Archetypes and Polytypes

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1 Introduction

As H.A. Simon (1982) famously observed, classical economics assumes that decisions are based on substantive rationality, and has little to say about the procedures by which decisions are reached. In this paper we examine data from a laboratory experiment in which individuals are required to select portfolios of three risky securities. Rather than focusing on the consistency of their behavior with standard models of portfolio choice, we study the fine-grained details of their decisions in search of clues to procedural rationality.

There are assumed to be three equally probable states of nature \(s = 1, 2, 3\) and an Arrow security for each state. An Arrow security for state \(s\) is defined to be a promise to deliver one dollar if state \(s\) occurs and nothing otherwise. Let \(\mathbf{x} = (x_1, x_2, x_3) \geq \mathbf{0}\) denote a generic portfolio of securities, where \(x_s\) denotes the number of units of security \(s\) chosen. A portfolio \(\mathbf{x}\) must satisfy the budget constraint \(\mathbf{p} \cdot \mathbf{x} \leq 1\), where \(\mathbf{p} = (p_1, p_2, p_3) \geq \mathbf{0}\) is the vector of security prices and \(p_s\) denotes the price of security \(s\). In the experiment, individuals are presented with a sequence of 50 decision problems. Each decision problem is represented graphically by a budget set on a computer screen, illustrated in Figure 1. The subject uses the mouse to select a portfolio from the feasible set by “pointing and clicking.” The budget sets are parameterized by a corresponding sequence of price vectors, which are i.i.d. random variables. After a subject has completed 50 decisions, one decision round is selected at random, the state of nature is realized, and the subject receives the payoff corresponding to the portfolio chosen in that round.

The data set consists of observations on 47 subjects. For each subject \(n = 1, ..., 47\) we have 50 observations \(\{\mathbf{p}^{(i,n)}, \mathbf{x}^{(i,n)}\}_{i=1}^{50}\), where \(\mathbf{p}^{(i,n)}\) is the price vector defining the \(i\)-th budget set of subject \(n\) and \(\mathbf{x}^{(i,n)}\) is the portfolio chosen from the \(i\)-th budget set by subject \(n\). Choi, Fisman, Gale and Kariv (2007; henceforth CFGK) analyze data from a similar experiment with only two securities and address a series of questions relating to the consistency and heterogeneity of the individual data. Like the data studied by CFGK, the data studied here shows a high degree of consistency with the Generalized Axiom of Revealed Preference (GARP), a necessary and sufficient condition for the existence of a well-behaved
preference ordering that represents the observed data. The data is also consistent with a parametric utility function that allows for loss aversion as well as expected utility (EU). Finally, like CFGK, we observe marked heterogeneity across individuals, both with respect to their measured risk aversion and the presence or absence of loss aversion.

In the present study, we take as given these features of the data and focus instead on more detailed patterns of individual choices that we call types. We identify a finite number of stylized behaviors, some of which can be rationalized by standard preferences but which collectively pose a challenge to decision theory. We call these basic behaviors archetypes. The five archetypes we identify are the center, edge, boundary, bisector and the Cobb-Douglas. The first four are named for their geometrical positions in the budget set; the fifth is named after a well known preference ordering from EU theory. In addition to the basic archetypes, we also find mixtures of archetypal behaviors, which we call polytypes. Although there is a large amount of heterogeneity, especially when one takes account of the mixing of types, the archetypes account for a large proportion of the data set and play a role in the behavior of most individuals. While some of the archetypes (e.g., center, boundary) can be explained in terms of standard preferences, others cannot (e.g., edge, bisector). Furthermore, the combinations of types defy any of the standard models of risk aversion.

The stand one takes on the interpretation of these findings will have important implications for experimental economics and decision theory. We discuss several competing interpretations in the concluding section.

The rest of the paper is organized as follows. In Section 2, we define and enumerate the archetypes. In Section 3, we discuss the evolution of behavior over the course of the experiment for a few individuals. The interpretation of the findings is discussed in Section 4. The experimental design, procedures and formal definitions of types are contained in the appendix.

2 Archetypes and polytypes

We begin by looking at the behavior of a small number of subjects whose patterns of behavior are particularly simple and regular. The transparency of these individual cases will help us to understand the more complicated behavior exhibited by the subject pool as a whole.

