Economics 101A
(Lecture 15, Revised)

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Outline

1 Health Club Attendance

- Health club industry study (DellaVigna and Malmendier, 2002)

- 3 health clubs

- Data on attendance from swiping cards

- Choice of contracts:

  - Monthly contract with average price of \$75

  - 10-visit pass for \$100

- Consider users that choose monthly contract. Attendance?

- Attend on average 4.8 times per *month*

- Pay on average over $17

- Average delay of 2.2 months ($185) between last attendance and contract termination

- Over membership, user could have saved $700 by paying per visit

- Health club attendance:
  - immediate cost c
  - delayed benefit b
- At sign-up (attend tomorrow):

$$NB^t = -\frac{\beta}{1+\delta}c + \frac{\beta}{(1+\delta)^2}b$$

- Plan to attend if $NB^t > 0$

$$c < \frac{1}{(1+\delta)}b$$

- Once moment to attend comes:

$$NB = -c + \frac{\beta}{(1+\delta)} b$$

- Attend if $NB > 0$

$$c < \frac{\beta}{(1+\delta)} b$$

- Interpretations?

- Users are buying a commitment device

- User underestimate their future self-control problems:
  - They overestimate future attendance
  - They delay cancellation

2 Production: Introduction

- Second half of the economy. **Production**

- Example. Ford and the Minivan (Petrin, 2002):
  - Ford had idea: "Mini/Max" (early '70s)
  - Did Ford produce it?
  - No!
  - Ford was worried of cannibalizing station wagon sector
  - Chrysler introduces Dodge Caravan (1984)
  - Chrysler: $1.5bn profits (by 1987)!

- Why need separate treatment?

- Perhaps firms maximize utility...

- ...we can be more precise:
  - Competition
  - Institutional structure

3 Production Function

- Nicholson, Ch. 11

- Production function: $y = f(\mathbf{z})$. Function $f : R_+^n \to R_+$

- Inputs $\mathbf{z} = (z_1, z_2, ..., z_n)$: labor, capital, land, human capital

- Output y: Minivan, Intel Pentium III, mangoes (Philippines)

- Properties of f:

  – no free lunches: $f(0) = 0$

  – positive marginal productivity: $f_i'(\mathbf{z}) > 0$

  – decreasing marginal productivity: $f_{i,i}''(\mathbf{z}) < 0$

- Isoquants $Q(y) = \{\mathbf{x} | f(\mathbf{x}) = y\}$

- Set of inputs $\mathbf{z}$ required to produce quantity y

- Special case. Two inputs:

  - $z_1 = L$ (labor)

  - $z_2 = K$ (capital)

- Isoquant: $f(L, K) - y = 0$

- Slope of isoquant $dK/dL = MRTS$

- Convex production function if convex isoquants

- Reasonable: combine two technologies and do better!

- Mathematically, $d^2K/d^2L =$

4 Returns to Scale

- Effect of increase in labor: f'_L

- Increase of all inputs: $f(t\mathbf{z})$ with t scalar, $t > 1$ [REVISED, notice $t > 1$]

- How much does input increase?

  – Decreasing returns to scale: for all $\mathbf{z}$ and $t > 1$,[REVISED, notice $t > 1$]

$$f(t\mathbf{z}) < tf(\mathbf{z})$$

  – Constant returns to scale: for all $\mathbf{z}$ and $t > 1$,[REVISED, notice $t > 1$]

$$f(t\mathbf{z}) = tf(\mathbf{z})$$

– Increasing returns to scale: for all $\mathbf{z}$ and $t > 1$,[REVISED, notice $t > 1$]

$$f(t\mathbf{z}) > tf(\mathbf{z})$$

- Example: $y = f(K, L) = AK^{\alpha}L^{\beta}$

- Marginal product of labor: $f'_L =$

- Decreasing marginal product of labor: $f''_L =$

- $MRTS =$

- Convex isoquant?

- Returns to scale: $f(tK, tL) = A(tK)^{\alpha}(tL)^{\beta} = t^{\alpha+\beta}AK^{\alpha}L^{\beta} = t^{\alpha+\beta}f(K, L)$

5 Two-step Cost minimization

- Nicholson, Ch. 12

- Objective of firm: Produce output that generates maximal profit.

- Decompose problem in two:
  - Given production level y, choose cost-minimizing combinations of inputs
  - Choose optimal level of y.

- *First step.* Cost-Minimizing choice of inputs

- Two-input case: Labor, Capital

- Input prices:
  - Wage w is price of L
  - Interest rate r is rental price of capital K

- Expenditure on inputs: $wL + rK$

- Firm objective function:

$$\begin{aligned} &\min wL + rK \\ &s.t. f(L, K) \geq y \end{aligned}$$

- Compare with expenditure minimization for consumers

- First order conditions:

$$w - \lambda f'_L = 0$$

and

$$r - \lambda f'_K = 0$$

- Rewrite as

$$\frac{f'_L\left(L^*, K^*\right)}{f'_K\left(L^*, K^*\right)} = \frac{w}{r}$$

- MRTS (slope of isoquant) equals ratio of input prices

- Graphical interpretation

- Derived demand for inputs:
  - $L = L^* (w, r, y)$
  - $K = K^* (w, r, y)$
- Value function at optimum is **cost function:**

$$c (w, r, y) = wL^* (r, w, y) + rK^* (r, w, y)$$

- *Second step.* Given cost function, choose optimal quantity of y as well

- Price of output is p.

- Firm's objective:

$$\max py - c(w, r, y)$$

- First order condition:

$$p - c'_y(w, r, y) = 0$$

- Price equals marginal cost – very important!

6 Next Lecture

- Continue Cost Minimization

- Solve an Example

- Cases in which s.o.c. are not satisfied

- Start Profit Maximization