leaves the skill premium unchanged. Hence the entry thresholds into the two sectors and the demand for unskilled labor from each active entrepreneur of the same skill are unaffected. Since the $L$-sector is less skill-intensive, it follows that the production of the $L$-good must fall, in order to allow the labor market to clear.

In the ‘non-classical’ region corresponding to case A2, there will be an additional effect of a change in $\mu$ on skill premia in the two sectors. What turns out to matter is the change in the relative skill premia in the two
sectors. An increase in $\mu$ tightens the labor market and thus tends to drive the unskilled wage higher. Since the $L$-sector is less skill-intensive, this tends to lower the skill premium in the $L$-sector by more than in the $H$-sector. However, the relative skill premium $\frac{\gamma_L}{\gamma_H}$ may still go up, if it was high enough to start with.\(^{26}\) In that case, we obtain a countervailing effect which can raise $\frac{X_L}{X_H}$.

To see this concretely, we consider the following example, where the density of the skill distribution does not fall too fast, and the production functions for both sectors have constant and equal elasticity of substitution between skilled and unskilled labor.

**Proposition 1** Consider Case A, and assume that $\frac{a^2g(a)}{d(a)}$ is increasing in $a$ and production function for $i = L, H$ exhibit constant, equal elasticity of substitution $\kappa (\geq 0)$:

$$X_i = A_i F_i(n_i, a_i) = A_i (k_i^{1/\kappa} a_i^{\frac{\kappa-1}{\kappa}} + n_i^{\frac{1}{\kappa}})^{x^{-1}}$$

with $k_H > k_L$ (to ensure Assumption ?? is satisfied). Then in the factor market equilibrium:

(i) If $\kappa \geq 1 - \frac{\log[k_H]}{\log[m_H]}$, an increase in $\mu$ has the effect of decreasing $X_L/X_H$ for any $p_L \in (p_L^3, p_L^1)$.

(ii) If $0 \leq \kappa < 1 - \frac{\log[k_H]}{\log[m_H]}$, the increase in $\mu$ has the effect of reducing $X_L/X_H$ for any $p \in (p_L^3, A_H/A_L)$. Moreover, there exists $\bar{p}_L \in (p_L^3, A_H/A_L)$ such that $X_L/X_H$ is increasing in $\mu$ for any $p > \bar{p}_L$.

The proof is provided in the Appendix. The Proposition shows that $\frac{X_L}{X_H}$ falls if the elasticity of substitution is large (case (i)) and otherwise for values of $p_L$ below $\frac{A_H}{A_L}$, but not for values of $p_L$ close enough to $p_L^1$. In the latter case the relative skill premium in the $L$-sector is sufficiently high to start with that it increases as a result of the increase in skill endowment. This is strong enough to cause the relative production of the less skill-intensive good to rise. Figure ?? provides an illustration of the effect on $\frac{X_L}{X_H}$. In the context of the open economy, this will provide an instance where the Leontief paradox appears, if a North and South country differ only in their skill endowments.

\(^{26}\)Specifically, the tighter labor market tends to lower $\gamma_H$, and the effect on the relative premium $\frac{\gamma_L}{\gamma_H}$ of a change in $\gamma_H$ is $\frac{d(\frac{\gamma_L}{\gamma_H})}{d\gamma_H} = \frac{1}{\gamma_H} [\frac{d\gamma_L}{d\gamma_H} - \frac{\gamma_L}{\gamma_H}]$ which is negative if $\frac{d\gamma_L}{d\gamma_H} < \frac{\gamma_L}{\gamma_H}$, i.e., the initial value of the relative premium is high enough.
4.3 Comparative Static Properties of the Factor Market Equilibrium: Validity of the Stolper-Samuelson Result

We are now in a position to consider the first key question of the paper: when does the Stolper-Samuelson relation hold? Specifically, what are the implications of changing product prices on returns to different factors? We focus on cases corresponding to lack of complete specialization in either good, i.e., where \( p_L \) lies between \( p_1^L \) and \( p_3^L \). Presuming that the South with a shortage of skill will have comparative advantage in the \( L \)-good, trade integration will induce a rise in \( p_L \) in that country.

Proposition 2 Consider Case A, where the \( L \)-good is more prone to moral hazard.

(a) If the economy is operating in Case A2, i.e., the \( L \)-good has a higher skill-premium than the \( H \)-good, a small increase in \( p_L \) will raise the skill premium in \( L \)-sector and lower the skill premium in sector \( H \).

(b) If the economy is operating in Case A3, i.e., the two sectors have the same skill premium, a small increase in \( p_L \) will lower the skill premia equally in both sectors.

Part (a) shows that the Stolper-Samuelson result is reversed in the ‘non-classical’ region where skill premia are unequal, while it continues to hold in
the classical region where they are equal. The relation between output price $p_L$ and skill premia in the two sectors is illustrated in Figure 5. Focusing on the former region, it is evident that a rise in $p_L$ shifts the skill premium relation characterizing price-cost equality in the two sectors to the left. Since the relation between them characterizing the labor market clearing condition is downward-sloping in case A2, it follows that the skill premium must rise in $L$-sector and fall in $H$-sector. The price-cost relations (23, 24) then imply that both $w$ and $\frac{p_L}{w}$ rise. Hence the wage rate expressed in units of the $H$-good rises, but expressed in units of the $L$-good falls.\textsuperscript{27}

The intuitive explanation of the increase in inequality in the $L$-sector is the following. The increase in $p_L$ induces initially a rise in profitability of the $L$-sector, lowering entry thresholds into the $L$-sector, which allows some entrepreneurs to move from the $H$ to the $L$-sector. This tightens the labor market, both because the $L$-sector employs more workers than the $H$-sector, and each $L$-sector firm employs more workers. The resulting upward pressure on the wage rate tends to reduce the skill premium in both sectors. The drop in $H$-sector profits will cause the labor market to slacken, as some low-ability $H$-sector entrepreneurs will switch to become production workers, and each $H$-sector firm hires fewer workers. But the decline in skill premium in $L$-sector firms caused by increased wages cannot reverse the initial increase caused by the increase in the product price. Otherwise a lower skill premium in the $L$-sector would slacken the labor market, accentuating the effect of...

\textsuperscript{27}The effect on utility of workers thus depends on relative preferences in their consumption for the two goods: if biased in favor of the $L$-good sufficiently, workers will be worse off.
the decline in the $H$-sector skill premium. For the labor market to clear, the $L$-sector skill premium must rise overall.