If \( \mathbf{x} = (x_1, x_2, x_3) \) is a portfolio and \( x_s \) denotes the number of units of security \( s \), then the token share of security \( s \) in the portfolio \( \mathbf{x} \) is defined to be

\[
\bar{x}_s = \frac{x_s}{x_1 + x_2 + x_3},
\]

for each \( s = 1, 2, 3 \). Let \( \bar{\mathbf{x}} = (\bar{x}_1, \bar{x}_2, \bar{x}_3) \) denote the vector of token shares for the portfolio \( \mathbf{x} = (x_1, x_2, x_3) \). We can represent the vectors of token shares as points in a triangular simplex illustrated in Figure 2. The vertices correspond to the vectors \( \mathbf{v}_1 = (1, 0, 0), \mathbf{v}_2 = (0, 1, 0), \) and \( \mathbf{v}_3 = (0, 0, 1) \), and each vector of token shares \( \bar{\mathbf{x}} \) is a convex combination of the three vertices,

\[
\bar{\mathbf{x}} = \bar{x}_1 \mathbf{v}_1 + \bar{x}_2 \mathbf{v}_2 + \bar{x}_3 \mathbf{v}_3.
\]
Geometrically, each token share $\bar{x}_s$ corresponds to the ratio of the length of the vector $v_s - \bar{x}$ to the length of the vector $v_s - z_s$ where $z_s$ is the projection of $v_s$ through $\bar{x}$ onto the opposite edge.

— Figure 2 about here —

The portfolio choices of five subjects (ID 905, ID 913, ID 1003, ID 1011, and ID 1001) are illustrated in Figure 3 below. For each subject, there is a scatter plot of the vectors of token shares corresponding to the fifty portfolios chosen by the subject.

— Figure 3 about here —

The striking property of the scatter plot for subject ID 905 is that all the observations lie on the edges of the simplex. These correspond to portfolios $\bar{x}$ in which $\bar{x}_s = 0$ for exactly one state $s$. This behavior is consistent with maximizing utility, even with maximizing expected utility if the subject’s risk aversion is low. Alternatively, one can look at this behavior as a form of bounded rationality, in which the subject simplifies his decision problem by eliminating one of the assets and dividing his wealth between the other two.

The behavior of the next subject, ID 913, is equally stylized. In this case, the observations are all concentrated on (or, in a few cases, adjacent to) the three vertices of the simplex. These correspond to portfolios $\bar{x}$ that satisfy $\bar{x}_s = 1$ for exactly one state $s$. In other words, the subject chooses to invest all of his wealth in a single security. Again, this behavior is consistent with maximizing the expected utility. For example, a risk neutral agent will typically maximize expected value of his portfolio by investing only in the security that has the highest expected return per dollar invested.

By contrast with the previous two examples, the behavior of subject ID 1011 seems quite strange. This individual chooses portfolios so that the vectors of token shares lie on the bisectors of the simplex, that is, the lines emanating from a vertex and passing through the midpoint of the opposite edge. Portfolios satisfying this condition have the property that $\bar{x}_s = \bar{x}_{s'}$ for two securities $s \neq s'$. Such behavior can be consistent with utility maximization, though not with expected-utility maximization, but it is not clear what kind of preferences would give rise to these choices.

Subject ID 1011 chooses all his portfolios so that the vectors of token shares lie at the center of the simplex, where the token shares are all equal to one-third. Clearly, choosing $x_1 = x_2 = x_3$, this subject receives the same payoff in each state and faces no risk. This behavior is consistent with expected utility maximization for someone whose risk tolerance is effectively zero (someone whose risk aversion is ‘infinite’).

Unlike the others, the choices of the final subject, ID 1001, does not seem to fit any neat geometrical pattern. In order to see the simplicity of this subject’s behavior we have to present it in another form. If the portfolio $x = (x_1, x_2, x_3)$ is chosen when the normalized price vector is $p = (p_1, p_2, p_3)$, the share of expenditure on security $i$ is

$$e_i = \frac{p_ix_i}{p_1x_1 + p_2x_2 + p_3x_3} = p_i x_i,$$
since we have normalized wealth to equal unity. The vector of expenditure shares is denoted by \( e = (p_1x_1, p_2x_2, p_3x_3) \). Figure 4 shows the scatter plots of the vectors of expenditure shares for the five subjects.

