Economics 101A
(Lecture 12, Revised)

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Outline

1. Mid-term questionnaire

2. Altruism and Charitable Donations

3. Introduction to probability

4. Expected utility

1 Mid-term questionnaire

2 Altruism and Charitable Donations

- Wendy maximizes

$$\max_{D} u(M_W - D) + \alpha u(M_M + D)$$

- First order condition:

$$-u'(M_W - D^*) + \alpha u'(M_M + D^*) = 0$$

- Second order conditions:

$$u''(M_W - D^*) + \alpha u''(M_M + D^*) < 0$$

- Assume $\alpha = 1$.

  - Solution?

  - $u'(M_W - D) = u'(M_M + D^*)$

  - $M_W - D^* = M_M + D^*$ or $D^* = (M_W - M_M)/2$

  - Transfer money so as to equate incomes!

  - Careful: $D < 0$ (negative donation!) if $M_M > M_W$

- Corrected maximization:

$$\max_D u(M_W - D) + \alpha u(M_M + D)$$
$$s.t. D \geq 0$$

- Solution $(\alpha = 1)$:

$$D^* = \begin{cases} (M_W - M_M)/2 & \text{if } M_W - M_M > 0 \\ 0 & \text{otherwise} \end{cases}$$

- Assume interior solution. $(D^* > 0)$

- Comparative statics 1 (altruism):

$$\frac{\partial D^*}{\partial \alpha} = -\frac{u'(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0$$

- Comparative statics 2 (income of donor):

$$\frac{\partial D^*}{\partial M_W} = -\frac{-u''(M_W + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0$$

- Comparative statics 3 (income of recipient):

$$\frac{\partial D^*}{\partial M_M} = -\frac{\alpha u''(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} < 0$$

3   Introduction to Probability

- So far deterministic world:

  - income given, known M

  - interest rate known r

- But some variables are unknown at time of decision:

  - future income M_1?

  - future interest rate r_1?

- Generalize framework to allow for uncertainty

  - Events that are truly unpredictable (weather)

  - Event that are very hard to predict (future income)

- Probability is the language of uncertainty

- Example:

  - Income M_1 at $t = 1$ depends on state of the economy

  - Recession $(M_1 = 20)$, Slow growth $(M_2 = 25)$, Boom $(M_3 = 30)$

  - Three probabilities: p_1, p_2, p_3

  - $p_1 = P(M_1) = P(\text{recession})$

- Properties:

  - $0 \le p_i \le 1$

  - $p_1 + p_2 + p_3 = 1$

- Mean income: $EM = \sum_{i=1}^{3} p_i M_i$

- If $(p_1, p_2, p_3) = (1/3, 1/3, 1/3)$,

$$EM = \frac{1}{3}20 + \frac{1}{3}25 + \frac{1}{3}30 = \frac{75}{3} = 25$$

- Variance of income: $V(M) = \sum_{i=1}^{3} p_i (M_i - EM)^2$

- If $(p_1, p_2, p_3) = (1/3, 1/3, 1/3)$,

$$
\begin{aligned}
V(M) &= \frac{1}{3}(20 - 25)^2 + \frac{1}{3}(25 - 25)^2 + \frac{1}{3}(30 - 25)^2 \\
&= \frac{1}{3}5^2 + \frac{1}{3}5^2 = 2/3 * 25
\end{aligned}
$$

- Mean and variance if $(p_1, p_2, p_3) = (1/4, 1/2, 1/4)$?

4 Expected Utility

- Nicholson, Ch. 8, pp. 198–206

- Consumer at time 0 asks: what is utility in time 1?

- At $t = 1$ consumer maximizes

$$\max U(c^1)$$
$$s.t. \ c_i^1 \leq M_i^1 + (1+r)\left(M^0 - c^0\right)$$

  with $i = 1, 2, 3$.

- What is utility at optimum at $t = 1$ if $U' > 0$?

- Assume for now $M^0 - c^0 = 0$

- Utility $U\left(M_i^1\right)$

- This is uncertain, depends on which i is realized!

- How do we evaluate future uncertain utility?

- **Expected utility**

$$EU = \sum_{i=1}^{3} p_i U\left(M_i^1\right)$$

- In example:

$$EU = 1/3U(20) + 1/3U(25) + 1/3U(30)$$

- Compare with $U(EC) = U(25)$.

- Agents prefer riskless outcome EM to uncertain outcome M if [REVISED]

$$
\begin{aligned}
1/3U(20) + 1/3U(25) + 1/3U(30) &< U(25) \text{ or} \\
1/3U(20) + 1/3U(30) &< 2/3U(25) \text{ or} \\
1/2U(20) + 1/2U(30) &< U(25)
\end{aligned}
$$

- Picture

- Depends on sign of U'', on concavity/convexity

- Three cases:

  - $U''(x) = 0$ for all x. (linearity of U)

    * $U(x) = a + bx$

    * $1/2 U(20) + 1/2 U(30) = U(25)$

  - $U''(x) < 0$ for all x. (concavity of U)

    * $1/2 U(20) + 1/2 U(30) < U(25)$

  - $U''(x) > 0$ for all x. (convexity of U)

    * $1/2 U(20) + 1/2 U(30) > U(25)$

- If $U''(x) = 0$ (linearity), consumer is indifferent to uncertainty

- If $U''(x) < 0$ (concavity), consumer dislikes uncertainty

- If $U''(x) > 0$ (convexity), consumer likes uncertainty

- Do consumers like uncertainty?

- Do *you* like uncertainty?

- **Theorem. (Jensen's inequality)** If a function $f(x)$ is concave, the following inequality holds:

$$f(Ex) \geq Ef(x)$$

where E indicates expectation. If f is strictly concave, we obtain

$$f(Ex) > Ef(x)$$

- Apply to utility function U.

- Individuals dislike uncertainty:

$$U(Ex) \geq EU(x)$$

- Jensen's inequality then implies U concave ($U'' \leq 0$)

- Relate to diminishing marginal utility of income