Economics 101A
Section Notes
GSI: David Albouy

Expected Utility and Insurance in a Two State Model

1 Expected Utility

1.1 The Basics

Expected Utility (EU) theory is a technique developed by Von Neumann and Morgenstern (1944) to deal with situations of quantifiable risk. It requires preferences to exhibit two additional axioms of *continuity* and *independence*, which are somewhat controversial. Assume that states of nature can be indexed by an $s = 1, ..., S$, each with a probability of occurring of $p_1, ..., p_S$, which as probabilities obey $p_s \geq 0$ and $\sum_{s=1}^{S} p_s = 1$.[1] Let x_s be the realization of some random variable, sometimes known as a **prospect** or **lottery**, x in state s, which yields utility $u(x_s)$. The **Expected Utility Theorem** states that if consumers have rational preferences that exhibit continuity and independence[2] then agents will behave *as if* they maximize the expected value of their utility or just **expected utility**

$$E[u(x)] = \sum_{s=1}^{S} p_s u(x_s) \qquad \text{(EU Utility)}$$

Similarly, firms can be assumed to maximize expected profits $E[\pi(x)]$ over various states of the world. The nature of the budget constraint will vary considerably upon the situation considered.

1.2 Two State Set Up and Indifference Curves

The easiest situation to set up is a 2 state set up with $p_1 = p$ and $p_2 = 1 - p$. Individuals maximize

$$E[u(x)] = pu(x_1) + (1 - p) u(x_2)$$

An interesting way to represent preferences in this case is with a standard consumer model over two goods, consumption in state 1 and consumption in state 2. As usual an indifference curve is given implicitly by setting utility to a fixed value and treating one variable (say x_2) as a function of the other (x_1) or formally

$$pu(x_1) + (1 - p) u\left(x_2^{IC}(x_1)\right) = \bar{u}$$

Differentiating this condition implicitly once gives the condition

$$-\frac{dx_2^{IC}}{dx_1} = MRS_{x_1 x_2} = \frac{p}{1-p} \frac{u'(x_1)}{u'(x_2)}$$

Along the 45 degree line of a graph of x_1 and x_2, where $x_1 = x_2$, then $u'(x_1) = u'(x_2)$ and so $MRS_{x_1 x_2} = p/(1-p)$ the **odds-ratio** (the proportion of state 1's to state 2's) no matter what the utility function looks like. This is one of the stronger implications of expected utility theory.

[1] This approach breaks down if the uncertainty is unquantifiable, i.e. you cannot attach numerical probabilities to each state. Risk is generally defined as quantifable uncertainty.

[2] Intuitively, **continuity** implies that very slight changes in probability will not affect a strict preference of a prospect x over a prospect y. **Independence** implies that if x is preferred to y and we mix x and y each with the same lottery z, so that x' = x with a 50% chance and z with a 50% chance (and y' defined similarly) then x' will be preferred to y'.

1.3 Risk Aversion

One way of thinking about **risk aversion** is to think that people have convex preferences over consumption in either state: they would rather have a moderate consumption in both states rather than low consumption in one state and high consumption in the other, just as people tend to prefer a mixed bundle of goods than lots of only one good. This means that under risk aversion people should exhibit diminishing marginal rates of substitution along an indifference curve. Taking the derivative of the MRS with respect to x_1 and not forgetting that x_2 is a function of x_1 along the indifference curve we get

$$\begin{aligned}
\frac{d}{dx_1} MRS_{x_1 x_2} &= \frac{p}{1-p}\left[\frac{d}{dx_1}\frac{u'(x_1)}{u'\left(x_2^{IC}(x_1)\right)}\right] \\
&= \frac{p}{1-p}\left[\frac{u''(x_1)\,u'(x_2) - u''(x_2)\,u'(x_1)\,\frac{dx_2^{IC}}{dx_1}}{\left[u'(x_2)\right]^2}\right] \\
&= \frac{p}{1-p}\left[\frac{u''(x_1)\,u'(x_2) + u''(x_2)\,u'(x_1)\,\frac{p}{1-p}\frac{u'(x_1)}{u'(x_2)}}{\left[u'(x_2)\right]^2}\right]
\end{aligned}$$

where the last line comes from substituting in the value for the MRS. A sufficient condition for declining MRS (i.e. for this expression to be negative) is that $u''(x)$ is negative which is one version of how risk aversion is defined. Similarly if the individual is risk neutral $u''(x) = 0$ and so $d(MRS)/dx_1 = 0$ implying indifference curves are parallel lines, indicating that consumption in one period is a perfect substitute for consumption in the other period.