— Figure 4 about here —

For the first two subjects, the patterns of expenditure shares are similar to the patterns of token shares, since a zero token share is equivalent to a zero expenditure share. For the next two subjects, the expenditure share patterns appear quite irregular. For the final subject, ID 925, the expenditure shares are all concentrated at the center of the simplex, implying that the expenditure shares are all equal

\[
p_1x_1 = p_2x_2 = p_3x_3 = \frac{1}{3}.
\]

Equal expenditure shares would result from maximizing a Cobb-Douglas utility function, which corresponds to maximizing the expected value of a logarithmic von Neumann-Morgenstern utility function. It could be that this subject has constant relative risk aversion, but this behavior could also result from a simple geometric rule. It corresponds to choosing portfolios so that the demand for each security is precisely one third of the height of the corresponding vertex. Because income is normalized to unity, the maximum amount of security \( s \) available in the budget set, denoted by \( X_s^{\text{max}} \), is defined by

\[
X_s^{\text{max}} \equiv \frac{1}{p_s}.
\]

If the share of expenditure on each security is \( \frac{1}{3} \), this implies that \( x_s = \frac{1}{3p_s} = \frac{1}{3}X_s^{\text{max}} \) for each \( s \). So the point that equalizes expenditure shares corresponds to an intuitive geometric construction. This construction is illustrated in Figure 5. It might occur to a subject who has never heard of the Cobb-Douglas utility function and does not know what constant relative risk aversion is.

— Figure 5 about here —

Whether we regard these very regular behaviors as the result of utility maximization or as the result of some behavioral rule, their regularity is striking. In fact, these examples were chosen because of their extreme conformity to an ideal type. There are other examples that conform more or less well to the same types. The full set of scatter diagrams displaying token shares and expenditure shares can be found in Online Data Appendix I. These ideal types or archetypes do not by any means exhaust all of the possible behaviors in our data set. Most subjects cannot be classified as belonging to any one of these archetypes. Nonetheless, the archetypes are useful in constructing a taxonomy of the observed behavior. Although most subjects exhibit behavior that is more complex than these simple archetypes, their behavior does contain elements of these archetypical behaviors and, in a sense, can be said to be composed of archetypal behaviors. A few examples illustrate what we mean.
In Figure 6, we see the token shares for five subjects with ID Nos. 1018, 1023, 914, 910, and 1008. None of these scatter plots looks like the archetypal scatter plots in Figures 2 and 3, but each has elements that are reminiscent of the earlier scatter plots. The token shares for subject ID 1018 lie either on the edges or on the bisectors of the simplex. The token shares for subject ID 1023 are concentrated on the center, the vertices, and on midpoints of the edges, that is, the points where the bisectors and edges intersect. The choices of subjects ID 914 and ID 910 choices are a little hard to classify, but most of them fall either on the bisectors, the edges or the vertices. The token shares of subject ID 1008 do not correspond exclusively to the archetypal patterns, but most of the choices are confined to the bisectors and the center.

If the patterns of token shares and budget shares in Figures 2 and 3 are archetypes, the patterns in Figure 6 can be considered polytypes, because they display aspects of two or more archetypes. These polytypes are interesting for several reasons. First, while it is easy to rationalize some of the archetypes as the result of maximizing behavior, it is much harder to see the preferences that would give rise to the behavior of the polytypes. This is not to say that this behavior cannot be rationalized; in fact, the behavior of most subjects is nearly consistent in the sense of satisfying the generalized axiom of revealed preference, which is the necessary and sufficient condition for being generated by some utility function. While it may be possible to rationalize the data in terms of maximizing behavior, the preferences required would seem to be a bit odd. The fact that choices are concentrated on a set of measure zero is by itself surprising.

A second interesting feature is the fact that the polytypes seem to be made up of simpler behaviors. Nothing in the theory of maximizing behavior leads us to expect this internal structure. In fact, it seems to suggest that the simpler behaviors play some role in the decision making process, rather than being a fortuitous result of the decision making process. Whatever interpretation one prefers, it would seem to be worth exploring in greater detail the role of archetypes in the behavior of the subject pool. To do this systematically, we need to find a way of cataloguing the appearance of each of the archetypes throughout the data set. The first step is to provide formal definitions of the archetypes.

