Economics 101A
(Lecture 11)

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Outline

1. Intertemporal choice II
2. Altruism and charitable donations

1 Intertemporal choice II

- Maximization problem:

$$\max U(c_0) + \frac{1}{1+\delta} U(c_1)$$
$$s.t.\ c_0 + \frac{1}{1+r} c_1 \leq M_0 + \frac{1}{1+r} M_1$$

- Lagrangean

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_0)}{U'(c_1)} = \frac{1+r}{1+\delta}$$

- Case $r = \delta$
  - $c_0^* \quad c_1^*$?
  - Substitute into budget constraint using $c_0^* = c_1^* = c^*$:
    $$\frac{2+r}{1+r} c^* = \left[M_0 + \frac{1}{1+r} M_1 \right]$$
    or
    $$c^* = \frac{1+r}{2+r} M_0 + \frac{1}{2+r} M_1$$
  - We solved problem virtually without any assumption on U!
  - Notice: $M_0 < c^* < M_1$

- Case $r > \delta$
  - $c_0^* \quad c_1^*$?

- Comparative statics with respect to income M_0

- Rewrite ratio of f.o.c.s as

$$U'(c_0) - \frac{1+r}{1+\delta} U'(c_1) = 0$$

- Substitute c_1 in using $c_1 = M_1 + (M_0 - c_0)(1+r)$ to get

$$U'(c_0) - \frac{1+r}{1+\delta} U'(M_1 + (M_0 - c_0)(1+r)) = 0$$

- Apply implicit function theorem:

$$\frac{\partial c_0^*(r, \mathbf{M})}{\partial M_0} = -\frac{-\frac{1+r}{1+\delta} U''(c_1)(1+r)}{U''(c_0) - \frac{1+r}{1+\delta} U''(c_1) * (-(1+r))}$$

- Denominator is always negative

- Numerator is positive

- $\partial c_0^* (r, \mathbf{M}) / \partial M_0 > 0$ — consumption at time 0 is a normal good.

- Can also show $\partial c_0^* (r, \mathbf{M}) / \partial M_1 > 0$

- Comparative statics with respect to interest rate r

- Apply implicit function theorem:

$$\frac{\partial c_0^*(r,\mathbf{M})}{\partial r} = -\frac{-\frac{1}{1+\delta}U'(c_1)}{U''(c_0)-\frac{1+r}{1+\delta}U''(c_1)*(-(1+r))} - \frac{-\frac{1+r}{1+\delta}U''(c_1)*(M_0-c_0)}{U''(c_0)-\frac{1+r}{1+\delta}U''(c_1)*(-(1+r))}$$

- Denominator is always negative

- Numerator: First term is negative (subsitution effect)

- Second term is income effect:

  - positive if $M_0 > c_0$

  - negative if $M_0 < c_0$.

2 Altruism and Charitable Donations

- Maximize utility = satisfy self-interest?

- No, not necessarily

- 2-person economy:

  – Mark has income M_M and consumes c_M

  – Wendy has income M_W and consumes c_W

- One good: c, with price $p = 1$

- Utility function: $u(c)$, with $u' > 0$, $u'' < 0$

- Wendy is altruistic: she maximizes $u(c_W) + \alpha u(c_M)$ with $\alpha > 0$

- Mark simply maximizes $u(c_M)$

- Wendy can give a donation of income D to Mark.

- Wendy computes the utility of Mark as a function of the donation D

- Mark maximizes

$$\max_{c_M} u(c_M)$$
$$s.t.\ c_M \leq M_M + D$$

- Solution: $c_M^* = M_M + D$

- Wendy maximizes

$$\max_{c_M, D} u(c_W) + \alpha u\left(M_M + D\right)$$
$$s.t.\ c_W \leq M_W - D$$

- Rewrite as:

$$\max_{D} u(M_W - D) + \alpha u\left(M_M + D\right)$$

- First order condition:

$$-u'(M_W - D^*) + \alpha u'\left(M_M + D^*\right) = 0$$

- Second order conditions:

$$u''(M_W - D^*) + \alpha u''\left(M_M + D^*\right) < 0$$

- Assume $\alpha = 1$.
  - Solution?
  - $u'(M_W - D) = u'(M_M + D^*)$
  - $M_W - D^* = M_M + D^*$ or $D^* = (M_W - M_M)/2$
  - Transfer money so as to equate incomes!
  - Careful: $D < 0$ (negative donation!) if $M_M > M_W$

- Corrected maximization:

$$\max_D u(M_W - D) + \alpha u(M_M + D)$$
$$s.t. D \geq 0$$

- Solution ($\alpha = 1$):

$$D^* = \begin{cases} (M_W - M_M)/2 & \text{if } M_W - M_M > 0 \\ 0 & \text{otherwise} \end{cases}$$

- Assume interior solution. ($D^* > 0$)

- Comparative statics 1 (altruism):

$$\frac{\partial D^*}{\partial \alpha} = -\frac{u'(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0$$

- Comparative statics 2 (income of donor):

$$\frac{\partial D^*}{\partial M_W} = -\frac{-u''(M_W + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0$$

- Comparative statics 3 (income of recipient):

$$\frac{\partial D^*}{\partial M_M} = -\frac{\alpha u''(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} < 0$$