Economics 101A
Section Notes

Bargaining Models

In this handout we will consider bargaining in the context of a labor union (represented by a single representative worker) bargaining with its employer, a typical firm. These bargaining models apply to many other contexts other than labor unions. Note that this is just the tip of the iceberg in the bargaining literature (for instance John Nash developed a very different bargaining model from the ones considered below).

1 Indifference Curves and Isoprofit Curves Revisited

One way of modelling a union's preferences is to pretend that it corresponds to the preferences of a representative worker. Typically we write a worker's utility as a function of consumption, C, and leisure, L, say $U(C, L)$ However it is of interest here to represent utility in terms of the wage w and labor, h. Assuming that the budget constraint holds, $C = wh$, (setting non-labor income to zero for simplicity) and because of the time constraint $L = 1 - h$, when the worker has 1 one unit of time. Therefore we can write the utility function as $\tilde{U}(h, w) = U(wh, 1-h)$ as function (h, w) rather than (C, L). An **indifference curve** in (h, w) −space is defined by setting utility to be a constant, say $\bar{u}$, which defines w implicitly as a function of h, $w^{IC}(h)$, allowing us to write $\tilde{U}\left(h, w^{IC}(h)\right) = U\left(w^{IC}(h) \cdot h, 1-h\right) = \bar{u}$. Differentiating implicitly the last equality with respect to h we can determine the slope of the indifference curve

$$\frac{\partial U}{\partial C} \cdot \left(\frac{dw^{IC}}{dh} h + w\right) + \frac{\partial U}{\partial L}(-1) = 0 \Rightarrow \frac{dw^{IC}}{dh} = \frac{1}{h}\left(\frac{\partial U/\partial L}{\partial U/\partial C} - w\right) = \frac{MRS_{LC} - w}{h}$$

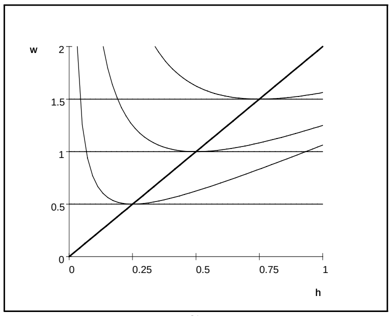
This implies that along the labor supply curve, where $MRS = w$ the the indifference curve will have zero slope. To the left of the labor supply curve, workers would like to work more and so $MRS < w$ and the indifference curve is downward sloping. Symmetrically, to the right $MRS > w$ and the indifference curve is upward sloping. In fact we can reinterpret the first order condition for finding labor supply as the worker finding the highest indifference curve in (w, h) subject to the constraint that w equals the offered wage, leading to the tangency shown below.

The firm's "preferences" are derived in a similar fashion as the isocost curve, although it is in fact more straightforward. We write profits directly as a function of wages and employment E: $\pi(E, w) = f(E) - wE$, setting the price of output to one for simplicity. Along an **isoprofit curve** we simply set profit equal to some constant $\bar{\pi}$ implying an implicit relationship between w and E, $w^{IP}(E)$, such that $f(E) - w^{IP}(E) \cdot E = \bar{\pi}$. Differentiating implicitly allows to find the slope of the isoprofit curve.

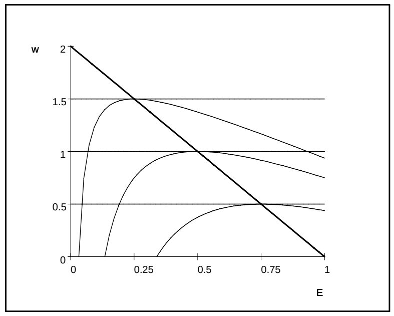
$$f'(E) - \frac{dw^{IP}}{dE} \cdot E - w = 0 \Rightarrow \frac{dw^{IP}}{dE} = \frac{f'(E) - w}{E} = \frac{MP_E - w}{E}$$

Along the labor demand curve $MP_E = w$, implying that isoprofit curves are flat when they cross the labor demand curve. To the left of the labor demand curve $MP_E > w$ and so the isoprofit curve is upward sloping, and to the right of the labor demand curve $MP_E < w$ and the isoprofit curve is downward sloping.