The result resembles that in an exogenous specific factor model. However, a key difference is that in our model entrepreneurs do move between sectors in response to output price changes, as observed empirically in the Ugandan and Mozambique contexts cited in the Introduction. The newly entering entrepreneurs are of lower skill than incumbents in the sector: for them to be able to function in the $L$-sector while meeting the moral hazard constraint, entrepreneurial returns must rise relative to worker earnings, since the latter serves as the punishment payoff associated with a loss of reputation.

The other difference from an exogenous specific factor model is that the Stolper-Samuelson result holds in the classical region where skill premia are equalized across sectors. The relation between skill premia in the two sectors consistent with labor market clearing is upward-sloping; hence a leftward shift in the relation between skill premia consistent with price-cost equality implies that skill premia must fall in both sectors. The logic is similar to that in the mobile-factor version of the Heckscher-Ohlin model, arising from the ability of (some) entrepreneurs to move freely between sectors. Entrepreneurs with skill above the threshold for the $L$-sector are indifferent between operating in the $L$ and $H$ sectors. A positive fraction of them are already operating in either sector. Hence it is possible for a subset of these high skill entrepreneurs to move into the $L$-sector out of the $H$-sector, without any change in the skill thresholds for sector $L$. Changes in skill premia result from a rise in the wage rate, which owes to the shift of entrepreneurs into the $L$-sector which is more labor intensive. Skill premia go down in step in both sectors.

The detailed distributional effects of a rise in $p_L$ in the anti-Stolper-Samuelson case A2 are illustrated in Figure ??.

This shows the distribution of income across agents with varying skills, and how it changes as a result of an increase in $p_L$. Agents with skill below the entry threshold $a_H$ for the $H$-sector earn the unskilled wage $w$. Between the thresholds $a_H$ and $a_L$ for the two sectors, the agents are $H$-sector entrepreneurs, earning $\gamma_H wa$. By definition of the threshold $a_H = \frac{mw}{\gamma H}$, it follows that the earning of a $H$-sector entrepreneur at this threshold equals $wm_H$, which strictly exceeds $w$ as $m_H > 1$. Hence there is a discrete upward jump in earnings at the entry threshold for entrepreneurship. There is a similar discrete jump in earnings at the threshold $a_L$ for entry into the $L$-sector, owing to the difference in skill premia between the two sectors. The highest incomes accrue to entrepreneurs in the $L$-sector, who manage the largest firms in the economy. They are followed by $H$-sector entrepreneurs, who manage smaller firms, and finally workers who work as unskilled employees in both sectors.

The distribution of income is altered following a rise in $p_L$ in the following way: a rise in incomes at the top ($L$-sector entrepreneurs) and the bot-
Figure 6: Income Distribution Changes Resulting from Increase in $p_L \in (p_L^2, p_L^1)$

tom (workers), and a fall in incomes in the middle (H-sector entrepreneurs). Within the L-sector, inequality in earnings between entrepreneurs and workers rises. On the other hand inequality falls within the H-sector. Note that these inequality effects appear within firms.

The output and distributive impact of a rise in $p_L$ depend on induced entry and exit effects of entrepreneurs, which in turn depends on the local behavior of the ability distribution. To illustrate this, consider the limiting case of a Leontief technology.

**Proposition 3** Suppose the production function in each sector $i$ exhibits perfect complementarity: $X_i = A_i \min \{n_i, a\}$ for the high-quality good, and $X_i = A_i \min \{n_i, z_i a\}$ for the low-quality good. Suppose also that case A2 applies. Then small increase in $p_L$ results in:

(i) no change in $w$ or outputs $X_L, X_H$, while $\gamma_L$ rises and $\gamma_H$ remains constant, if $g(\frac{m_L}{\gamma_L}) = 0$ while $g(\frac{m_H}{\gamma_H}) > 0$.

(ii) no change in $\gamma_L$, while $w$ rises and $\gamma_H$ falls, if $g(\frac{m_H}{\gamma_H}) = 0$ while $g(\frac{m_L}{\gamma_L}) > 0$.

This shows that relative rates of entry into the $H$ and $L$-sectors, which depend on relative densities at the corresponding thresholds, affect the distribution of benefits between entrepreneurs and workers. In case (i) where
there is no entry into the $L$-sector following a rise in $\gamma_L$ owing to ‘thinness’ of the ability distribution at the threshold $\frac{m_L}{\gamma_L}$, changes in $p_L$ will be associated with a zero output response, and none of the benefits of the rise in $p_L$ are passed on to workers. In case (ii) on the other hand, the entire benefits of rising $p_L$ are passed on to workers, as the rise in $w$ does not choke off the output expansion in the $L$-sector owing to shrinking entry into the $H$-sector at the entry threshold for that sector (combined with lack of substitution within firms owing to a Leontief technology).

4.4 Autarky Equilibrium

Now we close the model of the autarkic economy by specifying the demand side. There is a representative consumer with a homothetic utility function $U = U(D_H, D_L)$, where $D_H, D_L$ denote consumption of the two goods. The relative demand function is then given by

$$D_L/D_H = \phi(p_L)$$

where $\phi(p_L)$ is continuous and strictly decreasing in $p_L$. We assume that $\lim_{p_L \to 0} \phi(p_L) = \infty$ and $\lim_{p_L \to \infty} \phi(p_L) = 0$.

The economy-wide equilibrium is represented by equality of relative supply and relative demand:

$$D_L/D_H = \phi(p_L) = \frac{X_L}{X_H}$$

where the dependence of relative supply $\frac{X_L}{X_H}$ on $p_L$ is provided by the factor market equilibrium described in the previous section.

**Lemma 3** Consider Case A, where the $L$-good is more prone to moral hazard. An autarkic equilibrium always exists, and is unique. It must satisfy $p_L \in (p_L^3, p_L^1)$.

This follows from the fact that relative demand is continuous and strictly decreasing in $p_L$. An autarky equilibrium $(p_L, \gamma_L, \gamma_H, w)$ is characterized by conditions of profit-maximization ($\frac{\partial U}{\partial p_L} = 0$), ($\frac{\partial U}{\partial w} = 0$); the labor market clearing condition ($\frac{\partial U}{\partial w} = 0$), and the product-market clearing condition ($\frac{\partial U}{\partial w} = 0$). It is illustrated in Figure ??.

