5 Search With Bargained Prices

We now investigate how these two assumptions matter for the results, and in the process develop a baseline search and matching model for macroeconomic analysis. The model is basically that of Diamond (RES, 1982), Mortensen (AER, 1982), and Pissarides (AER, 1985). Pissarides (2004) gives a very accessible exposition. The results I present here are largely based on Hosios (RES, 1989), but the exposition is very different (much more standard here than in the original article). The importance of this model also stems from the fact that it is very closely related to the Mortensen-Pissarides model we will use to analyze unemployment fluctuations later in the class.

5.1 Environment and Preliminaries

The model is again continuous time, infinite horizon, and agents are risk neutral with discount rate r, i.e., maximizing

$$V(t) = \int_t^\infty e^{-rs} y(s)\, ds$$

where $y(t)$ is their net income at time t.

Let's assume for now that there is an exogenously given stock of buyers (firms) of measure N, who can employ workers productively. The productivity of each firm is determined as a draw from the distribution

$$x \sim F(x),$$

with support X, after the match between the firm and the worker (i.e., only ex post heterogeneity). This productivity remains constant throughout the life of the match.

All workers and firms are ex ante identical.

The population of workers, i.e., stock of sellers, is L. I also denote the number (measure) of unemployed workers by U, and the number of firms is N and the number of vacant firms looking for workers by V.

Frictions in the labor market are modeled by way of a matching function

$$M(U,V)$$

which determines the flow rate of matches per instant when the stock of unemployed workers is U and the stock of vacancies is V. We make a number of assumptions on this function:

1. Increasing: more vacancies and more unemployed workers result in more matches $M_U, M_V > 0$ (this could be modified to ≥ 0)

2. Externalities:
$$\frac{\partial \frac{M(U,V)}{U}}{\partial V} > 0, \frac{\partial \frac{M(U,V)}{U}}{\partial U} < 0, \frac{\partial \frac{M(U,V)}{V}}{\partial V} < 0, \text{ and } \frac{\partial \frac{M(U,V)}{V}}{\partial U} > 0,$$
meaning that when there are more vacancies, the matching probability of a given unemployed worker increases, and the matching probability of a given unfilled vacancy decreases, and similarly in response to an increase in unemployment.

One important point here is that although there are frictions, there is a sense in which the market is "regular" meaning that when there are more vacancies demanding matches, it's easier for unemployed workers to find matches and vice versa.

The second assumption clarifies that there are both negative and positive externalities in this world.

Digression on pecuniary and non-pecuniary externalities: why do I refer to those in (2) as externalities? Is it an externality if I demand one more apple? What is the difference between me demanding one more apple and a firm demanding one more worker by posting a vacancy?

A couple of other points are useful to note:

- If $M(U,V) \to \infty$, matching frictions are disappearing, so it will be interesting to investigate whether we approach a competitive model (some care needs to be taken here; frictionless matching corresponds to the case where at an instant $\min\{U,V\}$ jobs are created; as a flow rate this corresponds to $M(U,V) \to \infty$).

- A natural benchmark, which seems to be consistent with the data both in the US and the UK is that $M(U,V)$ exhibits constant returns to scale, so when there is a doubling in the number of unemployed workers and vacancies, the total number of matches within a given period also doubles (and matching probabilities remain unchanged). I do not impose constant returns to scale yet, but this will play an important role below.

Finally, let us assume that there are exogenous separations once a worker and the firm come together at the flow rate s.

The cost of a vacancy is γ, and when unemployed, workers receive income or benefit from leisure equal to b.

Let ask focus on steady-state equilibria here (non-steady-state equilibria are for the homework).

A steady state equilibrium in this economy will specify a wage rate function

$$w : X \to \mathbb{R}_+$$

assigning a wage for every level of productivity, and a pair of acceptance functions for the worker and the firm

$$\begin{aligned} a_w &: X \to [0,1] \\ a_f &: X \to [0,1] \end{aligned}$$

assigning a probability of accepting a job of productivity x after this productivity is realized following the match. We will see that the decision of the worker and the firm can be collapsed into a single function without loss of any generality below, so we can work with the simpler setup where there is only one function

$$a : X \to [0,1]$$

to be determined.