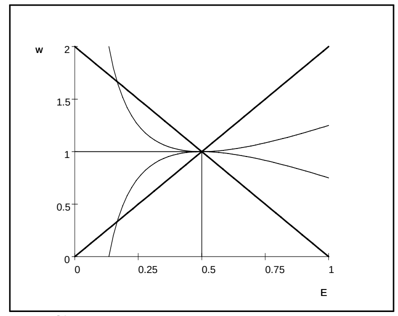
Example 1 *Consider the case where* $U = C - (1-L)^2$, $V = 0$, $f(E) = 2E - E^2$. *This results in a labor supply of* $h^S(w) = w/2$, *and indifference curves given by* $w^{IC}(h) = h + \bar{u}/h$ *with slope* $dw^{IC}/dh = 1 - \bar{u}/h^2$. *One can see that the indifference curve is flat along the labor supply function by first showing that* $\bar{u} = V(w) = U\left(wh^S(w), 1-h\right) = w^2/2 - w^2/4 = w^2/4$ *and so* $dw^{IC}/dh = 1 - \left(w^2/4\right)/\left(w^2/4\right) = 0$. *Labor demand is given by* $E^D = 1 - w/2$ *and isoprofit curves are given by* $w^{IP}(E) = 2 - E - \bar{\pi}/E$ *with slope* $dw^{IP}/dh = \bar{\pi}/E^2 - 1$. *The profit function here is* $\bar{\pi} = \pi(w) = f\left(E^D\right) - wE^D = 2(1 - w/2) - (1-w/2)^2 - w(1-w/2) = (1-w/2)^2$ *which can be used to show that the isoprofit curve is flat on the labor demand curve as* $dw^{IP}/dh = (1-w/2)^2/(1-w/2)^2 - 1 = 0$. *These are graphed below along with the competitive equilibrium where* $w^* = 1$ *and* $E^* = 1/2$ *(with just one worker so* $E = h$*). The efficiency of the competitive equilibrium is reflected in the tangency of the indifference and isoprofit curves at the equilibrium point.*



Labor Supply



Labor Demand



Competitive Equilibrium

2 Monopoly Union Bargaining

In the monopoly union model (aka the "right to manage" model, developed by Leontief in 1946) model the labor union first sets the wage w and the firm then sets the level of employment E. Since the firm will maximize profits it will simply set $VMP_E(E) = w$, so that E will lie on the firm's labor demand curve. The union foresees this behavior and therefore selects its preferred point along the firm's labor demand curve. If we pretend the union is just the worker so that $h = E$ its problem is then

$$\max_{E,w} U(wE, 1-E) \quad \text{st.} MP_E(E) = w$$

or substituting in the constraint $\max_E U(f'(E) \cdot E, 1-E)$, where we maximize with respect to E instead of w just because it is easier (conceptually it is equivalent since the union knows that setting w will determine E). Taking the FOC w.r.t E and using the fact that $f'(E) = w$

$$\frac{\partial U}{\partial C}\left(f''(E) \cdot E + f'(E)\right) - \frac{\partial U}{\partial L} = 0 \Rightarrow f''(E) = \frac{1}{E}\left(\frac{\partial U/\partial L}{\partial U/\partial C} - w\right) = \frac{MRS_{LC} - w}{E} = \frac{dw^{IC}}{dE}$$