Now consider the effect on the autarky equilibrium of increasing $\mu$, which will be helpful in determining patterns of comparative advantage when we

---

$^{28}$Relative supply is well-defined (owing to uniqueness of the factor market equilibrium) for $p_L \in (p_L^3, p_L^1)$, and over this range is continuous and strictly increasing in $p_L$. Moreover, as $p_L$ tends to $p_L^3$, relative supply of the $L$-good tends to 0 while relative demand is bounded away from zero. And as $p_L$ tends to $p_L^1$, relative supply of $L$ tends to $\infty$, while relative demand is bounded.
extend the model to an open economy setting. While the effects of varying $\mu$ on $\frac{X_L}{X_H}$ in the factor market equilibrium were seen above to be quite complicated, it turns out that the distributional effect on the autarkic equilibrium is quite simple: skill premia in both sectors fall.

**Lemma 4** Suppose Case A applies. A small increase in skill endowment $\mu$ lowers skill premia in both sectors, while $w$ and $\frac{w}{p_L}$ both rise.

### 4.5 Free Trade Equilibrium and Lack of Factor Price Equalization

Suppose there are two countries South $S$ and North $N$, the former corresponding to the less developed country. They are identical in all respects, except that country $N$ has a higher $\mu$ the proportion of skilled agents ($\mu^S < \mu^N$).\(^{29}\) Lemma ?? then implies that in autarky Northern country has a lower skill premium in both sectors.

In a free trade equilibrium (with zero transport costs), there will be a common equilibrium price $p_L^T$ in the two countries, determined by

$$\frac{D_L^S + D_L^N}{D_H^S + D_H^N} = \frac{X_L^S + X_L^N}{X_H^S + X_H^N}$$

(19)

where both relative demand and supplies in each country will depend on the common price. Once $p_L^T$ is determined, the respective factor market

\(^{29}\)Similar results obtain when $N$ has a higher relative TFP in the $H$-sector.
equilibrium of each country will determine the remaining variables in each country.

If the South has a comparative advantage in the $L$-good, trade integration will induce a rise in $p_L$ in the South, with distributive effects as described in Proposition ???. If skill premia differ across sectors, the skill premium will rise within the $L$-sector and fall in the $H$-sector in the South, and the opposite happens in the North. Hence the initial gap in skill premia in the $L$-sector across the two countries will be accentuated, while that in the $H$-sector will shrink. On the other hand, if both countries are operating in the classical region with equal skill premia in the $L$ and $H$-sectors, they will decline in the South and rise in the North: in this case factor prices tend to equalize.

We summarize these results below.

**Proposition 4** Suppose the $L$-good is more prone to moral hazard, and the South has a comparative advantage in the $L$-good.

(a) If skill premia differ across sectors (i.e., Case A2 applies) within both countries under autarky and trade integration, the gap between skill premia in the $L$-sector in the two countries grows while that between skill premia in the $H$-sector narrows as a result of trade integration. In this case free trade must be associated with unequal skill premia in each sector across countries.

(b) If skill premia are equal across the two sectors (i.e., Case A3 applies) under autarky and trade integration in both countries, the gap between skill premia in either sector across countries narrows as a result of trade integration. In this case free trade must be associated with equalization of skill premia across countries.

### 4.6 Welfare Effects of Trade

The effects of trade on the equilibrium outcomes of each country are represented by the effect of trade on relative product price $p_L$ and thereafter on the resulting factor market equilibrium. Hence it suffices to examine the welfare effect of changes in $p_L$, which can be shown to be equivalent to the following expression.

**Lemma 5** The aggregate welfare effect in country $j$ of a change in $p_L^j$ has the same sign as

$$ (X_L^j - D_L^j) + w^j \mu [(1 - m_H)g(a_H^j)da_H^j/dp_L^j - (\gamma_L^j - \gamma_H^j)a_L^jg(a_L^j)da_L^j/dp_L^j] \quad (20) $$
In addition to the standard allocative effect $X_L^j - D_L^j$, there is an additional set of welfare effects operating through the change in entry thresholds $a_L$ and $a_H$. This owes to the upward jumps in incomes at these thresholds, owing to the binding incentive constraints operating at these thresholds. A relaxation of these thresholds enables agents to transfer occupations (from being a worker to an entrepreneur, when $a_H$ falls) or sectors (from sector $H$ to sector $L$, when $a_L$ falls) and experience a discrete income gain. This explains the second and third terms in the expression above. When $a_H$ falls, the change in income is $-w_j(1 - m_H)$ for every agent at the threshold, who is switching from being a worker to a $H$-sector entrepreneur. When $a_L$ falls, the change in income is proportional to the difference in skill premia between the two sectors.

In general, it is difficult to sign the sum of these income effects resulting from a change in entry thresholds following a relaxation of trade barriers. For instance, consider the cases described above where country $S$ has a comparative advantage in the $L$-good, and it is operating in case A2. Trade causes an expansion in the $L$-sector (a fall in $a_L$) which raises aggregate entrepreneurial incomes. It also causes a contraction in the $H$-sector (a rise in $a_H$) which reduces aggregate income, as some $H$-sector entrepreneurs switch to becoming workers. The net effect is ambiguous. However, in the case where the moral hazard problem in the $H$-sector is negligible (i.e., $m_H$ approaches one), the pecuniary externality at the $a_H$ threshold vanishes. In that case the aggregate income effect of trade is positive for country $S$, which adds to the standard allocative benefits of trade. Conversely, the aggregate income effect is negative for country $N$, which subtracts from the allocative benefits. Hence starting from autarky, a small expansion of trade can be welfare-reducing for country $N$.

With different parameter values, these welfare results can get reversed. For instance, suppose that the moral hazard problem in the $H$-sector is non-negligible, and approximately the same as in sector $L$ (i.e., $m_H$ is bounded away from 1 and $m_L - m_H$ is negligible). Then only movements in the entry threshold $a_H$ will generate non-negligible income effects. If country $S$ has comparative advantage in the $L$-good, trade will generate negative income effects for country $S$ and positive income effects for country $N$.

### 4.7 Offshoring

If skill premia in the North are lower as a result of failure of factor prices to equalize with trade, Northern entrepreneurs will have an incentive to offshore production to the South. The incentive to offshore can be measured by the difference in profits between the two countries earned by an entrepreneur of
given ability.\footnote{Without loss of generality, a Northern entrepreneur producing in the South will sell in Southern markets. This is obvious if transport costs are high enough to render trade unprofitable. If transport costs are low enough to generate trade, the difference in prices of any good across countries will equal the transport cost, implying that entrepreneurs will be indifferent between selling in either country.} Our preceding results imply that trade integration will cause the incentive for North-South offshoring in the \( L \)-sector to go up, and in the \( H \)-sector to go down, when Case A2 applies and the South has a comparative advantage in the \( L \)-good.\footnote{We have shown in this case that the South-North difference in skill premia in the \( L \)-sector rises and the \( H \)-sector falls, as a result of trade integration. It is easy to check that the same property holds for the difference in profits in each sector: e.g., profits in sector \( L \) equals \( \gamma_L w = \frac{\gamma_L}{\gamma_L + \gamma_H} \) which rises in the South because \( \gamma_L \) rises while \( \gamma_H \) falls, and conversely falls in the North.} Hence our model predicts complementarity between trade integration and North-South offshoring in the \( L \)-sector, and substitutability in the \( H \)-sector.