I still need to specify the wage determination process, which will be done below, but for now note that wages will be "bargained" between the worker and the firm.

Let us define

$$q = \frac{M(U,V)}{V}$$

as the matching probability for an unfilled vacancy, and

$$p = \frac{M(U,V)}{U},$$

as the matching probability for an unemployed worker.

Imposing steady states, the asset value of a filled job of productivity x can be written as

$$rJ^F(x) = x - w(x) + s\left(J^V - J^F(x)\right), \tag{28}$$

where J^V is the value of an unfilled vacancy, and x is the output of the match. Now an immediate application of Theorem 2 implies that $J^F(x)$ is strictly monotonically increasing in x as long as $x - w(x)$ is increasing in x (i.e., when differentiable, $w'(x) < 1$ everywhere)—can you see why this will be true? Think of wage bargaining or see below. This implies that for the firm a reservation productivity rule will be optimal. One complication is that some matches may be acceptable to the firm, but not to the worker. We will see that this will not be the case, but this may need to be borne in mind.

Now the value of an unfilled vacancy is

$$rJ^V = -\gamma + q\int_X \max\left\{\left(J^F(x) - J^V\right); 0\right\} dF(x), \tag{29}$$

which imposes the decision that the firm will only accept a match if this yields a higher value $J^F(x)$ for it. This expression ignores the decision of the worker. More generally we should write

$$rJ^V = -\gamma + q \int_X \max\left\{a_w(x)\left(J^F(x) - J^V\right); 0\right\} dF(x),$$

which means that the firm can only choose to create the match if $a_w(x) = 1$, i.e., if the worker also want to create the match. Let us ignore this for now and work with (29)—we will see below, why it is okay to ignore this, and also why even if this were not the case, the analysis would not be much more complicated.

From the monotonicity of $J^F(x)$, (29) can be simplified to

$$rJ^V = -\gamma + q \int_{x^*}^{\infty} \left(J^F(x) - J^V\right) dF(x) \tag{30}$$

where x^* is the reservation productivity of the firm.

The value functions for the workers are similar

$$rJ^E(x) = w(x) + s\left(J^U - J^E(x)\right),$$

Now presuming that $w(x)$ is strictly increasing in x (again see below), Theorem 2 immediately implies that this is strictly increasing in x, so a reservation productivity rule will also be optimal for the worker in deciding whether to accept a job or not.

Imposing that this is the same threshold for the worker as for the firm, we can then write

$$rJ^U = b + p \int_{x^*}^{\infty} (J^E(x) - J^U) dF(x). \tag{31}$$

Naturally, both equations (30) and (31) will be valid even if the worker and the firm use different reservation productivity rules, with x^* corresponding to the maximum of these two thresholds. (Can you see why? Think of the case in which workers and firms use two different cutoff levels x^*_w and x^*_f; what would happen then?)

Now given this, we can write the law motion of the number of vacancies in unemployment as

$$\begin{aligned} \dot{U} &= s\,(L-U) - p\,(1-F(x^*))\,U \\ \dot{V} &= s\,(N-V) - q\,(1-F(x^*))\,V \end{aligned}$$

How are wages determined? Nash Bargaining.

Why do we need bargaining? Because of bilateral monopoly, or much more specifically: match-specific surplus (or as is sometimes called *quasi-rents*).

Think of a competitive labor market, at the margin the firm is indifferent between employing the marginal worker or not, and the worker is indifferent between supplying the marginal hour or not (or working for this firm or another firm). We can make both parties indifferent at the same time—no match-specific surplus.

In a frictional labor market, if we choose the wage such that $J^E(x) = 0$, we will typically have $J^F(x) > 0$ and vice versa. There is some surplus to be shared.

Nash solution to bargaining is a natural benchmark.