Example 1 *Let $u = \log(x)$ so expected utility is given by $E[u(x)] = p\log(x_1) + (1-p)\log(x_2)$. In this case indifference curves are given by*

$$\begin{aligned}
\bar{u} &= p\log(x_1) + (1-p)\log(x_2) \\
&\Rightarrow x_1^p x_2^{1-p} = e^{\bar{u}} \\
&\Rightarrow x_2 = e^{\bar{u}/(1-p)} x_1^{p/(1-p)}
\end{aligned}$$

The marginal rate of substitution is given by

$$MRS_{x_1 x_2} = \frac{p}{1-p}\frac{x_2}{x_1} = \frac{p}{1-p}\frac{e^{\bar{u}/(1-p)} x_1^{p/(1-p)}}{x_1} = \frac{p}{1-p}\frac{e^{\bar{u}/(1-p)}}{x_1^{1/(1-p)}}$$

which is declining in x_1.

Example 2 *Imagine a person faced with the prospect of a fair 50-50 bet. If the person with money I takes the bet $b > 0$ then $x_2 = I + b$ if he wins and $x_1 = I - b$ if he loses. With no bet $x_1 = x_2 = I$. The bet is fair since $E[b] = \frac{1}{2}b + \frac{1}{2}(-b) = 0$ and so the expected value of the prospect is I. The utility from not taking the bet is just $u(I)$, while the utility taking it is $\frac{1}{2}u(I+b) + \frac{1}{2}u(I-b)$. The person is will decline or accept the bet depending on whether*

$$u(I) \gtreqless \frac{1}{2}u(I+b) + \frac{1}{2}u(I-b)$$

A person who rejects the bet ($>$) is risk averse, takes the bet ($<$) is risk loving , and is indifferent about it ($=$) is risk neutral.

Generalizing to any prospect x we compare what the utility of its expected value of its expected utility $u[E(x)] \gtreqless E[u(x)]$; .$>$ implies risk aversion, $<$ risk loving, and $=$ risk neutrality. A mathematical fact known as *Jensen's Inequality* tells us that risk aversion is reflected in a $u(x)$ that is concave, i.e. $u''(x) < 0$ when x is a single variable. Similarly, risk loving implies a convex u, $u''(x) > 0$, and risk neutrality a linear u, $u''(x) = 0$.

2 Insurance

2.1 Efficient Insurance

Assuming individuals are risk averse and *actuarially fair insurance* exists (i.e. insurance with expected cost to the consumer of zero) then it can be shown that individuals will always choose to *insure fully* (i.e. eliminate all risk). Suppose an agent a utility function which depends only on income, which she has I to start out with. Let $p > 0$ be the the chance of an accident which causes a loss d of income. An agent can buy insurance contract (a, b) which has a premium b but pays out a net amount a in case the accident occurs. So the expected utility of an agent that buys such a contract is $pu(I - d + a) + (1-p)u(I-b)$. You can think of state 1 as the accident state and state 2 as the non-accident state with $x_1 = I - d + a$, and $x_2 = I - b$.

The expected profit of a firm which offers this contract is $\pi = p(-a) + (1-p)b$ (we leave out the expectations operator E) The presence of actuarially fair insurance can be justified by assuming insurance markets are *competitive* and firms are *risk neutral*: competition among firms will drive expected profits to zero $\pi = 0 \Rightarrow ap = b(1-p) \Rightarrow b = ap/(1-p)$. The effective "budget constraint" can be written by solving for x_2 as a function of x_1

$$
\begin{aligned}
x_2 &= I - b \\
&= I - a\frac{p}{1-p} \\
&= I - (x_1 + d - I)\frac{p}{1-p} \\
&= \frac{I - pd}{1-p} - \frac{p}{1-p}x_1
\end{aligned}
$$

Note that this budget constraint goes through the no insurance point $x_1 = I - d$, $x_2 = I$ and that it implies a relative price of consumption in period 1 and to consumption in period 2 as $p/(1-p)$.