We now examine the equilibrium implications of this type of offshoring, when there are zero costs to offshore, in addition to free trade in goods. The following proposition shows that the resulting equilibrium is identical to that in the completely integrated economy with \( \mu^G \equiv \mu^S + \mu^N \), with factor prices equal across the two economies.

**Proposition 5** With free trade and costless offshoring, the equilibrium is equivalent to that in the completely integrated economy with \( \mu^G \) proportion of skilled agents. In this equilibrium, the skill premium in each sector are equalized across countries. If the Southern country has comparative advantage in the \( L \)-good under autarky, complete integration relative to autarky causes skill premia to fall (resp. rise) in each sector in the South (resp. North).

In the integrated equilibrium, the absence of any trade or offshoring costs implies that entrepreneurs are indifferent which country to locate their operations. This implies that the structure of trade is indeterminate. This indeterminacy would be resolved in the presence of small trading and offshoring costs. Since the North has a higher endowment of skill, the net outsourcing from the North must be larger.

Proposition 5 indicates that the distributional effect of full integration differs sharply from trade integration when the latter is associated with factor price disequalization. If the South operates in Case A2 under autarky, trade integration raises the skill premium in the \( L \)-sector while complete integration lowers it. The reason is that in Case A2 there are restrictions on entry of entrepreneurs into sector \( L \), who must come from the pool of Southern entrepreneurs. These entry restrictions are relaxed under trade integration only if the skill premium in this sector increases. With complete integration
on the other hand, high skill entrepreneurs from the North can enter the \( L \)-sector in the South. So the Southern \( L \)-sector skill premium does not have to rise to induce this entry. The fact that it is higher than skill premia in the North motivates Northern entrepreneurs to offshore operations to the South, which drives down skill premia there.

The comparison of full integration with free trade is somewhat more complicated. Let us continue to suppose that under autarky the South has comparative advantage in the \( L \)-good, and Case A2 applies in both countries. Under autarky skill premia are higher in both sectors in the South; trade integration causes the skill premium in the Southern (resp. Northern) \( L \)-sector to rise (resp. fall) even further. Under complete integration the \( L \)-sector skill premium must be equalized, and lie between the autarkic skill premia in the two countries. Hence starting with free trade, offshoring must cause the Southern \( L \)-sector skill premium to fall. But the effect on the \( H \)-sector skill premium is ambiguous.

If relative product prices were to remain unchanged, offshoring would lower the \( H \)-sector skill premium in the South. In that case, offshoring unambiguously tends to reduce Southern inequality between unskilled and skilled agents. However, offshoring could have another effect, by causing a change in \( p_L \). This effect is not easy to sign. If terms-of-trade effects of offshoring are insignificant we can infer that it would generally improve Southern income distribution in favor of production workers in both sectors.

5 Extension to Case B, where \( H \)-good is More Prone to Moral Hazard

Now consider what happens if the \( H \)-good is more subject to moral hazard: \( m_H > m_L \). Now if skill premia differ, they will be higher in the \( H \)-sector. Consequently occupational patterns will be different: the most skilled entrepreneurs will locate in the \( H \)-sector, which will be associated with a higher entry threshold.

As we have seen previously, the relation between skill premia that ensures factor market clearing plays a central role in determining the effects of integration. The following cases can be distinguished:

**Case B1:** \( \gamma_L > \gamma_H \). Here the entry threshold in the \( H \)-sector is higher, while the \( L \)-sector is more lucrative, so all entrepreneurs with skill above \( \frac{m_L}{\gamma_L} \) enter the \( L \)-sector, and the rest become workers. The economy specializes in the \( L \)-good. The factor market clearing condition is the same as in case A1.

**Case B2:** \( \gamma_L = \gamma_H \). The entry threshold into the \( L \)-sector is lower but both sectors are equally profitable so entrepreneurs are indifferent between
the two sectors. Those with skills intermediate between the entry thresholds of the two sectors will enter the $L$-sector; others of higher skill will divide themselves between the two sectors so as to ensure that the labor market clears. The factor market clearing condition is similar to that in case A3.

**Case B3:** $\gamma_H > \gamma_L$, $\frac{m_L}{\gamma_L} < \frac{m_H}{\gamma_H}$. Now everyone with skill between $\frac{m_L}{\gamma_L}$ and $\frac{m_H}{\gamma_H}$ will enter the $L$-sector, and those above $\frac{m_H}{\gamma_H}$ will enter the $H$-sector. The skill premia must satisfy the condition

$$\mu_{\theta} \left[ \frac{d\left(\frac{m_L}{\gamma_L}\right)}{\theta} - d\left(\frac{m_H}{\gamma_H}\right) \right] + \mu \sigma \left(\frac{\gamma_H}{\gamma_L}\right) = (1 - \mu) + \mu G\left(\frac{m_L}{\gamma_L}\right).$$

(21)

**Case B4:** $\gamma_H > \gamma_L$, $\frac{m_L}{\gamma_L} > \frac{m_H}{\gamma_H}$. Everyone with skill above $\frac{m_H}{\gamma_H}$ enters the $H$-sector, everyone else becomes a worker, the economy specializes in producing the $H$-good. The factor market clearing condition is similar to that in case A4.