As the inverse demand curve is given by $w^D(E) = f'(E)$, the slope of the inverse demand is $\frac{dw^D}{dE} = f''(E)$ and so this expression says that the demand curve and the indifference curve will be tangent at the resulting contract, call it $\left(w^{MU}, E^{MU}\right)$. However this means that the indifference curve will have a negative slope while the isoprofit curve has zero slope (because $MP_E(E) = w$, $dw^{IP}/dE = 0$) and so the two curves cross, implying an inefficiency. Workers would be willing to work more at a slightly lower wage and firms would make profits hiring them. However even if unions do function this way, that does not mean they are necessarily bad - workers are made better off (they could pick the competitive wage but do not), but these gains are smaller than the losses to firms and consumers. If the value of the redistribution to workers (suppose they are very needy) is considered more important than the loss to the other parties then the union may still be a "good" thing. However it would be better for everyone if the union and firm could find a more efficient way of bargaining. [1]

[1] This problem is very similar to the monopoly problem where a firm moves first and sets a price for a good and a buyer moves second by deciding how much of the good to buy. In this way the "revenue" corresponds to $wE = f'(E)E$ while the "cost" is the disutility of labor (measured in consumption) so that the "marginal cost" is given by the MRS_{LC}. The marginal revenue equals marginal cost equation is given by the condition

$$\text{"}MR\text{"} = f''(E)E - f'(E) = MRS_{LC} = \text{"}MC\text{"}$$

Interestingly, if the employer moves first the "monopsony" wage and employment will be chosen. The firm takes its constrain that employees will be on their supply curve, so $w = MRS_{LC}$. Writing this out as the inverse supply curve we get $w^S(E) = MRS_{LC}$, and differentiating $dw^S/dE = dMRS_{LC}/dE$. The firm maximizes profits

$$\pi = f(E) - wE = f(E) - MRS_{LC} \cdot E$$

3 Efficient Contracts

Another model of unions assumes that union and firm will bargain in a way which will leads to an efficient outcome (without mentioning any specifics). Conceptually, any Pareto efficient outcome between two parties can be found by guaranteeing some level of profits to the firm, $\bar{\pi}$ (how high will determine the exact solution) and maximizing the union's (i.e. worker's) utility.

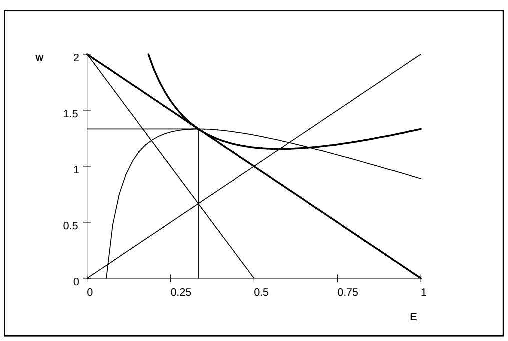
$$\max_{E,w} U\left(wE, 1-E\right) \qquad \text{st. } f\left(E\right) - wE = \bar{\pi}$$

Using the constraint to solve for $w = \left(f\left(E\right) - \bar{\pi}\right)/E$ and substituting into the original utility function this problem simplifies to finding $\max_E U\left(\left(f\left(E\right) - \bar{\pi}\right), 1-E\right)$. The first order condition is then

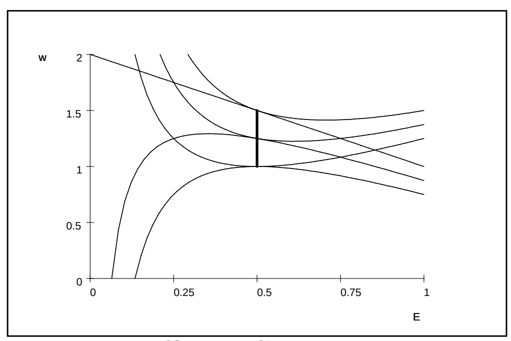
$$\frac{\partial U}{\partial C} \cdot f'\left(E\right) - \frac{\partial U}{\partial L} = 0 \Rightarrow f'\left(E\right) = \frac{\partial U/\partial L}{\partial U/\partial C} \Longleftrightarrow MP_E = MRS_{LC}$$