5.2 Digression: Nash's Solution to Bargaining

Nash's bargaining theorem considers the bargaining problem of choosing a point x from a set $X \subset \mathbb{R}^N$ for some $N \geq 1$ by two parties with utility functions $u_1(x)$ and $u_2(x)$, such that if they cannot agree, they will obtain respectively d_1 and d_2. The theorem is that if we impose the following four conditions: (1) $u_1(x)$ and $u_2(x)$ are Von Neumann-Morgenstern utility functions, i.e., concave, increasing and unique up to positive linear transformations; (2) Pareto optimality, the agreement point will be along the frontier; (3) Independence of Irrelevant Alternatives: suppose $X' \subset X$ and the choice when bargaining over the set X is $x' \in X'$, then x' is also the solution when bargaining over X'; (4) Symmetry: identities of the players do not matter, only their utility functions; then there is a unique bargaining solution which is

$$x^{NS} = \arg\max_{x \in X} \left(u_1(x) - d_1\right)\left(u_2(x) - d_2\right)$$

If we relax the symmetry assumption, so that the identities of the players can matter (e.g., worker versus firm have different "bargaining powers"), then we obtain:

$$x^{NS} = \arg\max_{x \in X} \left(u_1(x) - d_1\right)^{\beta}\left(u_2(x) - d_2\right)^{1-\beta} \tag{32}$$

where $\beta \in [0,1]$ is the bargaining power of player 1.

Next note that if both utility functions are linear and defined over their share of some pie, and the set $X \subset \mathbb{R}^2$ is given by $x_1 + x_2 \leq 1$, then the solution to (32) is given by

$$x_2 = (1-\beta)(1-d_1-d_2)+d_2$$

or

$$(1-\beta)(1-x_2-d_1) = \beta(x_2-d_2)$$

and $x_1 = 1-x_2$.

With Nash bargaining in place, we can already answer the question of why both firms and workers are using the same threshold x^*. Recall that $w(x)$ resulting from Nash bargaining satisfies "Pareto optimality". Suppose that there exists some x' such that $w(x')$ is the equilibrium wage, and $J^E(x') < J^U$ and $J^F(x') > J^V$. In that case, if $J^E(x') + J^F(x') > J^U + J^V$, we can find some $\tilde{w}(x') > w(x')$ such that both $J^E(x')$ and $J^F(x')$ are positive. Thus the two parties can agree to produce together, improving their welfare relative to disagreement, conflicting the presumption that $w(x')$ was part of an equilibrium (it would not have been Pareto optimal). Conversely, if $J^E(x') + J^F(x') < J^U + J^V$, then there should in fact be a separation at x'.

5.3 Back to the Model

Put differently, bargaining is going to ensure that separations are mutually beneficial, and thus x^* will be such that $J^E(x^*) + J^F(x^*) = J^U + J^V$.

Applied to our setting, let's assume that the worker has bargaining power β. Then, the Nash bargaining solution implies that the wage function $w(x)$

will be a solution to the equation:

$$\beta\left(J^F(x) - J^V\right) = (1-\beta)\left(J^E(x) - J^U\right) \tag{33}$$

Now using the value functions, we obtain

$$\begin{aligned} J^E(x) - J^U &= \frac{w(x) - rJ^U}{r+s} \\ J^F(x) - J^V &= \frac{x - w(x) - rJ^V}{r+s} \end{aligned} \tag{34}$$

then substituting into (33), we have

$$\begin{aligned} w(x) &= \beta\left(x - rJ^U - rJ^V\right) + rJ^U \\ &= \beta x + (1-\beta)rJ^U - \beta rJ^V \end{aligned} \tag{35}$$

This wage equation is very intuitive. The worker receives a fraction β of total surplus of the flow value of match, $x - rJ^U - rJ^V$, plus his outside option (more appropriately disagreement point), rJ^U.

Digression: when is the Nash solution the equilibrium of a well-specified bargaining game?

In addition, we have

$$rJ^U = b + p\int_{x^*}^{\infty}\left[\frac{w(x) - rJ^U}{r+s}\right]dF(x)$$

Now substituting for $w(x)$ from (35), gives

$$rJ^U = b + \frac{p\beta\bar{x}}{r+s} - \frac{p\beta\phi^*}{r+s}\left(rJ^U + rJ^V\right)$$

where

$$\bar{x} \equiv \int_{x^*}^{\infty} xdF(x)$$

Note that this is *not* the expectation of x conditional on $x \geq x^*$. That conditional expectation would be $E[x \mid x \geq x^*] = \int_{x^*}^{\infty} x dF(x) / [1 - F(x^*)]$, so $\bar{x}$ is the conditional expectation times the probability that x is indeed greater than x^*.