Case B3 is the region where both goods are produced and unequal skill premia across the two sectors. Unlike the case where the $L$-good is more prone to moral hazard, the slope of the skill-premium-frontier is ambiguous in general. The reason is that the effect of raising $\gamma_H$ on the tightness of the labor market is subject to two conflicting effects. Increasing $\gamma_H$ lowers the entry threshold into the $H$-sector, motivating the most able entrepreneurs previously operating in the $L$-sector to now enter the $H$-sector. Since the $L$-sector is less skill-intensive, this lowers the demand for production workers by an amount that depends on differences in skill intensity between the two sectors. On the other hand the rise in $\gamma_H$ induces each $H$-sector entrepreneur to hire more workers, which tightens the labor market; the strength of this effect depends on the elasticity of substitution between skilled and unskilled factors. To show that the effect can go either way, we specialize to the case of a CES production function considered earlier in Proposition ??, and a Pareto distribution for ability.\footnote{The following result extends to more general ability distributions, provided the associated $d$ function has a bounded elasticity. In such cases, the term $(2 - \delta)$, which is the corresponding (constant) elasticity for the $d$ function associated with the Pareto distribution, will be replaced by upper and lower bounds of the elasticity.}

**Lemma 6** Suppose $m_H > m_L$, the production function in each sector is given by (??), and the density of the ability distribution $g(a)$ is proportional to $a^{-\delta}$ for some parameter $\delta > 2$. The relation between skill premia $\gamma_L, \gamma_H$ in case B3 is downward-sloping if

$$\kappa > \left(\frac{k_H}{k_L} - 1\right)(\delta - 2)$$

(22)
and is upward-sloping if

\[ \kappa < \left[ \left( \frac{m_H}{m_L} \right)^{-\kappa} \frac{k_H}{k_L} - 1 \right] (\delta - 2) \]  

(23)

For sufficiently high elasticity of substitution (condition ??), the relation between skill-premia that ensures clearing of the labor market continues to be downward-sloping. In this case, all results of the preceding section continue to apply (with region B3 substituting for region A2, where skill premia are unequal and both goods are produced). The only difference is that skill premia are now higher in the \( H \)-sector, since the \( H \)-good is more prone to moral hazard. The key reason is that in this case a rise in the skill premium in either sector tightens the labor market. It continues to be true for the \( L \)-sector for the same reasons as before. Consider the effects of a ceteris paribus decline in \( \gamma_H \) (which may have been induced by rising wages, in turn the effect of rising \( p_L \) and the resulting tightening of the labor market). This induces some entrepreneurs to move from the \( H \)-sector into the \( L \)-sector.\(^{33}\) This movement tightens the labor market, since the \( L \)-sector is less skill intensive. On the other hand, a high elasticity of substitution implies a sharp drop in workers hired within \( H \)-sector firms, owing to the fall in \( \gamma_H \). This latter effect dominates under assumption (??), and the labor market slackens. In order to restore demand for labor, the \( L \)-sector skill premium must rise.

On the other hand for sufficiently low elasticity of substitution (condition ??), the relation between skill-premia is upward-sloping in region B3. An increase in \( p_L \) will lower the skill premium in both sectors, whenever both goods are produced. Hence the Stolper-Samuelson result will always hold in this case: trade integration will move skill premia closer together across countries.\(^{34}\) The movement of entrepreneurs from the \( H \)-sector to the \( L \)-sector following a decline in \( \gamma_H \) now ensures a tightening of the labor market, which causes the skill premium in the \( L \)-sector to decrease. However, factor prices will not get completely equalized with free trade, as long as at least one of the countries is operating in region B3.

\(^{33}\)This does not happen in Case A, where a fall in \( \gamma_H \) induces some low-ability entrepreneurs in the \( H \)-sector to shift to being a production worker. It happens in case B because now the highest skill premium is in the \( H \)-sector, so entrepreneurs in the \( H \)-sector that fall below the threshold for the \( H \)-sector enter the \( L \)-sector.

\(^{34}\)Some additional technical qualifications are necessary, however, for this statement to be true. First, note that factor market equilibrium need not be unique, as both the labor market clearing condition and the price-cost relation generate upward sloping relationship between skill premia. Hence comparative static propositions pertain to local effects of small changes in \( p_L \), starting with a locally unique equilibrium. Moreover, the statement is valid provided we start at a locally stable equilibrium, where the slope of the skill premium relationship corresponding to the price-cost condition is steeper than for the factor market clearing condition.
6 Concluding Comments

We have constructed a theory of middlemen or entrepreneurial rents in a general equilibrium model of trade, which provide incentives to maintain product quality reputations. Entry thresholds, occupational and sectoral choices of all agents are endogenously determined in an otherwise fully competitive model. The allocation of agents between production work and entrepreneurship is explained by their underlying endowment of entrepreneurial skill. In particular, the model explains why producers cannot directly sell to consumers, and must sell to intermediaries instead.

A novel feature of this model is that the extent of factor-specificity is determined endogenously. If the severity of moral hazard problem differs markedly between different goods, equilibrium skill premia must also vary in a corresponding way. The lack of equalization of skill premia is associated with restrictions on movement of entrepreneurs, and the distributive effects of trade liberalization end up resembling a Ricardo-Viner specific factor model. Otherwise, there is enough intersectoral mobility to ensure that classical results of the mobile factor Heckscher-Ohlin model results obtain. Empirical evidence from some African countries where rising export prices were accompanied by rising gaps between export and farmgate prices are consistent with the predictions of the model, suggesting the need for fuller empirical testing of the model in future research.

The model explains incentives for Northern countries to offshore their production to Southern countries, and predicts the distributive implications of such offshoring to be the opposite of trade liberalization. Normative implications for trade policy include the possibility of trade liberalization reducing welfare in the North owing to reduced entrepreneurial margins in import-competing sectors. Pass-through and output responsiveness to trade liberalization depends on underlying distribution of entrepreneurial ability which determines responsiveness of entry into entrepreneurship in response to increasing profit margins. The model suggests that policies encouraging entry responsiveness, such as regulatory reforms, or development of entrepreneurial abilities may thus enhance growth and pro-poor effects of globalization.

We abstracted from the realistic possibility that reputations may be market or country-specific in addition to being commodity-specific. For instance it may be harder to maintain a reputation in international markets compared with domestic markets, owing to the role of information networks that underlie word-of-mouth reputations. Such a model would create higher productivity thresholds for exports compared with domestic sales for any given commodity, providing an alternative to a number of recent explanations for export ‘premium’ in productivity and earnings. Yet another extension involves country-specific reputation thresholds, owing to differences in product
quality regulations or their enforcement across countries. Our model can be used to explore the general equilibrium implications of changes in regulatory policy.

References


Appendix: Proofs

Proof of Lemma ??: Implicitly differentiating (??) we obtain

\[
\frac{d\gamma_L}{d\gamma_H} = p_L \frac{A_L \theta^H_a}{\theta_a} = \frac{\theta^L_a + \gamma_L \theta^L_a + \theta^H_a \theta^H_a}{\gamma_L + \frac{\theta^L_a}{\theta_a}} > 1,
\]

with the second equality using (??), and the last inequality following from Assumption ??, \(\gamma_L \geq \gamma_H\) and the fact that \(\frac{\theta^L_a}{\theta_a}\) is non-decreasing in \(\gamma_L\). ■

Proof of Proposition ??

