Economics 101A
(Lecture 19, Revised)

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Outline

1. Elasticities
2. Comparative Statics of SR Equilibrium II
3. Response to Taxes

1 Elasticities

- Elasticity of x with respect to parameter p is

$$\varepsilon_{x,p} = \frac{\partial x}{\partial p}\frac{p}{x}$$

- Interpretation: Percent response in x to percent change in p :

$$\begin{aligned}\varepsilon_{x,p} &= \frac{\partial x}{\partial p}\frac{p}{x} = \lim_{dp \to 0} \frac{x(p+dp) - x(p)}{dp}\frac{p}{x} = \\ &= \lim_{dp \to 0} \frac{dx/x}{dp/p}\end{aligned}$$

where $dx \equiv x(p+dp) - x(p)$.

- Now, show

$$\varepsilon_{x,p} = \frac{\partial \ln x}{\partial \ln p}$$

- Notice: This makes sense only for $x > 0$ and $p > 0$

- Consider function

$$x = f(p)$$

- Rewrite as

$$\ln(x) = \ln f(p) = \ln f\left(e^{\ln(p)}\right)$$

- Define $\hat{x} = \ln(x)$ and $\hat{p} = \ln(p)$

- This implies

$$\hat{x} = \ln f\left(e^{\hat{p}}\right)$$

- Get

$$\begin{aligned}\frac{\partial \hat{x}}{\partial \hat{p}} &= \frac{\partial \ln x}{\partial \ln p} = \\ &= \frac{1}{f\left(e^{\hat{p}}\right)} \frac{\partial f\left(e^{\hat{p}}\right)}{\partial \hat{p}} e^{\hat{p}} = \frac{\partial x}{\partial p} \frac{p}{x}\end{aligned}$$

- Example with Cobb-Douglas utility function

- $U(x,y) = x^{\alpha} y^{1-\alpha}$ implies solutions

$$x^* = \alpha \frac{M}{p_x}, y^* = (1-\alpha)\frac{M}{p_y}$$

- Elasticity ε_{x,p_x} :

$$\varepsilon_{x,p_x} = \frac{\partial x^*}{\partial p_x}\frac{p_x}{x^*} = -\frac{\alpha M}{(p_x)^2}\frac{p_x}{\alpha \frac{M}{p_x}} = -1$$

- $\varepsilon_{x,p_y} = 0$

2 Comparative statics of SR equilibrium II

- Supply and Demand function of parameter α :
  - $Y_i^S(p_i, w, r, \alpha)$
  - $X_i^D(\mathbf{p}, \mathbf{M}, \alpha)$

- How does α affect p^* and Y^*?

- Comparative statics with respect to α

- Equilibrium:

$$Y_i^S(p_i, w, r, \alpha) = X_i^D(\mathbf{p}, \mathbf{M}, \alpha)$$

- Can write equilibrium as implicit function:

$$Y_i^S\left(p_i, w, r, \alpha\right) - X_i^D\left(\mathbf{p}, \mathbf{M}, \alpha\right) = 0$$

- Implicit function theorem:

$$\frac{\partial p^*}{\partial \alpha} = -\frac{\frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}}$$

- Use elasticities to rewrite response of p to change in α :

$$\frac{\partial p^*}{\partial \alpha}\frac{\alpha}{p} = -\frac{\left(\frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}\right)\frac{\alpha}{Y}}{\left(\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}\right)\frac{p}{Y}}$$

  or (using fact that $X^{D*} = Y^{s*}$)

$$\varepsilon_{p,\alpha} = -\frac{\varepsilon_{S,\alpha} - \varepsilon_{D,\alpha}}{\varepsilon_{S,p} - \varepsilon_{D,p}}$$

- We are likely to know elasticities from empirical studies.

3 Response to taxes

- Nicholson, Ch. 15, pp. 407–408
- Per-unit tax t
- Write price p_i as price including tax
- Supply: $Y_i^S(p_i - t, w, r)$
- Demand: $X_i^D(\mathbf{p}, \mathbf{M})$

$$Y_i^S(p_i - t, w, r) - X_i^D(\mathbf{p}, \mathbf{M}) = 0$$

- What is dp^*/dt?

- Comparative statics:

$$
\begin{aligned}
\frac{\partial p^*}{\partial t} &= -\frac{\frac{\partial Y^S}{\partial t}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}} = \\
&= -\frac{-\frac{\partial Y^S}{\partial p}\frac{p}{X}}{\left(\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}\right)\frac{p}{X}} = \\
&= \frac{\varepsilon_{S,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
\end{aligned}
$$

- How about price received by suppliers $p^* - t$?

$$
\begin{aligned}
\frac{\partial \left(p^* - t\right)}{\partial t} &= \frac{\frac{\partial Y^S}{\partial p}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}} - 1 = \\
&= \frac{\varepsilon_{D,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}
\end{aligned}
$$

- *Inflexible Supply.* Supply curve vertical ($\varepsilon_{S,p} = 0$)

- Producers bear burden of tax [REVISED]

- *Flexible Supply.* Supply curve horizontal ($\varepsilon_{S,p} \to \infty$)

- Consumers bear burden of tax [REVISED]

- *Inflexible demand.* Demand curve vertical ($\varepsilon_{D,p} = 0$)?

- Consumers bear burden [REVISED]

- General lesson: Most elastic side bears larger part of burden

- What happens with a subsidy ($t < 0$)?

- What happens to quantity sold?

- Use demand curve:

$$\frac{\partial X^{D*}}{\partial t} = \frac{\partial X^{D*}}{\partial p^*}\frac{\partial p^*}{\partial t}$$

  and use expression for $\partial p^* / \partial t$ above.