Moreover, let that probability be denoted by

$$\phi^* \equiv 1 - F(x^*).$$

Similarly

$$rJ^V = -\gamma + \frac{q(1-\beta)\bar{x}}{r+s} - \frac{q(1-\beta)\phi^*(rJ^U + rJ^V)}{r+s}$$

This implies that the sum of the disagreement points for a firm and a worker is:

$$\Rightarrow rJ^U + rJ^V = \frac{(r+s)(b-\gamma) + q(1-\beta)\bar{x} + p\beta\bar{x}}{r+s+q(1-\beta)\phi^* + p\beta\phi^*} \tag{36}$$

The preceding argument already establishes that in equilibrium

$$x^* = rJ^U + rJ^V$$

(to derive this equation, can use (34) together with the fact that $J^U + J^V = J^E(x^*) + J^F(x^*)$; alternatively, use $w(x^*) = rJ^U$ and $x^* - w(x^*) = rJ^V$)

Combining this with (36), we have

$$x^* = \frac{q(1-\beta)\bar{x} + p\beta\bar{x} + (r+s)(b-\gamma)}{r+s+q(1-\beta)\phi^* + p\beta\phi^*} \tag{37}$$

This equation defines x^* implicitly. (Recall that $\bar{x} \equiv \int_{x^*}^{\infty} x dF(x)$, $\phi^* \equiv 1 - F(x^*)$).

In addition, the equation (37) may be rewritten as follows:

$$
\begin{aligned}
(r+s)\left(x^{*}-(b-\gamma)\right) &= \left(q(1-\beta)+p\beta\right)\left(\bar{x}-\phi^{*}x^{*}\right) \\
&= \left(q(1-\beta)+p\beta\right)\int_{x^{*}}^{\infty}(x-x^{*})dF(x)
\end{aligned}
$$

From the point of view of an individual worker or firm, who takes q and p as given, this equation characterises a unique value for x^{*}. The left hand side is increasing in x^{*}. Holding q and p constant

$$
\frac{dRHS}{dx^{*}} = -\left(q(1-\beta)+p\beta\right)\left(1-F(x^{*})\right) < 0
$$

which establishes the result. The result means that all workers and firms in the economy deduce the same threshold x^{*} from observing U and V. For the economy as a whole, changing the value of x^{*} will cause changes in q and p. Characterizing uniqueness in this context is more demanding. In particular, as we will see below, steady-state unemployment is

$$
U = \frac{sL}{s+p\left(1-F\left(x^{*}\right)\right)}
$$

and steady-state vacancy level is

$$
V = \frac{sN}{s+q\left(1-F\left(x^{*}\right)\right)}
$$

Inverting these equations, we have

$$
p = \frac{s\left(L-U\right)}{U\left(1-F\left(x^{*}\right)\right)} = \frac{M\left(U,V\right)}{U} \text{ and } q = \frac{s\left(N-V\right)}{V\left(1-F\left(x^{*}\right)\right)} = \frac{M\left(U,V\right)}{V},
$$

which jointly solve for U and V as functions of x^{*}, and thus pin down p and q as functions of x^{*}. If we make further assumptions on $M\left(U,V\right)$, we can derive conditions under which x^* will be uniquely determined in general equilibrium. But I will not pursue this here.

Now continuing with the analysis, we also have

$$\begin{aligned} rJ^U &= b + \frac{\beta p\bar{x} - (b-\gamma)\beta p\phi^*}{r+s+q(1-\beta)\phi^* + p\beta\phi^*} \\ rJ^V &= -\gamma + \frac{(1-\beta)q\bar{x} - (b-\gamma)(1-\beta)q\phi^*}{r+s+q(1-\beta)\phi^* + p\beta\phi^*} \end{aligned} \tag{38}$$

which completes the description of the equilibrium (recall that $\bar{x} \equiv \int_{x^*}^{\infty} xdF(x)$).