We define the five archetypes as follows:

- **Vertex**: \( V_s = \{ x \in \Delta : x_s = 0, \ x_{s'} > 0, \ s' \neq s \} \).
- **Edge**: \( E_s = \{ x \in \Delta : x_s = 0, \ x_{s'} > 0, \ s' \neq s \} \).
- **Bisector**: \( B_{ss'} = \{ x \in \Delta : x_s = x_{s'} \} \).
- **Center**: \( C = \{ x \in \Delta : x_1 = x_2 = x_3 \} \).
- **Cobb-Douglas**: \( CD = \{ e \in \Delta : e_1 = e_2 = e_3 \} \).
There are two difficulties that arise when trying to use these definitions to classify the observed data. The first is that they define sets of measure zero, which does not allow for mistakes of the ‘trembling hand’ variety or for the fact that subjects are actually choosing points from a finite grid rather than a continuum. So we define a neighborhood around each set and treat each data point in the neighborhood as if it belongs to the set itself. The neighborhood is defined to consist of all points whose distance from the set is less than or equal to some fixed number \( r \) that we call the radius of the neighborhood. The second problem is that the sets defined above overlap each other, so we have to adopt some priority rule when assigning points to sets. The rule we have adopted is to assign points first to the sets with the smallest dimension and only then assign points to the sets with larger dimension. Thus, we first assign points to the Vertices and the Center. These sets are mutually exclusive if the radius \( r \) is small. Then, from the remaining data points, we choose any that belong to the Edge. From the remaining points, we select any that belong to the Bisectors. Finally, from the remaining data points, we choose those that belong to the Cobb-Douglas points. The precise definitions of these sets are contained in the appendix.

The results of this classification of the data points are summarized by the stacked bar chart in Figure 7. This chart shows for each subject the relative frequency of each of the archetypes in the subject’s behavior. The radius \( r \) is chosen equal to 0.5 tokens for this illustration.

— Figure 7 about here —

Several points are notable from the figure. First, the number of archetypes observed is quite high. Even for a relatively small radius \( r = 0.5 \), the mean proportion of archetypal data points is 0.556 and for many subjects it is substantially higher. Secondly, there is a great deal of heterogeneity, both in the number of archetypal data points per subject and in the composition of those archetypal points. The Edge is the most common, followed closely by the Bisector. The Center is also relatively common. The Cobb-Douglas and the Vertex are relatively uncommon for the subject pool as a whole, although they are frequently used by a few individuals.

While Figure 7 gives a quick summary view of the incidence of what we have called archetypal behaviors, we have to recognize that the interpretation of the data requires some caution. In the first place, the data are presumably sensitive to our somewhat arbitrary choice of the radius \( r = 0.5 \). Secondly, as the radius \( r \) increases, there is an increased likelihood that subjects’ choices could land in one of the defined neighborhoods by chance. We address these concerns in two ways. First, to determine the sensitivity of the classification, we repeated the exercise for several different values of \( r \). Secondly, we simulated choice behavior using a model that allows for a mixture of random and maximizing behavior to provide a benchmark for the empirical results. The simulation is based on a quantal response model of choice. We assume that the underlying risk preferences are represented by a von Neumann-Morgenstern utility function with constant relative risk aversion equal to one-half. Then the expected
utility of a portfolio \( \mathbf{x} = (x_1, x_2, x_3) \) is defined by

\[
u(\mathbf{x}) = \frac{2}{3} \left\{ (x_1)^{\frac{1}{2}} + (x_2)^{\frac{1}{2}} + (x_3)^{\frac{1}{2}} \right\}.
\]

The subjects’ choices are restricted to satisfy the budget constraint \( \mathbf{p} \cdot \mathbf{x} = 1 \), so let \( B(p) = \{ \mathbf{x} : \mathbf{p} \cdot \mathbf{x} = 1 \} \) denote the set of feasible choices. To allow for randomness in the choice of portfolio, we adopt the familiar logit specification, where the probability density on \( \mathbf{x} \) is denoted by \( f(\mathbf{x}; \lambda, \mathbf{p}) \) and defined by

\[
f(\mathbf{x}; \lambda, \mathbf{p}) = \frac{\exp\{\lambda u(\mathbf{x})\}}{\int_{B(p)} \exp\{\lambda u(\mathbf{x}')\} d\mathbf{x}'}.
\]

According to this formulation, the agent’s behavior is purely random when \( \lambda = 0 \) and becomes optimal as \( \lambda \to \infty \).

