Economics 101A
(Lecture 5, Revised)

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Outline

1. Properties of Preferences (continued)

2. From Preferences to Utility (and viceversa)

3. Common Utility Functions

4. (Utility maximization)

1 Properties of Preferences (ctd)

- Indifference relation $\sim$: $x \sim y$ if $x \succeq y$ and $y \succeq x$

- Strict preference: $x \succ y$ if $x \succeq y$ and not $y \succeq x$

- Exercise. If $\succeq$ is rational,
  - $\succ$ is transitive
  - $\sim$ is transitive
  - Reflexive property of $\succeq$. For all x, $x \succeq x$.

- Other features of preferences

- Preference relation $\succeq$ is:

  - *monotonic* if $x \geq y$ implies $x \succeq y$.

  - *strictly monotonic* if $x \geq y$ and $x_j > y_j$ for some j implies $x \succ y$.

  - *convex* if for all x, y, and z in X such that $x \succeq z$ and $y \succeq z$, then $tx + (1 - t)y \succeq z$ for all t in $[0, 1]$

2 From preferences to utility

- Nicholson, Ch. 3

- Economists like to use utility functions $u : X \to R$

- $u(x)$ is 'liking' of good x

- $u(a) > u(b)$ means: I prefer a to b.

- **Def.** Utility function u represents preferences $\succeq$ if, for all x and y in X, $x \succeq y$ if and only if $u(x) \geq u(y)$.

- **Theorem.** If preference relation $\succeq$ is rational and continuous, there exists a continuous utility function $u : X \to R$ that represents it.

- Proof for case $X = R^2_+$ and $\succeq$ strongly monotonic.
  - Define $u(x) =$?
  - Consider the points in the diagonal, (t, t)
  - Set $\{t : (t, t) \succeq x\}$ is non-empty by monotonicity
  - Set $\{t : x \succeq (t, t)\}$ is non-empty by monotonicity
  - Both sets are closed by continuity
  - (Connected set X: $A \subset X$ closed, $B \subset X$ closed, and $A \cup B = X \Longrightarrow A \cap B$ non-empty)
  - By connectedness of R, the two sets have non-empty intersection $\Longrightarrow \exists\, t_x$ such that $(t_x, t_x) \sim x$. Define $u(x) = t_x$.

– Does u represent $\succeq$?

– $x \succeq y$ implies $(u(x), u(x)) \sim x \succeq y \sim (u(y), u(y)) \Longrightarrow$ [by transitivity] $(u(x), u(x)) \succeq (u(y), u(y)) \Longrightarrow$ [by monotonicity] $u(x) \geq u(y)$

– Similarly can prove other direction (exercise!)

– (We do not prove continuity of $u(x)$)

- Utility function representing $\succeq$ is not unique

- Take $\exp(u(x))$

- $u(a) > u(b) \Longleftrightarrow \exp(u(a)) > \exp(u(b))$

- If $u(x)$ represents preferences $\succeq$ and f is a strictly increasing function, then $f(u(x))$ represents $\succeq$ as well.

- If preferences are represented from a utility function, are they rational?
  - completeness
  - transitivity

- Indifference curves: $u(x_1, x_2) = \bar{u}$

- They are just implicit functions! $u(x_1, x_2) - \bar{u} = 0$

$$\frac{dx_2}{dx_1} = -\frac{U'_{x_1}}{U'_{x_2}} = MRS$$

- Indifference curves for:
  - monotonic preferences;
  - strictly monotonic preferences;
  - convex preferences

3 Common utility functions

- Nicholson, Ch. 3, pp. 80–84

1. Cobb-Douglas preferences: $u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$

   - $MRS = -\alpha x_1^{a-1} x_2^{1-\alpha} / (1-a) x_1^{\alpha} x_2^{-\alpha} = \frac{\alpha}{1-\alpha} \frac{x_2}{x_1}$

2. Perfect substitutes: $u\left(x_1, x_2\right) = \alpha x_1 + \beta x_2$

   - $MRS = -\alpha / \beta$

3. Perfect complements: $u\left(x_1, x_2\right) = \min\left(\alpha x_1, \beta x_2\right)$

   - MRS discontinuous at $x_2 = \frac{\alpha}{\beta} x_1$

4. Constant Elasticity of Substitution: $u\left(x_1, x_2\right) = \left(\alpha x_1^\rho + \beta x_2^\rho\right)^{1/\rho}$

   - $MRS = -\frac{\alpha}{\beta}\left(\frac{x_1}{x_2}\right)^{\rho - 1}$
   - if $\rho = 1$, then...
   - if $\rho = 0$, then...
   - if $\rho \rightarrow +\infty$, then...