Econ 101A – Midterm 2
Th 6 November 2003.

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. I will collect the exams at 12.30 sharp. Show your work, and good luck!

Problem 1. Production. (43 points) In this exercise, we consider a firm producing a product using only one input, labor L. The production function f is as follows:

$$f(L) = \begin{cases} \left(L - \bar{L}\right)^{\alpha} & \text{if} \quad L \geq \bar{L} \\ 0 & \text{if} \quad 0 \leq L < \bar{L} \end{cases},$$

that is, the firm produces a positive quantity of output only if there are at least $\bar{L}$ workers. Assume that the wage of a worker is w. Assume $\bar{L} > 0$ and $\alpha > 0$.

1. Draw a picture of the production function assuming $\alpha = .5$ and $\bar{L} = 1$. (1 point)

2. For which values of α (if any) does the function exhibit (weakly) decreasing returns to scale ($f(tL) \leq tf(L)$ for all $t > 1$ and all $L \geq 0$)? Does the function exhibit (weakly) increasing returns to scale ($f(tL) \geq tf(L)$ for all $t > 1$ and all $L \geq 0$) for $\alpha \geq 1$? Provide an analytical proof if you can. (7 points)

3. Consider now the first step of the cost minimization problem. The firm solves

$$\begin{aligned} &\min wL \\ &s.t. f(L) \geq y \end{aligned}$$

for $y > 0$. What is the solution for $L^*\left(w, y|\bar{L}, \alpha\right)$? (This notation stresses that the solution depends also on the parameters α and $\bar{L}$. Hint: You are better off not using Lagrangeans...) (5 points)

4. Write down the implied cost function $c\left(w, y|\bar{L}, \alpha\right)$. (2 points)

5. Derive an expression for the average cost $c\left(w, y|\bar{L}, \alpha\right)/y$ and the marginal cost $c'_y\left(w, y|\bar{L}, \alpha\right)$ for $y > 0$. Graph the average cost and marginal cost for $\alpha = .5$, $w = 1$ and $\bar{L} = 1$. Graph the supply function for the same values of the parameters. [remember, y is on the horizontal axis]. (7 points)

6. Now that we graphically solved for the supply function, we also derive it formally. Consider the second step of cost minimization

$$\max_{y} py - c\left(w, y|\bar{L}, \alpha\right).$$

Write down the first order condition and the second order conditions. Solve for $y^*\left(w, p|\bar{L}, \alpha\right)$. For what values of α is the second order condition satisfied? (5 points)

7. Assume now $\alpha < 1$ and write the condition under which firms are making positive profits, that is, under which $py^*\left(w, p|\bar{L}, \alpha\right) - c\left(w, y^*|\bar{L}, \alpha\right) > 0$. If you can, solve for the value of price $\bar{p}$ such that firms produce if the price p is larger than $\bar{p}$. (5 points)

8. Taking into account the answers to points 6 and 7, write down the supply function, that is, an expression for $y^*\left(w, p|\bar{L}, \alpha\right)$. Keep assuming $\alpha < 1$. (3 points)

9. Why do these conditions imply that the supply function is the portion of the marginal cost curve above average cost as long in its upward sloping portion, and zero otherwise (4 points)?

10. Keep assuming $\alpha < 1$. What happens to the supply function as $\bar{L}$ increases? What if the wage w increases? You can respond using the graphs or using the solution in point 8. Give an economic interpretation. (4 points)

Problem 2. Uncertainty (17 points). So far, in class we have assumed that the utility function depends only on consumption c. Now, we consider the case of Prospectus, whose utility function also on a reference point r (Kahneman and Tversky, 1979). Prospectus has utility function

$$U(c|r) = \begin{cases} (c-r)^{1/2} & \text{if} \quad c \geq r \\ -(r-c)^{1/2} & \text{if} \quad c < r \end{cases}$$

You can see this utility function plotted in the attached Figure.

Prospectus needs to choose between jobs G and I. Job G is in a government agency and guarantees consumption of \$40,000 per year. Job I is in an investment bank and is very risky: with probability .5 the job will go well and Prospectus will consume \$70,000 per year, but with probability .5 Prospects will be fired and have a yearly consumption of \$10,000. Your task is to help Prospectus decide which job to take.

1. Assume that Prospectus just came out of school and has a low reference point, that is, r equals \$10,000. Write down the expected utility $EU(c|r)$ from accepting job G and the expected utility $EU(c|r)$ from accepting job I. What would you recommend that Prospectus should do in order to maximize expected utility? (6 points)
2. Assume that Prospectus just quit a consulting job and has a high reference point, that is, r equals \$70,000. Write down the expected utility $EU(c|r)$ from accepting job G and the expected utility $EU(c|r)$ from accepting job I. In this case, what would you recommend that Prospectus should do in order to maximize expected utility? (5 points)
3. Can you give an intuition for why the best job choice for Prospectus depends on the reference point? (Hint: use the Figure and possibly Jensen's inequality) (6 points)

Problem 3. Altruistic workers. (20 points) Consider a worker in a firm that has to decide how hard to work. The worker's effort e, with $0 \leq e \leq 1$, has a disutility for the worker $-e^2/2$. The worker gets wage $0 < w < 1$. The worker's utility is therefore

$$U = w - e^2/2,$$

that is, the wage net of the effort cost. The production of the firm is e and the good is sold at price 1.

1. The worker chooses e to maximize own utility (subject to the constraint $0 \leq e \leq 1$). What is the optimal choice of effort e? (3 points)
2. The firm does not like the above outcome and decides to monitor the worker with probability p. With probability $p*(1-e)$ the worker is caught shirking and is fired, and therefore gets 0 wage. With probability $p*e$ the worker is found at work and gets the wage w. With probability $(1-p)$ there is no monitoring and the worker gets wage w. The worker maximizes the expected wage payment minus the effort cost. Write down the expected utility of the worker and solve for the utility maximizing effort e. What is the comparative statics with respect to p? Provide intuition. (6 points)
3. The firm now attempts to maximize profits given by revenue e minus the expected wage payment minus monitoring costs cp^2. (Hint: The firm pays the wage w with probability $1-p+pe$) Solve for the profit-maximizing level of monitoring p^*. (5 points)
4. We now go back to point 1, that is, there is no monitoring. Assume now, however, that the worker is altruistic toward the firm. That is, the worker maximizes $w - e^2/2 + \alpha e$, where the last term is the product of the altruism coefficient α and the production of the firm. Solve for the utility maximizing effort in this case assuming no monitoring. What is the comparative statics with respect to the altruism coefficient α? (3 points)
5. A firm wants to improve productivity e. The firm has the choice between increasing the costly monitoring or selecting altruistic workers. What should the firm do? (3 points)