We simulated 5000 choices. In each case, the budget set was randomly generated just as it was in the experiment and a random choice was made using the model described above. Table 1 contains the results of the simulations for \( \lambda = 0, 1.0, 2.5, 5.0 \) and \( r = 0.25, 0.5, 1.0 \).

— Table 1 about here —

The first thing to note is that the proportion of archetypes is much lower for the simulated data than it is for the empirical data. For the radius \( r = 0.5 \) the incidence of archetypes in the empirical data is \( 0.556 \div 0.141 = 3.943 \) times as great as it is in the simulated data with \( \lambda = 0 \) (purely random). When \( \lambda = 5 \), a value that ensures an important role for maximizing behavior, the incidence of archetypes in the empirical data is \( 0.556 \div 0.076 = 7.316 \) times as great as in the simulated data. It seems very unlikely that such an outcome could occur by accident or is consistent with expected utility maximization for empirically relevant risk preferences.

The second point to note is that the simulated data contains fewer archetypal data points the higher the value of \( \lambda \) is. As the behavior of the simulated agent becomes less random, the chance of observing points close to the edges becomes smaller. Inspection of the simulated results indicates that most of the change in the total number of archetypal data points comes from the change in the number of Edge points. A smaller change is observed in the number of Bisector points. So purely random behavior would seem to be a fairly conservative benchmark by which to judge the empirical results.

Obviously, the larger the value of \( r \) the more data points will be included in the different archetype categories. However, the relationship between the simulated and empirical data moves in the opposite direction. Comparing the empirical data with the simulated data for the case \( \lambda = 0 \), the likelihood ratios are \( 0.084 \div 0.442 = 0.190 \), \( 0.141 \div 0.556 = 0.254 \), and \( 0.274 \div 0.666 = 0.411 \) for the values \( r = 0.25, 0.5, \) and \( 1.0 \), respectively. Thus, as the value of \( r \) increases, the probability of archetypal behavior in the simulated data increases much faster than the probability of archetypal behavior in the empirical data. This suggests that
the smaller value of $r$ provides a stronger test of whether the empirical data could have occurred by chance.

Another point illustrated by the data in Table 1 is that the distribution of the different archetypes is sensitive to the choice of $r$. For ease of comparison, Table 2 shows the percentage of archetypes belonging to each category for the empirical data when the distance $r$ takes three values. As $r$ increases from 0.25 to 1.0, the values for the Center and Vertex rise and values for the Bisector and the Edge decline. This is explained by the priority rules that we adopted to deal with overlaps. An increase in the neighborhood around the Center and Vertices takes space from the neighborhoods of the Edge and Bisector.

<table>
<thead>
<tr>
<th>Radius</th>
<th>Center</th>
<th>Vertex</th>
<th>Cobb-Douglas</th>
<th>Bisector</th>
<th>Edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 0.25$</td>
<td>0.256</td>
<td>0.0158</td>
<td>0.00453</td>
<td>0.346</td>
<td>0.378</td>
</tr>
<tr>
<td>$r = 0.5$</td>
<td>0.273</td>
<td>0.0288</td>
<td>0.00162</td>
<td>0.326</td>
<td>0.350</td>
</tr>
<tr>
<td>$r = 1.0$</td>
<td>0.278</td>
<td>0.0480</td>
<td>0.00450</td>
<td>0.317</td>
<td>0.314</td>
</tr>
</tbody>
</table>

**Table 2**

Distribution of archetypes for different values of radius $r$

To conclude this section, there appears to be evidence that subjects are choosing portfolios belonging to the set of what we have called archetypes more often than can be explained by chance or maximizing behavior. Although the classification is sensitive to the definition of the neighborhood, the evidence for the existence of archetypal behavior seems robust. For small values of $r$, the simulated data provide much lower incidence of archetypal behavior than the empirical data, whereas for higher values of $r$, which make allowance for the careless mistakes likely to be found in experimental data, the incidence of archetypal behavior accounts for as much as two-thirds of the observations.

## 3 Stationarity

The stationarity of observations is an important question in any experimental study. In this case, there are some interesting violations of stationarity. Some correspond to switching between archetypes and some correspond to transitions from more complex behavior to a precise archetype or vice versa. Figure 8 illustrates these possibilities.

