Economics 250a
Search Theory 3 - Search and Matching

References
Daron Acemoglu - extract of book, "search-notes.pdf"
Christopher Flinn (2006). "Minimum Wage Effects on Labor Market Outcomes Under Search, Matching and Endogenous Contract Rates" 74 (July 2006), pp. 1013-1062.

Flinn's search-matching model with a minimum wage
Notation:
-θ = value of the match, d.f. $G(\theta)$; otherwise all workers and firms homogenous
-ρ = discount rate
-η = job destruction rate
-b = flow utility while searching
-λ = arrival rate of offers – will be endogenized later
-no on the job search.

Start by assuming no minimum wage:
-firm profit if employing a worker with match θ at wage $w = \theta - w$, value to firm $= \frac{\theta - w}{\rho + \eta}$.
-value functions for worker $V_n, V_e(w)$ if searching, unemployed
-reservation wage w^* will satisfy $V_n = V_e(w^*)$
-Bellman equations for a worker:

$$\begin{aligned} \rho V_n &= b + \lambda \int_{w^*} (V_e(w) - V_n) f(w) dw \\ (\rho + \eta) V_e(w) &= w + \eta V_n \end{aligned}$$

Where $f(w)$ is the density of wages. Some manipulations establish:

$$\begin{aligned} \rho V_n &= w^* \\ V_e(w) - V_n &= \frac{w}{\rho + \eta} - \frac{\rho}{\rho + \eta} V_n = \frac{w - w^*}{\rho + \eta} \end{aligned}$$

What happens when a searching worker meets a firm and the value of the match is θ? Assume they conduct Nash bargaining: choose a wage to maximize

$$\begin{aligned} & (V_e(w) - V_n)^{\alpha} \left(\frac{\theta - w}{\rho + \eta} \right)^{1-\alpha} \\ = & \left(\frac{w - w^*}{\rho + \eta} \right)^{\alpha} \left(\frac{\theta - w}{\rho + \eta} \right)^{1-\alpha} \end{aligned}$$

which leads to a "split the rents" model with worker share α :

$$w = w^* + \alpha(\theta - w^*)$$

The lowest match that will ever be considered is $\theta = \theta^* = w^* = \rho V_n$. So we can also write the wage when the match value is θ as:

$$w = w^* + \alpha(\theta - w^*).$$

Now from the relations above:

$$V_e(w) - V_n = \frac{w - w^*}{\rho + \eta} = \frac{\alpha(\theta - w^*)}{\rho + \eta}$$

Finally we can rewrite the expression for V_n :

$$\begin{aligned} w^* &= \rho V_n = b + \lambda \int_{w^*} (V_e(w) - V_n) f(w) dw \\ &= b + \frac{\lambda \alpha}{\rho + \eta} \int_{w^*} (\theta - w^*) dG(\theta) \end{aligned}$$

which can be solved for w^*, given $b, \lambda, \alpha, \rho, \eta$ and $G(\theta)$. Notice that the expression under the integral is like an option value, expressing the "option value" to the worker of matches that pay better than w^*. If we define

$$O(w^*) = \int_{w^*} (\theta - w^*) dG(\theta)$$

then $O(w^*)$ is a decreasing, positive function, with $O(0) = E[\theta]$, and $O(\theta^{\max}) = 0$. Graphically the determination of w^*, is shown in Figure 1. Note that higher values of b, α, λ all lead to higher values of w^*.

Endogenizing λ

We want to make λ endogenous. To do that, we will introduce 3 things: (1) vacancies; (2) the "matching function"; (3) the vacancy creation function. Let

- L = labor force (exogenous)
-U = total number (mass) of unemployed
- V = total number (mass) of vacancies
- $k = U/V$ = ratio of searchers to job openings
- $m(U, V)$ = "matching function" = flow rate of new matches.
- Standard CRTS. assumption on $m(.)$:

$$m(U, V) = Vq(U/V) = Vq(k)$$

for some increasing function $q(.)$. With this assumption we get the arrival rate of offers (to unemployed workers who are searching) is:

$$\lambda = \frac{m(U, V)}{U} = \frac{q(k)}{k}$$

and the "job filling" rate is:

$$\frac{m(U, V)}{V} = q(k).$$

In the S/M literature, $k = U/V$ is a measure of labor market "tightness". Let $u = U/L$ represent the unemployment rate, and let $v = V/L$ represent the number of vacancies per person in the labor force. Then

$$k = \frac{U}{V} = \frac{u}{v}$$

In a steady state, job destruction = job creation:

$$L(1-u)\eta = Lu\lambda(1 - G(w^*)$$

which implies

$$u = \frac{\eta}{\eta + \lambda} = \frac{\eta}{\eta + \frac{q(k)}{k}(1 - G(w^*)}$$

which is the "Beveridge curve": a downward-sloping relation between u and v that represents the balance between job creation and job destruction. For intuition think of the case where

$$m(U,V) = \mu U^{1/2} V^{1/2} \Longrightarrow q(k) = \mu k^{1/2}$$

What determines V? Assume firms can create a vacancy for cost ψ. The expected value of a vacancy is

$$\rho V_v = -\psi + q(k)(1 - G(m))(J - V_v)$$

where J = the expected profits of a consumated match. If we assume $V_v = 0$ (vacancies are created until the net profit is 0), we get

$$\psi = q(k)(1 - G(m))J$$

Now we have to determine J. For any given match the firm's expected discounted profit is $J(\theta)$,where

$$(\rho + \eta)J(\theta) = (\theta - w(\theta)) + \eta V_v.$$

With $V_v = 0$, we get

$$\begin{aligned} J(\theta) &= \frac{\theta - w(\theta)}{\rho + \eta}, \theta \geq w^* \\ &= (1-\alpha)\frac{\theta - w^*}{\rho + \eta}, \theta \geq w^* \end{aligned}$$

and $J(\theta) = 0$ for $\theta < w^*$, since in that case any potential wage is below the worker's reservation wage. Thus:

$$J = \frac{1-\alpha}{\rho + \eta} \int_{w^*} (\theta - w^*) \frac{dG(\theta)}{1 - G(w^*)}$$

and combining this with the vacancy creation equation, we get:

$$\begin{aligned}
\psi &= q(k)(1-G(m))J \\
&= q(k)\frac{1-\alpha}{\rho+\eta}\int_{w^*}(\theta-w^*)dG(\theta)
\end{aligned}$$