The choice of optimal insurance is found by finding the optimal x_1

$$\max_{x_1} pu(x_1) + (1-p)u\left(\frac{I-pd}{1-p} - \frac{p}{1-p}x_1\right)$$

Taking the FOC with respect to a

$$pu'(x_1) - (1-p)u'\left(\frac{I-pd}{1-p} - \frac{p}{1-p}x_1\right)\left(\frac{-p}{1-p}\right) = 0$$

which rearranging and cancelling out $(1-p)$ and p implies

$$u'(x_1) = u'\left(\frac{I-pd}{1-p} - \frac{p}{1-p}x_1\right)$$

If agents are risk averse then concavity implies u' is decreasing and so $u'(x_1) = u'(x_2)$ implies $x_1 = x_2$ and so agents will have the same effective income in either state, they are fully insured. We can solve for the premium as

$$I - d + a^* = I - a^*\frac{p}{1-p} \Rightarrow a^* = (1-p)d$$

and so $b^* = a^*p/(1-p) = pd$ and the optimal contract will be $(a^*, b^*) = ((1-p)d, pd)$. In each state income is $x_1^* = x_2^* = I - pd$. This is shown in a diagram since the zero profit condition implies that the relative price $p/(1-p)$ equals the marginal rate of substitution along the 45 degree line (see above) where the two allocations are equal.

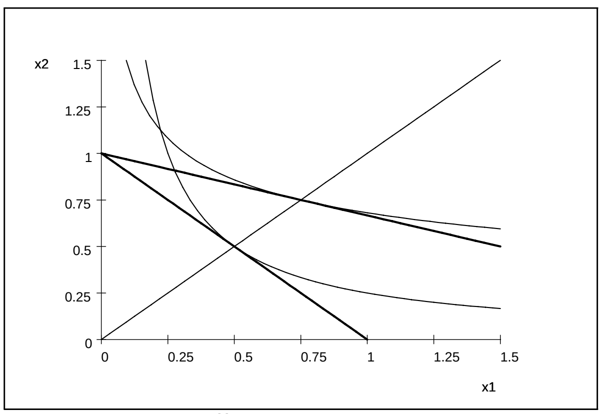
2.2 Adverse Selection and Insurance

The problem of adverse selection in insurance markets was laid about by Rothschild and Stiglitz (1976). Take the same setup as above except assume there are two types of individuals - low risk types L and high risk types H with accident probabilities p_L and p_H respectively, where $p_L < p_H$, but where I, u, and d are the same for both types. Competition among firms will have some interesting implications as it implies firms will always offer contracts which can make at least zero profits. Say proportion λ are low risk types and so $(1-\lambda)$ are high risk types. If insurance companies can tell the two apart then they can just offer each type the efficient contract $(a_L^*, b_L^*) = ((1-p_L)\,d, p_L d)$ and $(a_H^*, b_H^*) = ((1-p_H)\,d, p_H d)$. Low risk individuals will then consume $I - p_L d$ and high risk individuals will consume $I - p_H d$.

Example 3 *Let $p_L = \frac{1}{4}$ and $p_H = \frac{1}{2}$ and assume that the proportion of L types is $\lambda = 1/2$ while the proportion of H types is $1-\lambda = 1/2$. Each has income $I = 1$, and if they get into an accident suffer damages suffers $d = 1$. Expected utility takes the form*

$$p_i \log x_1^i + (1-p_i) \log x_2^i \quad i = L, H$$

where x_1^i is remaining income in the accident state and x_2^i is remaining income in the nonaccident state. Using the above solution we can show that $x_{1L}^* = x_{2L}^* = 3/4$, $x_{1H}^* = x_{2H}^* = 1/2$, $(a_L^*, b_L^*) = ((1-p_L)\,1, p_L 1) = (3/4, 1/4)$ and $(a_H^*, b_H^*) = ((1-p_H)\,1, p_H 1) = (1/2, 1/2)$.



Efficient Insurance

2.2.1 Failure of Efficient Insurance

If the types are not observable by the insurance firms then the efficient contracts no longer work as firms cannot prevent one type from taking the other type's efficient contract. The high types H all want to pretend to be low types L as the accident benefit is higher as $a_L^* = (1-p_L)\,d > (1-p_H)\,d = a_H^*$ and premium costs are lower $b_L^* = p_L d < p_H d = b_L^* d$. Since firms compete to get the low-risk types $\pi_L = -p_L a_L^* + (1-p_L)\,b_L^* = 0$ and so overall profits when high types take the efficient low risk contract will turn negative:

$$\begin{aligned}\lambda \pi_L + (1-\lambda)\,\pi_H &= 0 + (1-\lambda)\left[(1-p_H)\,b_L^* - p_H a_L^*\right] \\ &= (1-\lambda)\left[(1-p_H)\,p_L d - p_H\,(1-p_L)\,d\right] \\ &= d\,(1-\lambda)\,[p_L - p_H] < 0\end{aligned}$$

where the inequality comes from the fact that $p_L < p_H$. Therefore the efficient contracts cannot be an equilibrium, and some other suboptimal equilibrium must be found. There are two main possibilities to consider: (i) where firms offer a one-size-fits-all or "pooling" contract (a^P, b^P) which both types will take and (ii) where firms offer "separating" contracts, one for low risk types $\left(a_L^S, b_L^S\right)$ and one for high risk types $\left(a_H^S, b_H^S\right)$ which are designed so that each type voluntarily self-selects into buying the contract made for it.

2.2.2 Pooling Contracts

Say a firm tries to institute a pooling contract (a^P, b^P) so that everyone buys it. The question is can this contract can work as an equilibrium. (The answer is no) If a firm can offer a profitable contract which at least one type will take, then the equilibrium will fall apart. In fact, for any pooling equilibrium there is always a contract (a', b') that is better for the low risk types and is profitable. Therefore any pooling equilibrium will fall apart.

The overall probability of accident $p = \lambda p_L + (1-\lambda) p_H$ and so $p_L < p < p_H$. The zero profit condition is that $\pi^P = -pa^P + (1-p) b^P = 0$ and thus $b^P = a^P p/(1-p)$. Therefore the relative price of consumption in either state is $\frac{p}{1-p}$. Now at any point the indifference curves of high types and low types will cross, as high types will have a higher $MRS^H_{x_1x_2}$.

$$MRS^H_{x_1x_2} > MRS^L_{x_1x_2} \Leftrightarrow \frac{p^H}{1-p^H}\frac{u'\left(x_1^P\right)}{u'\left(x_2^P\right)} > \frac{p^L}{1-p^L}\frac{u'\left(x_1^P\right)}{u'\left(x_2^P\right)} \Leftrightarrow \frac{p^H}{1-p^H} > \frac{p^L}{1-p^L} \Leftrightarrow p^H > p^L$$

which makes sense since high types will value consumption in the accident state 1 more than in the non-accident state 2. Because p is between p^H and p^L it also follows that $MRS^H_{x_1x_2} > \frac{p}{1-p} > MRS^L_{x_1x_2}$.

Since a firm offering a pooling contract have to make up for losses with hightypes using profits from the low types, a contract which attracts low types away from the pooling contract will put such firms out of business. It turns out that profitable (non-equilibrium) contracts can always be offered to low types. These are located above the indifference curve for low types and below the indifference curve for high types (implying only low types take this contract) and beneath the zero profit constraints for low types (implying positive profits). Therefore pooling equilibria cannot exist as we assumed that any profitable contract would be offered, and so profitable contracts attracting low types away from the pooling contract will be offered, making any pooling contracts unprofitable. A technical proof is contained in the footnote.[3]

Example 4 $\left(a^P, b^P\right) = (5/8, 3/8)$ *is a one possibility for a pooling contract (one that implies full insurance for both types). The overall probability of an accident is* $p = \frac{1}{2}p_L + \frac{1}{2}p_H = \frac{1}{2}\frac{1}{4} + \frac{1}{2}\frac{1}{2} = \frac{3}{8}$. *The profit made will be* $\pi = -pa^P + (1-p) b = -\frac{3}{8}\frac{5}{8} + \frac{5}{8}\frac{3}{8} = 0$ *satisfying the zero profit condition. However the profits made off of the low types is positive* $\pi_L = -p_L a^P + (1-p_L) b = -\frac{1}{4}\frac{5}{8} + \frac{3}{4}\frac{3}{8} = \frac{9-5}{32} = \frac{4}{32} = \frac{1}{8}$ *and so* $\pi_H = -\frac{1}{8}$. *Another firm can offer an insurance contract with slightly less coverage that will attract the low types only and make a profit. Calculating marginal rates of substitution we can see that they will not be equal*

$$MRS^L_{x_1x_2} = \frac{p_L}{1-p_L}\frac{1/x_1}{1/x_2} = \frac{1/4}{3/4}\frac{1/(5/8)}{1/(5/8)} = \frac{1}{3}, \quad MRS^L_{x_1x_2} = \frac{p_L}{1-p_L}\frac{1/x_1}{1/x_2} = \frac{1/2}{1/2}\frac{1/(5/8)}{1/(5/8)} = 1$$

implying that indifference curves will cross as shown in the diagram below. Contracts which can disrupt pooling contracts are located in an area below.