Step 1

(i) If \(\kappa \geq 1\), \(d[\gamma_L/\gamma_H]/d\gamma_H = d[\lambda(\gamma_H; p_L, \frac{A_H}{A_L})]/d\gamma_H > 0\) for any \(\gamma_H\) so that \(\lambda(\gamma_H; p_L, \frac{A_H}{A_L})/\gamma_H \geq 1\)

(ii) If \(\kappa < 1\), \(d[\gamma_L/\gamma_H]/d\gamma_H = d[\lambda(\gamma_H; p_L, \frac{A_H}{A_L})]/d\gamma_H > 0\) if and only if \(p_L < A_H/A_L\) (and equivalently \(\gamma_L/\gamma_H < (\frac{A_H}{A_L})^{\frac{1}{\kappa}}\)).

Proof of Step 1

From (??),

\[
d[\gamma_L/\gamma_H]/d\gamma_H = (1/\gamma_H)[\frac{\gamma_L + \frac{\theta^L_a}{\theta_a}}{\gamma_L + \frac{\theta^H_a}{\theta_a}} - \frac{\gamma_L}{\gamma_H}],
\]

which means that \(d[\gamma_L/\gamma_H]/d\gamma_H > 0\) if and only if

\[
\frac{\gamma_L \theta^L_a(\gamma_L)}{\theta^L_a(\gamma_L)} < \frac{\gamma_H \theta^H_a(\gamma_H)}{\theta^H_a(\gamma_H)}.
\]

Under this production function in the proposition,

\[
\frac{\theta^L_a(\gamma_i)}{\theta^H_a(\gamma_i)} = (\gamma_i)^{-\kappa} k_i
\]

and

\[
p_L = \frac{A_H}{A_L} \frac{\theta^L_a(\gamma_L) + \gamma_L \theta^L_a(\gamma_L)}{\theta^H_a(\gamma_H) + \gamma_H \theta^H_a(\gamma_H)} = \frac{A_H}{A_L} \frac{k_L \gamma_L^{1-\kappa} + 1}{k_H \gamma_H^{1-\kappa} + 1} \frac{1}{1-\kappa}.
\]

In the case of \(\kappa \geq 1\) and \(\gamma_L \geq \gamma_H\),

\[
\frac{\gamma_L \theta^L_a(\gamma_L)}{\theta^L_a(\gamma_L)} = (\gamma_L)^{-\kappa} k_L < (\gamma_H)^{-\kappa} k_H = \frac{\gamma_H \theta^H_a(\gamma_H)}{\theta^H_a(\gamma_H)}
\]

implying \(d[\gamma_L/\gamma_H]/d\gamma_H > 0\). In the case of \(\kappa < 1\),

\[
\frac{\gamma_L \theta^L_a(\gamma_L)}{\theta^L_a(\gamma_L)} < \frac{\gamma_H \theta^H_a(\gamma_H)}{\theta^H_a(\gamma_H)},
\]

if and only if \(\gamma_L/\gamma_H < (\frac{A_H}{A_L})^{\frac{1}{\kappa}}\) which is equivalent to \(p_L < A_H/A_L\). ■
Step 2

(i) If $\kappa \geq 1 - \frac{\log[k_H/k_L]}{\log[m_H/m_L]}$, 
$$d[\lambda(\gamma_H; p_L, A_H/A_L)/\gamma_H]/d\gamma_H > 0$$
holds for $p_L \in [p^2_L, p^1_L)$.

(ii) If $0 \leq \kappa < 1 - \frac{\log[k_H/k_L]}{\log[m_H/m_L]}$, for any $p_L < A_H/A_L$, 
$$d[\lambda(\gamma_H; p_L, A_H/A_L)/\gamma_H]/d\gamma_H > 0$$
and for any $p_L > A_H/A_L$, 
$$d[\lambda(\gamma_H; p_L, A_H/A_L)/\gamma_H]/d\gamma_H < 0.$$

Proof of Step 2

First suppose that $m_L/m_H \leq [k_H/k_L]^{1/(1-\kappa)}$ and $\kappa < 1$, which are equivalent to $1 > \kappa \geq 1 - \frac{\log[k_H/k_L]}{\log[m_H/m_L]}$. If $p_L \in (p^2_L, p^1_L)$, since $m_L/m_H > \gamma_L/\gamma_H \geq 1$ is satisfied in an equilibrium of supply-side, it implies $\gamma_L/\gamma_H < (k_H/k_L)^{1/(1-\kappa)}$ (or $p_L < A_H/A_L$). From (ii) of Step 1, this means that 
$$d[\lambda(\gamma_H; p_L, A_H/A_L)/\gamma_H]/d\gamma_H > 0$$
holds for $p_L \in [p^2_L, p^1_L)$. From (i) of Step 1, this inequality also holds for $\kappa \geq 1$. This completes the proof of (i).

Next take $0 \leq \kappa < 1 - \frac{\log[k_H/k_L]}{\log[m_H/m_L]}$. From (ii) in Step 1, for any $p_L < A_H/A_L$, 
$$d[\lambda(\gamma_H; p_L, A_H/A_L)/\gamma_H]/d\gamma_H > 0$$
and for any $p_L > A_H/A_L$, 
$$d[\lambda(\gamma_H; p_L, A_H/A_L)/\gamma_H]/d\gamma_H < 0.$$

This completes the proof of (ii).

Step 3

Taking $p_L \in (p^1_L, p^2_L)$ as given, let’s consider the effect of $\mu$ on 
$$X_L/X_H = \frac{A_L}{A_H} \frac{\theta_H^\mu(\gamma_L)}{\theta_L^\mu(\gamma_H)} \frac{d(a_L)}{d(a_H) - d(a_L)}$$
We can use the following relationship.