We can also calculate the number of unemployed workers (or the unemployment rate in this economy) now. Notice that we could characterize x^* without worrying about unemployment. This is a common feature of many search models, sometimes referred to as "block recursiveness".

The unemployment evolution is given by

$$\dot{U} = s(L-U) - p(1-F(x^*))U \tag{39}$$

where the first term is separations from existing jobs, of which there are $L-U$, and the second term is job creation, which happens at the flow rate $p(1-F(x^*))$. Thus in steady-state

$$U = \frac{sL}{s+p(1-F(x^*))},$$

or defining the unemployment rate as $u = U/L$,

$$u = \frac{s}{s+p(1-F(x^*))}.$$

It is natural to look towards comparative statics now.

First, consider an increase in b, the level of utility or benefits in unemployment. For given q and p, a higher b would increase x^* from (37), reducing the probability of job creation conditional on a match, and increasing unemployment. However, the effect of b on overall unemployment also needs to take into account the changes in q and p. Can you derive this effect? Can you derive the effect of γ on unemployment?

Also note that an increase in β, the bargaining power of the workers, and an increase in r, the discount rate, have ambiguous effects. Why?

Now consider an extended model where new agents can enter at the per period cost c_W for workers and c_F for firms, and the initial stocks of workers and firms is small enough (why is this caveat necessary?). What is the equilibrium of this extended model? It is straightforward to see that as long as $c_W < rJ^U$, there will be entry. Similarly for firms. Then in equilibrium we also have

$$c_W = rJ^U; \quad c_F = rJ^V.$$

In fact, the standard Mortensen-Pissarides search model, which will be analyzed in greater detail later in the class, is one where L is constant, $c_W = \infty$, $N = 0$ and $c_F = 0$. Now in this case, we have another equilibrium condition, given by

$$rJ^V = -\gamma + \frac{(1-\beta)q\bar{x} - (b-\gamma)(1-\beta)q\phi^*}{r + s + q(1-\beta)\phi^* + p\beta\phi^*} = 0.$$

Calculating the comparative statics in this model is not as easy as it seems. This is because when the matching function has decreasing or increasing returns to scale, there can be difficulties in establishing comparative statics. We will do much more of these comparative statics when we look at the Mortensen-Pissarides model in the context of understanding unemployment fluctuations later, but these models will impose constant returns to scale matching. Here intuitively, we expect

$$\frac{\partial U}{\partial b} > 0, \ \frac{\partial U}{\partial \gamma} > 0, \ \frac{\partial U}{\partial \beta} > 0, \text{ and } \frac{\partial U}{\partial r} > 0,$$

but for now, you will be asked to derive these results only in the case with constant returns to scale matching and free entry in the homework (you can think about the intuition more generally, however, if you want).

Now we can investigate what happens to equilibrium as the amount of frictions are diminished, i.e., $M(U,V) \to \infty$. For this case, seem that N and W are constant (i.e., no free entry). This implies that $p, q \to \infty$, and $p/q \to P$ finite. In this case, also assumed that there is bounded support on x, in particular, let the upper bound be x^{sup}. Taking limits, we have from (37)

$$x^* \to \frac{(1-\beta) + P\beta}{(1-\beta) + P\beta} \frac{\int_{x^*}^{x^{\text{sup}}} x dF(x)}{1 - F(x^*)} = \frac{\int_{x^*}^{x^{\text{sup}}} x dF(x)}{1 - F(x^*)} = E\left[x \mid x \geq x^{\text{sup}}\right]$$

which is only possible if $x^* = x^{\text{sup}}$ (why?). Thus exactly as in a competitive equilibrium, only the most productive jobs are active in equilibrium (homework question: what happens to wages in the limit?).

How does this equilibrium compare to the "second-best", that is the solution to the planner's problem where the constraints are the same as those imposed on the decentralized economy. Therefore, the planner's problem is to maximize output subject to search constraints.

The following current value Hamiltonian describes the problem of the planner. To simplify, I have already imposed the cutoff rule that all jobs above some $\tilde{x}^*$ will be active (otherwise, the program has to follow each x and choose $a(x)$ again as the probability of a match conditional on productivity x).