[3] Let $K = \left(MRS^L_{x_1x_2} + MRS^L_{x_1x_2}\right)/2$ be the average of the $MRS_{x_1x_2}$ for both types. Now for some small $\varepsilon > 0$, consider the following contract that offers just slightly less coverage ε, but requires a lower premium $K\varepsilon$, $(a', b') = (a^P - \varepsilon, b^P - K\varepsilon)$, implying $x_1' = x_1^P - \varepsilon$, and $x_2' + K\varepsilon$ The low types will take this contract since it will yield a higher utility then $\left(a^P, b^P\right)$, which can be shown using a differential argument from calculus

$$U_L\left(x_1^P, x_2^P\right) - U_L(a', b') = p_L\left[u\left(x_1^P - \varepsilon\right) - u\left(x_1^P\right)\right] + (1-p_L)\left[u\left(x_2^P + K\varepsilon\right) - u\left(x_2^P\right)\right] \simeq p_L u'\left(x_1^P\right)(-\varepsilon) + (1-p_L) u'\left(x_2^P\right)(K\varepsilon)$$

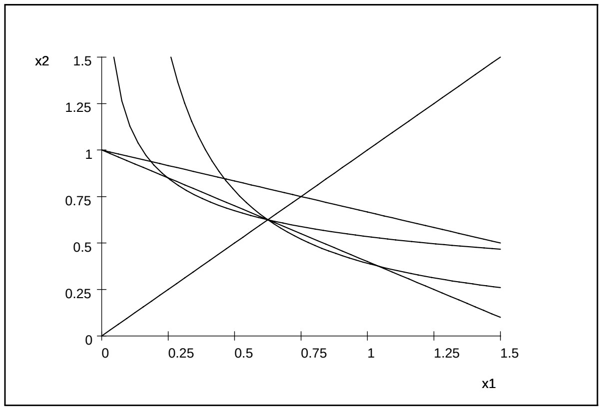
This quantity is positive since

$$(1-p_L) u'\left(x_2^P\right)(K\varepsilon) - p_L u'\left(x_1^P\right)(\varepsilon) > 0 \Leftrightarrow K > \frac{p^L}{1-p^L}\frac{u'\left(x_1^P\right)}{u'\left(x_2^P\right)} = MRS^L_{x_1x_2}$$

For the same reason high risk types will not like this contract since $K < MRS^H_{x_1x_2}$ because the reduction in coverage is not made up enough for them by the reduction in the premium. Firms will want to offer such a contract as they can make a positive profit from it

$$\begin{aligned}\pi' &= -p_L\left(a^P - \varepsilon\right) + (1-p_L)\left(b^P - K\varepsilon\right) = \left[-p_L a^P + (1-p_L) b^P\right] + \varepsilon\left[p_L - (1-p_L) K\right] \\ &= \left[-p_L a^P + (1-p_L)\frac{p}{1-p}a^P\right] + \varepsilon\left[p_L - (1-p_L) K\right] = a^P\left[\frac{p-p_L}{1-p}\right] + \varepsilon\left[p_L - (1-p_L) K\right]\end{aligned}$$

The first term is always positive while the second term is negative. The firm will just pick an $\varepsilon > 0$ small enough the it can make the second term negligibly small and assure itself positive profits.



Pooling Contract

2.2.3 Separating Contracts

We know that no pooling contract will ever work as it will lose out to a separating contract. However that separating contract is not an equilibrium since the former pooling contract which serves the high types no longer works. An equilibrium pair of separating contracts $\left(a_L^S, b_L^S\right)$ and $\left(a_H^S, b_H^S\right)$ must be stable for both types. As we saw earlier it is impossible for both types to get their respective efficient contracts as high risk types prefer the low risk efficient contract to their own optimal contract. However, it is possible for high risk types to get their efficient contract will low types get an *inefficient* contract, since low types do not want the high risk efficient contract. In, fact competition amongst firms for the high types business will assure that the high risk types will get their efficient contract $\left(a_H^S, b_H^S\right) = \left(a_H^*, b_H^*\right)$ in a separating equilibrium.