Subtracting w and dividing by E on both sides this implies that $\frac{VMP_E - w}{E} = \frac{MRS_{LC} - w}{E}$ or that the isoprofit and indifference curves are tangent. In general it cannot be resolved which (E, w) combination will be chosen as there are several points - the locus of all of these points is the **contract curve**. The amount of profit given to the firm $\bar{\pi}$ will determine the exact solution $E\left(\bar{\pi}\right)$ and $w\left(\bar{\pi}\right)$. Some information on profit and utility functions is necessary to determine whether the contract curve of the efficient contracts is downward or upward sloping, or vertical (the **strongly efficient** case).

Example 2 *A little calculation shows $MRS_{LC} = 2\left(1-L\right) = 2h$ and $MP_E = 2 - 2E$ and $f''\left(E\right) = -2$. The monopoly solution is found by setting $dw^{IC}/dE = f''\left(E\right)$ or $\left(2E - w\right)/E = -2$ where $w = MP_E = 2 - 2E$ which implies $2 - 4E = 2E$. Solving gives the solution is $w^{MU} = 4/3$ and $E^{MU} = 1/3$. The efficient solution is found by setting $MP_E = MRS_{LC} \Rightarrow 2 - 2E = 2E$ so $E^*\left(\bar{\pi}\right) = 1/2$. The efficient wage can be found from the constraint $\bar{\pi} = 2\left(1/2\right) - \left(1/2\right)^2 - w\left(1/2\right) \Rightarrow w^*\left(\bar{\pi}\right) = 3/2 - 2\bar{\pi}$. Assuming the firm can shutdown if it makes negative profits and that workers can find other jobs at the competitive wage, this solution can vary from $w = 1$ (the competitive case, when $\bar{\pi} = 1/4$) to $w = 3/2$ ($\bar{\pi} = 0$). Note that because $E^*\left(\bar{\pi}\right)$ does not depend on the profits, and hence the wage, the contract curve will be upward sloping and contracts are said to be "strongly efficient."*



Monopoly Union



Efficient Contracts

4 Rubinstein's Sequential Bargaining Model

This model considers the simple problem of how a union and a firm (e.g. the shareholders) can split a fixed amount of profits, which we set to one ($\bar{\pi} = 1$) for simplicity. More generally this can be thought of a

which gives an FOC which imples both a tangency condition between the isoprofit curve and labor supply curve as well as the interesection of MP_E and MC_E (the marginal cost of employment)

$$f'\left(E\right) - w - \frac{dMRS_{LC}}{dE}E = 0 \Rightarrow \frac{dw^{IP}}{dE} = \frac{f'\left(E\right) - w}{E} = \frac{dMRS_{LC}}{dE} \text{ and } MP_E = f'\left(E\right) = w + \frac{dw^S\left(E\right)}{E} = MC_E$$

"pie-splitting" model between 2 pie-lovers who want as much of the pie as possible (feel free to imagine other contexts). In the first round, player 1 proposes a split $(s_1, 1-s_1)$ with s_1 going to player 1 and $1-s_1$ to player 2. Player 2 then decides whether to accept or refuse this offer. If she accepts, bargaining stops each gets their share. If she refuses, bargaining continues for a second round with the player 2 getting to propose a new split $(s_2, 1-s_2)$, with share s_2 going to player 1, and $1-s_2$ going to player 2, except that now the whole pie is now only of size $\delta < 1$ (which can be thought of as the time-costs of bargaining)[2] so that each player gets only δs_2 or $\delta(1-s_2)$. Player 1 can then accept or refuse this offer.

4.1 Three Round Model

In this model bargaining can only proceed for 2 rounds as described above. If player 1 refuses the split proposed by player 2 in round 2, then each player gets a default "status-quo" share of the pie in round 3 $(\bar{s}, 1-\bar{s})$. However because two round have elapsed the pie is now of size δ^2, so each player gets only $\delta^2\bar{s}$ or $\delta^2(1-\bar{s})$.