— Figure 8 about here —

The figure shows the token shares for the first ten (left hand diagram) and the last 20 observations (right hand diagram) for Subject ID Nos. 902, 920, 1007, and 1015. Subject 902 starts out as a Bisector and ends as an Edge. Subject ID No. 920 is hard to classify in the beginning but is clearly a combination of Center and Vertex by the end. Subject ID No. 1007 is a Center initially, but becomes a combination of Center and Bisector. Subject ID No. 1015 is also a Center initially, but his behavior is hard to classify in the last 20 periods. What can we make of the changes we observe over time? Clearly the behavior is not stable, but neither does it appear to represent ‘learning’ in the usual sense. It is more
like the subject is changing his mind about how to approach the decision problem. In some cases, this means moving from a complex to a simpler decision rule. In other cases, we see the reverse movement from a simple to a more complex rule. In any case, this behavior seems to reinforce the idea that subjects are switching between alternative behaviors rather than maximizing a single preference ordering.

If we look at budget shares, we see similar transformations over time. There are only a few subjects who make use of the Cobb-Douglas archetype, but their behavior is interesting and instructive. Figure 9 shows the budget shares for Subject ID Nos. 1002 and 1015, divided between the first ten periods (the left hand diagram) and the last twenty periods (the right hand diagram). Subject ID No. 1002 appears to be choosing points on the bisectors and edges in the first ten periods, but in the last twenty is concentrating most of his choices around the Cobb-Douglas point. We encountered Subject ID No. 1015 in Figure 8, where we saw his choices conformed to the Center archetype in the first ten periods. Here we can see the choices in the last 20 periods are fairly tightly grouped around the Cobb-Douglas point. Again, the transition from Center to Cobb-Douglas suggests switching between distinct, simple, behavioral rules rather than maximizing a single preference ordering.

— Figure 9 here —

4 Discussion

How are we to interpret the peculiar features of the data? As we have mentioned, for most subjects, the data are very close to being consistent, in the sense of satisfying GARP, so that Afriat’s theorem implies the existence of a preference ordering that generates most of the data. The data can therefore be interpreted as revealing the true preferences of the individual subjects. These preferences will not necessarily satisfy the axioms of expected utility theory, but they are represented by a well behaved utility function. The peculiarities of the utility functions, e.g., kinks that give rise to concentrations at the center and bisectors of the simplex, can be interpreted as evidence of loss aversion, disappointment aversion, or other behavioral “effects,” but these are simply different ways of describing the features of a preference ordering.

Note that the existence of a utility function representing the preferences revealed in this experiment does not guarantee that subjects’ behavior can be predicted by these preferences in another experiment. The symmetry of the decision problem and the geometrical representation of the choice set may contribute in some measure to the peculiar features of the data. To the extent that they do, we cannot expect the revealed preferences to be robust to changes of the experimental “frame.”

An alternative approach assumes that, although a subject has some underlying objective that he seeks to maximize, the need to simplify the problem leads him to adopt strategies that approximate the maximization of the objective by means of short cuts or rules of thumb. In this interpretation, the peculiar features of the data are the result of individual subjects choosing among a restricted set of decision rules in order to implement a second-best
optimization of the underlying objectives. Thus, rather than optimize over the entire budget set, the individual first chooses to locate his choice on the edge or bisector and then adjusts the choice within that subset in order to exploit the tradeoff between risk and reward.

We do not yet have a theoretical model that will provide a coherent account procedural rationality, but there are reasons for thinking that this will be a productive line of enquiry. First, it holds out the hope (yet to be fulfilled) that it will be possible to extract underlying preferences from the behavior and use these as a robust basis for predicting behavior in other settings. Second, to the extent that the peculiarities of the data are related to the frame, this may be the only way to obtain robust predictions from this data.

Whether we treat individuals as substantively or procedurally rational has far-reaching consequences. If we assume that individuals are procedurally rational, e.g., that they use archetypes to simplify their decision problems, we can no longer use the methods of revealed preference theory to infer their “true” preferences. Instead, we have to model the process by which they choose the archetypes that determine their choices. The underlying preferences that motivate the choice of archetypes may be robust, but the archetypes are likely dependent on the frame of the decision problem. So we have to take into account the possibility that the procedures used in one situation will not transfer to another.