The low types can only get the most efficient contract $\left(a_L^S, b_L^S\right)$ that high risk types will *not* want to take. We model this by making *high types indifferent about both contracts*, assuming they take the one for the high types. Let $\left(x_{1H}^*, x_{2H}^*\right)$ be the amounts in each state from the efficient high type separating contract and $\left(x_{1L}^S, x_{2L}^S\right)$ be that for the low types. Since utility for the high types is $U_H^* = p_H u\left(x_{1H}^*\right) + (1-p_H) u\left(x_{2H}^*\right) = p_H u\left(I - p_H d\right)$ then

$$u\left(I - p_H d\right) = p_H u\left(x_{1L}^S\right) + (1-p_H) u\left(x_{2L}^S\right) \qquad \text{(Cond 1)}$$

is the implicit restriction on $\left(x_{1L}^S, x_{2L}^S\right)$: be careful to note that it is the high risk probabilities for the low risk contract. The contract $\left(a_L^S, b_L^S\right)$ that will result in a separating equilibrium will satisfy the above condition and satisfy the *zero profit condition* for the low risk types

$$b_L^S = a_L^S p_L / \left(1 - p_L\right) \qquad \text{(Cond 2)}$$

The two conditions can be used to solve for the separating contract. Substituting in (Cond 2) into (Cond 1) we have

$$\begin{aligned} u\left(I - p_H d\right) &= p_H u\left(I - d + a_L^S\right) + (1-p_H) u\left(I - a_L^S p_L / \left(1 - p_L\right)\right) \\ &= p_H u\left(x_{1L}^S\right) + (1-p_H) u\left(\frac{I - p_L d}{1 - p_L} - \frac{p_L}{1 - p_L} x_{1L}\right) \end{aligned}$$

This one equation implicitly defines the one variable a_L^S in a single equation, which you may be able solve for. Note that this contract cannot be efficient: if we try to substitute in the efficient contract for L types, with $a_L^S = \left(1 - p_L\right) d$ then we get

$$u\left(I - p_H d\right) = u\left(I - p_L d\right)$$

which cannot hold unless $p_L = p_H$, i.e. low and high types are identical, contrary to our starting assumption. Therefore this contract is inefficient for low types. A little more work would show that the resulting insurance will be too low.

So far we have found a set of equilibrium contracts that Because the contract for low types is inefficient there are lots of possible contracts that could preferred by low risk types that would increase profits. However a more efficient contract for the low risk types will also attract the high risk types. The question is whether there are pooling contracts which could also sustain high risk types *and* still make a profit. If λ is big and there are relatively few high profit types then such a pooling contract exists. Then the separating equilibrium we just solved is not a true equilibrium as some firm can offer the pooling contract which will cause both types to abandon the separating contracts. In this case there will be *no market equilibrium whatsoever*. Any potential pooling equilibrium will be abandoned for separating contracts and any potential separating equilibrium will be abandoned for a pooling one!

This contract $(\tilde{a}, \tilde{b})$ has to make at least zero profits so

$$p\tilde{b} = (1-p)\,\tilde{a}$$

and it has to be better for low risk types (which automatically makes it better for high risk types) which substituting in the above means

$$p_L u\left(I - d + \tilde{a}\right) + (1-p_L)\,u\left(I - \tilde{a}p/\left(1-p\right)\right) > p_L u\left(I - d + a_L^S\right) + (1-p_L)\,u\left(I - a_L^S p_L/\left(1-p_L\right)\right)$$

So rearranging, if there there exists an $\tilde{a}$ that satisfies this above condition then there will be no separating equilibrium either. Graphically this can be checked by seeing if the zero profit constraint for a pooled contract lies beneath the indifference curve for low types at the separating equilibrium. If it is beneath then the above condition is not satisfied and the separating contracts are an equilibrium. If it crosses this indifference curve then there is no equilibrium: a separating contract can disrupt any pooling contract and a pooling contract and disrupt any separating contracts. An interesting conclusion of this paper is that if there are only a few high risk types (λ close to one) then the pooled zero profit is likely to cross L type's indifference curve, causing the insurance market to fall apart.