\[
\frac{d}{d\mu} \left[ \frac{\theta^H_a(\gamma_H)}{\theta^L_a(\gamma_L)} \right] = \frac{\theta^H_a(\gamma_H)}{\theta^L_a(\gamma_L)} \frac{\theta^H_a(\gamma_H)}{\theta^L_a(\gamma_L)} - \frac{\theta^L_a(\gamma_H)}{\theta^L_a(\gamma_L)} \lambda_1(\gamma_H; p_L, A_H/A_L) d\gamma_H/d\mu
\]

\[
= \frac{\theta^H_a(\gamma_H)}{\theta^L_a(\gamma_L)} \frac{\theta^L_a(\gamma_H)}{\theta^L_a(\gamma_L)} \left[ \gamma_L \frac{\theta^H_a(\gamma_H)}{\theta^L_a(\gamma_L)} \right] - \lambda_1(\gamma_H; p_L, A_H/A_L) d\gamma_H/d\mu
\]

\[
= \frac{\theta^H_a(\gamma_H)}{\theta^L_a(\gamma_L)} \frac{\theta^L_a(\gamma_H)}{\theta^L_a(\gamma_L)} \left[ \frac{\gamma_L \theta^H_a(\gamma_H)}{\theta^L_a(\gamma_L)} \right] - \lambda_1(\gamma_H; p_L, A_H/A_L) d\gamma_H/d\mu
\]

\[
< \frac{\theta^H_a(\gamma_H)}{\theta^L_a(\gamma_L)} \frac{\theta^L_a(\gamma_H)}{\theta^L_a(\gamma_L)} \left[ \frac{\lambda_1(\gamma_H; p_L, A_H/A_L)}{\gamma_H} \right] - \lambda_1(\gamma_H; p_L, A_H/A_L) d\gamma_H/d\mu < 0
\]

if \( d[\lambda(\gamma_H; p_L, A_H/A_L)/\gamma_H] / d\gamma_H > 0 \). This relationship is using the fact that

\[
\frac{\gamma_1 \theta^I_a}{\theta^I_a} = -\kappa / (\gamma_1 \theta^I_a + 1)
\]

and \( d[\lambda(\gamma_H; p_L, A_H/A_L)/\gamma_H] / d\gamma_H > 0 \) if and only if \( \frac{\gamma_1 \theta^I_a(\gamma_L)}{\theta^I_a(\gamma_L)} < \frac{\gamma_1 \theta^I_a(\gamma_H)}{\theta^I_a(\gamma_H)}. \) Similarly, we obtain

\[
\frac{d}{d\mu} \left( \frac{a_L}{a_H} \right) = \frac{d}{d\mu} \left( \frac{a_H}{a_L} \right) - \frac{a_H}{a_L} g(\gamma_H) \lambda_1(\gamma_H, p_L) d\gamma_H/d\mu
\]

\[
> \frac{d}{d\mu} \left( \frac{a_L}{a_H} \right) \frac{a_H}{a_L} \left[ \frac{\lambda_1(\gamma_H; p_L, A_H/A_L)}{\gamma_H} \right] - \lambda_1(\gamma_H; p_L, A_H/A_L) d\gamma_H/d\mu > 0
\]

if \( d[\lambda(\gamma_H; p_L, A_H/A_L)/\gamma_H] / d\gamma_H > 0 \). This is using the assumption that \( \frac{a_H}{a_L} g(a) \) is increasing in \( a \). This implies that

\[
d(X_L/X_H) / d\mu < 0.
\]

for \( p_L \in [p_L^2, p_L^3] \) if \( \kappa \geq 1 - \frac{\log[1/k]}{\log[1/k(H)]} \) and for \( p_L \in [p_L^2, A_H/A_L] \) if \( 0 \leq \kappa < 1 - \frac{\log[1/k(H)]}{\log[1/k]} \).

**Step 4**

Next suppose \( p_L \in (p_L^2, p_L^3). \gamma_L = \gamma_H = \gamma^* \) is determined by

\[
p_L = \frac{A_H}{A_L} \left( \frac{\theta^L a(\gamma^*)}{\theta^L a(\gamma^*)} + \gamma^* \frac{\theta^L a(\gamma^*)}{\theta^L a(\gamma^*)} \right).
\]

\( \gamma^* \) is independent of \( \mu \). This means that \( d\gamma^*/d\mu = 0 \). We have only the direct effect of \( \mu \) on \( X_L/X_H \), which is negative.

From step 3 and this step, this completes the proof of (i) and the first half of (ii) in the proposition.

**Step 5**

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Finally let us show the last part of (ii). Suppose that there does not exist ¯p_L ∈ (A_H/A_L, p^2_L) so that X_L/X_H is increasing in µ for any p ∈ (¯p_L, p^2_L). Then p^1_L has to be non-decreasing in µ. However

\[ dp_L^1/dµ = (p_L^1/γ_H)\left\{ \frac{m_Hγ_H^1θ_H^L(m_Hγ_H^1)}{θ_H^L(m_Hγ_H^1)} + \frac{m_Hγ_H^1θ_H^L(m_Hγ_H^1)}{θ_H^L(m_Hγ_H^1)} - \frac{γ_H^1θ_H^L(γ_H^1)}{θ_H^L(γ_H^1) + γ_H^1θ_H^L(γ_H^1)}\right\} dγ_H^1/dµ \]

is negative from step 2(ii). This is the contradiction.

\[ \text{Proof of Proposition ??} \]

With perfect complementarity in production, factor intensities within firms are independent of the skill premium in that sector. Moreover, d(a) is locally constant if g(a) = 0. Then from the factor market clearing condition in case A2, we obtain

\[ dγ_L/dγ_H = - \left[ \frac{θ_H^L(a_H + 1)}{θ_H^L(a_H) + θ_H^L(a_H)m_H} \right] \frac{dγ_H^1/dµ}{dγ_L/dγ_H}. \]

This shows that γ_H and hence w does not change in case (i), while γ_L does not change in case (ii). The rise in γ_L in case (i) generates no change in X_L because g(\(\frac{m_L}{a_L}\)) = 0.

\[ \text{Proof of Lemma ??} \]

Suppose that an increase in µ raises p_L that the initial price level is in (p^3_L, p^2_L). Then as explained previously, taking p_L as given, the increase in µ causes γ_H and γ_L to decrease in the factor market equilibrium. On the other hand, the increase in p_L causes γ_H to fall and γ_L to rise. Therefore the total effect on γ_H is negative. Since equilibrium p_L rises, the equilibrium level of X_L/X_H must be lower. However the right hand side of

\[ X_L/X_H = \frac{A_L}{A_H} \frac{θ_H^L(γ_H)}{θ_H^L(γ_L)} \frac{d(a_L)}{d(a_H) - d(a_L)} \]

increases with a decrease in γ_H, which implies that the total effect on γ_L must be negative. From the price-cost relations, the effect on w and w/p_L must be positive. On the other hand, if the price level is in (p^3_L, p^2_L), the increase in µ does not have a direct effect on γ_H and γ_L for given p_L, and the effect on both through the increase in p_L is negative.