5 Appendix:

5.1 Design

The experimental task we study can be interpreted as follows. There are three states of nature denoted by \( s = 1, 2, 3 \) and three associated Arrow securities, each of which promises a unit payoff in one state and nothing in the others. Let \( \pi_s \) denote the probability of state \( s \), and let \( x_s \) denote the demand for the security that pays off in state \( s \) and \( p_s \) denote its price. We normalize the prices by the individual’s wealth so that \( p_1 x_1 + p_2 x_2 + p_3 x_3 = 1 \). The individual can choose any portfolio \( x = (x_1, x_2, x_3) \geq 0 \) that satisfies this budget constraint.

An example of a budget constraint defined in this way is a triangular hyperplane in the three-dimensional graph, which is drawn in Figure 1. The axes measure the future values of portfolios in each of the three states. For example, the point \( A \) (resp. \( B \) and \( C \)) represents a portfolio in which all wealth is invested in the security that pays off in state 1 (resp. 2 and 3). By contrast, the point \( D \), which lies on a line that is at 45 degrees to all axes, corresponds to a portfolio whose value is certain. The line \( EF \) whose slope \( -p_1/p_2 \) gives the rate at which the decision maker, starting from a position of certainty \( C \), can trade a security that pays off in state 1 with a security that pays off in state 2. If the slope of the line \( EF -p_1/p_2 \) is flatter than \(-\pi_1/\pi_2\), positions along \( CF \) have a higher payoff in state 1, a lower payoff in state 2, and a higher expected portfolio return than point \( C \).

[Figure 1 here]

If the decision maker only concerns the distribution of monetary payoffs and if his preferences over portfolios \( x = (x_1, x_2, x_3) \) can be represented by a function \( U(\cdot) \), then such a
decision maker will choose a portfolio to maximize $U(\cdot)$ subject to the budget constraint. Clearly, there is no need to assume the Savage (1954) axioms in order to investigate rational behavior in general under uncertainty. Any consistent preference ordering over portfolios is admissible. The standard approach for studying choice under uncertainty was to assume the existence of some such preferences, while imposing minimal restriction on their precise form, and to ask what further restrictions can be placed on $U(\cdot)$. Our simple state-of-nature representation of uncertainty provides a test of rational decision making under uncertainty by testing for consistency with utility maximization using revealed preference axioms.

5.2 Procedures

The experiments were conducted at the Experimental Social Science Laboratory (X-Lab) at UC Berkeley. The subjects were recruited from all undergraduate classes and administrative staff at UC Berkeley. Each experimental session consisted of 50 independent decision problems. In each decision problem, a subject was asked to allocate tokens between three accounts, labeled $x$, $y$ and $z$. Each of these accounts corresponds to an axis in a three-dimensional graph. Each choice involved choosing a point on a budget set of possible token allocations. Each decision problem started by having the computer select such a budget set randomly from the collection of budget sets that intersected with at least one of the axes at 50 or more tokens, but with no intercept exceeding 100 tokens. The budget sets were selected independently across subjects and across decision problems.

The axes of the graph were scaled from 0 to 100 tokens. The resolution compatibility of the budget sets was 0.2 tokens. At the beginning of each decision problem, the experimental program dialog window went blank and the entire setup reappeared. The appearance and behavior of the pointer were set to the Windows mouse default and the pointer was automatically repositioned randomly on the budget line at the beginning of each decision problem. Subjects were always informed of the exact allocation that the pointer is located. To choose an allocation, subjects used the mouse or the arrows on the keyboard to move the pointer on the computer screen to the desired allocation. Subjects could either left-click or press the Enter key to record their allocation. No subject reported difficulty understanding the procedures or using the computer interface. The computer program dialog window is shown in the experimental instructions reproduced in Appendix I.

The payoff in each round was determined by the number of tokens in each account. At the end of the round, the computer selected one of the accounts, $x$, $y$ or $z$ in a random manner to be explained shortly. Each subject received the number of tokens allocated to the account that was chosen. Subjects were not informed of the account that was actually selected at the end of each decision problem. At the end of the experiment, the experimental program randomly selected one decision problem from each subject to carry out. Each round had an equal probability of being chosen, and the subject was paid the amount he had earned in that round. Payoffs were calculated in terms of tokens and then converted into dollars, where each token was worth $0.50. Subjects received this payoff, which averaged about $\_\_\_\_\$, and $5 show up fee privately as they left the experiment.
We studied a symmetric treatment (subjects ID 901-924 and 1001-1023) where the three accounts were equally likely \( \pi_x = \pi_y = \pi_z = \frac{1}{3} \). The treatment was held constant throughout a given experimental session.

