

Economics 137
Professor Craine

Expected Interest rate Parity or the Forward Discount
 Bodie-Kane-Marcus Chapter 23

Notation

S_t	spot exchange rate , price of foreign currency (#\$/yen)
F_t	forward rate (#\$/yen for $t+1$) contracted today
$1+I_t$	gross interest rate
$1+I^*_t$	gross foreign interest rate
s_t	$\ln(S_t)$
f	$\ln(F)$
i	$\ln(1+I)$
i^*	$\ln(1+I^*)$

Covered Interest rate Parity

Invest \$1 in the US at the risk free interest rate and the payoff a year from now, at $t+1$, is $1+I_t$. Now consider taking a \$ and investing it in Japan at the Japanese risk free rate. The payoff in a year in yen is $1+I^*_t$. Enter a forward contract to convert yen to \$s and the payoff in \$s is certain. There is no risk. No arbitrage profit requires the same return on either investment.

Covered interest rate parity says,

$$1 + I_t = \frac{1}{S_t} (1 + I^*_t) F_t, \text{ or in logs} \tag{1.1}$$

$$i_t - i^*_t = f_t - s_t$$

that the interest rate differential, $i - i^*$, equals the contracted currency depreciation, $f - s$. Or, if a country pays a higher interest rate, then the investor loses on the contracted currency exchange.

Expected Interest rate Parity

Invest \$1 in the US at the risk free interest rate and the payoff a year from now is $1+I_t$. Now consider taking a \$ and investing it in Japan at the Japanese risk free rate. The payoff in yen is $1+I^*_t$. At time $t+1$ convert the yen payoff to \$s at the exchange rate S_{t+1} .

This is a risky bet. If the currency depreciation is greater than the interest rate differential, then one loses. Or, if depreciation is less one wins. Suppose the exchange rate is lognormally distributed (a good approximation). Then, expected interest rate parity says,

$$i_t - i^*_t = E_t s_{t+1} - s_t \tag{1.2}$$

that the interest rate differential, $i - i^*$, equals the expected currency depreciation, $E s_{t+1} - s$.

Empirical

Testable Implications

By definition the realization of the exchange rate,

$$s_{t+1} = E_t s_{t+1} + e_{t+1} \quad (1.3)$$

equals the conditional expectation plus an error that is uncorrelated with the conditioning information. Substitute the realization in the expected interest rate parity condition (1.3),

$$s_{t+1} - s_t = a + b(i_t - i^*_t) + e_{t+1} \quad (1.4)$$

If the expected interest rate parity model is correct, then $a = 0$, $b = 1$, and the error is uncorrelated with the interest rates.

Notice that covered interest rate parity allows one to substitute forward rates for the interest rate differential,

$$s_{t+1} - s_t = a + b(f_t - s_t) + e_{t+1} \quad (1.5)$$

Project: Test the Expected Interest Rate Parity Model

Get monthly data from for the US, Japan, and another country. I thought Global Financial Data had all the data, but I couldn't find it today (and their documentation is terrible.) Datastream has the data.

1. Test the model, ie run the regression.
2. Calculate the excess return from borrowing in the low interest rate country and investing in the high interest rate country.