To write this Hamiltonian, reason as follows. The planner creates $M(U,V)$ matches at every instant when there are U unemployed workers and V vacancies. Only a fraction $1-F(\tilde{x}^*)$ of the matches are turned into jobs. Each job is worth on average $E[x \mid x \geq \tilde{x}^*]$ conditional on being created. Finally, a job of productivity x has a discounted net present value equal to $x/(r+s)$, because of discounting and potential future separations. Thus the net return to the planner during an instant can be written as

$$\frac{E[x \mid x \geq \tilde{x}^*]}{r+s}(1-F(\tilde{x}^*))M(U,V) + bU - \gamma V$$

where the last two terms are the net income flows from unemployed workers and vacant firms. This is simply equal to

$$\frac{\int_{\tilde{x}^*}^{\infty} x dF(x)}{r+s} M(U,V) + bU - \gamma V$$

Thus the objective function of the planner is to maximize

$$\int_0^{\infty} e^{-rt}\left[\frac{\int_{\tilde{x}^*}^{\infty} x dF(x)}{r+s} M(U,V) + bU - \gamma V\right]$$

where I suppressed time dependence to save on notation.

Now adding the constraints with corresponding multipliers, the Hamiltonian is

$$\begin{aligned} H &= \left[\frac{\int_{\tilde{x}^*}^{\infty} x dF(x)}{r+s} M(U,V) + bU - \gamma V\right] \\ &\quad + \lambda \left[s(L-U) - (1-F(\tilde{x}^*))M(U,V)\right] \qquad \text{search constraint} \\ &\quad + \mu \left[N - V - L + U\right] \qquad \text{adding up constraint.} \end{aligned}$$

Here the control variables are $\tilde{x}^*, V$, and the stock variable is U (recall the constraint (39)).

In addition, the multipliers are:

λ: social value of one more match.

μ: social value of one more vacancy. (Why? Why not the value of one more worker?)

This is a standard optimal control problem, with necessary conditions

$$\begin{aligned} \frac{\partial H}{\partial \tilde{x}^*} &= 0 \\ \frac{\partial H}{\partial U} &= r\lambda - \dot{\lambda} \\ \frac{\partial H}{\partial V} &= 0 \end{aligned}$$

As in the equilibrium, let us focus on steady states: $\dot{\lambda} = 0$.

The first-order conditions are:

With respect to $\tilde{x}^*$

$$\left(-\frac{\tilde{x}^* f(\tilde{x}^*)}{r+s}\right) M(U,V) + \lambda f(\tilde{x}^*) M(U,V) = 0.$$

Or rearranging:

$$\tilde{x}^* = (r+s)\lambda \tag{40}$$

Thus the cutoff threshold has to be proportional to the shadow value of one more unemployed worker appropriately discounted. What is the intuition?

With respect to U :

$$\frac{\int_{\tilde{x}^*}^{\infty} xdF(x)}{r+s} M_U + b - \lambda(1 - F(\tilde{x}^*))M_U - \lambda(r+s) + \mu = 0 \tag{41}$$

Finally, with respect to V :

$$\frac{\int_{\tilde{x}^*}^{\infty} xdF(x)}{r+s} M_V - \gamma - \lambda(1 - F(\tilde{x}^*))M_V - \mu = 0 \tag{42}$$

Now adding (41) and (42) to eliminate μ, we obtain:

$$\frac{\int_{\tilde{x}^*}^{\infty} xdF(x)}{r+s}(M_U + M_V) + b - \gamma - \lambda(1 - F(\tilde{x}^*))(M_U + M_V) - \lambda(r+s) = 0$$

or rearranging to solve for λ,

$$\lambda = \frac{\left(\frac{\int_{\tilde{x}^*}^{\infty} xdF(x)}{r+s}\right)(M_U + M_V) + b - \gamma}{r + s + \tilde{\phi}^* M_U + \tilde{\phi}^* M_V} \tag{43}$$

where I have defined

$$\tilde{\phi}^* \equiv 1 - F(\tilde{x}^*)$$

For future reference, we also have

$$\mu = -\gamma + \frac{\left(\int_{\tilde{x}^*}^{\infty} xdF(x)\right) M_V - (b - \gamma)\tilde{\phi}^* M_V}{r + s + \tilde{\phi}^* M_U + \tilde{\phi}^* M_V} \tag{44}$$

Now combining this with (40), we have

$$\tilde{x}^* = \frac{\left(\int_{\tilde{x}^*}^{\infty} xdF(x)\right)(M_U + M_V) + (r+s)(b - \gamma)}{r + s + \tilde{\phi}^* M_U + \tilde{\phi}^* M_V} \tag{45}$$

When will the decentralized allocation be efficient?