5.3 Definition of archetypes

We first define a set of archetypes that are easily identifiable in the data. The definitions of archetypes are in terms of tokens, rather than token shares. Since we allow small error in the classification of archetypes, it implies that we allow for a constant error rather than a constant proportional error in classifying each archetype. One could alternatively define the archetypes in terms of token shares, in which case the error in the definition is proportional to the size of portfolios.

The first archetype is called the center (\( C \)), which chooses an equal amount of three assets. With a classification error parameter \( \varepsilon \), the center archetype is defined to be

\[
C_\varepsilon = \{ x : d(x, C) \leq \varepsilon \},
\]

where \( C = \{ (x_1, x_2, x_3) : x_1 = x_2 = x_3 \} \). In the definitions of archetypes, we use a distance function \( d(\cdot, \cdot) \) that can serve as either the Euclidean distance, if it measures the distance between two points, or the Hausdorff distance, if it measures the distance between a point and a set. The second archetype is called vertex (\( V \)), which chooses to spend all wealth to one asset. Again with an error parameter \( \varepsilon \), the vertex archetype is defined as

\[
V_\varepsilon = \{ x : d(x, V) \leq \varepsilon \},
\]

where \( V = \{ (x_1, x_2, x_3) : x_i = x_j = 0, \text{ for some } i \neq j \} \). Another archetype is called the edge (\( E \)), which eliminates one asset and chooses portfolios between two remaining assets. The edge archetype is classified with error \( \varepsilon \) as

\[
E_\varepsilon = \{ x : d(x, E) \leq \varepsilon \} \cap V^C_\varepsilon,
\]

where \( E = \{ (x_1, x_2, x_3) : x_i = 0, \text{ for some } i \} \). When archetypes \( B_\varepsilon \) and \( E_\varepsilon \) are overlapped in the \( \varepsilon \)-neighborhood of the vertex portfolio choice, we define the edge archetype as excluding the vertex. The next archetype is called the bisector (\( B \)), which chooses an equal amount between two assets. The bisector archetype can be defined with an error \( \varepsilon \) as

\[
B_\varepsilon = \{ x : d(x, B) \leq \varepsilon \} \cap (C_\varepsilon \cup V_\varepsilon \cup E_\varepsilon)^C,
\]

where \( B = \{ (x_1, x_2, x_3) : x_i = x_j, \text{ for some } i \neq j \} \). By the definition of the bisector archetype, it can overlap the center, vertex, and edge archetypes. We define the bisector archetype as excluding the center, vertex, and edge portfolios. Finally, we consider a portfolio allocation whose budget shares among three assets are equal to each other. This choice behavior is consistent with the Cobb-Douglas utility maximization. We call it the Cobb-Douglas (\( CD \)) archetype and it can be formally classified with an error \( \varepsilon \) as

\[
CD_\varepsilon = \{ x : d(x, CD) \leq \varepsilon \} \cap (C_\varepsilon \cup B_\varepsilon)^C,
\]

where \( CD = \{ (x_1, x_2, x_3) : p_i x_i = \frac{1}{3}, \text{ for all } i \} \). We also define the Cobb-Douglas archetype as excluding the center and bisector portfolios.
References


Figure 2
Token shares as convex combinations of the vertices $v_1$, $v_2$, $v_3$.

In this diagram, the point $P$ is represented as a convex combination of the vertices $v_1$, $v_2$, and $v_3$, where the weights on the vertices are the token shares $x_1$, $x_2$, and $x_3$, respectively. Geometrically, the token share $x_2$ is the ratio of the distance $v_2P$ to the distance $v_2Q$. 
Figure 3
Token Shares for Subject ID Nos. 905, 913, 1003, 1011, and 10001.
Figure 4
Budget Shares for Subject ID Nos. 905, 913, 1003, 1011, and 1001.
Figure 5
Token Shares for Subject ID Nos. 1018, 1023, 914, 910.
Figure 6
Illustration of geometric construction of the Cobb-Douglas point
Figure 7
Bar chart showing classification of data points for each subject among different archetypes.
Table 1  
Sensitivity of the classification of data points as the radius $r$ is varied.
Figure 8
Token Shares for Subject ID Nos. 902, 920, 1007, and 1015.
Figure 9
Budget shares for Subject ID Nos. 1002 and 1015.