Next, consider the case where an increase in µ is associated with a fall in p_L. By Proposition ?? this is possible only if p_L ∈ (p^2_L, p^1_L). Then the direct effect of µ taking p_L as given is negative for both γ_L and γ_H. On the other hand, the indirect effect through the decrease in p_L is negative for γ_L and positive for γ_H. Hence the total effect on γ_L is negative. A symmetric argument to that in the previous paragraph also implies that the total effect on γ_H is negative.

\[ \text{Proof of Lemma ??} \]

Let the indirect utility function for country j be V(p^j_L, I^j) where national income

\[ I^j = p^j_LX^j_L + X^j_H. \]
By Roy’s identity $D_L^j = -\frac{V_p(p_L^j, I^j)}{V_I(p_L^j, I^j)}$, 

$$dV(p_L^j, I^j)/dp_L^j = \frac{\partial V(p_L^j, I^j)}{\partial p_L^j} + \frac{\partial V(p_L^j, I^j)}{\partial I} \frac{dI}{dp_L^j}$$

$$= \frac{\partial V(p_L^j, I^j)}{\partial I} [-D_L^j + \frac{dI}{dp_L^j}]$$

$$= \frac{\partial V(p_L^j, I^j)}{\partial I} [(X_L^j - D_L^j) + p_L^j dX_L^j/dp_L^j + dX_H^j/dp_L^j]$$

Since $\frac{\partial V(p_L^j, I^j)}{\partial I} > 0$, the sign of the welfare effect of the change in $p_L^j$ is the same as the sign of

$$(X_L^j - D_L^j) + p_L^j dX_L^j/dp_L^j + dX_H^j/dp_L^j$$

Owing to linear homogeneity of the production function and the common $n_L/a$ chosen by all entrepreneurs in a given sector, $X_L^j = A_L F_L(n_L^j, h_L^j)$ where $n_L^j$ (resp. $h_L^j$) is the total amount of unskilled (resp. skilled) labor used in the $L$-sector of country $j$. Since $p_L^j dX_L^j/dn_L^j = w^j$ and $\gamma_L^j = \frac{\partial p_L(n_L^j, h_L^j) / \partial h_L^j}{\partial p_L(n_L^j, h_L^j) / \partial n_L^j}$

$$p_L^j dX_L^j/dp_L^j = p_L^j A_L [\partial F_L(n_L^j, h_L^j) / \partial n_L^j [dn_L^j / dp_L^j] + \partial F_L(n_L^j, h_L^j) / \partial h_L^j [dh_L^j / dp_L^j]]$$

Similarly

$$dX_H^j / dp_L^j = A_H [\partial F_H(n_H^j, h_H^j) / \partial n_H^j [dn_H^j / dp_L^j] + \partial F_H(n_H^j, h_H^j) / \partial h_H^j [dh_H^j / dp_L^j]]$$

Therefore

$$p_L^j dX_L^j / dp_L^j + dX_H^j / dp_L^j = w^j [\gamma_L^j dX_H^j / dp_L^j + \gamma_H^j (\gamma_L^j - \gamma_H^j) dh_L^j / dp_L^j].$$

Since $h_L^j = \mu d(a_L^j)$ for $p_L^j \in \{p_L^{j1}, p_L^{j2}\}$ and $\gamma_L^j = \gamma_H^j$ for $p_L^j \in \{p_L^{j3}, p_L^{j4}\}$,

$$(\gamma_L^j - \gamma_H^j) dh_L^j / dp_L^j = \mu (\gamma_L^j - \gamma_H^j) d(a_L^j) / dp_L^j$$

holds for $p_L^j \in \{p_L^{j3}, p_L^{j4}\}$. With $n_L^j + n_H^j = \mu G(a_L^j) + (1 - \mu)$ and $h_L^j + h_H^j = \mu d(a_L^j)$ for $p_L^j \in \{p_L^{j3}, p_L^{j4}\},$

$$\gamma_L^j (\gamma_L^j - \gamma_H^j) dX_H^j / dp_L^j + (\gamma_L^j - \gamma_H^j) dX_H^j / dp_L^j = w^j \mu [\gamma_H^j d(a_L^j) / dp_L^j]$$

Proof of Proposition ??

Suppose that $w^S \neq w^N$ with free trade and costless offshoring. If $w^S < w^N$, all entrepreneurs would hire only workers in country $S$. However production workers in country $N$ do not have the option to become entrepreneurs, and would thus be unemployed, implying $w^N = 0$, a contradiction. Similarly, we cannot have $w^S > w^N$. With a common product price ratio $p_L$ and the common unskilled wage, skill premia must be equalized in each sector across the two countries. These premia must clear the market for production
workers in the integrated economy, i.e., satisfy (??) with \(\mu^G\) representing the proportion of skilled agents.

As shown in the autarky equilibrium, in the region that \(\gamma_L > \gamma_H\) holds in the equilibrium, the autarky levels of \(\gamma_H\) and \(\gamma_L\) are decreasing in \(\mu\) regardless of its impact on \(p_L\). Hence \(\mu^S < \mu^G < \mu^N\) implies a fall (resp. rise) in skill premia in each sector in the South (resp. North). 

Proof of Lemma ??

It is evident that increasing \(\gamma_L\) tightens the labor market clearing condition (??). So the relation with \(\gamma_H\) ensuring labor market clearing is downward-sloping if and only if an increase in \(\gamma_H\) also tightens the labor market, i.e.,

\[
\phi'_L(\gamma_L)d\left(\frac{m_H}{\gamma_H}\right) - [\phi_H(\gamma_H) - \phi_L(\gamma_L)]d\left(\frac{m_H}{\gamma_H}\right) > 0
\]

where \(\phi_i(\gamma_i) \equiv \frac{\partial \theta_i(\gamma_i)}{\partial \gamma_i}\), which reduces to the condition

\[
\frac{\phi_L - 1}{\phi_H} \frac{\phi_H}{\phi'_H \gamma_H} < \frac{\gamma_H}{m_H} \left(\frac{d\left(\frac{m_H}{\gamma_H}\right)}{d'\left(\frac{m_H}{\gamma_H}\right)}\right)
\]

With the CES production function, we have \(\phi_i = (\gamma_i)^{\kappa - 1} k_i\), with \(k_H > k_L\). Hence it reduces to

\[
\left(\frac{\gamma_L}{\gamma_H}\right)^\kappa \frac{k_H}{k_L} - 1\epsilon < \kappa
\]

where \(\epsilon\) denotes the elasticity of \(d\) evaluated at \(\frac{m_H}{\gamma_H}\). This elasticity equals \(\delta - 2\) in the case of the Pareto distribution. The result now follows, upon observing that in region B3, the skill premium ratio \(\frac{\gamma_L}{\gamma_H}\) varies between \(\frac{m_L}{m_H}\) and 1. 

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