Notice that for a given cost of vacany creation and a given w^* this equation determines k. If ψ is bigger, k will have to be bigger to ensure a faster rate of vacancy filling and offset the higher cost of creating a vacancy. Likewise if w^* is higher k will have to be higher. We have a simple 2-equation equilibrium model:

$$\begin{aligned}
w^* &= b+\frac{q(k)}{k}\frac{\alpha}{\rho+\eta}O(w^*) \\
\psi &= q(k)\frac{1-\alpha}{\rho+\eta}O(w^*)
\end{aligned}$$

which determines w^* and k as functions of b,ψ,α and the "option value", which depends on $G(\theta)$.

Add a minimum wage

With a minimum wage m the worker's value of search is $V_n(m)$, which we will have to solve for. As before assume there is a rent-splitting wage process. Ignoring the minimum wage, the wage when the value of the match is θ would be:

$$w=\alpha\theta+(1-\alpha)\rho V_n(m).$$

Define $\widehat{\theta}$ as the value such that

$$m=\alpha\widehat{\theta}+(1-\alpha)\rho V_n(m)$$

For $\theta>\widehat{\theta}$ the minimum wage is not a problem. But for a range of lower values the minimum is binding. Assuming $\widehat{\theta}>m$ there is a range of $\theta's$ that are "efficient" (ie, have match value at least as big as the minimum) but under the "ordinary" wage model would be paid less than the minimum. Flinn assumes these matches are consumated and the wage is set to m, generating a "spike" at the minimum wage. Very nice idea!

The value of unemployment is now:

$$\begin{aligned}
\rho V_n(m) &= b+\frac{\lambda}{\rho+\eta}\int_m^{\widehat{\theta}}(m-\rho V_n(m))dG(\theta) \qquad \text{(Flinn eq 4)} \\
&\quad +\frac{\lambda\alpha}{\rho+\eta}\int_{\widehat{\theta}}(\theta-\rho V_n(m))dG(\theta)
\end{aligned}$$

The presence of the minimum wage creates a wedge between $V_n(m)$ and $V_e(m)$.

The lowest-wage job is now more valuable than unemployment (whereas in a standard model the job that is just acceptable has the same value as continuing to search). This is an interesting feature of the model to think about.

Equilibrium with a min. wage

Flinn adds an endogenous labor force rate to make the model interesting. Let

-ρV_0 = value of non-participation, distributed across the population with d.f. $Q(\rho V_0)$

-ℓ = fraction of workers who participate (either work or search)

$$\ell = Q(\rho V_n(m))$$

Now the lowest wage anyone can receive is m, and the expected value of a vacancy is

$$\rho V_v = -\psi + q(k)(1 - G(m))(J - V_v)$$

where as before J = the expected profits of a consumated match. If we continue to assume $V_v = 0$ we get

$$\psi = q(k)(1 - G(m))J. \qquad \text{(Flinn eq 6)}$$

As before, the value of a filled job with match value θ is:

$$(\rho + \eta)J(\theta) = (\theta - w(\theta)) + \eta V_v.$$

and with $V_v = 0$, we get

$$\begin{aligned} J(\theta) &= \frac{\theta - w(\theta)}{\rho + \eta} \\ &= \frac{\theta - w(\theta)}{\rho + \eta} \text{ if } \theta \leq \widehat{\theta} \\ &= (1 - \alpha)\frac{(\theta - \rho V_n(m))}{\rho + \eta} \quad \text{if } \theta > \widehat{\theta} \end{aligned}$$

and

$$J = E[J(\theta)|\theta \geq m].$$

Finally, what is U? Since the job loss rate is η and the job finding rate is $\lambda(1 - G(m)) = \frac{q(k)}{k}(1 - G(m)),$. the unemployment rate is:

$$u = \frac{\eta}{\eta + \frac{q(k)}{k}(1 - G(m))}.$$

If the size of the labor force is ℓ then

$$U = u\ell = \frac{\eta}{\eta + \frac{q(k)}{k}(1 - G(m))}Q(\rho V_n(m)) \qquad \text{(Flinn eq 7)}$$

So now we are ready to discuss the equilibrium. The primitives are

$$\rho, b, \eta, \alpha, G(.), Q(.), q(.), \psi, m$$

The endogenous variables are

$$\ell, u, v, \rho V_n(m)$$

So note that in contrast to the 'partial equilibrium' case, we now have to solve for u, v. Flinn notes that there is a simple recursive algorithm:

1. choose a value for λ
2. using equation 4 (above) solve for $x = \rho V_n(m)$
3. given x find $\ell = Q(x)$, and solve for J
4. using equation 7 (above) solve for $\widetilde{u}$
5. using equation 6 (above) solve for v
6. this generates a new value of $\lambda = q(\frac{\widetilde{u}}{v})/(\frac{\widetilde{u}}{v})$.