In the model without entry, we only need $x^* = \tilde{x}^*$ where these two thresholds are given by (37) and (45). (Why?)

In the model with entry, we need two more conditions to ensure optimal entry. To see what these are, note that the planner will add unemployed workers in vacancies up to the point where

$$\frac{dH}{dL} = c_W \text{ and } \frac{dH}{dN} = c_F$$

Thus for full constrained efficiency, we need the following three conditions:

(a)

$$x^* = \tilde{x}^*$$

(b)

$$rJ^U = \frac{dH}{dL} = s\lambda - \mu$$

(c)

$$rJ^V = \frac{dH}{dN} = \mu$$

Now comparing dH/dL and dH/dN as implied from (43) and (44) with (38) and recalling that $p \equiv M/U, \quad q \equiv M/V$, we obtain the following simple conditions for the equilibrium to coincide with the constrained efficient allocation.

(a)

$$M_U + M_V = \beta\frac{M}{U} + (1-\beta)\frac{M}{V}$$

(b)

$$\frac{M_U\left(\int_{\tilde{x}^*}^{\infty} x dF(x)\right)}{r+s+\tilde{\phi}^* M_U+\tilde{\phi}^* M_V}=\frac{\beta p \bar{x}}{r+s+\phi^*(1-\beta)q+\phi^*\beta p}$$

(c)

$$\frac{M_V\left(\int_{\tilde{x}^*}^{\infty} x dF(x)\right)}{r+s+\tilde{\phi}^* M_U+\tilde{\phi}^* M_V}=\frac{(1-\beta) q \bar{x}}{r+s+\phi^*(1-\beta)q+\phi\beta p}$$

First, suppose $M(U,V)$ exhibits increasing returns to scale or decreasing returns to scale. Then (b) + (c) are jointly impossible. Why? Part of the homework exercise...

Next, suppose that $M(U,V)$ exhibits constant returns to scale. Then (a), (b), (c) all hold true if and only if

$$\beta=\frac{M_U\cdot U}{M}\qquad\left(\text{or}\quad 1-\beta=\frac{M_V\cdot V}{M}\right)$$

(This is not obvious, you need to play with the equations to convince yourself).

This is the famous *Hosios condition*. It requires the bargaining power of a factor to be equal to the elasticity of the matching function with respect to the corresponding factor.

What is the intuition?

It is not easy to give an intuition for this result, but here is an attempt: as a planner you would like to increase the number of vacancies to the point where the marginal benefit in terms of additional matches is equal to the cost. In equilibrium, vacancies enter until the marginal benefits in terms of their bargained returns is equal to the cost. So if β is too high, they are getting too small a fraction of the return, and they will not enter enough. If β is too high, then they are getting too much of the surplus, so there will be excess entry. The right value of β turns out to be the one that is equal to the elasticity of the matching function with respect to unemployment (thus $1-\beta$ is equal to the elasticity of the matching function with respect to vacancies, by constant returns to scale).

The acceptance externalities are then easy to understand, since turning down a job is just like entering this economy by paying some cost.

[Important observation: Job Acceptance externalities ((a)) are easier to internalize than entry externalities ((b) + (c)). Why?]

Does the Hosios result imply that the decentralized equilibrium is going to be efficient? Possible, but unlikely unless the planner chooses β.

Other important observations:

- No Scale Effects, unless the matching technology is Increasing Returns to Scale.
- Inefficiencies look more like distorted prices (very neo-classical).

6 Frictions and Investment

In the above model, there are investment-like activities; workers and firms decide to enter before the matching stage. Nevertheless, these are limited to the extensive margin, and somewhat miraculously, the Hosios condition