Economics 101A
(Lecture 17, Revised)

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Outline

1. Geometry of Cost Curves
2. Supply Function
3. Short-run Cost Minimization
4. One-step Profit Maximization
5. Introduction to Market Equilibrium

1 Geometry of cost curves

- Nicholson, Ch. 12, pp. 307–312 and Ch. 13, pp. 342–346.

- Marginal costs $MC = \partial c / \partial y \rightarrow$ Cost minimization

$$p = MC = \partial c\left(w, r, y\right) / \partial y$$

- Average costs $AC = c/y \rightarrow$ Does firm break even?

$$\begin{aligned} \pi &= py - c\left(w, r, y\right) > 0 \text{ iff} \\ \pi / y &= p - c\left(w, r, y\right) / y > 0 \text{ iff} \\ c\left(w, r, y\right) / y &= AC < p \end{aligned}$$

- **Supply function.** Portion of marginal cost MC above average costs.(price equals marginal cost)

- Assume only 1 input (expenditure minimization is trivial)

- **Case 1.** Production function. $y = L^{\alpha}$

  - Cost function? (cost of input is w):

$$c(w, y) = wL^{*}(w, y) = wy^{1/\alpha}$$

  - Marginal cost?

$$\frac{\partial c(w, y)}{\partial y} = \frac{1}{\alpha} wy^{(1-\alpha)/\alpha}$$

  - Average cost $c(w, y)/y$?

$$\frac{c(w, y)}{y} = \frac{wy^{1/\alpha}}{y} = wy^{(1-\alpha)/\alpha}$$

- **Case 1a.** $\alpha > 1$. Plot production function, total cost, average and marginal. Supply function?

- **Case 1b.** $\alpha = 1$. Plot production function, total cost, average and marginal. Supply function?

- **Case 1c.** $\alpha < 1$. Plot production function, total cost, average and marginal. Supply function?

- **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

- **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?

2 Supply Function

- Supply function: $y^* = y^*(w, r, p)$

- What happens to y^* as p increases?

- Is the supply function upward sloping?

- Remember f.o.c:

$$p - c'_y(w, r, y) = 0$$

- Implicit function:

$$\frac{\partial y^*}{\partial p} = -\frac{1}{-c''_{y,y}(w, r, y)} > 0$$

as long as s.o.c. is satisfied.

- Yes! Supply function is upward sloping.

3 Short-run Cost Minimization

- So far, we assumed flexibility in choose of all inputs
- Is this realistic?
  - In long-run, yes. Can adjust machines, land,...
  - But... in the long-run, we are all dead! (Keynes)
  - In short-run, no. Capital and land are fixed

- Short-run cost minimization: K fixed at $\overline{K}$.
- Firm's objective function:

$$\min_{L} wL + r\overline{K}$$
$$s.t.\ f\left(L, \overline{K}\right) \geq y$$

- Capital $\overline{K}$ is a constant

- Solution:

$$L^* = L^*_{SR}\left(r, w, y|\overline{K}\right)$$

- Short-run cost function

$$c_{SR}\left(r, w, y|\overline{K}\right) = wL^*_{SR}\left(r, w, y|\overline{K}\right) + r\overline{K}$$

- Exercise: Show $c_{SR}\left(r, w, y|\overline{K}\right) > c\left(r, w, y\right)$

- Graphically,

4 One-step Profit Maximization

- Nicholson, Ch. 13, pp. 346–350.

- One-step procedure: maximize profits

- Perfect competition. Price p is given
  - Firms are small relative to market
  - Firms do not affect market price p_M

  - Will firm produce at $p > p_M$?
  - Will firm produce at $p < p_M$?
  - $\Longrightarrow p = p_M$

- Revenue: $py = pf\left(L, K\right)$

- Cost: $wL + rK$

- Profit $pf\left(L, K\right) - wL - rK$

- Agent optimization:

$$\max_{L,K} pf(L,K) - wL - rK$$

- First order conditions:

$$pf'_L(L,K) - w = 0$$

and

$$pf'_K(L,K) - r = 0$$

- Second order conditions? $pf''_{L,L}(L,K) < 0$ and

$$\begin{aligned} |H| &= \begin{vmatrix} pf''_{L,L}(L,K) & pf''_{L,K}(L,K) \\ pf''_{L,K}(L,K) & pf''_{K,K}(L,K) \end{vmatrix} = \\ &= p^2 \left[f''_{L,L} f''_{K,K} - \left(f''_{L,K} \right)^2 \right] > 0 \end{aligned}$$

- Need $f''_{L,K}$ not too large for maximum

- Comparative statics with respect to to p, w, and r.

- What happens if w increases?

$$\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix} -1 & pf''_{L,K}(L,K) \\ 0 & pf''_{K,K}(L,K) \end{vmatrix}}{\begin{vmatrix} pf''_{L,L}(L,K) & pf''_{L,K}(L,K) \\ pf''_{L,K}(L,K) & pf''_{K,K}(L,K) \end{vmatrix}} < 0$$

and

$$\frac{\partial L^*}{\partial r} =$$

- Sign of $\partial L^*/\partial r$ depends on $f''_{L,K}$.