Example 5 *The separating contracts for out example are given by*

$$\left(a_H^P, b_H^P\right) = \left(a_H^*, b_H^*\right) = \left(\frac{1}{2}, \frac{1}{2}\right) \text{ and } \left(a_L^P, b_L^P\right) = \left(\frac{3}{2} - \sqrt{\frac{3}{2}}, \frac{1}{2} - \frac{1}{\sqrt{6}}\right) \cong (0.275, 0.092)$$

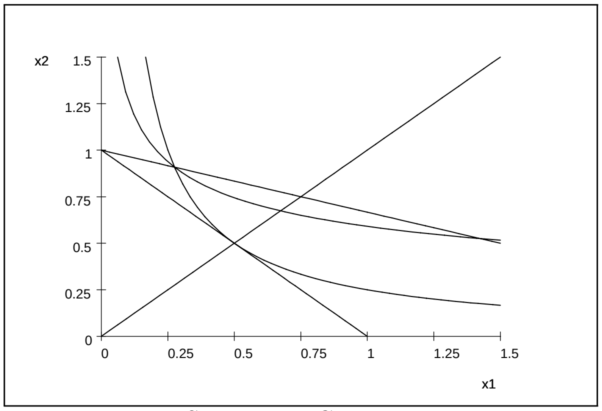
The utility for the H types is simply $\log(1/2) = -\log 2$. *They have to be indifferent about between their contract and L's contract so* $\frac{1}{2}\log a_L^P + \frac{1}{2}\log\left(1 - b_L^P\right) = \log\frac{1}{2} \Rightarrow \left(a_L^P\right)^{1/2}\left(1 - b_L^P\right)^{1/2} = \frac{1}{2} \Rightarrow a_L^P\left(1 - b_L^P\right) = \frac{1}{4} \Rightarrow a_L^P = \frac{1}{4\left(1 - b_L^P\right)}$ *(or* $x_{1L} = \frac{1}{4x_{2L}}$*). The firm's zero profit condition implies*

$$p_L a_L^P = (1-p_L)\,b_L^P \Rightarrow a_L^P = \frac{1-p_L}{p_L} b_L^P = 3b_L^P$$

(or $x_{2L} = \frac{I - p_L d}{1-p_L} - \frac{p_L}{1-p_L}x_{1L} = 1 - \frac{1}{3}x_{1L}$*) Combining the two conditions and using the quadratic formula*

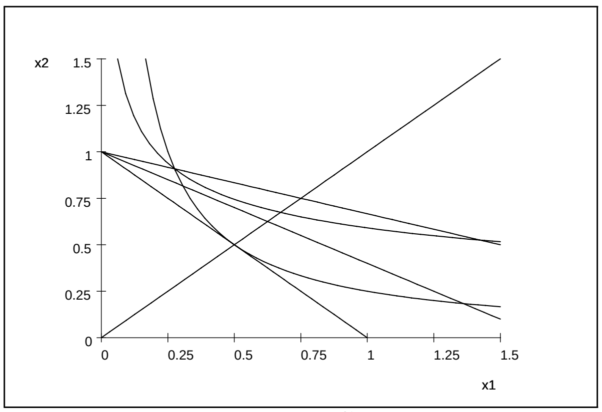
$$3b_L^P = \frac{1}{4\left(1 - b_L^P\right)} \Rightarrow 12b_L^P\left(1 - b_L^P\right) = \frac{1}{12} \Rightarrow \left(b_L^P\right)^2 - b_L^P - \frac{1}{12} = 0 \Rightarrow b_L^P = \frac{1 \pm \sqrt{1 - \frac{1}{3}}}{2} = \frac{1}{2} \pm \frac{\sqrt{\frac{2}{3}}}{2} = \frac{1}{2} \pm \frac{1}{\sqrt{6}}$$

(or $1 - \frac{1}{3}x_{1L} = \frac{1}{4x_{1L}} \Rightarrow 4\left(x_{1L}\right)^2 - 12x_{1L} + 3 = 0$*) Only the smaller answer makes sense (the other one overinsures and gives L-types lower utility): so* $b_L^P = \frac{1}{2} - \frac{1}{\sqrt{6}}$*.and so* $a_L^P = 3b_L^P = \frac{3}{2} - \frac{3}{\sqrt{6}} = \frac{3}{2} - \sqrt{\frac{3}{2}}$*, which then implies* $x_{1L} = \frac{1}{2} + \frac{1}{\sqrt{6}} \cong 0.908$ *and* $x_{2L} = \sqrt{\frac{3}{2}} - \frac{1}{2} \cong 0.725$.



Separating Contracts

The separating equilibrium does exist. If we draw the zero profit condition we see that it lies below the indifference curve for the L-types. Therefore there is no pooling contract that can make money and draw both types disrupting the separating contract.



No Disrupting Pooling Contract Possible