The three round model has four decisions (1) player 1's split, s_1, (2) player 2's decision to accept or refuse, (3) player 2's split, s_2 and (4) player 1's decision to accept or refuse

which can be solved for by backward induction, starting with the last move first and going backwards.

(4) Player 1 in round 2 should clearly refuse if he gets more in round 3 than with player 2's split, so he refuses if $\delta s_2 < \delta^2\bar{s}$ or $s_2 < \delta\bar{s}$.

(3) Knowing what player 1 will refuse, player 2 in round 2, will propose a split which player 1 can accept, but maximizes his own slice of the pie so he proposes $s_2^* = \delta\bar{s}$, (we assume a player will accept if he is indifferent), and so player 2 gets $1-s_2^* = 1-\delta\bar{s}$. If bargaining proceeds this far then player 2 will propose this split and player 1 will accept it, resulting in payoffs $\left(\delta^2\bar{s}, \delta(1-\delta\bar{s})\right)$ which is just as good as the "status-quo" for player 1 and clearly better for player 2 as $\delta(1-\delta\bar{s}) = \delta - \delta^2\bar{s} > \delta^2 - \delta^2\bar{s} = \delta^2(1-\bar{s})$.

(2) In round 1, player 2 knows that he can get a payoff of $\delta(1-\delta\bar{s})$ if he refuses and therefore will not accept a split which will not give him at least as much. Therefore player 2 refuses if $1-s_1 < \delta(1-\delta\bar{s})$ or $s_1 > 1-\delta(1-\delta\bar{s})$.

(1) Knowing what player 2 will refuse, player 1 in round 1 will pick $s_1^* = 1-\delta\left(1-\bar{\delta}s\right)$ so player 2 gets $1-s_1^* = \delta(1-\delta\bar{s})$. Player 2 will accept this offer (he is indifferent) and so payoffs are given by $(1-\delta(1-\delta\bar{s}), \delta(1-\delta\bar{s}))$. This is the ultimate outcome of the game. Both players are better off relative to the "status-quo" as you can check for yourself.

4.2 Infinite Round Model

Now consider the situation where bargaining in round 3 repeats itself as in round 1 with player 1 proposing a split and player 2 getting a chance to accept or refuse. Each round this happens with the player 1 proposing in odd rounds and player 2 in even rounds, with the pie diminishing by a factor δ each round. Backward induction cannot be used to solve this model as there is no final round. However because the game repeats itself essentially every 2 round with the pie being δ^2 times smaller we can take a short-cut (not fully justified here). As in the three round model, bargaining will end in the first round with a split $s_1^*, 1-s_1^*$. Now imagine bargaining were to start (say "by accident") in the third round - since the game is essentially identically with just a smaller pie, the same split $s_1^*, 1-s_1^*$ would succeed in ending the bargaining. Therefore we can see s_1^*,$1-s_1^*$ to be the same as the "status-quo" split considered in the three round model, and so we can set $\bar{s} = s_1^*$. Using this condition with the three round solution we can find the solution for the infinite-round model

$$s_1^* = 1-\delta(1-\delta s_1^*) \Rightarrow s_1^* = \frac{1}{1+\delta},\ 1-s_1^* = \frac{\delta}{1+\delta}$$

From this model we can see that player 1 gets more of the pie the higher is the time-cost of bargaining. In fact player 1 gets the whole pie if $\delta = 0$. On the other-hand if the time-cost of bargaining is essentially zero, then as $\delta \to 1$ we get a 50-50 split of $s_1^* = 1/2 = 1-s_1^*$.

[2] For instance the time-cost of bargaining could be the loss in foregone interest in investing these profits, so $\delta = 1/(1+r)$ where r is the interest rate. It could also represent some other kind of loss, like the costs of a strike.