

Econ 250a  
 Problem Set 4

1. Consider a simple search model with no on-the-job search, no job destruction, and similar utility while working or searching. As shown in Lecture, the reservation wage  $w^*$  satisfies the equation:

$$w^* = b + \frac{\lambda}{r} \int_{w^*}^{\infty} (w - w^*) f(w) dw$$

Assume that the offered wage distribution is discrete,  $w \in \{w_1, \dots, w_n\}$  with d.f.:

$$\begin{aligned} P(w = w_1) &= \Phi\left(\frac{w_1 - \mu}{\sigma}\right), \\ P(w = w_2) &= \Phi\left(\frac{w_2 - \mu}{\sigma}\right) - \Phi\left(\frac{w_1 - \mu}{\sigma}\right) \\ &\dots \\ P(w = w_{n-1}) &= \Phi\left(\frac{w_{n-1} - \mu}{\sigma}\right) - \Phi\left(\frac{w_{n-2} - \mu}{\sigma}\right) \\ P(w = w_n) &= 1 - \Phi\left(\frac{w_{n-1} - \mu}{\sigma}\right), \end{aligned}$$

where  $\Phi$  is the standard normal d.f., and  $(\mu, \sigma)$  are parameters. Assume that time is measured in months, and that  $b = 1000$ ,  $r = 0.02$ . Set up a numerical procedure to find the optimal reservation wage, using a grid with  $n = 200$ , and setting  $w_1 = 0$  and  $w_n = 8000$ . Assuming  $\lambda = 0.1$  and  $\mu = 1800$ , find  $w^*$  for  $\sigma = 100, 500, 1000, 1500, 2000$ . Find the expected duration of job search  $d$  for each  $\sigma$ . Graph the relationships between  $\sigma$ ,  $w^*$  and  $d$ .

2. Consider the simplified model with on the job search described in Lecture 9, where wage offers are distributed on the interval  $[0, \bar{w}]$  according to a given d.f.  $F(w)$ , and the two value functions are:

$$U(w) = \frac{w - c}{r + \delta} + \frac{\delta}{r + \delta} V + \frac{\lambda(1 - \delta)}{r + \delta} \int_w^{\bar{w}} (U(\tilde{w}) - U(w)) f(\tilde{w}) d\tilde{w} \quad (1)$$

and:

$$V = \frac{b}{r} + \frac{\lambda(1 - \delta)}{r} \int_{w^*}^{\bar{w}} (U(\tilde{w}) - V) f(\tilde{w}) d\tilde{w} \quad (2)$$

where  $w^* = b + c$  is the reservation wage.

a) Find the derivative  $U'(w)$ . What is  $U'(\bar{w})$ ? What is  $U'(w^*)$ ? Draw a picture of  $U(w)$  and  $V$ .

b) Suppose  $b = 800$  (per month),  $c = 0$ ,  $\lambda = 0.4$ ,  $r = 0.02$ , and  $\delta = 0.25$ , and  $F(w) = N(1200, 400)$ . Assume that with this distribution  $\bar{w} = 2500$  (which is almost true). Develop a numerical procedure to solve for  $U(w)$  and  $V$ .

3. Read Card-Chetty-Weber, QJE 2008. The model is discrete time, with variable search intensity and endogenous asset accumulation. Each worker has a fixed wage  $w$ ; all jobs last indefinitely (no job destruction); and utility is additively separable in consumption and search effort. Some details:

discount rate =  $\delta$ ; interest rate =  $r$  (non-random)  
 flow utility =  $u(c_t) - \psi(s_t)$ ;  $c_t = \text{consumption}$ ;  $s_t = \text{search effort}$   
 Beginning-of-period: start with assets  $A_t$ , choose  $s_t$   
 If successful ( $\text{prob} = s_t$ ) start working, receive  $w$ ,  
 if not successful ( $\text{prob} = 1 - s_t$ ), receive benefit  $b$ ,  
 End-of-period: choose  $c_t^e$  or  $c_t^u$  depending on search outcome

Value function at the end of period  $t$  for an individual who finds a job:

$$V_t(A_t) = \max_{A_{t+1} \geq L} u(A_t - A_{t+1}/(1+r) + w_t) + \frac{1}{1+\delta} V_{t+1}(A_{t+1}),$$

Value function at end of period  $t$  for an individual who does not find a job:

$$U_t(A_t) = \max_{A_{t+1} \geq L} u(A_t - A_{t+1}/(1+r) + b) + \frac{1}{1+\delta} J_{t+1}(A_{t+1})$$

Beginning-of-period value function:  $J_t(A_t) = \max_{s_t} s_t V_t(A_t) + (1 - s_t) U_t(A_t) - \psi(s_t)$ .

a) Find the first order condition for optimal search intensity,  $s_t^*$ . Show that the marginal utility of effort depends on the gap  $V_t(A_t) - U_t(A_t)$ .

b) Using your answer in (a) and the derivatives of  $V_t$  and  $U_t$ , find  $\partial s_t^* / \partial A_t$ ,  $\partial s_t^* / \partial w_t$ , and  $\partial s_t^* / \partial b_t$  and discuss the conditions under which  $\partial s_t^* / \partial A_t = 0$ .

c) Suppose that benefits  $b$  are available indefinitely, that  $u(c_t) = \log c_t$ , that  $\psi(s_t) = a s_t^2$ , and that  $\delta = r$ . Suppose in addition that  $L = 0$  (i.e., assets have to be positive). Can you derive an algorithm to compute  $U_t(A_t)$  and the optimal search intensity choice  $s_t^*(A_t)$ ?

HINTS: find  $V_t(A_t) = V(A_t)$ . Now think about how to compute  $U_t(A_t) = U(A_t)$ .